The error analysis of TDOA based ultrasonic position

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Abstract

How to enhance the accuracy and range is a typical issue in the ultrasonic position technology. The position accuracy depends on the hardware architecture and the detecting methods. This paper designs a hardware platform called Hexagon-Buck with six transceivers around. For the inherent property of TDOA (Time of Difference of Arrival) ranging, the RF synchronous time and embedded software processing time are two inevitable factors that leads to the distance error. At the same time, the ultrasonic velocity is sensitive to the environment temperature. The paper considers both the temperature compensation and Least Squares Linear Regression to enhance the ranging accuracy. By taking the physical test data to identify the linear regression model, we can estimate the real distance. The validity of the method has been proved by the final experiment.

Keywords: ultrasonic ranging, TDOA, least squares linear regression, hexagon-buck

1 Introduction

Each node how to locate themselves in the large sensor network systems has become a challenging subject. According to the measuring distance and the angle information, the wireless location technology can be classified into two types: the range-based technology and the range-free technology. Although range-free technology has advantages in the power consumption and hardware cost, its measured accuracy can not meet the requirement of WSN (Wireless Sensor Networks), such as DV-Hop [1] and APIT [2]. On the contrary, range-based technology like TOA (Time of Arrival) [3], TDOA (Time of Difference of Arrival) [4, 5], AOA(Angle of Arrival) [6, 7], RSSI(Received signal strength indication) [8, 9] and TOF(Time of Fly) [10] is more commonly used in engineering.

TDOA technology is used to range the distance between the sender and receiver by the time difference, which has been widely proposed as a necessary ingredient in WSN self localization systems. Compared with other range technologies, TDOA works without the network time synchronization, which can significantly reduce the resource calculation cost and hardware expenditures. In order to ensure the ranging accuracy, the paper takes TDOA to calculate the distance between two nodes and adopts the Least Squares Linear Regression model to revise the measured error. The hardware called Hexagon-Buck with six transceivers around the board is also implemented to test the ranging performance.

The ultrasonic based TDOA leads to the nodes distance but not the nodes position. This paper also discusses the multi-dimensional scaling positioning algorithm based on the nodes' distance matrix [11]. Theoretically, if the nodes ranging is zero-defection, then the position is zero-error. So we are pursuing an error eliminating method, and concerns how to enhance the ranging accuracy by means of software algorithm optimizing as well as the hardware improvement.

2 System architecture

In this paper, the Hexagon-Buck which is a novel ultrasonic platform featured with six transceivers is presented. The hardware architecture is shown as Figure 1.

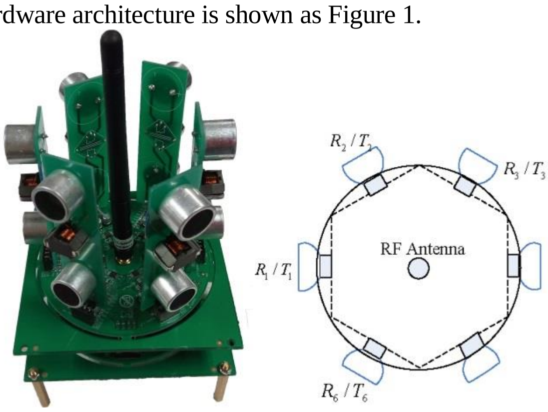

FIGURE 1 Hexagon-Buck platform with six transceivers around (the board has a diameter of 8 centimeters)

Since the radio transmitting time is measured by the local clock of the host node, and the arrival time is calculated on the ultrasound board, so it is important that the clock drift between the two microcontrollers keeps minimal. Thus the time synchronization among the sending nodes and the receiver nodes is necessary. For our hardware platform, the ad-hoc network is constructed by two layers: the physical layer with IEEE 802.15.4 and the network layer with Zigbee. The core of Hexagon-Buck is a dedicated micro-controller to control the ultrasonic transmission, receive signals, as well as manage the Zigbee stack and network application. The MCU of Hexagon-Buck is JN5148 produced by NXP. It is combined with an IEEE802.15.4 compliant transceiver on chip. JN5148 also integrates a temperature sensor. By providing the absolute measure of

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the device temperature, we can easily amend the sound velocity according to Equation (3).

Typical ultrasonic transceiver is directional. In order to realize the omni-directional detection, we equip six transceivers around to solve the limited beam angle problem, just like the previous platforms (e.g. the SpiderBat nodes [7] and the AHLoS nodes [12]). 40C16T/R-1 Ultrasonic Sensor produced by OSENON is designed as the transceiver. The centre frequency is 40kHz and the bandwidth is 2kHz. The ultrasonic transmission is cone shaped with the angle of $60 \pm 15^{\circ}$. So six 40C16T/R-1 transceivers can cover 360° omni-direction. Further more, the separate ultrasound transmitters and receivers can reduce the complexity without switching the circuits between receive and transmit mode.

1) The transmission circuit is driven by the square wave with 50% duty ratio and 40kHz frequency. The square wave is operated by the Pulse Width Modulation (PWM) of the micro controller. DC-DC module as power amplifier provides $5V$ and $\pm 10V$ voltage to 40C16T/R-1.

2) Since the receiver signal is only a few millivolt, it needs to be amplified for reliable detection. As shown in Figure 2, the first amplification stage provides 40dB amplification. The second amplification stage equipped with a digital potentiometer is to adjust the detection threshold and prevent the saturation of the sampled signal. This detection signal is then to compare with a reference voltage, which is also can be adjusted by a digital potentiometer. If the signal exceeds the reference voltage, the comparator will trigger an electrical level change captured by the micro controller, which leads to hardware interrupt.

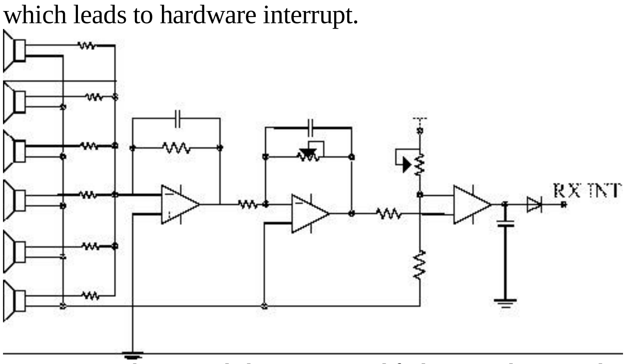

FIGURE 2 The received ultrasonic is amplified twice and compared with a reference voltage

3 Temperature and frequency feature of ultrasonic

3.1 TEMPERATURE FEATURE

Like other acoustic wave, the ultrasonic velocity is about $340.2\,m/s$ in the air at the condition of normal pressure and temperature (NPT). But actually it is affected by the status of transmission medium such as the moisture, pressure and density. Compared with the moisture and pressure, sound velocity is more sensitive to the temperature. The propagation velocity in normal atmosphere can expressed as bellow:

$$v(t) = \sqrt{\gamma RT/\mu}, T = 273 + t, \quad (1)$$

where $\gamma = 1.4$ is the air specific heat ratio. R is the perfect gas constant, and the value in the air is $R = 8.134 kg \cdot mol^{-1} \cdot K^{-1}$. $\mu = 0.00283 kg \cdot mol^{-1}$ is the air molar mass and t is the centigrade temperature. The Taylor Series Expansion of Equation (2) is as bellow:

$$v(t) = \sum_{n=0}^{\infty} \frac{v^{(n)}(0)}{n!} t^n = v(0) + v'(0)t + \frac{1}{2!} v''(0)t^2 + \cdots + \frac{1}{n!} v^{(n)}(0)t^n + \frac{1}{(n+1)!} v^{(n+1)}(\xi) t^{n+1}, \quad (2)$$

where $\xi \in (0,t)$. Omit the higher order term:

$$v(t) \approx 331.5 + 0.607t(m/s). \quad (3)$$

According to Equation (3), the velocity increases about $0.607\,m/s$ along with every additional centigrade. Later we will take Equation (3) to correct the ultrasonic velocity.

3.2 FREQUENCY FEATURE

On one hand, the sound intensity is damping along with the transmission for the energy keeps decreasing during the wave diffusion, reflection and scattering. When the frequency is higher, the intensity declines faster.

On the other hand, the directivity of ultrasonic also changes with the frequency, which is described by the index of half-power beam width (the angle of two half-power point with -3dB damping). The sound energy dumps along with the axial direction and fades out on both sides. If the frequency is higher, the half-power beam width is smaller and the directivity is better.

So the balance between a better directivity and a lower sound intensity decline must be considered when choose the ultrasonic frequency during the hardware design. This paper selects 40 kHz ultrasonic for its better robust and anti-interference.

4 Accuracy deterioration factors

The ultrasound ranging is unidirectional. The sender node initiates a single measurement by broadcasting a radio packet and indicates a time stamp. While the nodes nearby listen for the incoming ultrasound waves. During the procedure, there are several factors which may cause the accuracy deterioration.

4.1 SYNCHRONOUS TIME ERROR

In Figure 3, the coordinator and router are taken as the sender and receiver respectively to show the error schematic. t_{rf} is the RF transmission time and δt_{rp} is the synchronization signal processing time at the receiver. After receiving a synchronization signal, the receiver sends an ACK signal back to the sender and starts a hardware timer to count the ultrasonic fly time. As soon as the sender node gets this ACK, it starts a PWM wave to drive 40C16T/R to generate an ultrasonic. The processing time at the sender is marked as δt_{sp}. The measured fly time of ultrasonic counted by receiver is marked as t_{uf}, and the theoretical fly time is t'_{uf}.

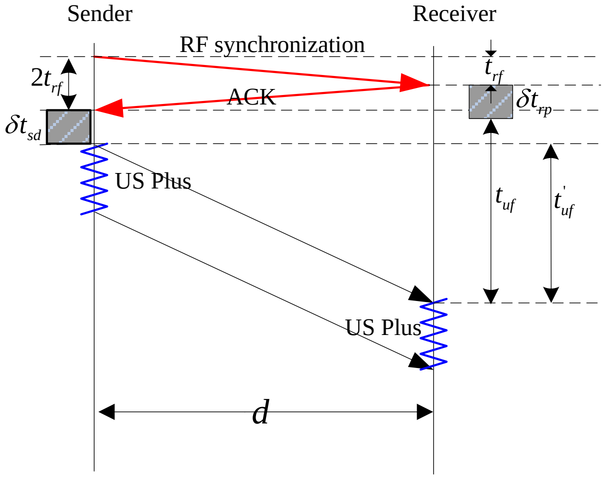

FIGURE 3 Schematic of time synchronization error

As shown in Figure 3, there is the equation: $2t_{rf}+\delta t_{sp}+t'_{uf}=t_{rf}+\delta t_{rp}+t_{uf}$.

Define $\Delta t=t'_{uf}-t_{uf}$, then the Equation above can be rewritten as:

$$\Delta t=\delta t_{rp}-\delta t_{sp}-t_{rf}. \quad (4)$$

Because of the tremendous transmission velocity of RF signals and the span limitation of $20m$, its maximum transmission time is only $0.07us$, during which the ultrasonic only travels $0.0024cm$. So t_{rf} can be omitted during the calculation. But no matter the sender node or the receiver node, the software procession time and hardware start-up time are inevitable and immeasurable. So t'_{uf} must be estimated by t_{uf}:

$$t'_{uf}=t_{uf}+\Delta t\approx t_{us}+(\delta t_{rp}-\delta t_{sp}). \quad (5)$$

4.2 ULTRASONIC DETECTION ERROR

At the receiver node, the ultrasonic should firstly be transmitted into the voltage signal and then be amplified and filtrated. Next the voltage signal will be compared with a specified threshold to identify if the ultrasonic has reached the receiver node. When the voltage amplitude is greater than the threshold, the comparator triggers a hardware interrupt to tell the MCU that an ultrasonic pulse has been detected and ready to enter the interrupt service routine.

The ultrasonic detection error is because there is time delay t_{sw} before the ultrasonic amplitude reaches the threshold. This delay changes along with the ranging distance. For example, this time is nearly $112.3us$ in our platform and the corresponding distance is about $3.8cm$.

4.3 THE SHAPE ERROR OF HEXAGON-BUCK

As previous mentioned Hexagon-Buck has six parts of transceivers around. In Figure 4, d' is the theoretically transmitting distance. But we need the central to central distance d in practice. So the shape caused error is: $d_{hw}=d-d'$.

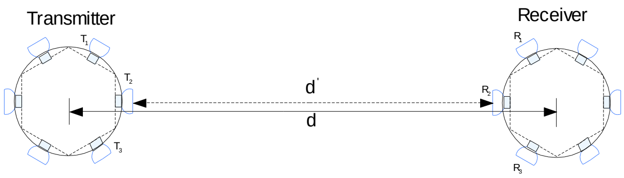

FIGURE 4 Schematic of shape caused error

4.4 ENVIRONMENT EFFECT

If the environment moisture is random distribution in 10~90%RH, the maximum normalized velocity error is only 1.5×10^{-3}, which means that there is only $3cm$ bias at the maximum measure range (about $20m$). This accuracy is enough for the implementation, so need no compensate.

As we discussed in section 3.1, the ultrasonic velocity is more sensitive to the temperature. Define Δv_{temp} be ultrasonic velocity error caused by the temperature. In the following section, the velocity compensation is only based on temperature without considering the environment moisture.

5 Ranging errors correction

5.1 TEMPERATURE COMPENSATION

Use the following equation to calculate the ultrasonic velocity compensation for the distance measurement:

$$d^*=\left(v+\Delta v_{temp}\right)\cdot\left(t_{us}+(\delta t_{rp}-\delta t_{sp})\right)+d_{hw}. \quad (6)$$

Combined the Equation (5), there is: $d^*=v_{temp}\Delta t+d_{hw}$.

The experiments are designed to test the ranging performance of Hexagon-Buck with only temperature compensation. The distance moves every $1m$ and is corrected according to Equation (6). Take the average value of 50 data at each distance. Figure 5 shows the results of ranging error.

The maximum range is about $15.8m$ and the ranging data is stable at $13m$. The average error is about $28.6cm$ and the extreme is $33.6cm$. The phenomenon is because there is no consideration on the RF synchronization error t'_{uf}, the ultrasonic detection error t_{sw}, and the shape error d_{hw}. In order to enhance the accuracy, use Least Square method to correct to error affected by other factors.

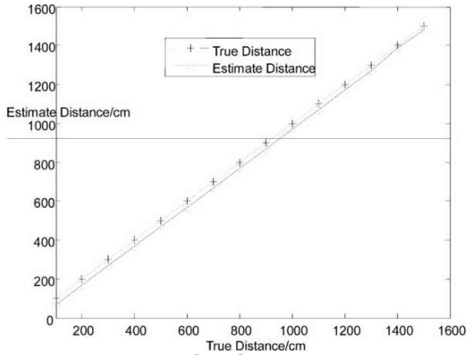

FIGURE 5 Ranging error based on temperature compensation

5.2 LSF CORRECTION

The ranging data correction is a typical liner regression problem: $D_n(x) = \mathbf{AX} = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$.

Define $\{(x_i, D_n(x_i)) \mid i = 1, 2 \ldots m\}$ to approximate the actual ranging data $\{(x_i, y_i) \mid i = 1, 2 \ldots m\}$, where x_i is the measured ranging data corrected by the temperature compensation, and y_i is the actual distance between the sender and receiver. Considering the CPU processing ability, take first-order least square regression to estimate the value of constant a_0 and a_1.

$$E = \sum_{i=1}^{m} (y_i - a_1 x_i - a_0)^2 . \quad (7)$$

In order to minimize the variance E, calculating the partial differential of Equation (7):

$$\begin{cases} \dfrac{\partial E}{\partial a_1} = -2\sum_{i=1}^{m} x_i (y_i - a_1 x_i - a_0) = 0 \\ \dfrac{\partial E}{\partial a_0} = -2\sum_{i=1}^{m} (y_i - a_1 x_i - a_0) = 0 \end{cases} .$$

Solve the equation, there is:

$$a_0 = \frac{\sum_{i=1}^{m} x_i \sum_{i=1}^{m} x_i y_i - \sum_{i=1}^{m} x_i^2 \sum_{i=1}^{m} y_i}{\left(\sum_{i=1}^{m} x_i\right)^2 - m\sum_{i=1}^{m} x_i^2} , \quad (8)$$

$$a_1 = \frac{\sum_{i=1}^{m} x_i \sum_{i=1}^{m} y_i - m\sum_{i=1}^{m} x_i y_i}{\left(\sum_{i=1}^{m} x_i\right)^2 - m\sum_{i=1}^{m} x_i^2} , \quad (9)$$

where x_i is the average value of 50 measured data at specific distance, and the distance is increased every 1*m*. Because the maximum range is about 15.8*m*, so the regression only effective in 15*m*, etc. *m*=15. The regression model expression is:

$$D(x) = 0.9922d^* + 0.347 . \quad (10)$$

Figure 6 is the comparison of the measured distance calculated by regression model and the actual distance. The average bias is only about 3*cm*.

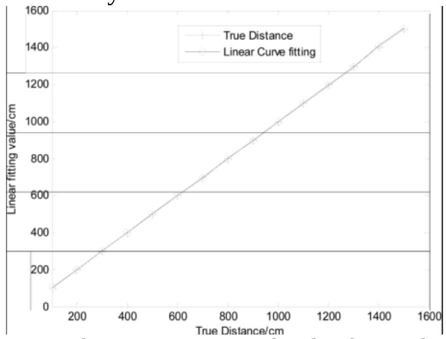

FIGURE 6 The regression is very closed to the actual value

5.3 EXPERIMENT VALIDATION

A further experiment is implemented to evaluate the validity of the linear regression model with the ultrasonic velocity compensation. The experiments are operated twice: first only with the temperature compensation (UN-Calibration) and second considering both the temperature compensation and LSF correction (Calibration). The ranging is also in the range of 15*m* and changes every 1*m*. Figure 7 shows the results. In the condition of only velocity calibration, the average bias is 29.8*cm* and the extreme is 35.3*cm*. However, it is noteworthy that the LSF correction with velocity calibration can greatly reduce the ranging error. The average bias is only 3.5*cm* and the extreme is 8.4*cm* (at the distance of 9*m*).

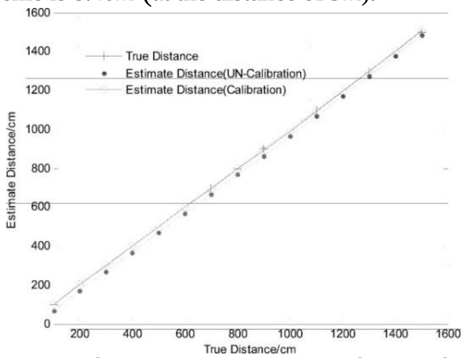

FIGURE 7 The ranging error correction with two methods

6 Conclusions

The paper designs and implements the Hexagon-Buck to enforce the node ranging and position. Considering the ultrasonic velocity error caused by the temperature and the RF synchronous time error caused by TDOA, the paper adjusts temperature compensation and least squares linear regression model to enhance the ranging accuracy. First design the physical experiment and liner regression model to match the unknown parameter, and then calculate the real distance. The experiments show the validity of this method.

Beyond these, the further study will continue consider the impact of ranging error distribution as well as the WSN nodes distribution to the position estimating.

Acknowledgments

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