Epidemic spreading in weighted homogeneous networks with community structure

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Abstract

Community structure has been proven to have great impact on epidemic spread in weighted networks. To understand the epidemic propagation in weighted homogeneous networks with community structure, a model of pseudo-random network is presented with adjustable community structure. By changing the number of edges connecting to the nodes in the same community and the weight of edges connecting to the nodes in the same community, we investigate the epidemic spreading in weighted homogeneous networks with different community structure. Simulations show that both the number of within-community edge and the weight of within-community edge have great impact on epidemic spreading behaviour.

Keywords: epidemic spread, weighted network, homogeneous network, community structure

1 Introduction

In the past few years, the study of complex network has attracted the increasing interest since the small-world phenomenon introduced by Watts and Strogatz [1] and the scale-free phenomenon proposed by Barabási and Albert [2]. The ultimate goal of these studies on topology is to understand and explain the dynamic process up these networks, for instance, to understand how the topology of social network affects large-scale epidemics such as H1N1 [3] and so on. It is of great importance to control the epidemic spreading taking place in real world networks. A great deal of models has been proposed to investigate the feature of epidemic spreading where the node is classified in three states: susceptible (which will not infect others but may be infected), infected (which is infective) and recovered (which has recovered from the disease and has immunity). The SI [4-6], SIS [7-9], and SIR [10-12] models are proposed based on the discrete states of the nodes. To investigate the dynamical behaviours in the very early stage of epidemic outbreaks when the effects of recovery and death can be ignored, we shall focus on the susceptible-infected (SI) model in which individuals can be in two discrete states, either susceptible or infected. Each individual is represented by a node of the network and the edges are the links between individuals along which the infection may spread. An infected node can infect any of its neighbourhood nodes with a fixed probability λ at each time step and the infected nodes remain always infective. At the beginning time, I_0 nodes are randomly selected to infect the rest of the network, the dynamical process being affected by the topology of the network. The behaviours of the SI model are not only of theoretical interest, but also of practical significance.

The previous studies on networks have been principally focused on the unweighted network where edges between nodes are either present or not. However, lots of real world systems such as the scientific collaboration networks [13], the world-wide airport network [14], the mobile networks [15] and the Internet [16] have proved to be specified not only by the topology but also by the edge weight.

Accompany with the continuing study of complex networks, another common feature of many real world systems, the community structure, is founded [17-21]. Community structure is the tendency for nodes to divide into subsets within which node-node connections are dense, but between which connections are sparser. There have also been some studies to investigate the impact of community structure upon epidemic spreading in scale-free networks [22-24].

However, there are few studies to combine weight and community structure well to investigate the epidemic spreading in homogeneous networks. Indeed, the weight distribution of the edges would impact the community structure and the epidemic spreading in weighted homogeneous networks with community structure. In this paper, we proposed a model of pseudo-random network with adjustable community structure. By changing the number of edges connecting to the nodes in the same community and the within-community edge weight in the same community, we investigate the epidemic spreading in weighted homogeneous networks with different community structure.

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This paper is organized as follows. In section 2 we describe the models, followed by the experimental evaluations in section 3. The conclusions are given in section 4.

2 Models

In order to study the dynamical behaviours in the very early state of epidemic outbreaks, we focus on the traditional SI model in which nodes are either susceptible or infected. If $s(t)$ and $i(t)$ are the density of susceptible and infected nodes at time t respectively, then $s(t)+i(t)=1$. Denote the spreading rate as λ at which each susceptible node acquires infection from an infected neighbour during one time step. In homogeneous networks, each node has approximately the same degree which makes it possible to use mean-field theory to obtain approximate results. In this case the system, we have

$$\frac{di(t)}{dt} = \lambda < k > i(t)[1-i(t)] . \quad (1)$$

Equation (1) states that the growth rate of infected nodes is proportional to the spreading rate λ, the density of susceptible nodes that may become infected $s(t)=1-i(t)$, and the number of infected nodes in contact with any susceptible one. The homogeneous mixing hypothesis considers that the last term is the product of node degree $<k>$ and the average density of infected nodes $i(t)$.

For our weighted SI model, We assume that transmit probability through the edge with weight w, λ_w, is equivalent to the infected probability that w infected nodes simultaneously influence the susceptible nodes [25,26], which is

$$\lambda_w = 1-(1-\lambda)^w . \quad (2)$$

We also employ the pseudo-random network model to investigate the epidemic spreading since all other properties such as average node degree will be equivalent to fully random networks except the controllable varying strength of community structure. These networks are comprised of n nodes which are split into *mods* communities of *n/mods* nodes each. Each node has on average Z_{in} edges connecting it to nodes of the same community and Z_{out} edges to nodes of other communities. While Z_{in} is varied, the value of Z_{out} is chosen to keep the total average degree constant, and set to $<k>$. And we assign different weights to the different kinds of edges: between-community edges are given a fixed weight of w_{out} (which is often set to 1 for simplicity), while within-community edges are given the weight $w_{in} = w$. As Z_{in} and w are increasing, the communities become better defined and easier to identify.

To know the influence of the accuracy of community structure identification on information transfer capacity, we employ the modularity measure. **A** is the adjacency matrix where $\mathbf{A}_{ij}=1$ if nodes i and j are connected and 0 otherwise. Let c_i be the community, which node i belongs to. The modularity measure, Q, is defined as follows [18]:

$$Q = \frac{1}{2m}\sum_{ij}\left[A_{ij} - \frac{k_i \cdot k_j}{2m}\right]\delta(c_i, c_j) , \quad (3)$$

where $m = \frac{1}{2}\sum_{ij} A_{ij}$ is the number of edges in the network, $\delta(u,v)$ is 1, if $u=v$ and 0 otherwise. The higher the modularity Q_{max} is the stronger community structure the network has. In practice values for such networks typically fall in the range from about 0.3 to 0.7. Higher modularity values are very rare.

3 Simulations and analysis

At first, we check the impact of within-community edge weight w on the maximum modularity Q_{max} using pseudo-random networks with n=128 nodes which are divided into *mods*=4 communities with 32 nodes in each community. The average degree $<k>$ is set to 16. Simulation results are shown in Figure 1.

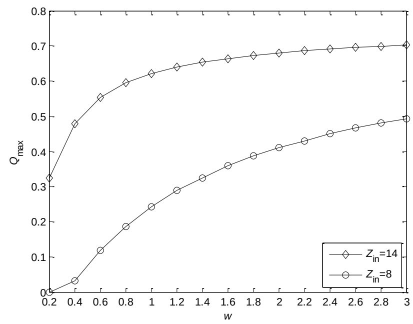


FIGURE 1 Q_{max} vs w, n=128, *mods*=4, $<k>$=16

From Figure 1 we can obtain that the increasing of edge weight will result in the increase of community structure especially in the traditional random network. In the pseudo-random network which has more with-community edges (Z_{in}=14 in Figure 1), the network has pronounced community structure even though the within-community edge weight is smaller. And the greater the number or the weight of within-community edge is, the higher the maximum modularity Q_{max} is.

Then we check the impact of the number of within-community edge Z_{in} on the maximum modularity Q_{max}. Simulation results are shown in Figure 2.

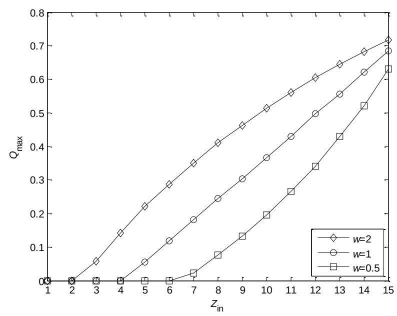
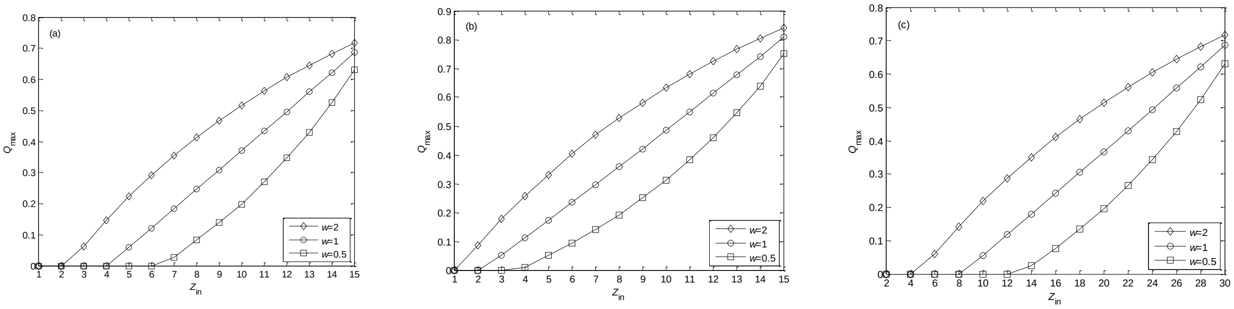


FIGURE 2 Q_{max} vs Z_{in}, n=128, *mods*=4, $<k>$=16

Figure 2 also proves that the greater the number or the weight of within-community edge is, the higher the maximum modularity $Q_{\max}$ is.

Then we check the impact of the total node number n, the community number *mods*, and the average node degree $<k>$ on the maximum modularity $Q_{\max}$. Simulation results are shown in Figure 3.

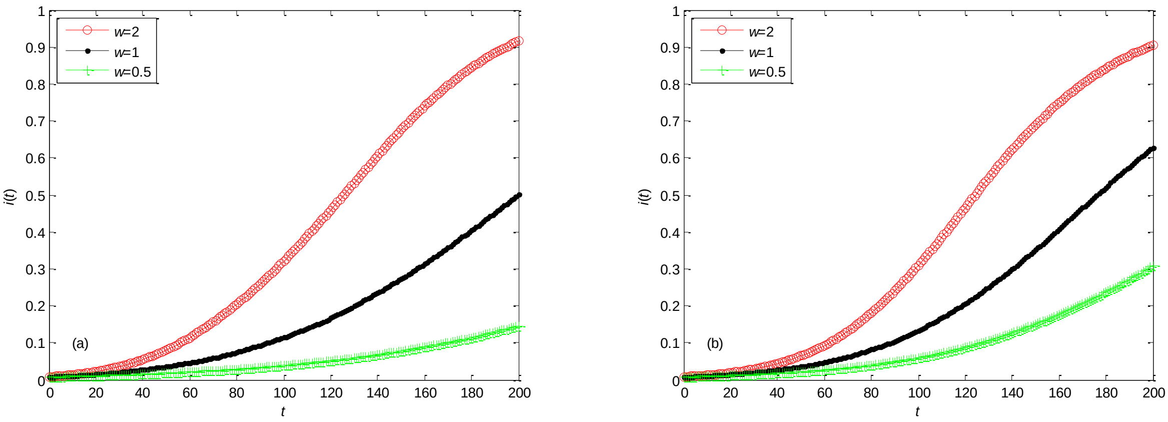


a) n=256, *mods*=4, $<k>$=16 b) n=128, *mods*=8, $<k>$=16 c) n=128, *mods*=4, $<k>$=32

FIGURE 3 $Q_{\max}$ vs Z_{in}

Comparing with Figure 2, we double the total node number, the community number and the average node degree to get the results shown in Figure 3a, 3b and 3c correspondingly. From Figure 3 we can obtain that the maximum modularity $Q_{\max}$ is also increasing accompany with the number and the weight of within-community edge. Only the changing of the community number affects the absolute value of maximum modularity. The changing of the total node number and the average node degree will barely affect the result of maximum modularity.

Now we focus on the epidemic spreading in weighted homogeneous networks. We set the the number of within-community edge Z_{in} as 14 for the network which has stronger community structure and 8 for the network which is a traditional random network. (In the scenario where average degree is 32, it changes to 28 and 16 accordingly.) At the initial age, we select a node randomly and make it an infected node. At each time step, the infected nodes will interact with their neighbours with probability λ_w which is defined in Equation (2). Simulations of different scenarios are shown in Figures 4, 5, 6 and 7.

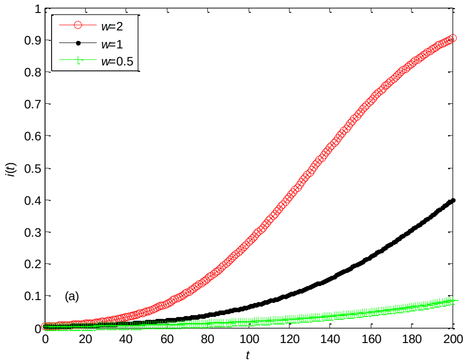


a) Z_{in}=14 b) Z_{in}=8

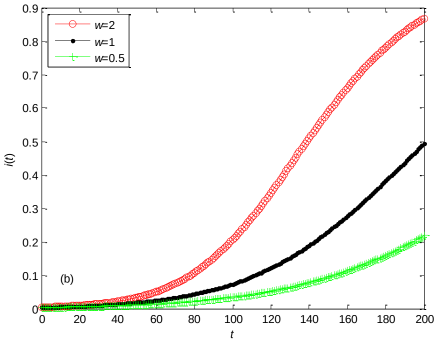
FIGURE 4 Epidemic spreading in weighted networks, n=128, *mods*=4, λ=0.002, $<k>$=16

In Figure 4, we report the density of infected nodes versus time in weighted homogeneous networks with different community structure. As shown in each figure, the epidemic propagation velocity is higher in networks with greater weight of within-community edge. However, when the weight of within-community edge is less than the weight of the between-community edge (w=0.5), less within-community edge will unexpectedly accelerate the epidemic propagation.

Then we also double the total node number, the community number and the average node degree to get the results shown in Figures 5, 6 and 7 correspondingly.

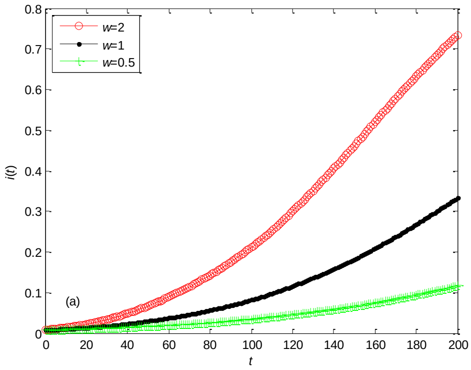


a) Z_{in}=14

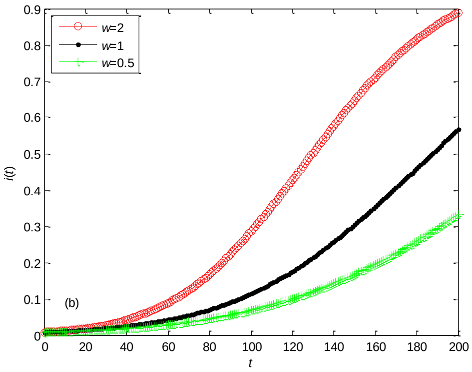


b) Z_{in}=8

FIGURE 5 Epidemic spreading in weighted networks, n=256, $mods$=4, λ=0.002, $<k>$=16

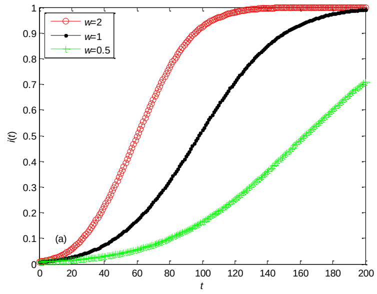


a) Z_{in}=14

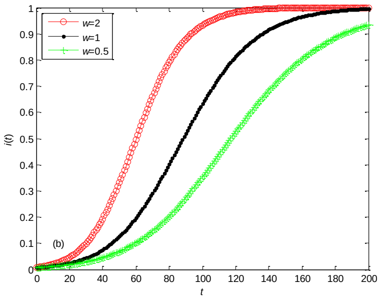


b) Z_{in}=8

FIGURE 6 Epidemic spreading in weighted networks, n=128, $mods$=8, λ=0.002, $<k>$=16



a) 3Z_{in}=28

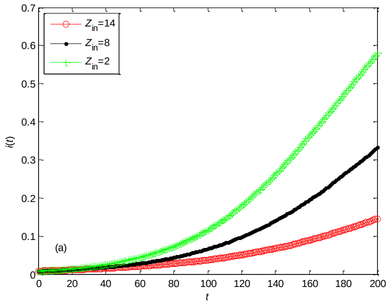


b) Z_{in}=16

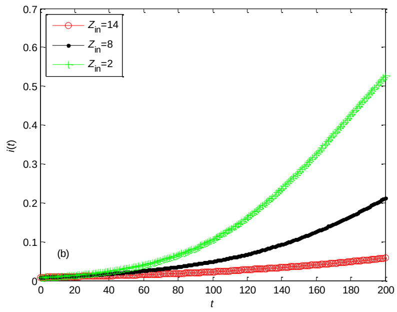
FIGURE 7 Epidemic spreading in weighted networks, n=128, $mods$=4, λ=0.002, $<k>$=32

As shown in each Figure, the increasing of weight of within-community edge will result in the acceleration of the epidemic propagation and less within-community edge will also accelerate the epidemic propagation when the weight of within-community edge is less than the weight of the between-community edge.

We utilize different within-community edge number Z_{in} to check the impact on epidemic spreading as shown in Figure 8.



a) w=0.5



b) w=0.25

FIGURE 8 Epidemic spreading in weighted networks, n=128, $mods$=4, λ=0.002, $<k>$=16

In both scenarios where the weight of within-community edge is less than the weight of the between-community edge, less within-community edge can accelerate the epidemic propagation.

4 Conclusions

How the community structure impact on the epidemic spreading in weighted homogeneous networks is studied in this paper. With our weighted SI model and the computer-generated pseudo-random networks model, the epidemic propagation velocity is studied in difference scenarios. Both increasing the number of within-community edge and increasing the weight of within-community edge can enhance the community structure. And increasing the weight of within-community edge will result in the acceleration of the epidemic propagation. Furthermore, we also proposed that less within-community edge will accelerate the epidemic propagation when the weight of within-community edge is less than the weight of the between-community edge. This study will shed light on how to restrain the epidemic spreading in weighted homogeneous network.

Acknowledgments

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