Inversion strategy research of travel time tomography with sparse rays

Yali Guo*, Yan Han, Linmao Liu

National key laboratory of Electronic test Technology in North University of China, Taiyuan, Shanxi, China

Received 1 October 2014, www.cmnt.lv

Abstract

The incomplete data travel time tomography with sparse rays can result in the ill-posed inverse problem in practical engineering, so the inversion strategy is very important in order to obtain reasonable inversion result. In this paper, the generalized inverse theory is taken and the influences are discussed which the system layout, initial model and prior information will impose on the inversion. The indexes of system optimal layout, the selection principle of initial model and regularization methods are presented in this paper. A velocity model of explosion is imitated and the inversion results are compared. A conclusion can be gained that system optimal layout, initial model rational selection and regularization methods utilization can help to improve inversion precision farthest in practical project.

Keywords: sparse rays, inversion strategy, regularization methods

1 Introduction

Computerized tomography is one of the non-destructive testing methods [1]. We can achieve the distribution of physics parameter existing in detected target by laying sensors outside the detected target and resolve the engineering and technology problems [2-4]. Computerized tomography technology has already used in medical and industrial fields widely.

Elastic wave travel time tomography is one of the important methods of engineering physics survey, it has played an important role in industry testing and resources prospecting [5]. It is necessary that there are enough detecting rays through the target in order to achieve high accuracy image information inside the target, however, which will increase the practical project difficulty and improve the cost. The data of elastic wave computerized tomography in practical project is usually incomplete owing to the insufficient driving sources and detectors. This engineering problem results in the incomplete tomography data inevitably.

Incomplete data tomography has the problems of sparse inverse data and low inversion accuracy and the inversion result is determined by a good many factors [6]. This paper aiming at the problems of travel time tomography with sparse rays, analyses the influences that the system layout, initial model and prior information imposed on the inversion by numerical experiment. In practical project, we can improve inversion accuracy by system optimal layout, initial model rational selection and utilization of regularization methods.

2 Inversion strategy analysis

For travel time tomography, there is the equation below:

$$DS = T, \tag{1}$$

where $T=(t_1,t_2\cdots t_m)'$ is the m-dimension column vector of travel time; $S=(s_1,s_2\cdots s_n)'$ is an unknown n-dimensional column vector, it expresses unknown discrete element slowness value in discrete cell; D is the distance matrix of $m\times n$ and its element is d_{ij}.

2.1 SYSTEM LAYOUT OPTIMIZATION

2.1.1 Mesh generation

The tested area is divided into numbers of regular meshes and each mesh has a uniform wave velocity. The more meshes are generated, the higher resolution of computed tomography will be achieved and the more uncertain solutions will be achieved too. Mesh generation should accord to the tested area size, the prior information (such as velocity distribution characteristic, abnormal body size, sampling position etc.), reconstruction accuracy, the number of driving sources and detectors.

2.1.2 Optimal distribution of sensors and judgment indexes

When designing sensors position, we should meet the following principles: extensive coverage and uniform distribution of rays, reduced number of zero elements in distance matrix. In order to make the tested area covered by rays as much as possible and achieve the effective detection, we should have a rational distribution of the sensors according to the follow factors: the ray density, orthogonality and the condition number of matrix D.

The ray density represents the number of rays passing through each mesh. The ray orthogonality is measured by maximal sine value of angle between rays [7]. The greater

***Corresponding author** e-mail:guoyali@nuc.edu.cn

the ray density is and the better the orthogonality is, the smaller inversion error will be achieved.

The tomography inversion is to solve the ill - posed Equation (1). The inversion stability is determined by the condition number of matrix *D*. The bigger condition number can result in poor inversion stability. Supposing that observed data *T* has a minor change, the variation of solution is δS Equation (1) has the relation:

$$D(S+\delta S)=T+\delta T \ . \tag{2}$$

Then:

$$\delta S = D^{-1}\delta T \ . \tag{3}$$

According to the property of subordinate norm, there is the relation: $\|\delta S\| \le \|D^{-1}\|\|\delta T\|$ and $\|T\| \le \|D\|\|S\|$, so $\frac{\|\delta S\|}{\|D\|\|S\|} \le \frac{\|D^{-1}\|}{\|T\|}\|\delta T\|$. That is:

$$\frac{\|\delta S\|}{\|S\|} \le cond(D)\frac{\|\delta T\|}{\|T\|} \ . \tag{4}$$

Then,

$$cond(D)=\|D\|\|D^{-1}\|, \tag{5}$$

where $cond(D)$ is the condition number of matrix *D*. Rational distribution of the sensors can reduce the condition number and we can receive the more stable solutions.

2.2 THE INITIAL MODEL SELECTION

When the rays through the target are sparse, the inverse results depend on the initial model severely. If the deviation of initial model and true model is small, we can obtain better results, otherwise, the results aren't true.

This paper calculates the correlation degree of initial model and true model by correlation coefficient; analyses the change trend of correlation coefficient with inversion error and obtains the dependence degree of inversion result on initial model.

For two groups data of x_i and y_i , $i=1,2,\cdots n$, the correlation coefficient is defined as[8]:

$$r=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}} \ . \tag{6}$$

The absolute value of *r* is closer to 1, the two groups data have higher correlation degree. We can set reasonable initial model by prior information and experience. We should analyse the character of model according to prior information before inversion. Experience formula is also a good way to set initial model. In a word, we should improve the correlation degree of initial model and true model as far as possible, only thus can we improve the inversion accuracy.

2.3 APPLICATION OF REGULARIZATION METHODS

Incomplete data travel time tomography with sparse rays makes the $n>m$ in Equation (1) and represents underdetermined problem in mathematics. In this condition, the inversion results have multiple solutions and credibility is poor. An effective way to resolve the underdetermined inversion problem of multiple solutions is adopting regularization methods to constrain the inversion process and the inversion results by prior information. At the same time, regularization methods make an important role in perfecting ill-conditioned problem.

The application of regularization methods includes: damping the asymmetrical covering of rays, damping the inaccuracy of observed data, tight constraint for true data, setting the value range of some parameters in the iterative procedure and so on. The ways of regularization adding have addition and multiplication. This paper presents some regularization methods as described below.

2.3.1 Damping the asymmetrical covering of rays

The ray coverage will affect the inverse results. The damping regularization methods aiming at the asymmetrical covering of rays is model covariance matrix, which weights different value in different mesh according to ray coverage. Generally, rays is denser, the inversion results are more accurate. Therefore, the meshes through by more rays have more information and should weight a larger value.

2.3.2 Damping the inaccuracy of observed data

The observed data contains noises. The data in different acquisition channel has different noises and the noises will be amplified in inversion. So, the data should have different weighting according to its accuracy. The damping matrix is data covariance matrix. For travel time tomography, the waveform signal is weakening with the transmission distance. The ray path is shorter, the signal to noise ratio is higher, so the smaller travel time should weight a larger value.

2.3.3 Tight constraint of true data

In practical application, we can measure the velocity of this position by individual sampling point in interested area and treat it as the prior information. Equivalent to increase the constraint equations in the linear Equation (1) with:

$$WS=S' , \tag{7}$$

where *W* is a $h\times n$ matrix. *h* is the number of sampling point. And the $w_i=[000\cdots10]$, the position of constrained parameter is 1, others are 0. S' is the observed data of sampling points. So the Equation (1) is rewritten as:

$$\begin{bmatrix} D_0 \\ W \end{bmatrix} \cdot S = \begin{bmatrix} T_0 \\ S' \end{bmatrix}, \tag{8}$$

where D_0, T_0 are the distance matrix and travel time matrix without prior information. Rewrite above equation becoming $DS = T$ and make calculation.

2.3.4 Setting the value range of some parameters

We can obtain the value range of some parameters by the prior information sometimes. Constraining the results by the value range of some parameters in the iterative procedure helps to improve the inversion precision.

3 Weighted generalized inversion algorithm

Solving the discrete inversion Equation (1) by matrix is the same as solving inverse matrix of D. However, in practical project, the problem of incomplete data owing to the insufficient number of driving sources and detectors makes Equation (1) become into a sparse, morbid, underdetermined, incompatible linear equation set [9]. The coefficient matrix-D in general is a singular matrix and its inverse matrix does not exist obviously. So it is necessary to adopt the generalized inverse theory to solve matrix-D.

Thinking of the regularization methods, we adopt model covariance matrix and data covariance matrix to damp the asymmetrical covering of rays and the inaccuracy of observed data. The weighted generalized inversion method is the generalized inversion algorithm combining with the regularization methods.

Given $A \in C^{m\times n}$, P and Q are positive definite matrix of $m\times m$ and $n\times n$ respectively. If $X \in C^{n\times m}$, satisfying:

$$\begin{cases} AXA = A, XAX = X \\ (PAX)^T = PAX \\ (QXA)^T = QXA \end{cases}. \tag{9}$$

Then X is defined as weighted generalized inverse of A: A^+_{mn}, A^+_{mn} is expressed as [10]:

$$A^+_{mn} = Q^{-1}(PAQ^{-1})^+ P. \tag{10}$$

The data covariance matrix and model covariance matrix are P and Q, the diagonal element of P and Q is defined as:

$$diag\{P\} = T^{-1}, \tag{11}$$

$$diag\{Q\} = K. \tag{12}$$

T is the M-dimension vector of travel time of current model, the element of which is $t_i = \sum_{j=1}^{n} d_{ij}\cdot s_j$, $i = 1,2,3,\cdots,m$. K is the N-dimension vector, the element of K is $k_j = \sum_{i=1}^{m} d_{ij}\cdot v_j$ ($j = 1,2,3,\cdots,n$). v_j is velocity in mesh of No. j.

Consequently, travel time tomography based on weighted generalized inverse is rewritten as:

$$S = Q^{-1}(PDQ^{-1})^+ P\cdot T. \tag{13}$$

4 Numerical simulation experiments

4.1 TESTING MODEL

The model is a velocity model of explosion field as shown in Figure1. The tested area is divided into 10×10 meshes, and bomb is placed in the centre of test area as shown in Figure 2. With the principle of symmetry, we only need make velocity inversion in the 1/4 area. The sensor number is no more than 20. This is a typical model of travel time tomography with single driving sources and sparse rays.

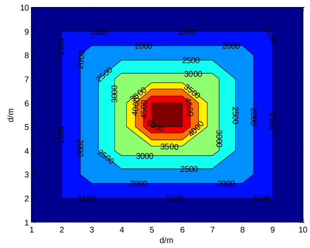

FIGURE 1 The velocity model of explosion field

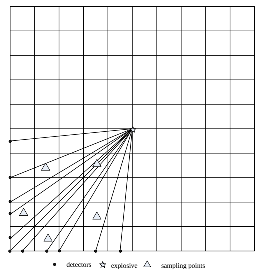

FIGURE 2 The tested area layout

Defining, the number of regular meshes is n, The relative velocity error in the No. j mesh is defined:

$$\delta_j = \frac{v_j - v_j^{'}}{v_j^{'}}, \tag{14}$$

where $v_j^{'}$ is the true velocity value in No. j grid, v_j is the inversion result.

The average relative error in all meshes is defined:

$$\bar{\delta} = \frac{1}{N}\sum_{j=1}^{N}|\delta_j| = \frac{1}{N}\sum_{j=1}^{N}\frac{|v_j - v_j^{'}|}{v_j^{'}}. \tag{15}$$

4.2 SIMULATION OF SENSORS DISTRIBUTION

We select the sensors number is 13; two different sensor layouts are presented in Figure 3. The first sensor layout is symmetrical as shown in Figure 3a, the second is optimized layout by the judgment indexes in this paper as shown in Figure 3b.

The ray density and orthogonality distribution in the whole tested area in the two layouts are in Figure 4 and Figure 5. The condition number of matrix D in the two layouts are $2.74\cdot 10^{16}$ and 35.61 respectively. We can see that the second layout has more reasonable ray distribution, the better orthogonality and smaller condition number. The initial model is in Figure 6. We make velocity inversion and the relative errors in each mesh are in Figure 7, the average relative errors are 8.47% and 3.32%. The inversion results are in Figure 8. We can see that the inversion errors are smaller and inversion result is approaching the true model with the optimized sensors layout in this paper.

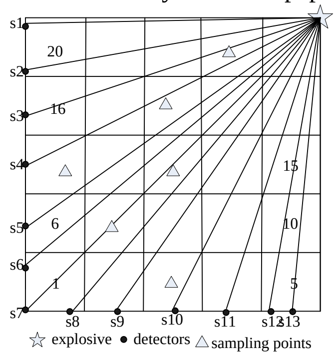

FIGURE 3a The first layout

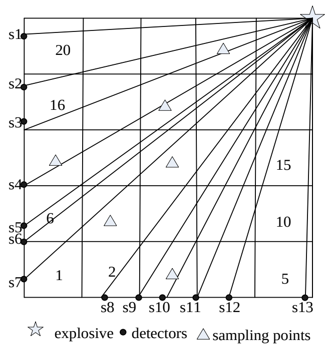

FIGURE 3b The second layout

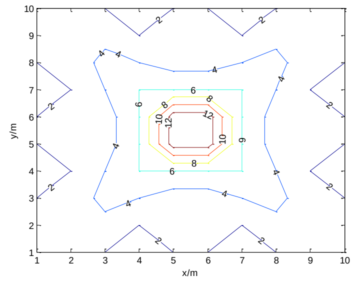

FIGURE 4a The first density distribution

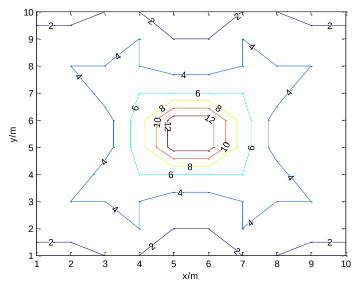

FIGURE 4b The second density distribution

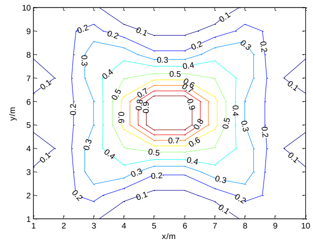

FIGURE 5a The first orthogonality distribution

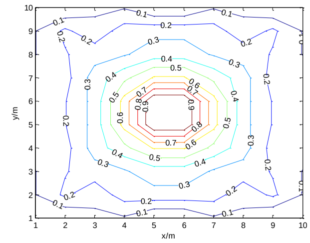

FIGURE 5b The second orthogonality distribution

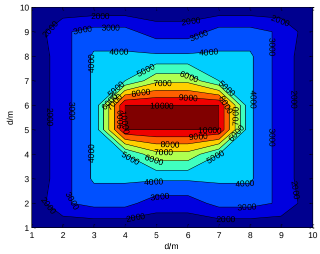

FIGURE 6 The initial model

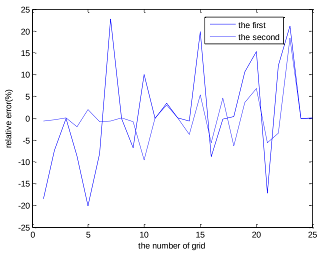

FIGURE 7 The relative error in each mesh

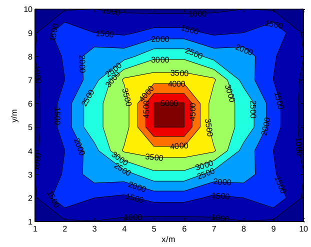

FIGURE 8a The first inversion results

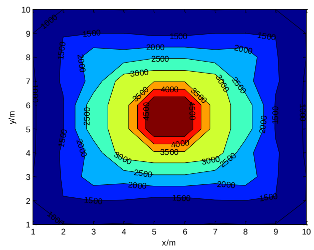

FIGURE 8b The second inversion results

4.3 INFLUENCE OF INITIAL MODEL

We generate the initial model by adding a certain proportion of random noise to true model and make the correlation coefficient of initial model and true model change from 0 to 1. We adopt the same weighted generalized inverse algorithm to simulate above model with different initial model. The relationship curve of correlation coefficient with inverse average relative error is in Figure 9. We can see that the correlation coefficient of initial model and true model is closer to 1, the inverse average relative error is smaller. Figure 10 is the relative error in each mesh when the correlation coefficient is 0.97, 0.75, 0.54 and the initial model is even velocity.

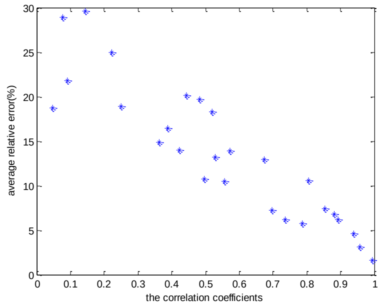

FIGURE 9The relationship curve of correlation coefficient with inverse average relative error

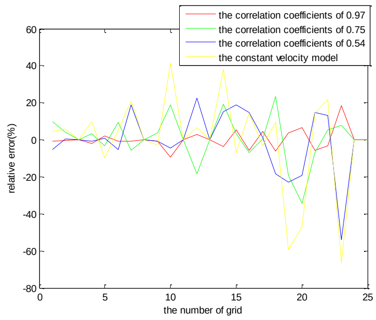

FIGURE 10 The relative error in each mesh with different correlation coefficient

From above analysis, we can conclude that inverse results depend on the initial model severely in the same condition. Selection reasonable initial model is very important for incomplete data travel time tomography with sparse rays.

4.4 INFLUENCE OF PRIOR INFORMATION

We make velocity inversion by generalized inversion algorithm with regularization methods. Figure 11 shows the different results of generalized inversion algorithm with regularization methods and without regularization methods. The average relative errors in all meshes are 4.2% and 19.81% respectively.

Figure 12 shows the influence of tight constraint of true data on inversion. When the number of sampling points is respectively 3, 5, 6, the relative errors in each meshes are shown in Figure 12. We can see that the relative error decreases with the number increase of sampling points.

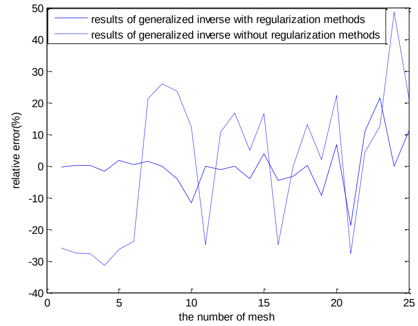

FIGURE 11 Inversion relative error in each mesh

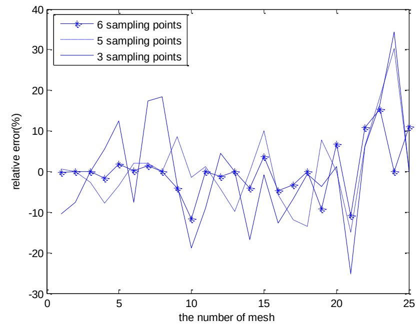

FIGURE 12The influence of tight constraint of true data on inversion

5 Conclusions

The particularity and complexity of the detected target result in the incomplete distribution of sensors. The travel time tomography owing to the insufficient driving sources and detectors is an incomplete data travel time tomography with sparse rays. This paper aiming at the problems of travel time tomography with sparse rays, analyses the influences that the system layout, initial model and prior information imposed on the inversion. We obtain the conclusions as follows:

In the condition of sparse rays, system optimal layout helps to improve the inversion accuracy. The indexes of system optimal layout are presents;

When the rays through the target are sparse, the correlation degree of initial model and true model is higher, the results are more accurate. We can set reasonable initial model by prior information and experience.

We can adopt regularization methods by prior information to overcome the underdetermined inversion problem of multiple solutions.

In practical project we can improve inversion accuracy by system optimal layout, initial model rational selection and utilization of regularization methods.

Acknowledgements

The authors would like to thank the reviewers and editors for their valuable comments and sincere helps, which contribute much to the improvement of this paper. This work was supported by the National Natural Science Foundation of China under Grant 61171179.

References

[1] Li G, Li T 2012 *Journal of Yan'an Vocational &Technical Institute* **26**(3) 90-3 *(in Chinese)*

[2] Liu H, Xue X 2003 Journal of Engineering Geology **11**(4) 435-9

[3] Watanabe T and Sassa K 1996 *International journal of rock mechanics and mining sciences & geomechanics abstracts* **33**(5) 467-77

[4] Wéber Z 2001 *Physics of the Earth and Planetary Interiors* **124** 33-43

[5] Liu G, Wang Z, Sun J 2003 *China Civil Engineering Journal* **36**(5) 76-81 *(in Chinese)*

[6] Cheng G, Ma Z, Zhang B 2003 *Progress in Geophysics* **18**(3) 512-8 *(in Chinese)*

[7] Pei Z, Yu Q, Di B 2006 *Geophysical and Geochemical Exploration* **26**(3) 218-24 *(in Chinese)*

[8] Chen J 1995 *Probability and statistics* Higher Education Press Beijing *(in Chinese)*

[9] Liu Y, Dong L, Xia J 2007 *Oil Geophysical Prospecting* **6**(42) 682-5 *(in Chinese)*

[10] Berryman J G 1989 *Physical Review Letters* **62**(25) 2953-6

Authors

Yali Guo, bbborn in June, 1980, Taiyuan, China

Current position, grades: instructor at North University of China.
University studies: master's degree in Signal Processing at North University of China.
Scientific interests: signal processing, reconstruction and inversion.
Publications: 2 patents, 10 papers.

Yan Han, born in June, 1957, Taiyuan, China

Current position, grades: professor and doctoral supervisor at North University of China.
University studies: doctor's degree in Signal Processing at Beijing Institute of Technology.
Scientific interests: signal processing, non-destructive testing, reconstruction and inversion.
Publications: 30 patents, 200 papers.

Linmao Liu, born in December, 1980, Taiyuan, China

Current position, grades: instructor in North University of China.
University studies: master's degree in Signal Processing at North University of China.
Scientific interests: signal processing, reconstruction and inversion.
Publications: 5 papers.