Similarity measure based on characteristic values for intuitionistic trapezoidal fuzzy numbers and its multicriteria decision-making method

Shapu Ren*

Department of Electrical and Information Engineering, Shaoxing University, Shaoxing, Zhejiang Province 312000, P.R. China

Received 1 September 2014, www.cmnt.lv

Abstract

A similarity measure and a weighted similarity measure based on the distance between characteristic values for intuitionistic trapezoidal fuzzy numbers are proposed in this paper. Then an intuitionistic trapezoidal fuzzy multicriteria decision-making method is established based on the weighted similarity measure between the characteristic values, in which the preference values of alternatives on criteria are the form of intuitionistic trapezoidal fuzzy numbers and the criteria weights are known information. By means of the ideal alternative, the weighted similarity measure between an alternative and the ideal alternative based on the intuitionistic trapezoidal fuzzy numbers is presented to derive the optimal evaluation for each alternative. The ranking of alternatives and the best one can be determined according to the values of the weighted similarity measure for all alternatives. Finally, an illustrative example demonstrates the effectiveness of the proposed method.

Keywords: Intuitionistic fuzzy number; Intuitionistic trapezoidal fuzzy number; Characteristic value; Measuring distance; Similarity measure; Multi-criteria decision-making

1 Introduction

A similarity measure is an important tool for determining the degree of similarity between two objects. Since Atanassov [1] proposed intuitionistic fuzzy sets (IFSs), many different similarity measures between IFSs have been proposed in literature. Li and Cheng [2] discussed some similarity measures on IFSs and then proposed a suitable similarity measure between IFSs, which is the first one to be applied to pattern recognition problems. Later, Liang and Shi [3] proposed several similarity measures to differentiate different IFSs, in which they discussed more about the relationships between these measures. Furthermore, Mitchell [4] interpreted IFSs as ensembles of ordered fuzzy sets from a statistical viewpoint to modify Li and Cheng's measures. On the other hand, Hung and Yang [5] proposed another method to calculate the distance between IFSs based on the Hausdorff distance. They used this distance to generate several similarity measures between IFSs that are suited to be used in linguistic variables. Huang and Young [6] proposed several reasonable measures to calculate the degree of similarity between IFSs, in which the proposed measures are induced by Lp metric. Ye and Lian [7] proposed an improved similarity measure for multicriteria decision-making of mechanical transmission schemes, and also Jiang and Ye [8] proposed the method of multicriteria fuzzy decision-making based on an improved similarity measure of vague set. However, intuitionistic fuzzy sets and vague sets are the same as fuzzy sets, the domains of which are discrete sets, intuitionistic fuzzy sets and vague sets are used to indicate the extent to which the criterion does or does not belong to some fuzzy concepts. The notion of a fuzzy number and the operation on fuzzy numbers were introduced by Dubois and Prade [9, 10]. Nehi and Maleki [11] introduced intuitionistic trapezoidal fuzzy numbers and some operators for them based on the intuittionistic fuzzy numbers defined by Grzegrorzewski [12], which are the extending of intuitionistic triangular fuzzy numbers. Intuitionistic triangular fuzzy numbers and intuitionistic trapezoidal fuzzy numbers are the extending of intuitionistic fuzzy sets in another way, which extends discrete set to continuous set, and they are the extending of fuzzy numbers. Nehi [13] proposed a characteristic value of an intuitionistic fuzzy number based on the characteristic value for fuzzy number proposed by Chiao [14] and an ordering method for ranking intuitionnistic fuzzy numbers according to the comparisons between characteristic values of membership and nonmembership in intuitionistic fuzzy numbers. Also Grzegorzewski [12] proposed a distance and ordering method for intuitionistic fuzzy numbers by using the expected interval of an intuitionistic fuzzy number. Then Wang and Zhang [15] defined some aggregation operators, including intuittionistic trapezoidal fuzzy weighted arithmetic averaging operator and weighted geometric averaging operator, and proposed an intuitionistic trapezoidal fuzzy multicriteria decision-making method with known weights based on expected values, score function, and accuracy function of intuitionistic trapezoidal fuzzy numbers. Furthermore, Ye [16] presented an expected value method for intuitionistic trapezoidal fuzzy multicriteria decision-making problems with intuitionistic trapezoidal fuzzy weights. Also Ye [17, 19] proposed vector similarity measures and distances-based similarity measures for trapezoidal intuitionistic fuzzy numbers and their multicriteria group decision-making methods with intuittionistic trapezoidal fuzzy weights.

* ***Corresponding author's*** e-mail: renshapu@163.com

This paper proposes a similarity measure and a weighted similarity measure of intuitionistic trapezoidal fuzzy numbers based on the distance between characteristic values for intuitionistic trapezoidal fuzzy numbers. Then an intuitionistic trapezoidal fuzzy multi-criteria decision-making method based on the weighted similarity measure is proposed to identify the best alternative in the application of multicriteria decision-making problems, in which the evaluation values of alternatives on criteria are represented by the form of intuitionistic trapezoidal fuzzy numbers and the criteria weights are known information. By means of the ideal alternative, the weighted similarity measure between an alternative and the ideal alternative based on the intuitionistic trapezoidal fuzzy numbers is established to derive the optimal evaluation for each alternative. The ranking of alternatives and the best one can be determined on the basis of the weighted similarity measure for alternatives. Finally, an illustrative example shows the applicability of the proposed method.

2 Preliminaries

This section introduces some definitions and basic concepts related to IFSs, similarity measure between the two IFSs, fuzzy numbers, intuitionistic fuzzy numbers, and intuitionistic trapezoidal fuzzy numbers.

Definition 1 [1]. Let X be a universe of discourse. Then an intuitionistic fuzzy set A in X is given by

$$A=\left\{\left\langle x,\mu_A\left(x\right),\nu_A\left(x\right)\right\rangle \middle| x\in X\right\}\ , \tag{1}$$

where $\mu_A(x)$: $X\to[0,1]$ and $\nu_A(x)$: $X\to[0,1]$, with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$. The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A. For each A in X, we can compute the intuitionistic index of the element x in the set A, which is defined as follows:

$$\pi_A\left(x\right)=1-\mu_A\left(x\right)-\nu_A\left(x\right),\ x\in X\ , \tag{2}$$

where $\pi_A(x)$ is also called a hesitancy degree of x to A. It is obvious that $0 \le \pi_A(x) \le 1$, $x \in X$.

Of course, a fuzzy set is a particular case of the intuitionistic fuzzy set with $\nu_A(x) = 1-\mu_A(x)$. Atanassov has also defined two kinds of α-cuts for intuitionistic fuzzy sets. Namely

$$A_\alpha= \{x \in X \mid \mu_A(x) \ge \alpha\}, \tag{3}$$

$$A^\alpha= \{x \in X \mid \nu_A(x) \le \alpha\}. \tag{4}$$

In the following, we introduce the definition of similarity measures between IFSs [2, 4, 6].

Definition 2. A mapping S: IFSs(X) × IFSs(X) → [0, 1], where IFSs(X) denotes the set of all IVFSs on X. $S(A, B)$ is said to be the degree of similarity between A and $B \in$ IFSs(X), if $S(A, B)$ satisfies the following properties (P1)–(P4):

(P1) $0 \le S(A, B) \le 1$;

(P2) $S(A, B) = 1$ if and only if $A = B$;

(P3) $S(A, B) = S(B, A)$;

(P4) $S(A, C) \le S(A, B)$ and $S(A, C) \le S(B, C)$ if $A \subseteq B \subseteq C$, $C \in$ IFSs(X).

Definition 2 [9]. Let A be an fuzzy number in the set of real numbers R, its membership function is defined as

$$\mu_A\left(x\right)=\begin{cases}0,\ x<a_1,\\ f_A\left(x\right),\ a_1\le x\le a_2\\ 1,\ a_2\le x\le a_{3,}\\ g_A\left(x\right),\ a_3\le x\le a_4,\\ 0,\ a_4<x.\end{cases}, \tag{5}$$

where a_1, a_2, a_3, $a_4 \in R$, $f_A : [a_1, a_2] \to [0, 1]$ is a nondecreasing continuous function, $f_A(a_1) = 0$, $f_A(a_2) = 1$, which is called the left side of the fuzzy number A, and g_A: $[a_3, a_4] \to [0, 1]$ is a nonincreasing continuous function, $g_A(a_3) = 1$, $g_A(a_4) = 0$, which is called the right side of the fuzzy number A.

Definition 3 [12]. Let A be an intuitionistic fuzzy number in the set of real numbers R, its membership function is defined as

$$\mu_A\left(x\right)=\begin{cases}0,\ x<a_1,\\ f_A\left(x\right),\ a_1\le x\le a_2\\ 1,\ a_2\le x\le a_{3,}\\ g_A\left(x\right),\ a_3\le x\le a_4,\\ 0,\ a_4<x.\end{cases}; \tag{6}$$

while its nonmembership function is defined as

$$\nu_A\left(x\right)=\begin{cases}1,\ x<b_1\\ h_A\left(x\right),\ b_1\le x\le b_2,\\ 0,\ b_2\le x\le b_3,\\ k_A\left(x\right),\ b_3\le x\le b_4,\\ 1,\ b_4<x.\end{cases}, \tag{7}$$

where $0 \le \mu_A(x) + \nu_A(x) \le 1$ and a_1, a_2, a_3, a_4, b_1, b_2, b_3, $b_4 \in R$ such that $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$, and four functions f_A, g_A, h_A, k_A: $R\to [0, 1]$ are called the side of a fuzzy number. The functions f_A and k_A are increasing continuous functions and the functions g_A and h_A are decreasing continuous functions.

It is worth noting that each intuitionistic fuzzy number $A=\left\{\left\langle x,\mu_A\left(x\right),\nu_A\left(x\right)\right\rangle \middle| x\in R\right\}$ is a conjunction of two fuzzy numbers: A^+ with a membership function $\mu_{A+}(x) = \mu_A(x)$ and A^- with a membership function $\mu_{A^-}(x) = 1-\nu_A(x)$. It is seen that $supp\, A^+ \subseteq supp\, A^-$.

A useful tool for dealing with fuzzy numbers is their α-cuts. Every α-cut of a fuzzy number is a closed interval and a family of such intervals describes completely a fuzzy number under study. In the case of an intuitionistic fuzzy number it is convenient to distinguish following α-cuts: $(A^+)_\alpha$ and $(A^-)_\alpha$. It is easily seen that

$$(A^+)_\alpha = \{x \in R \mid \mu_A(x) \ge \alpha\} = A_\alpha\ , \tag{8}$$

$$(A^-)_\alpha = \{x \in R \mid 1 - \nu_A(x) \ge \alpha\} =$$

$$\{x \in \mathrm{R} \mid \nu_A(x) \le 1 - \alpha\} = A^{1-\alpha}. \tag{9}$$

According to the definition it is seen that every α-cut $(A^+)_\alpha$ or $(A^-)_\alpha$ is a closed interval. Hence we have $\left(A^+\right)_\alpha = \left[A_L^+(\alpha), A_U^+(\alpha)\right]$ and $\left(A^-\right)_\alpha = \left[A_L^-(\alpha), A_U^-(\alpha)\right]$, respectively, where

$$A_L^+(\alpha) = \inf\{x \in R \mid \mu_A(x) \geq \alpha\}, \tag{10}$$

$$A_U^+(\alpha) = \sup\{x \in R \mid \mu_A(x) \geq \alpha\}, \tag{11}$$

$$A_L^-(\alpha) = \inf\{x \in R \mid \nu_A(x) \leq 1-\alpha\}, \tag{12}$$

$$A_U^-(\alpha) = \sup\{x \in R \mid \nu_A(x) \leq 1-\alpha\}. \tag{13}$$

If the sides of the fuzzy number A are strictly monotone, then by (6) and (7) one can see easily that $A_L^+(\alpha)$, $A_U^+(\alpha)$, $A_L^-(\alpha)$, and $A_U^-(\alpha)$ are inverse functions of f_A, g_A, h_A, and k_A, respectively. Generally, we may adopt the convention that $f_A^{-1}(\alpha) = A_L^+(\alpha)$, $g_A^{-1}(\alpha) = A_U^+(\alpha)$, $h_A^{-1}(\alpha) = A_L^-(\alpha)$ and $k_A^{-1}(\alpha) = A_U^-(\alpha)$.

Particularly, if the increasing functions f_A and k_A and decreasing functions g_A and h_A are linear, then we have intuitionistic trapezoidal fuzzy numbers, which are preferred in practice.

Definition 4 [11]. An intuitionistic trapezoidal fuzzy number A with parameters $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ is denoted as $A = \langle(a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4)\rangle$ in the set of real numbers R. In this case, its membership function and nonmembership function can be given as

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_2 \leq x \leq a_3, \\ \dfrac{x-a_4}{a_3-a_4}, & a_3 \leq x \leq a_4, \\ 0, & a_4 < x. \end{cases}, \tag{14}$$

$$\nu_A(x) = \begin{cases} 1, & x < b_1 \\ \dfrac{x-b_2}{b_1-b_2}, & b_1 \leq x \leq b_2, \\ 0, & b_2 \leq x \leq b_3, \\ \dfrac{x-b_3}{b_4-b_3}, & b_3 \leq x \leq b_4, \\ 1, & b_4 < x. \end{cases}. \tag{15}$$

If $b_2 = b_3$ (hence $a_2 = a_3$) in an intuitionistic trapezoidal fuzzy number A, the intuitionistic triangular fuzzy numbers are considered as special cases of the intuitionistic trapezoidal fuzzy numbers.

The following properties for intuitionistic trapezoidal fuzzy numbers have been given in [11].

Let $A_1 = \langle(a_{11}, a_{12}, a_{13}, a_{14}), (b_{11}, b_{12}, b_{13}, b_{14})\rangle$ and $A_2 = \langle(a_{21}, a_{22}, a_{23}, a_{24}), (b_{21}, b_{22}, b_{23}, b_{24})\rangle$ be two intuitionistic trapezoidal fuzzy numbers and r be a positive scalar number. Then,

$$A_1+A_2 = \langle(a_{11}+a_{21}, a_{12}+a_{22}, a_{13}+a_{23}, a_{14}+a_{24}), (b_{11}+b_{21}, b_{12}+b_{22}, b_{13}+b_{23}, b_{14}+b_{24})\rangle, \tag{16}$$

$$rA_1 = \langle(ra_{11}, ra_{12}, ra_{13}, ra_{14}), (rb_{11}, rb_{12}, rb_{13}, rb_{14})\rangle. \tag{17}$$

3 Similarity measure for intuitionistic trapezoidal fuzzy numbers

In this section we introduce a definition and some concepts of the characteristic values of intuitionistic fuzzy numbers and then propose a measuring distance between the characteristic values and similarity measure based on the distance.

Definition 5 [13]. Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in R\}$ be an intuitionistic fuzzy number and let $s(r,k) = \frac{1}{2}(k+1)r^k$ be a regular reducing function with positive parameters $k \in [0, \infty)$ and $r \in [0, 1]$. Then the characteristic values of membership and nonmembership for an intuitionistic fuzzy number A with parameters $k \in [0, \infty)$ and $r \in [0, 1]$ are defined as

$$C_\mu^k(A) = \frac{(k-1)}{2}\int_0^1 r^k [f_A^{-1}(r) + g_A^{-1}(r) dr, \tag{18}$$

$$C_\nu^k(A) = \frac{(k-1)}{2}\int_0^1 r^k [h_A^{-1}(r) + k_A^{-1}(r) dr. \tag{19}$$

when $f_A(x) = \frac{x-a_1}{a_2-a_1}$, $g_A(x) = \frac{x-a_4}{a_3-a_4}$, $h_A(x) = \frac{x-b_2}{b_1-b_2}$, and $k_A(x) = \frac{x-b_3}{b_4-b_3}$, $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in R$, Then, their inverses for these shape function for any $r \in [0, 1]$ are obtained by

$$f_A^{-1}(r) = a_1 + (a_2 - a_1)r, \tag{20}$$

$$g_A^{-1}(r) = a_4 + (a_3 - a_4)r, \tag{21}$$

$$h_A^{-1}(r) = b_1 + (b_2 - b_1)(1-r), \tag{22}$$

$$k_A^{-1}(r) = b_4 + (b_3 - b_4)(1-r). \tag{23}$$

If $A = \langle(a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4)\rangle$ is an intuitionistic trapezoidal fuzzy number in the set of real numbers R. Then the characteristic values of membership and nonmembership for A are denoted by

$$C_\mu^k(A) = \frac{(k-1)}{2}\int_0^1 r^k [f_A^{-1}(r) + g_A^{-1}(r) dr = \frac{a_2+a_3}{2} + \frac{(a_1-a_2)+(a_4-a_3)}{2(k+2)}, \tag{24}$$

$$C_\nu^k(A) = \frac{(k-1)}{2}\int_0^1 r^k [h_A^{-1}(r) + k_A^{-1}(r) dr = \frac{b_1+b_4}{2} + \frac{(b_2-b_1)+(b_3-b_4)}{2(k+2)}, \tag{25}$$

where $k \in [0, \infty)$. The larger the value k, the less influence of the left and right function of membership and nonmembership on the characteristic values of the intuitionistic trapezoidal fuzzy number.

If A is a symmetrical intuitionistic trapezoidal fuzzy number, the two characteristic values are simplified as follows:

$$C_\mu^k(A) = \frac{a_2 + a_3}{2}, \tag{26}$$

$$C_\nu^k(A) = \frac{b_1 + b_4}{2}. \tag{27}$$

Hence, a measuring distance between characteristic values for intuitionistic trapezoidal fuzzy numbers is proposed in the following definition.

Definition 6. Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in R\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in R\}$ be two intuitionistic fuzzy numbers. The measuring distance $d(A, B)$, indexed by a parameter $1 \le p < \infty$, is defined as

$$d(A,B) = \left\{\frac{1}{2}[|C_\mu^k(A) - C_\mu^k(B)|^p + |C_\nu^k(A) - C_\nu^k(B)|^p]\right\}^{1/p}. \tag{28}$$

It is known that distance measures and similarity measures are dual concepts. According to the measuring relation of distance measures and similarity measures [6] the similarity measures between intuitionistic fuzzy numbers A and B is proposed as follows:

$$S(A,B) = 1 - \left\{\frac{1}{2}[|C_\mu^k(A) - C_\mu^k(B)|^p + |C_\nu^k(A) - C_\nu^k(B)|^p]\right\}^{1/p}. \tag{29}$$

Obviously, the similarity measure $S(A, B)$ satisfies four properties (P1–P4) in Definition 2.

Then, the value of the parameter k might be determined by the decision maker according to the preference of the grade values of fuzzy utilities.

Weighted similarity measure for intuitionistic trapezoidal fuzzy multicriteria decision-making method

In this section, we present a handling method for intoitionistic trapezoidal fuzzy multicriteria decision-making problems with weights.

Let $A = \{A_1, A_2,\ldots, A_m\}$ be a set of alternatives and let $C = \{C_1, C_2,\ldots, C_n\}$ be a set of criteria. The preference value of an alternative A_i ($i = 1, 2, \ldots, m$) on a criterion C_j ($j = 1, 2,\ldots, n$) is an intuitionistic trapezoidal fuzzy number $t_{ij} = \langle(a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4},), (b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4})\rangle$ ($j = 1, 2,\ldots, n$; $i = 1, 2,\ldots, m$), $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in R$, and $0 \le b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4 \le 1$, which indicates the degree that the alternative A_i satisfies or does not satisfies the criterion C_j given by decision makers or experts according to some evaluated criteria. Therefore, we can elicit a decision matrix $D = (t_{ij})_{m\times n}$, which is represented by intuitionistic trapezoidal fuzzy numbers.

In multicriteria decision making environments, the concept of ideal point has been used to help the identification of the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct to evaluate alternatives.

Generally, the evaluation criteria can be categorized into two kinds, benefit and cost. Let H be a collection of benefit criteria and L be a collection of cost criteria. Then we define an ideal intuitionistic trapezoidal fuzzy number for a benefit criterion in the ideal alternative A^* as $t_j^* = \langle(1,1,1,1,), (1,1,1,1)\rangle$ for $j \in H$; while for a cost criterion, we define an ideal intuitionistic trapezoidal fuzzy number as $t_j^* = \langle(0,0,0,0,), (0,0,0,0)\rangle$ for $j \in L$.

The weighting vector of criteria for the different importance of each criterion is given as $W = (w_1, w_2,\ldots, w_n)$, where $w_j \ge 0$ and $\sum_{j=1}^n w_j = 1$. Thus a weighted similarity measure between an alternative A_i and the ideal alternative A^* based on the intuitionistic trapezoidal fuzzy numbers is defined as

$$S_i(A^*, A_i) = 1 - \sum_{j=1}^n w_j \left\{\frac{1}{2}[(|C_\mu^k(t_j^*) - C_\mu^k(t_{ij})|^p + (|C_\nu^k(t_j^*) - C_\nu^k(t_{ij})|^p]\right\}^{1/p}, \tag{30}$$

which provides the global evaluation for each alternative regarding all criteria. From Eq. (30), the larger the similarity measure $S_i(A^*, A_i)$, the better the alternative A_i ($i = 1, 2,\ldots, m$). Through the weighted similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well.

The decision procedure for the proposed method can be summarized as follows:

Step 1. Calculate the weighted similarity measure between an alternative A_i ($i = 1, 2,\ldots, m$) and the ideal alternative A^* by using Eqs. (24), (25), and (30).

Step 2. Rank the alternatives and select the best one(s) in accordance with each weighted similarity measure S_i ($i = 1, 2,\ldots, m$).

4 Illustrative example

In this section, an example for a multicriteria decision-making problem of alternatives is used as a demonstration of the application of the proposed multicriteria decision-making method in a realistic scenario.

Suppose there is an investment company, which wants to invest a sum of money in the best option (adapted from Herrera and Herrera-Viedma [18] and Ye [16]). There is a panel with four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The investment company must take a decision according to the following three criteria: (1) C_1 is the social benefit; (2) C_2 is the economical benefit; (3) C_3 is the environmental impact, where C_1 and C_2 are benefit criteria, and C_3 is a cost criterion. The weight rector of the three criteria is given by $W = (0.35, 0.3, 0.35)$. The preference values of each alternative A_i ($i = 1, 2, 3, 4$) are to be evaluated by the decision makers under the above three criteria and are represented by using the intuitionistic trapezoidal fuzzy numbers, as listed in the following decision matrix D [16]:

$$D = \begin{bmatrix} \langle(0.26,0.36,0.46,0.56),(0.16,0.36,0.46,0.66)\rangle & \langle(0.34,0.44,0.54,0.64),(0.24,0.44,0.54,0.74)\rangle & \langle(0.12,0.22,0.32,0.42),(0.04,0.22,0.32,0.50)\rangle \\ \langle(0.50,0.60,0.70,0.80),(0.42,0.60,0.70,0.88)\rangle & \langle(0.50,0.60,0.70,0.80),(0.42,0.60,0.70,0.88)\rangle & \langle(0.34,0.44,0.54,0.64),(0.24,0.44,0.54,0.74)\rangle \\ \langle(0.38,0.48,0.58,0.68),(0.28,0.48,0.58,0.78)\rangle & \langle(0.54,0.64,0.74,0.84),(0.46,0.64,0.74,0.92)\rangle & \langle(0.26,0.36,0.46,0.56),(0.16,0.36,0.46,0.66)\rangle \\ \langle(0.66,0.76,0.86,0.96),(0.64,0.76,0.86,0.98)\rangle & \langle(0.62,0.72,0.82,0.92),(0.58,0.72,0.82,0.96)\rangle & \langle(0.18,0.28,0.38,0.48),(0.08,0.28,0.38,0.58)\rangle \end{bmatrix}$$

Obviously, there are all symmetrical intuitionistic trapezoidal fuzzy numbers in the decision matrix $D = (t_{ij})_{4\times 3}$.

Then, we utilize the proposed approach to get the most desirable alternative(s).

Step 1. When $p = 1$, by using Eqs. (26), (27), and (30) we can compute $S_i\ (A^*, A_i)$ (i = 1, 2, 3, 4) as follows:

$S_1\ (A^*, A_1) = 0.546$, $S_2\ (A^*, A_2) = 0.601$, $S_3\ (A^*, A_3) = 0.599$, and $S_4\ (A^*, A_4) = 0.749$.

When $p = 2$, in the same way we have the same as the above results:

$S_1\ (A^*, A_1) = 0.546$, $S_2\ (A^*, A_2) = 0.601$, $S_3\ (A^*, A_3) = 0.599$, and $S_4\ (A^*, A_4) = 0.749$.

Step 2. The order of quality for the four alternatives is A_4, A_2, A_3 and A_1, obviously, amongst them A_4 is the best alternative.

In this example, all the intuitionistic trapezoidal fuzzy numbers are symmetrical. Therefore, the calculation of the characteristic values is not relevant to the parameter k. Then in some cases, the value of the parameter k might be determined by the decision maker according to the preference value to satisfy practical decision-making requirements.

5 Conclusion

This paper presented a method to measure the similarity between intuitionistic trapezoidal fuzzy numbers. First, we adopt the concept of characteristic values of intuitionistic trapezoidal fuzzy numbers to define the measuring distance between the characteristic values. Then we used this distance to generate a similarity measure and a weighted similarity measure between intuitionistic trapezoidal fuzzy numbers and established an intuitionistic trapezoidal fuzzy multicriteria decision-making method based on the weighted similarity measure of the characteristic values. By means of the ideal alternative, the weighted similarity measure between an alternative and the ideal alternative was given based on the characteristic values for intuitionistic trapezoidal fuzzy numbers. Thus, the ranking of alternatives can be easily obtained according to the weighted similarity measure for each alternative. In the information integration for intuitionistic trapezoidal fuzzy numbers, the advantage of the proposed method is to maintain the integrity of information as intuitionistic trapezoidal fuzzy numbers are continuous sets; while discrete sets my be loss partial information in the information integration. Therefore, the continuous sets are superior to the discrete sets. Finally, an illustrative example for the multicriteria decision-making problem of alternatives in an investment company illustrates the applicability of the proposed method.

References

[14] Atanassov K 1986 Intuitionistic fuzzy sets, *Fuzzy Sets Syst.* **20**, 87-96.

[15] Li D and Cheng C 2002 New similarity measures of intuitionistic fuzzy sets and application to pattern recognition, *Pattern Recogn. Lett.* **23**, 221–5.

[16] Liang Z and Shi P 2003 Similarity measures on intuitionistic fuzzy sets, *Pattern Recogn. Lett.* **24**, 2687–93.

[17] Mitchell H B 2003 On the Dengfeng–Chuntian similarity measure and its application to pattern recognition, *Pattern Recogn. Lett.* **24**, 3101–4.

[18] Hung W L and Yang M S 2004 Similarity measures of intuitionistic fuzzy sets based on Hausdorff distance. *Pattern Recogn. Lett.* **25**, 1603–11.

[19] Hung W L and Yang M S 2007 Similarity measures on intuitionistic fuzzy sets based on Lp metric, *INT. J. Appro. Reason.* **46**, 120–36.

[20] Ye J and Lian J Y 2006 Decision-making method of mechanical transmission schemes based on an improved similarity measure between vague sets, *Proceedings of the International Conference on Mechanical Transmissions*, Chongqing, China, p. 1694-8.

[21] Jiang W J and Ye J 2009 Decision-making method based on an improved similarity measure between vague sets, *IEEE 10th International Conference on Computer-Aided Industrial Design & Conceptual Design*, Wenzhou, China, p. 2086-90.

[22] Dubois D and Prade H 1978 Operation on fuzzy number, *Int. J. Syst. Sci.* **9**, 613-26.

[23] Dubois D and Prade H 1987 The mean value of a fuzzy number, *Fuzzy Sets Syst.* **24**, 279-300.

[24] Nehi H M and Maleki H R 2005 Intuitionistic fuzzy numbers and its applications in fuzzy optimization problem, *Proceedings of the 9th WSEAS International Conference on Systems, Athens, Greece*, p. 1-5.

[25] Grzegrorzewski P 2003 The hamming distance between intuitionistic fuzzy sets, *Proceedings of the 10th IFSA World Congress*, Istanbul, Turkey, p. 35-8.

[26] Nehi H M 2005 An order method for intuitionistic fuzzy numbers. *Proceedings of the 9th WSEAS International Conference on Systems*, Athens, Greece, p. 1-4.

[27] Chiao K P 2001 Characteristic value of fuzzy number defined with parameter integral form, *Proceedings of the Ninth National Conference on Fuzzy Theory and Its Application*, Taiwan, p.1-6.

[28] Wang J Q and Zhang Z 2009 Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multicriteria decision making problems, *J. Syst. Eng. Electron.* **20**(2), 321-6.

[29] Ye J 2011 Expected value method for intuitionistic trapezoidal fuzzy multicriteria decision-making problems. *Expert Syte, Appl.* **38**, 11730-4.

[30] Ye J 2012 Multicriteria group decision-making method using vector similarity measures for trapezoidal intuitionistic fuzzy numbers. *Group Decis. Negot.* **21**(4), 519-30.

[31] Herrera F and Herrera-Viedma E 2000 Linguistic decision analysis: steps for solving decision problems under linguistic information, *Fuzzy Sets Syst.* **115**, 67-82.

[32] Ye J 2012 Multicriteria group decision-making method using the distances-based similarity measures between intuitionistic trapezoidal fuzzy numbers, *Int. J. Gen.l Syst.* **41**(7), 729-39

Authors

Shapu Ren, Aug. 24, 1960, Shaoxing, China P.R.

Current position, grades: Associate Professor.
University studies: 1988-1991 Lanzhou Institute of Seismology, China Seismological Bureau, M.E. degree(1991)
1980-1984 China University of Geosciences, B.E. degree(1984)
Scientific interest: Multi-criteria Decision-Making, Pattern Recognitions and Embedding System Design.
Publications: More than fifteen papers in journals.
Experience: More than 20 years of experience Shaoxing University. 2008 - Visiting scholar in Hangzhou Dianzi University.