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Editors' Remarks

The Winners

("The Story of the Gadsbys")

What the moral? Who rides may read.
When the night is thick and the tracks are blind
A friend at a pinch is a friend, indeed,
But a fool to wait for the laggard behind.
Down to Gehenna or up to the Throne,
He travels the fastest who travels alone.

White hands cling to the tightened rein,
Slipping the spur from the booted heel,
Tenderest voices cry " Turn again!"
Red lips tarnish the scabbarded steel,
High hopes faint on a warm hearth-stone--
He travels the fastest who travels alone.

One may fall but he falls by himself--
Falls by himself with himself to blame.
One may attain and to him is pelf--
Loot of the city in Gold or Fame.
Plunder of earth shall be all his own
Who travels the fastest and travels alone.

Wherefore the more ye be helpen and stayed,
Stayed by a friend in the hour of toil,
Sing the heretical song I have made--
His be the labour and yours be the spoil.
Win by his aid and the aid disowns--
He travels the fastest who travels alone!

Rudyard Kipling[1] (1865-1936)

The No.2 of the 8th volume pays attention to some problems of computer modelling and special devices for transport technologies.

The present issue is the continuation of our publishing activities. We hope our journal will be interesting for research community, and we are open for collaboration both in research and publishing.

EDITORS

Yu. N. Shunin

I.V. Kabashkin

[1] **Rudyard Kipling (1865-1936)** was born in India at the time of the British Empire. His father was the Headmaster of a school in Bombay, and Kipling was sent at age six to a boarding school in England. He returned to India in 1882, and began work as a journalist on the 'Civil and Military' Gazette, while quickly gaining a reputation as a great writer. Kipling stands as a literary giant with a whole host of classic books to his name such as *Kim*, and *The Jungle Book.* Of his many poems his most famous is probably *'If'* which even today the encapsulation of a mini-degree course is in human psychology. He received the Nobel Prize for literature in 1907, and when he died was buried in Poet's corner at Westminster Abbey.

Computer Modelling & New Technologies, 2004, Volume 8, No.2, 7-13
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PRODUCTIVITY ESTIMATION OF *UNIX* OPERATING SYSTEM

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1. Introduction

The high **WWW** growth and a huge interest in distributing an information and computation resources through a network services creates a network servers as an important applications, which are used in the computer systems. In the recent days a lot of computer systems were isolated and worked for local data computation. Today, a lot of computers are involved in the network computations and a biggest part of data computation, which is performed by computer systems, involves the using of resources, which are given by another computers. The computer systems, which are giving particular services, are called servers. Usually, the service provides an access to the particular resource, which is managed by the server. There are a lot of resources, such as physical – disk space, CPU cycles, or, more general resources, such as functional capabilities, which are generated by the server, or specific information, which is stored by the server. There are a lot of examples of these resources, such as file server, which provides an access to file holders; a mail server, providing a reception and sending an electronic mail; WWW server, providing an access to the information from the different sources. The special programs, which are named server applications, carry the functionality of the server.

With the reliability increasing of remote servers, the speed of computation for the clients depends on the server capability to support the effective service. Also, computation possibilities (this means a diapason of tasks, which can be implemented by the computer) depend on functional possibilities and reliability of servers, which are accessible to computation systems. Unfortunately, with speed increasing of hardware components and networks, the server's speed has increased in small values. Also, the server systems have disadvantages in a lot of criteria's, which are a necessity for a complete functionality for the biggest infrastructure in our days – Internet.

For the better productivity are interest as a client and a server. A lot of researches, which are addressed to the tasks, such as increasing WWW caching [1, 2], the improving of **HTTP** protocol [3], the effective algorithms for HTTP servers [4], the improving of network functions in the kernel of operating systems [5], which works as a platform for an internet server, have the main goal to increase the productivity of WWW server.

The one of aspects, which influences on the Internet server works, is the choice of input/output interface, which has been chosen by the developers during the development work. In the UNIX operating system socket's interface is built on the processes management principles. It means that all the interfaces in the system, including input/output interfaces, use the file descriptors.

2. Internet server architecture

WWW clients and WWW servers communicate with each other by using an HTTP protocol and "client-server" technology. For data downloading, which are locate on WWW server, an HTTP client (i.e. WEB browser) should establish a **TCP** [6] connection. When the new request comes to the server, the kernel of operating system transfers this request to server application by using system call **accept()**. Then, server application waits a new incoming request from the client. After that, when this request arrives on the server, he passes the processing and after the processing server application sends the answer to the client. Usually, WEB servers receive answer from the file system, instead of it; **proxy** servers can receive the answer from other network resources, but both of them can use the caching for increasing the speed of answers.

The modern high-speed HTTP servers work as a set of concurrent processes and consisting from main process and set of slave processes. The main process serves the incoming requests and distributes them between slave processes in regular intervals.

3. OS UNIX network part

Server applications very intensive use network resources and a lot of time spend in the kernel context of operating system. So, the quality of network components in the kernel has a big pressure to server application productivity. In the UNIX operating system network part is built on BSD sockets technology.

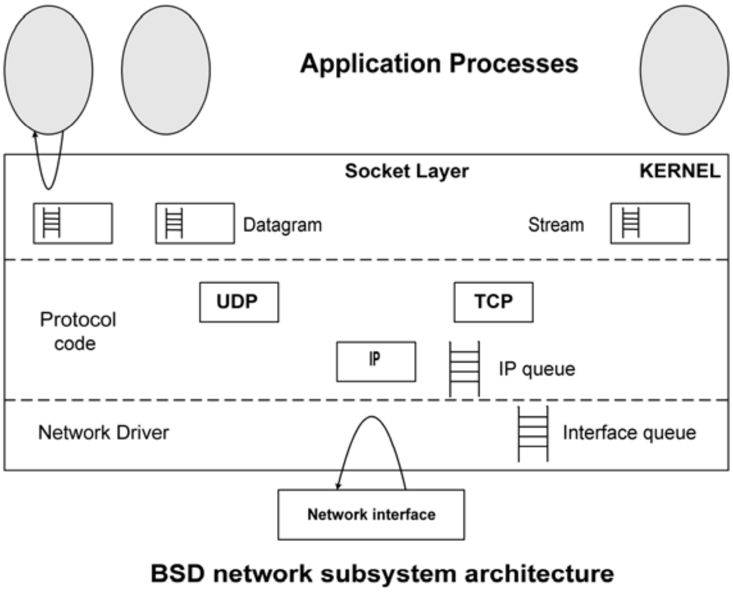

Figure 1

Kernel network part consists of three parts. The first part is a sockets layer, which is accessible to server applications through file descriptors. The second part of the system includes a network interface driver, which is a middle chunk between hardware components and the operating system kernel. Different interrupters perform a hardware part of the network. The third part includes a realization of network protocols such as **TCP, UDP** and **IP**. This part realizes a "software interrupts". Software interrupt is like a hardware interrupt, but the first means that the software event has occurred. For example, the network interface driver signals that a new **IP** [7] packet has come by sending a software interrupt to the module, which serves an IP protocol. The software interrupts are supported by interrupts handlers, which are run in the kernel context in the asynchronous mode. In other words, the software interrupts can be looked as a main mechanism, which manages an input Internet traffic on the server machine.

4. Research

Based on the information that has been received from the previous paragraph, all the kernel input/output interfaces have been discovered, all the input/output models have been built and the appropriate checking has been done. The checking has been done in real conditions with S-Client methodology. Based on the received results the analysis has been performed and as a result of this analysis the optimal input/output model for Internet server on UNIX platform has been found.
Based on the completed analysis of network part of UNIX kernel we can show up three input/output interfaces:

- ***Simple I/O interface***
- ***Multiplexing I/O interface***
- ***Multithreaded I/O interface***

The following servers do build based on these interfaces:

- ***Callback server***
- ***Simple-process multiplexing server***
- ***Multithreaded server***

Each of these interfaces has the own structures in the operating system kernel, specific-performing functions for these structures and specific system calls interfaces. So, what is the best of these interfaces? For receiving the answer for this question we have to build up all the founded models for all the founded interfaces.

All the input/output models do mean two stages, which has a process-server:

1. Waiting the data which should come to the socket;
2. Copying the data from kernel space into the process-server space.

After the completion of this event the process-server notification is performing, which means that all the data that process-server waits are locating in the process's buffer. The mechanism of coping packets from server to client works in the same way. Do show all the founded during the analysis models.

5. Simple I/O model

All the sockets are blocking ones. That means that this model is the most used. The usage of this model during server development means that the main process (which accepts the incoming requests) is being blocking all the time from the moment when the function (i.e. **recvfrom()**), which accepts incoming data has been called. When the data comes to the buffer the process wakes up and continues its work. The principle, which has been showed above, locates on Figure 2.

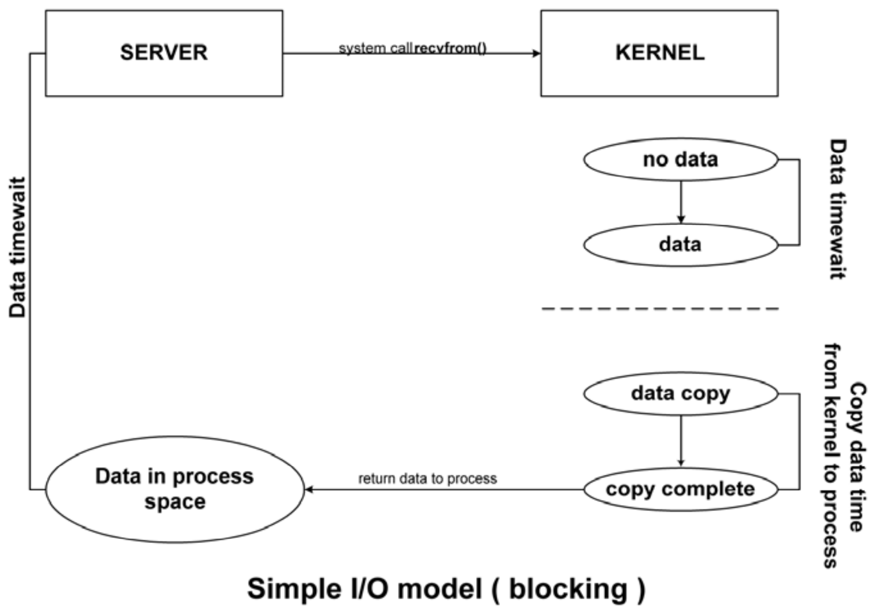

Figure 2

There is also a subtype of this model – the model without blocking mechanism. As the investigation has showed, the socket has a set of states, which allow change the socket behavior. One of these states is on/off of the process blocking during function call, which works with the socket. In other words, if the input/output operation cannot be completed without process transfer into the sleeping state, the process isn't gone to this state, but immediately returns the return code. This operation is performed until all the required data will be in the process-server buffer. This schema has received the following name – **pooling**. After that has been founded that the schemas like this aren't effective because of maximum usage of central processor on the server machine. But, on other hand, has been founded that if the server is used for two / three tasks only, than this schema has very good results of usage. The advantage – the absence of blocking mechanism and, as a result, ability perform big arrays of input requests. The principle, which has been showed above, locates on Figure 3.

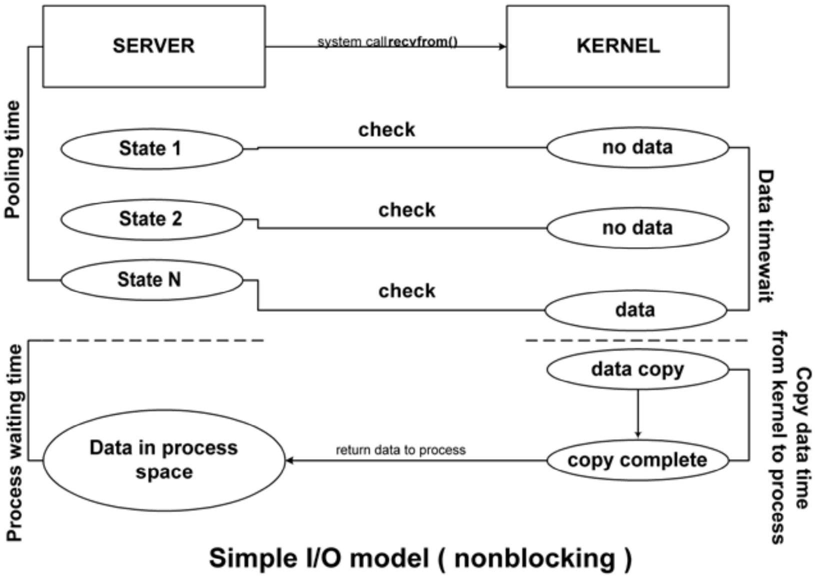

Figure 3

6. Multiplexing I/O model

This model allows simultaneously support a set of sockets by blocking process not in the system call, which supports a concrete socket, but in the system call, which supports all the set of sockets. These sockets are divided on three types: sockets for reading, sockets for writing and sockets for out-of-band operations. When the new request comes, the process-server add newly allocated descriptor into the descriptors set (descriptors for writing) and sets the bit, which describes this descriptor, in one. That means, that during the next **select()** call the new descriptor will be checked. As the other functionality of this mechanism, the descriptor that has been added in the set will be deleted, when the connection is closed. This schema is the same for descriptors for reading and for descriptors for out-of-band operations. Also, this model has an ability to setup a counter for the sockets, which are interesting for process-server. That means, that the data checking performed by the server when the counter is down. If requires, there is an ability to perform a **"polling"** by counter resetting in zero. But, there is a disadvantage, because CPU loading will be higher that in case of **Simple I/O model**, where there is only one connection. As a result, **"polling"** is not recommended for usage on the machines with average power. The principle, which has been showed above, locates on Figure 4.

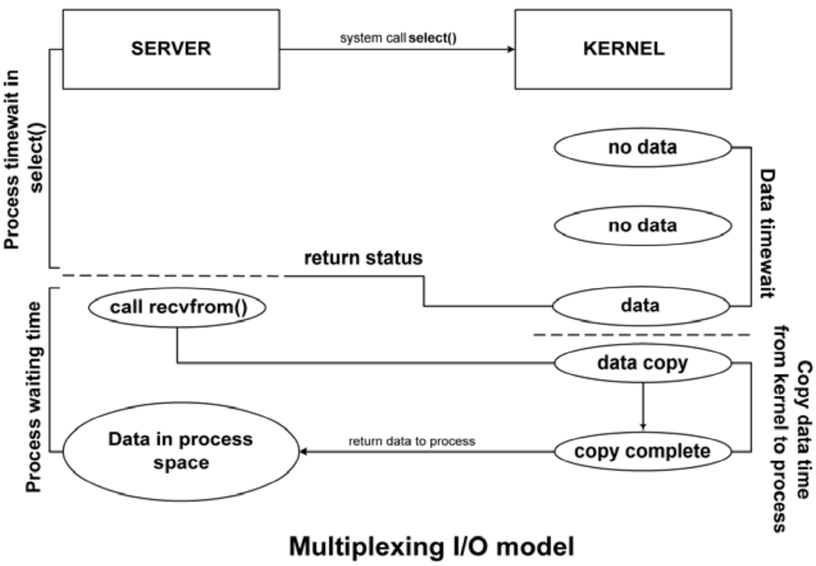

Figure 4

7. Multithreaded I/O model

The process performs the system call **fork ()** for creating a process child, which performs a job with a particular socket in all the models that have been listed above. In other words the parent process accepts a connection, creates a child and this child works with the client. The main disadvantage is bad usage of resources. The system call **fork ()** is very expensive. If the server uses the technology that has been listed above, the operating system cannot serve a big number of incoming requests. On other side, this model doesn’t require the creation of new process. When the new connection arrives, new thread is created and new allocated descriptor is passed to this thread. The communication between client and server is performed in the context of new created thread. In author’s opinion, this model should be the most effective in case of huge number of incoming requests. The principle, which has been showed above, locates on Figure 5.

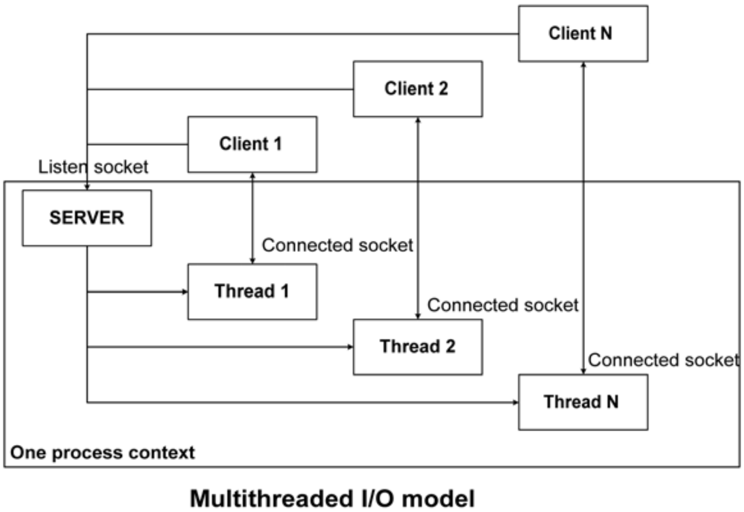

Figure 5

These models were tested in the real conditions for receiving the best one. The **S-Client** [8] technology has been used for productivity measuring. This technology allows generate an HTTP traffic, which is bounded to real traffic what has a big plus when a measuring of Internet server is performing. As a criterion of productivity, the number of simultaneous connections per second has been discovered. The average time, during which the client waits an answer from the server, can be as the second criterion of Internet server productivity. We have to say, that the Internet server productivity has a lot of criterions, such as file system work, the effective hardware components collaboration, network components work, etc. The investigation of all these criterions is out of scope of this paper, because the main goal was the productivity of input/output works in network components in the kernel. The receiving of practical results has been the following. For each model the Internet server prototype has ran. Respectively, for each model the ten tests were ran with 100, 1000 and 2000 pseudo-clients. The results of testing are showed below onto the graphics and in the Table 1.

TABLE 1. Connections per second

Simple I/O model			**Multiplexing I/O model**			**Multithreaded I/O model**		
100	**1000**	**2000**	**100**	**1000**	**2000**	**100**	**1000**	**2000**
14.3	15.8	15	125.7	95.7	93.7	145.7	95.7	91.5
13.8	13	13.8	125.9	95.8	94.1	143.9	95.1	93.2
14.1	14	14.5	125.4	95.1	93.1	145.7	94.6	92.5
14.6	13.3	14.8	125.7	95.3	93.3	144.7	95.1	92.6
13.9	15.3	14.6	126.3	95.7	93.8	145.6	94.5	93.5
14.8	13.4	13.6	126.2	95.6	93.7	145.7	95.3	92.9
15	15.2	14.1	125.2	95.2	93.2	145.3	95	93
13.9	13.7	14.4	127.7	96.7	95	146	94.2	92.4
14.1	14.4	15	127.7	96.7	94.9	145.4	95.4	92.3
14.9	13.4	14.1	126.7	96	94.1	145.1	93.6	92.2

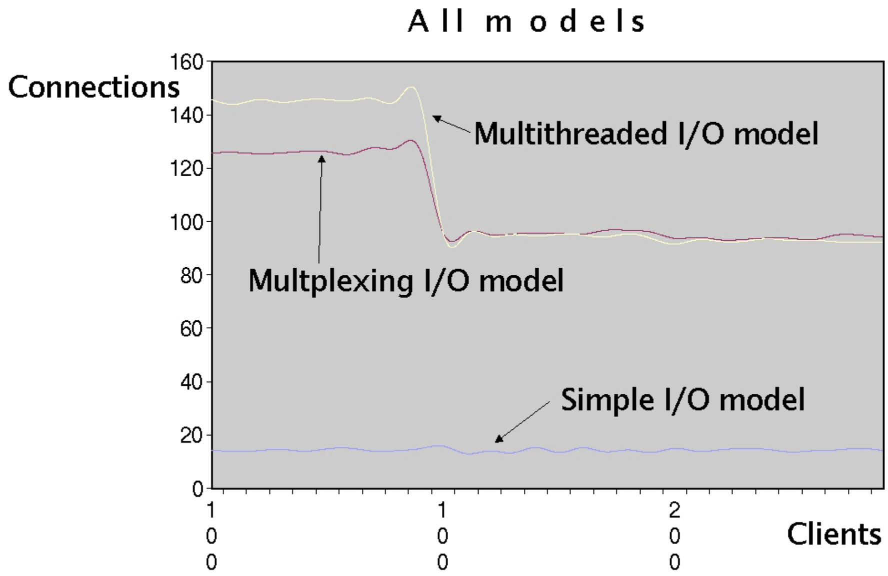

Figure 6

As the picture shows, the multiplexing input/output model is much better than the others two. The statistical checking has been performed for receiving that the data that has been received by practical way are really true data.

The number of connections per second was as a characteristic for checking. The task was to check a null hypothesis $H_0 : \overline{X} = \overline{Y}$ about equality of average sets VS concurrent hypothesis $H_1 : \overline{X} > \overline{Y}$ about non-equality.

The statistical analysis was performed in statistical software product STATGRAPHICS **Plus** 2.1. During the analysis the Fisher criterion was checked. The analysis has showed that the results for multiplexing input/output model statistically better than the results for other models. So, as a result, data that has been received practically have been defended theoretically.

8. Conclusion

This work researches the problem of productivity of Internet server, which runs on UNIX operating system. During the research, the network protocol have been discovered, the network part of the kernel has been researched, the models, which are made a base of Internet server algorithms, have been built.

The productivity practical estimation of these models has been done. The multiplexing I/O model has becomes the most effective one. For acknowledgment, the statistical analysis has been done. The result of analysis has shown the optimality of multiplexing input/output model.

The work results can be useful for Internet applications developers and also for whom, who engages with problems of productivity in computation systems in the Internet.

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CHANCE CONSTRAINED ORIENTED DISPATCHING RULES FOR FLEXIBLE JOB-SHOP SCHEDULING

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1. The backgrounds

We consider a job-shop manufacturing cell of n jobs (orders) J_i, $1 \le i \le n$, and m machines M_k, $1 \le k \le m$. Each job-operation $O_{i,j}$ (the j-th operation of job J_i) has its random time duration $t_{i,j}$ with the mean value $E(t_{i,j})$ and the variance $V(t_{i,j})$. Each job J_i has its arrival date A_i and its due date D_i. The penalty cost C_i^d for not completing the job on time (to be paid once to the customer), an additional penalty C_i^v to be paid for each time unit of delay, and the confidence probability (chance constraint) p_i^* of the job's accomplishment on time are pregiven too. If job J_i is accomplished before the due date D_i it has to be stored until the due date with the expenses C_i^s per time unit storage. The same storage expenses per time unit are paid in case when job J_i is idle, i.e., is ready to be operated on a certain machine and has to stay and wait in a line for that machine. Each machine M_k, $1 \le k \le m$, has to be delivered to the manufacturing cell at a certain moment T_k^* which has to be determined beforehand, i.e., before the job-shop starts manufacturing. Each machine is released at the moment when the last job is finished operating on that machine. The cost of hiring and maintaining each machine M_k per time unit C_k^m is pregiven. The problem is to determine the machines' delivery schedule $\{T_k^*\}$ in order to minimize the average scheduling expenses within the time period of processing all the jobs subject to their chance constraints.

It can be well recognized from recent publications (see, e.g. [1, 6]) that optimal analytical models in job-shop stochastic scheduling have been developed only for specific cases. The general job-shop problem with random operations has not yet obtained an analytical solution even for ($m \cdot n$)-models of small and medium-size. Several job-shop problems for flexible manufacturing cells with random operations have been solved [2-4] via heuristic decision-making based on pair-wise comparison [1]. The results obtained centre on developing heuristic rules in situations when several jobs are ready to be served on one and the same machine and the problem is to choose one of them to be passed to that machine. The decision-making rules developed in [2,3] are applied to a set of jobs (orders) J_i, $1 \le i \le n$, with different priority indices η_i, to be operated on m different machines M_k, $1 \le k \le m$, with random job-operations and due dates D_i. Ref. [4] considers a cost-optimization problem to determine quasi-optimal earliest possible time moments S_i to start processing jobs J_i in order to minimize the expected total value of penalty expenses to be paid for the jobs' delays (after the due dates) and storage expenses (for jobs being accomplished before their due dates). However, chance constraints have not been implemented in the model.

The paper under consideration presents a generalized version of the cost job-shop model. The optimized variables are planned moments T_k^*, $1 \le k \le m$, for each machine M_k to be delivered to the job-shop, subject to the chance constraints, in order to minimize the total average expenses within the makespan.

2. Notation

J_i	-	the i-th job (order) to be processed, $1 \le i \le n$ (n - number of jobs);
M_k	-	the k-th machine, $1 \le k \le m$ (m - number of machines);
$O_{i,j}$	-	the j-th operation of the i-th job, $1 \le j \le m_i$ (m_i - number of operations of job J_i);
$m_{i,j}$	-	the index of the machine to process operation $O_{i,j}$, $1 \le m_{i,j} \le m$ (pregiven);
A_i	-	the arrival-date of job J_i (pregiven);
D_i	-	the due date of job J_i (pregiven);
p_i^*	-	the confidence probability (chance constraint) of job J_i to be accomplished on time (pregiven);
$S_{i,j}$	-	the actual moment operation $O_{i,j}$ starts (a random value);
$t_{i,j}$	-	the random duration of $O_{i,j}$ with mean value $E(t_{i,j})$ and variance $V(t_{i,j})$;
F_i	-	the moment job J_i is accomplished (a random value);
T_k^*	-	the planned time of hiring and delivering machine M_k to the job-shop (an optimal variable, to be determined beforehand);
T_k^{**}	-	the time of releasing machine M_k (a random value);
C_i^d	-	the penalty cost for not accomplishing job J_i on time (pregiven), $1 \le i \le n$;
C_i^v	-	the penalty cost per time unit of delay, i.e., within the period $[D_i, F_i]$, (pregiven);
C_i^s	-	the expenses per time unit storage in case when J_i has been accomplished before its due date or in cases of waiting in lines for a machine (pregiven);
C_k^m	-	the cost of idleness of machine M_k (hiring and maintaining expenses) per time unit within the period $[T_k^*, T_k^{**}]$ (pregiven), $1 \le k \le m$;
$E(C)$	-	the average scheduling expenses throughout the cell's life cycle;
$E(C^m)$	-	the average idleness of machines expenses throughout the cell's life cycle;
$E(C^p)$	-	the average delay penalty expenses;
$E(C^s)$	-	the average job storage expenses throughout the manufacturing cell's life cycle.

3. The model

The dependent expenses (function of $\{T_k^*\}$) are:

- the average idleness of machines' expenses throughout the manufacturing cell's life cycle

$$E(C^m) = \sum_{k-1}^{m} \{ (T_k^{**} - T_k^* - \sum_{i=1}^{n} \sum_{j=1}^{m_i} [t_{i,j} d_{i,j}^k]) C_k^m \} , \tag{1}$$

where

$$T_k^{**} = \underset{(i,j)}{Max}(F_{i,j}\, d_{i,j}^k)\ , \tag{2}$$

$$d_{i,j}^k = \begin{cases} 1, & \text{if} \quad m_{i,j} = k; \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

- the average delay penalty expenses,

$$E(C^p) = \sum_{i=1}^{n} [C_i^d + C_i^v (F_i - D_i)]\delta(J_i)\ , \tag{4}$$

where

$$\delta(J_i) = \begin{cases} 1 \ \text{if} \ \ F_i > D_i; \\ 0 \ \text{otherwise}, \ 1 \le i \le n. \end{cases} \tag{5}$$

- the average job storage expenses throughout the manufacturing cell's life cycle

$$E(C^s) = \sum_{i=1}^{n} C_i^s \{(D_i - F_i)[1 - d(J_i) + (F_i - A_i^* - \sum_{j=1}^{m_i} t_{i,j})]\} \tag{6}$$

where

$$A_i^* = Max(A_i, T_k^* \delta_{i,1}^k)\ . \tag{7}$$

The problem is to determine values $T_k^*,\ 1 \le k \le m$ to minimize objective function

$$E(C) = \underset{\{T_k^*\}}{Min}[E(C^m) + E(C^p) + E(C^s)] \tag{8}$$

subject to

$$S_{i,1} = Max[A_i, T_{m_i,1}^*] \tag{9}$$

$$Pr\{F_i \le D_i\} = \bar{p}_i^*,\ 1 \le i \le n \tag{10}$$

Objective function (8) minimizes the scheduling job-shop expenses, while addendums (1), (4) and (6) denote the random job-shop's operational expenses. Restrictions (9) are obvious and denote that any job J_i cannot start processing before the machine needed to undertake the first operation O_{i_1} would be delivered and, of course, before its arrival to the manufacturing cell, while restrictions (10) are the chance constraints.

Model (1-10) is a stochastic optimization problem that is much more complicated and generalized than the model outlined in [4]. The model cannot be solved in the general case and allows only a heuristic solution.

4. Decision-making rule

The problem is solved via simulation, on the basis of the developed heuristic decision-making rule to choose a job from the line of jobs ready to be served on one and the same machine. The rule is an essential modification of the previously developed heuristics based on pair-wise comparison [2-4] of long-term forecasting to calculate for a routine job J_i at a certain moment t the probability $P_t(J_i)$ of meeting the due date on time. Developing a new decision rule for a routine couple (J_{i_1}, J_{i_2}) is the backbone of the paper. We suggest comparing the expected future scheduling expenses in order to choose one job from the couple by summarizing the penalty and storage expenses for the couple (J_{i_1}, J_{i_2}), beginning from moment t and finishing with the moment the last job from the couple will be accomplished. If $E(C_{i_1}^{(1)}) + E(C_{i_2}^{(1)}) \le E(C_{i_1}^{(2)}) + E(C_{i_2}^{(2)})$, that means that choosing J_{i_1} results in less expected scheduling expenses for two comparative jobs, than choosing J_{i_2} first. Thus, J_{i_1} is the winner? Otherwise J_{i_2} wins the competition. The winner is then compared with job J_{i_3}, etc., until only one winner is left, and is chosen for the machine. Implementing the decision-making rule it enables both the job-shop to be controlled in real time and simulating the job-shop by random sampling the job-operations' durations. In both cases, decision-making has to be introduced in multiple decision points t, when more than one job are standing in the line for one and the same machine. Simulating the job-shop many times enables evaluation of the job-shop's expected scheduling expenses (8).

It can be well-recognized that determining delivery schedule values T_k^*, $1 \le k \le m$, defines fully all the parameters of the manufacturing process. Determining $\{T_k^*\}$ results as well in determining values $\{A_i^*\}$ by (7), which, in turn, enables simulation of the job-shop in order to test the fitness of the suggested model. Thus, values T_k^*, $1 \le k \le m$, at each search point are input values for the simulation model.

5. Simulation model

The simulation model:

a) calculates at the beginning of each simulation run values A_i^*, $1 \le i \le n$, via (3) and (7);
b) determines lines of jobs ready to be processed and seeking one and the same machine;
c) chooses jobs from lines to the machines according to the decision-making rules via pairwise comparison;
d) calculates the scheduling job-shop expenses C according to (1-8) within a simulation run;
e) monitors the jobs' technological processes according to the initial data matrix;
f) simulates the duration $t_{i,j}$ of operation $O_{i,j}$ at moment $S_{i,j}$;
g) calculates the timing of essential moments t, when either an operation terminates, or there is a free machine to operate jobs which stay in a line for that machine, or a new job J_i at moment A_i enters the job-shop to start processing;
h) simulates random time values T_k^{**} to release machines M_k, $1 \le k \le n$;
i) simulates random time values F_i when job-shop J_i is accomplished;
j) calculates values $\overline{p}_i$, $1 \le i \le n$, and $E(C)$ on the basis of simulated statistics via numerous simulation runs.

Thus, by means of simulation, one can obtain the set of optimal values $\{T_k^*\}$ to minimize the expected scheduling expenses $E(C)$. Implementing the coordinate descent search algorithm [5] into the simulation model carries out optimization. Extensive experimentation have been undertaken to check the fitness of the model.

6. Conclusions

1. The newly developed optimization model for a general job-shop problem with a cost objective function enables an efficient solution to be obtained. The model covers practically all job-shop expenses and is more realistic than the previously developed models.
2. The optimization algorithm comprises an optimization sub-algorithm based on a cyclic coordinate descent method, and a simulation model comprising a modified decision-making rule for cost objectives under chance constraints.
3. The newly developed job-shop optimization model is the first publication in the area of job-shop scheduling under chance constraints. Thus, we are unable to compare the optimization model with other results.
4. Future research can be undertaken in the area of other job-shop optimal problems, e.g. stochastic scheduling with release dates and due dates, etc.

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THE TOTAL ERROR ESTIMATE OF ONE REGULARIZATION ITERATIVE METHOD FOR THE SOLUTION OF THE FIRST KIND OPERATOR EQUATIONS WITH INEXACT BOTH OPERATORS AND RIGHT-HAND SIDES

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1. Introduction

In the present work we consider the concrete iterative method for the solution of the such first kind operator equations in the abstract separable Hilbert Space and we estimate its total degree of convergence.

In this paper we consider an operator equation of the first kind

$$Az = u, \qquad u \in U,\ z \in F, \tag{1.1}$$

where F and U are separable Hilbert Spaces, the operator $A: F \to U$ is linear, self-conjugate, positive and completely continuous, $u \in U$ is a given element and $z \in F$ is the desired solution.

Let it is known a'priori information that the equation (1.1) has the unique exact solution z_{ex} under the given exactly initial data $\{A_{ex}; u_{ex}\}$ where $A = A_{ex} : F \to U$ is the exact operator, and $u = u_{ex} \in U$ is the exact right-hand side of the equation (1), i.e. $A_{ex} z_{ex} = u_{ex}$.

In the work $[1]$ is considered the method for construction of the approximate solution of the equation (1.1) which is stable against the small changes of the initial information when only right side of the equation (1.1) was inexact, but the operator was assumed exactly known, i.e. instead of $u = u_{ex}$ there was $\{u^{\delta}; \delta\}$ such that $\left\| u_{ex} - u^{\delta} \right\|_H \le \delta$.

The following Theorem 1.1 and Theorem 1.2 are proved in $[1]$:

THEOREM 1.1. *Let z_{ex} be an exact solution of (1.1) for $u = u_{ex} \in F$. Then the iterative process*

$$\begin{cases} z_{n+1}^{\delta} = \left(E - \theta \cdot A_{ex}\right) z_n^{\delta} + \theta \cdot u^{\delta},\ n \ge 0 \\ \\ z_0^{\delta} = 0. \end{cases} \tag{1.2}$$

converges to the exact solution z_{ex} of the equation (1.1) in the spectral norm of space F, if the number of iterations n is chosen from the condition $\sqrt{n} \cdot \delta \to 0$ as $n \to \infty, \delta \to 0$. Moreover, under the condition

$0 < \theta \le \dfrac{4}{3 \cdot \|A_{ex}\|_H}$ *the following error estimate hold for the iterative process (1.2)*

$$\|z_{ex} - z_n^\delta\|_A \le (2 \cdot n \cdot \theta \cdot e)^{-\frac{1}{2}} \cdot \|z_{ex}\|_A + \left(\frac{4}{3 \cdot m} \cdot \theta\right)^{\frac{1}{2}} \cdot \left[\sqrt{n} \cdot \delta\right],\ n \ge 1, \tag{1.3}$$

where $\|x\|_A = \sqrt{(Ax, x)}$.

THEOREM 1.2. *Under the additional conditions* $E_m z_{ex} = 0$ *and* $E_m z^\delta = 0$ *,where* $E_m = \int_0^m dE_\lambda$ *,* $m \in (0, \|A_{ex}\|)$ *– some fixed number, and under the conditions of Theorem 1.1 it is valid the following estimated deviation:* $\|z_{ex} - z_n^\delta\|_F \le (2 \cdot n \cdot m \cdot \theta \cdot e)^{-\frac{1}{2}} \cdot \|z_{ex}\|_F + \left(\frac{4}{3 \cdot m} \cdot \theta\right)^{\frac{1}{2}} \cdot \left[\sqrt{n} \cdot \delta\right],\ n \ge 1$.

2. The iterative process for solution of some class first kind operator equation with inexact both operator and right-hand sides

In this paper, in contrast to $[1]$, we will consider a case when instead of exacts initial data $\{A_{ex}; u_{ex}\}$ there are approximately initial data $\{A^h, h;\ u^\delta, \delta\}$. There data are characterized to the positive data δ and h in the following way: δ characterizes the error of the right-hand side of the equation as $\|u_{ex} - u^\delta\|_U \le \delta$, and h characterizes the error of the operator A^h as $\|A_{ex} - A^h\|_F \le h$. When we have such information about the equation (1.1) then we could find only approximate solution of the equation (1.1). Besides this approximate solution converges to the exact solution z_{ex} as δ, h converge to zero independently.

Thus it is necessary to find the approximate solution of the equation

$$A^h z = u^\delta, \tag{2.1}$$

convergent to the exact solution z_{ex} as $\delta \to 0$ and $h \to 0$ if it is known that

$$\|u_{ex} - u^\delta\|_U \le \delta, \quad \|A_{ex} - A^h\|_F \le h, \tag{2.2}$$

and is known the data $\{\delta; h\}$ also.

In this work we offer the similar to (1.2) iterative process for solution of problems (2.1)-(2.2).

THEOREM 2.1. *Let* z_{ex} *be an exact solution of (1.1) for* $u = u_{ex} \in U$ *, i.e.* $A_{ex} z = u_{ex}$. *Let there are the additional following conditions:* $E_m z_{ex} = 0$, $E_m z^\delta = 0$ *, where* $E_m = \int_0^m dE_\lambda$ *,* $m \in (0, \|A_{ex}\|)$ *– some fixed number. Then the iterative process*

$$\begin{cases} z_{n+1}^{\delta,h} = (E - \theta \cdot A^h) z_n^{\delta,h} + \theta \cdot u^\delta,\ n \ge 0 \\ z_0^{\delta,h} = 0. \end{cases} \tag{2.3}$$

determines the approximate solution of the problem (2.1)-(2.2) and converges to the exact solution z_{ex} *of the equation (1.1) in the norm of given Hilbert space* ***F****, if the number of iterations* $n = n(\delta, h)$ *is chosen from the conditions* $\sqrt{n} \cdot \delta \to 0$ *as* $n \to \infty, \delta \to 0$ *and* $n^2 \cdot h \to 0$ *as* $n \to \infty, h \to 0$.
Moreover, under the condition
$0 < \theta \le \frac{4}{3 \cdot \|A_{ex}\|_F}$ *the following total error estimate hold for the iterative process (2.3)*

$$\left\| z_{ex} - z_n^{\delta,h} \right\|_F \le \left(2 \cdot n \cdot m \cdot \theta \cdot e\right)^{-\frac{1}{2}} \cdot \|z_{ex}\|_F + \left(\frac{4}{3 \cdot m} \cdot \theta\right)^{\frac{1}{2}} \cdot \left[\sqrt{n} \cdot \delta\right] + \tag{2.4}$$

$$+\frac{\theta^2}{2} \cdot \left(n \cdot (n-1) \cdot h\right) \cdot \left\| u^\delta \right\|_U + O\left(h^2 \cdot n^3\right). \quad n \ge 1$$

PROOF

We must investigate the total error estimate of the iterative process (2.3) with regard to errors in the right-hand side and operator of our equation. Let us estimate the norm $\left\| z_{ex} - z_n^{\delta,h} \right\|_F$ from the above:

$$\left\| z_{ex} - z_n^{\delta,h} \right\|_F \le \left\| z_{ex} - z_n^{\delta} \right\|_F + \left\| z_n^{\delta} - z_n^{\delta,h} \right\|_F , \tag{2.5}$$

where z_n^δ is determined by the iterative process (1.2) and it is the approximate solution of the equation (1.1) with given inexact right-hand side $u = u^\delta$, i.e. $\left\| u_{ex} - u^\delta \right\|_U \le \delta$. The Theorem 1.1 asserts that the upper bound (1.3) for z_n^δ is correct. Then from inequality (2.5) it is clear that we must estimate upper the norm $\left\| z_{ex} - z_n^{\delta,h} \right\|_F$. To this end let us introduce new error function $Y_n^{\delta,h} \overset{def}{\equiv} z_n^{\delta,h} - z_n^\delta$ and new error operator $B \overset{def}{\equiv} A_{ex} - A^h$. Let's take into account this designation in (2.3):

$$z_{n+1}^{\delta,h} = z_n^{\delta,h} + \theta \cdot \left[u^\delta - A^h z_n^{\delta,h}\right] = z_n^{\delta,h} - z_n^\delta + z_n^\delta +$$

$$+\theta \cdot \left[u^\delta - A^h z_n^{\delta,h}\right] = Y_n^{\delta,h} + z_n^\delta + \theta \cdot \left[u^\delta - A^h \left(Y_n^{\delta,h} + z_n^\delta\right)\right] =$$

$$= Y_n^{\delta,h} + z_n^\delta + \theta \cdot \left[u^\delta - A^h z_n^\delta\right] - \theta \cdot A^h Y_n^{\delta,h} = Y_n^{\delta,h} + z_n^\delta +$$

$$+\theta \cdot \left[u^\delta - \left(A_{ex} - B\right) z_n^\delta\right] - \theta \cdot A^h Y_n^{\delta,h} = \left\{z_n^\delta + \theta \cdot \left[u^\delta - A_{ex} z_n^\delta\right]\right\} +$$

$$+\left\{Y_n^{\delta,h} - \theta \cdot A^h Y_n^{\delta,h}\right\} + \theta \cdot B z_n^\delta = z_{n+1}^\delta + \theta \cdot B z_n^\delta \left[E - \theta \cdot A^h\right] Y_n^{\delta,h}$$

Thus, we have got

$$z_{n+1}^{\delta,h} = z_{n+1}^\delta + \theta \cdot B z_n^\delta + \left[E - \theta \cdot A^h\right] Y_n^{\delta,h} \quad . \tag{2.6}$$

On the other hand on definition we have got $z_{n+1}^{\delta,h} - z_{n+1}^\delta \overset{def}{\equiv} Y_{n+1}^{\delta,h}$.

Granting this in (2.6) we will get

$$Y_{n+1}^{\delta,h} = \theta \cdot Bz_n^{\delta} + \left[E - \theta \cdot A^h\right] Y_n^{\delta,h} \quad , \quad n \geq 0 . \tag{2.7}$$

As since $z_0^{\delta,h} = z_0^{\delta} = 0$ then $z_1^{\delta} = \left(E - \theta \cdot A_{ex}\right) z_0^{\delta} + \theta \cdot u^{\delta} = \theta \cdot u^{\delta}$ and $z_1^{\delta,h} = \left(E - \theta \cdot A_{ex}\right) z_0^{\delta,h} + \theta \cdot u^{\delta} = \theta \cdot u^{\delta}$, i.e. $z_1^{\delta,h} = z_1^{\delta}$. Therefore $Y_0^{\delta,h} = Y_0^{\delta} = 0$. Granting this in (2.7) we get

$$Y_{n+1}^{\delta,h} = \theta \cdot Bz_n^{\delta} + \left[E - \theta \cdot A^h\right]\left[\theta \cdot Bz_{n-1}^{\delta} + \left[E - \theta \cdot A^h\right] Y_{n-1}^{\delta,h}\right] =$$

$$= \theta \cdot Bz_n^{\delta} + \theta \cdot \left[E - \theta \cdot A^h\right] Bz_{n-1}^{\delta} + \left[E - \theta \cdot A^h\right]^2 Y_{n-1}^{\delta,h} =$$

$$= \theta \cdot \left\{\left[E - \theta \cdot A^h\right]^0 Bz_n^{\delta} + \left[E - \theta \cdot A^h\right]^1 Bz_{n-1}^{\delta}\right\} +$$

$$+ \left[E - \theta \cdot A^h\right]^2 \left\{\theta \cdot Bz_{n-2}^{\delta} + \left[E - \theta \cdot A^h\right] Y_{n-2}^{\delta,h}\right\} = \ldots =$$

$$= \theta \cdot \sum_{j=0}^{n-1} \left[E - \theta \cdot A^h\right]^j Bz_{n-j}^{\delta} + \left[E - \theta \cdot A^h\right]^n Y_1^{\delta,h} =$$

$$= \theta \cdot \sum_{j=0}^{n-1} \left[E - \theta \cdot A^h\right]^j Bz_{n-j}^{\delta} \quad .$$

Thus, the introduced function $Y_n^{\delta,h}$ is determined by means of the following recurrence relation:

$$Y_{n+1}^{\delta,h} = \theta \cdot \sum_{j=0}^{n-1} \left[E - \theta \cdot A^h\right]^j Bz_{n-j}^{\delta} \quad . \tag{2.8}$$

Let's call to mind that $z_n^{\delta,h} - z_n^{\delta} \overset{def}{\equiv} Y_n^{\delta,h}$. This function has a following interpretation: it is an error of between the iterative process (2.3) and (1.2), and it registers how much differ the iterative methods (2.3) and (1.2) when we have got the approximate operator A^h instead of the exact operator A_{ex}. In this connection the formula (2.8) demonstrates dependence of this function on the regularization solution of the problem (2.1)-(2.2) at an explicit form.

Now let us estimate ${Y^{\delta,h}}_n$ from the above. It is evident that

$$\left\|E - \theta \cdot A^h\right\|_F \leq \left\|E - \theta \cdot A_{ex}\right\|_F + \left\|\theta \cdot B\right\|_F \leq 1 + \theta \cdot h \cdot$$

Properly from (2.8)

$$\left\|Y_{n+1}^{\delta,h}\right\|_F \leq \theta \cdot h \cdot \sum_{j=0}^{n-1} \left(1 + \theta \cdot h\right)^j \cdot \left\|z_{n-j}^{\delta}\right\|_F \cdot \tag{2.9}$$

Let's rewrite (1.2) in the following form:

$$z_n^{\delta} = \theta \cdot \sum_{j=0}^{n-1} \left[E - \theta \cdot A_{ex}\right]^j Bu^{\delta} \quad , \quad n \geq 1 \cdot$$

From here

$$\left\| z_{n-j}^{\delta} \right\|_F \le \theta \cdot \sum_{i=0}^{n-j-1} \left\| E - \theta \cdot A_{ex} \right\|^i \cdot \left\| u^{\delta} \right\|_U \le \theta \cdot \left\| u^{\delta} \right\|_U \cdot (n-j) \cdot \tag{2.10}$$

Here substituting (2.10) in (2.9) we get

$$\left\| Y_{n+1}^{\delta,h} \right\|_F \le \theta^2 \cdot h \cdot \left\| u^{\delta} \right\|_U \cdot \sum_{j=0}^{n-1} (n-j) \cdot (1+\theta \cdot h)^j =$$

$$= \theta^2 \cdot h \cdot \left\| u^{\delta} \right\|_U \cdot \left\{ n \cdot \sum_{j=0}^{n-1} (1+\theta \cdot h)^j - \sum_{j=0}^{n-1} j \cdot (1+\theta \cdot h)^j \right\} =$$

$$= \theta^2 h \left\| u^{\delta} \right\|_U \cdot \left\{ n \cdot \frac{(1+\theta \cdot h)^n - 1}{\theta \cdot h} - (1+\theta \cdot h) \cdot \left[\sum_{j=1}^{n-1} (1+\theta \cdot h)^j \right]'_{(1+\theta \cdot h)} \right\} =$$

$$= \theta^2 \cdot h \cdot \left\| u^{\delta} \right\|_U \cdot \left\{ n \cdot \frac{(1+\theta \cdot h)^n - 1}{\theta \cdot h} - (1+\theta \cdot h) \cdot \frac{(1+\theta \cdot h)^{n-1} - 1}{\theta \cdot h} - \right.$$

$$\left. - (1+\theta \cdot h)^2 \cdot \frac{(n-1) \cdot (1+\theta \cdot h)^{n-2} \cdot \theta \cdot h - (1+\theta \cdot h)^{n+1} + 1}{\theta^2 \cdot h^2} \right\} =$$

$$= \frac{(1+\theta \cdot h)^{n+1} - (n+1) \cdot \theta \cdot h - 1}{h} \cdot \left\| u^{\delta} \right\|_U .$$

Thus, finally we have got $\left\| Y_n^{\delta,h} \right\|_F \le \frac{(1+\theta \cdot h)^n - n \cdot \theta \cdot h - 1}{h} \cdot \left\| u^{\delta} \right\|_U$,

or in the another form

$$\left\| z_n^{\delta,h} - z_n^{\delta} \right\|_F \le \frac{(1+\theta \cdot h)^n - n \cdot \theta \cdot h - 1}{h} \cdot \left\| u^{\delta} \right\|_U \cdot \tag{2.11}$$

Now let us designate

$$P(h,n) \overset{def}{\equiv} \frac{(1+\theta \cdot h)^n - n \cdot \theta \cdot h - 1}{h} \cdot \left\| u^{\delta} \right\|_U \cdot$$

Then $P(h,n) = \left\{ \frac{n \cdot (n+1)}{2} \cdot \theta^2 \cdot h + O\left(h^2 \cdot n^3\right) \right\} \cdot \left\| u^{\delta} \right\|_U \cdot$

From here

$$\lim_{h \to 0} \frac{P(h,n)}{h} = \frac{n \cdot (n+1)}{2} \cdot \theta^2 \cdot \left\| u^{\delta} \right\|_U \cdot \tag{2.12}$$

Thus the bound (2.11) and equality (2.12) respecting to the error h of the operator A^h has an order $O(h)$, i.e. $\left\| z_n^{\delta,h} - z_n^{\delta} \right\|_F \sim O(h)$.

Therefore, we have the following inequality:

$$\left\|z_{ex}-z_n^{\delta,h}\right\|_F \le \left(2\cdot n\cdot m\cdot\theta\cdot e\right)^{-\frac{1}{2}}\cdot\left\|z_{ex}\right\|_F+\left(\frac{4}{3}\cdot\frac{\theta}{m}\right)^{\frac{1}{2}}\cdot\left(\sqrt{n}\cdot\delta\right)+$$

$$+\frac{\theta^2}{2}\cdot\left(n\cdot(n-1)\cdot h\right)\cdot\left\|u^{\delta}\right\|_U+O\left(h^2\cdot n^3\right),\ n\ge 1$$

From here it is seen that the first summand of the left-hand side of last inequality converges to zero as $n\to\infty$ independently on values of δ and h, but the second and the third summands of the left-hand side of the bound (3.1) converge to zero if the following conditions hold:

$$\begin{cases}\sqrt{n}\cdot\delta\to 0 \quad as \quad n\to\infty,\ \delta\to 0,\\ n^2\cdot h\to 0 \quad as \quad n\to\infty,\ \delta\to 0.\end{cases}$$

Let's note that the condition $\lim\limits_{\substack{n\to\infty\\ h\to 0}} n^2\cdot h=0$ guarantee that the summand $O\left(h^2\cdot n^3\right)$ converges to zero. Really,

$$\lim_{\substack{n\to\infty\\ h\to 0}} n^k\cdot h^{k-1}=\lim_{\substack{n\to\infty\\ h\to 0}}\left(n^2\cdot h\right)^{\frac{k}{2}}\cdot h^{\frac{k}{2}-1}=0 \quad \text{for} \quad \forall\ k=2,3,\ldots\ .$$

Theorem 2.1 is proved. ∎

3. Determinations of the optimal number of iterations and the bound order

Now let us find such $n=n(\delta,h)$, depending on both the given error h of the operator and the given error δ of the right-hand side of (1.1), for which the estimate (2.4) becomes minimal. Differentiating the right-hand side of (2.4) on the parameter n and equating it to zero we obtain after some elementary transformation

$$n^{optimal}=\left[\sqrt{\frac{3}{8\cdot e}}\,\frac{1}{\theta\cdot\delta}\cdot\left\|z_{ex}\right\|_F\right]+ \qquad (3.1)$$

$$+\left[\frac{4\cdot\theta^2\cdot h+3\cdot\theta+\sqrt{7\cdot\theta^4\cdot h^2-15\cdot\theta^3\cdot h+9\cdot\theta^2}}{3\cdot\theta^2\cdot h}\cdot\left\|u^{\delta}\right\|_U\right],$$

where $[x]$ denotes the integer part of x.

Then, substituting this pseudo-optimal parameter in (2.4) we can obtain a pseudo-optimal error bound for iterative process (2.3) in the initial norm of Hilbert Space F.

4. Some remarks concerning obtained results

REMARK 4.1. *The bound (2.4) depends not only on an iterative number n but also depends on an iterative parameter θ. Therefore we can set a goal to find this optimal iterative parameter $\theta^{optimal}$ at which the estimation (2.4) decreases.*

REMARK 4.2. *The operator* A_{ex} *is assumed to be self-conjugate and positive. If the operator* A_{ex} *is not self-conjugate or positive, then equivalent equation*

$$\left(A_{ex}\right)^* A_{ex} z = \left(A_{ex}\right)^* u, \quad z \in F, \; u \in U \; , \tag{4.1}$$

should be used instead of equation (1.1), where the operator $\left(A^p\right)^*$ is conjugate to A^p. Then we have got the following iterative process:

$$\begin{cases} z_{n+1}^{\delta,h} = z_n^{\delta,h} + \theta \cdot \left(A^h\right)^* A^h \left(u^\delta - \left(A^h\right)^* A^h z_n^{\delta,h}\right), \; n \ge 0 \\ z_0^{\delta,h} = 0. \end{cases}$$

All obtained results are valid in this case.

REMARK 4.3. All the above obtained results take place if zero does not belong to the spectrum of the operator A_{ex}. If zero point belongs to the spectrum of the operator A_{ex}, then the equation (1.1) (or equation (4.1)) has infinitely many solutions. Above described approach and all obtained results in this paper are valid also in this case. Above described method guarantees convergence to normal solution, i.e. to the solution with minimal norm (see, $[2]$).

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DECISION SUPPORT SYSTEMS IN LITHUANIA

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1. Decision support systems

1.1. DECISION SUPPORT SYSTEMS AND MULTIPLE CRITERIA ANALYSIS

Quite a number of people when purchasing a house pay attention to its price. Others concentrate on operating and maintenance costs comprising of the house's heating and maintenance costs, the repair work, insurance, taxes and other related expenses. A third group of customers are concerned with the development rates of the surrounding area's infrastructure (i.e. schools, hospitals, theatres, concert halls, stores and communication lines, etc.), the environment's level of contamination and neighbours, etc. A fourth group of purchasers' puts emphasis on the issues of comfort and convenience.

Most customers identify comfort and convenience with the number of rooms, their size and height, layout and planning, functional efficiency, proportional distribution of rooms, size of the kitchen, total area of the apartment, absence of hazardous substances, thermal and acoustic insulation of walls and level of the engineering equipment, etc.

However, most often purchasers try to carry out a complex evaluation of both positive and negative characteristics of the house. It should be also noted that different people when choosing between alternatives and depending on their needs and possibilities, usually use different systems of criteria and attach different values and weights to the similar criteria.

With the aim of providing customers with substantial assistance in choosing the most effective house, the decision support system (DSS) should have comprehensive historical information covering analogous buildings life cycle. This data can be both objective and subjective.

Objective data is the price of a building, its dimensions, year of construction, interest rate of the loan taken for purchasing or construction the building, the thermal and acoustic insulation of external walls, levels of contamination with hazardous substances and fluctuations of this level. Subjective data is related to the aesthetic issues of the building's exterior, the surrounding area, comfort and convenience and neighbours, etc. As a rule, people have quite different opinions on these rather subjective issues. Such opinions may change in time, which is not bad because such opinions represent people's goals and their possibilities to changes.

When analysing possible alternatives, the customer should have the possibility to quickly receive comprehensive information (i.e. quantitative and qualitative criteria, their values and weights with the necessary explanations) that describes the project under consideration. For example, the customer should receive updated and detailed information on the substances hazardous to human health, changes on their impact in terms of time; a building's exterior finishing materials, their prices, quality, heavy-duty specification; and possible building insurance alternatives under different conditions, etc. Besides, depending on their experiences, needs and available resources, customers should be provided with the possibility to update and upgrade the criteria, their values and weights that describe the possible alternatives.

As we can see, the customer, on the basis of digital, textual, graphic, audio and video information provided by the database management system as well as the model-base management system, can generate and comprehensively analyse the possible alternatives, i.e. carry out a complex evaluation of the system of criteria, their values and weights that describe specific alternatives and help to make the necessary decisions.

The decision support system should comprise of the following four major constituent parts. These parts are: a data (database and its management system), models (model base and its management system), a user interface and a message management system.

There are several interpretations of databases (DB). The first DB is an aggregate of the interrelated and jointly stored data, i.e. information objects intended for computer processing. The concept of the second DB is wider and identifies DB with the data and a set of programs that process the same data.
Database Management Systems (DBMS) are developed for defining, creating, maintaining, controlling, managing and using databases. Special software is required for enabling the user to operate and communicate with databases. The database management system provides access to data as well as to all the control programs necessary to receive data in the form that is appropriate for an object under consideration to be analysed without too much effort from the user who is programming.

The major functions of the Database Management System are as follows: designing of the database's structure; enlargement, collection and editing of the database; maintenance, search, sorting and other handling of data.

Some people view databases as being more or less independent systems and databanks, i.e. treating databanks as a system of information, mathematics, linguistics, organizational, software and hardware facilities.

The model-base management system performs a similar task for models in the DSS. It keeps track of all the possible models that might be run during the analysis, as well as controls for running the models. The model-base management system also links between models so that the output of one model can be input into another model.

The user interface represents all the mechanisms whereby information is input into the system and is output from the system. The system includes all the input and screens by which users can request data, models and output screens and through which users can obtain their results.

The message management system allows for the use of electronic mail as another source of providing data.

DSS provides a framework through which decision-makers can obtain the necessary assistance for a decision through an easy-to-use menu or command system. Generally, a DSS will provide help in formulating alternatives, accessing data, developing models and interpreting their results, selecting options, or analysing the impacts of a selection.

1.2. MODEL DIMENSIONS

The decision support system can include many models. These models can exist both inside and outside the DSS. The following three dimensions define the models: representation, time dimension and methodology.

Accordingly, the representation models can be divided further into quantitative and qualitative ones. The qualitative (i.e. expert and multiple criteria) models are based on judgments, subjective estimates, opinions and the expert's evaluations. When different experts evaluate the same qualitative characteristics of the same option, they often get different results. This can be explained by the different experiences, educational background, goals and available tools, etc. that may be used. The achieved results can be made more objective by applying expert evaluation methods.

The quantitative models (i.e. statistics and accounting) represent objective features of the options, irrespective of the expert's subjective evaluations and judgments. Objective features are represented directly by physical measurement units such as monetary units, kilograms, meters, degrees, percents and ratios, etc.

Both quantitative and qualitative models have their advantages and drawbacks. The quantitative models represent their options in an objective way, but usually not thoroughly and comprehensively enough. On the contrary, the qualitative models represent reality subjectively, and more thoroughly and comprehensively. Therefore, the application of quantitative or qualitative methods usually depends on the concrete decision-making situation. Very often a complex method of application of both quantitative and qualitative methods should be applied when making a decision. For example, when analysing the general level of a building's comfort, the best way would be to apply qualitative research methods. However, when making an assessment of the funds that will be spent during the period of the existence of a particular building (e.g. building's purchase, construction, maintenance, repair work and insurance expenses, etc.), it would be best to apply the quantitative methods.

Time dimension models are divided into static and dynamic ones. The static models support the position that feature the options, in the course of time that do not change, whereas the dynamic models take into consideration the changing nature of the options also in the course of time.

The methodology addresses how the data will be collected and processed. According Sauter [Sauter], there are five general methodologies: complete numeration, algorithmic, heuristic, simulations and analytical ones.

When applying the complete numeration method we collect and evaluate information about all the feasible options. This method is highly time-consuming, costly, often impractical and is used for example when conducting a general census.

The algorithmic models are best represented by operations research methods and are applied when counting from the beginning till the end (i.e. from the moment of the initial data's entering until the gaining of the wanted results or goals).

The heuristic models are applied for settling problems that cannot be solved algorithmically. All heuristic models involve searching, evaluating and finding a good solution. The heuristic models help to diminish the number of search options and aim at providing a solution and other findings. Heuristics is the most important part of the artificial intelligence and expert systems.

Simulation settles problems that cannot be accurately and precisely examined on the basis of a mathematical analysis. When applying these models we can create an adequate and typical situation of the options. Simulation models simplify the relationships and interdependencies of the alternatives being considered and provide information about conditions from which can be find a rational solution. Repeating the possible states of the option provides the possibility to experiment and reveals ways of improving the system's functioning. Such a method of simulation is often used when examining problems related to storage and the servicing of reserves, the demand for products, raw materials arrivals.

At the beginning of the analytical modelling, a general analysis of the option is carried out. Thereafter, the option is divided into separate parts for their examination. Later we have to determine the relations and dependencies of the elements that comprise of the option. A statistical analysis serves as a perfect example of analytical modelling.

2. Decision support systems as created by Vilnius Gediminas Technical University

Many internet-based systems are processing and submitting only economic information for decisions. Alternatives under consideration have to be evaluated not only from the economic position, but take into consideration qualitative, technical and other characteristics. Therefore, the efficiency of e-business and Web-based systems may be increased by applying multiple criteria decision support systems.

Web-based decision support systems created by authors in cooperation with their associates are described in various publications:

- Multiple Criteria On-Line Export Decision Support System (Kaklauskas A., 2002 a, 2002 b).
- Multiple Criteria Decision Support Web-Based System for Facilities Management (Zavadskas E. , 2002 a, 2002 b).
- Ethical Multiple Criteria Decision Support Web-Based System (Kaklauskas A., 2002 c).
- Internet Based DSS for Real Estate (Kaklauskas A., 2002 d, Zavadskas E. 2001).
- Multiple Criteria Decision Support On-Line System for Construction Products (Kaklauskas A., 2002 e, Zavadskas E. 2002 c).

The above decision support systems comprise of the following constituent parts: a data (database and its management system), models (model base and its management system) and a user interface.

When creating the Web-based decision support systems the authors based their work on the following major principles and methods:

- Method of complex analysis. The use of a complex analysis makes it possible to carry out economic, technical, qualitative, technological, environmental, managerial and other kinds of optimisation throughout the life cycle of a project.
- Method of functional analysis. The expenditures associated with project functions are usually determined by taking into account the benefits of a function and the cost of its realization.
- Principle of cost-benefit ratio optimisation. Efforts are made to get maximum benefit (economic, qualitative, environmental and social, legal, etc.) at minimum project's life cycle expenses, i.e. to optimise the cost-benefit ratio.
- Principle of interrelation of various sciences. The problem of cost-benefit ratio may be successfully solved only when the achievements of various sciences, such as management. economics, law, engineering, technology, ethics, aesthetics and psychology, etc. are used.

- Methods of multi-variant design and multiple criteria analysis. These methods allow us to take into consideration the quantitative and qualitative factors, as well as cutting the price of the project and better satisfying the needs of all interested parties.
- Principle of close interrelation between project's efficiency and interested parties and their aims.

Presentation of information in databases may be in conceptual (digital, textual, graphical, photographic, video) and quantitative forms.

Conceptual information means a conceptual description of alternatives, the criteria and ways of determining their values and weight. Conceptual information is needed to make more complete and accurate analysis of the alternatives considered. In this way, the above DSS enable the decision maker to receive conceptual and quantitative information on alternatives from a database and a model-base allowing him/her to analyse the above factors and form an efficient solution.

Quantitative information presented involves criteria systems and subsystems, units of measurement, values and initial weight fully defining the variants provided. Quantitative information of alternatives is submitted in the form of grouped decision-making matrix, where the columns mean n alternatives under analysis, and rows include quantitative information.

The databases were developed providing a multiple criteria analysis of alternatives from economical, legislative, infrastructure, social, qualitative, technical, technological and other perspectives. This information is provided in a user-oriented way. To design the structure of a database and perform its completion, storage, editing, navigation, searching, and browsing, a database management system was used in this research.

Then the brief study of authors above developed (construction and facilities management) Web-based DSS follows.

2.1. MULTIPLE CRITERIA DECISION SUPPORT ON-LINE SYSTEM FOR CONSTRUCTION PRODUCTS

Today there are a great number of directories and electronic commerce systems, in the world related to construction products. Some of the well known Web-sites are: www.needproducts.com, www.4specs.com, www.buildscape.com, www.commerce.net and www.sri.com.

This section deals with Multiple Criteria Decision Support On-Line System for Construction (OLSC) developed by authors with V.Trinkunas. At the present moment the developed OLSC allows the performance of the following functions:

1. Search of construction products. A consumer may perform a search of alternatives from catalogues of different suppliers and producers. This is possible since the forms of data submitted are standardized into specific levels. Such standardization creates conditions to use special intelligent agents who perform a search of the required construction products from various catalogues, and gather information about the products. One or several regions may limit such search.
2. Finding out alternatives and making comparative tables. Consumers specify requirements and constraints and the System queries the information of specific construction products from a number of online vendors and returns a price-list and other characteristics that best meets the consumer's desire. The System performs the tedious, time-consuming, and repetitive tasks of searching databases, retrieving and filtering information and delivering the information back to the user. Results of a search of specific construction products are submitted in tables, which may include direct links to a Web page of a supplier or producer. By submission such a display, of the multiple criteria comparisons can become more effectively supported. The results of the search of a concrete construction product are often provided in one table where one can sometimes find direct links to the Web page of the supplier or manufacturer.
3. Evaluation stages of alternatives (i.e. multiple criteria analysis of alternatives and selection of most efficient ones). While going through the purchasing decision process a customer must examine a large number of alternatives, each of which is surrounded by a considerable amount of information (price, discounts given, thermal insulation, sound insulation, rate of harm to human health of the products, aesthetic, weight and technical specifications, physical and moral longevity). Following on from the gathered information the priority and utility degree of alternatives is then calculated. The utility degree is directly proportional to the relative effect of the values and weights of the criteria considered on the efficiency of the alternative. It helps consumers to decide what product best fits their requirements.
4. Analysis of interested parties (competitors, suppliers, contractors, etc.).

5. The after-purchase evaluation stage. A consumer evaluates the usefulness of the product in the after-purchase evaluation stage, etc.

2.2. MULTIPLE CRITERIA DECISION SUPPORT SYSTEM FOR FACILITIES MANAGEMENT

An analysis of multiple criteria decision support systems (see first section) and facilities management Web-based automation applications (calculators [1-5], analysers [6-8, software [9-13], expert [14] and decision support [15-17] systems, etc.) that were developed by researchers from various countries assisted the authors to create one of their own Multiple Criteria Decision Support Web-Based System for Facilities Management (DSS-FM). The DSS-FM developed by authors with M.Gikys and A.Gulbinas is presented in this section. The following tables form the DSS-FM's database:

- Initial data tables. These contain information about the facilities (i.e. building, complexes, alternative facilities management organisations).
- Tables assessing facilities management solutions. These contain quantitative and conceptual information about alternative facilities management solutions: space management, administrative management, technical management and management of other services, complex facilities management, market, competitors, suppliers, contractors, renovation of walls, windows, roof, etc.

The tables assessing facilities management solutions are used as a basis for working out the matrices of decision-making. These matrices, along with the use of a model-base and models, make it possible to perform a multiple criteria analysis of alternative facilities management projects, resulting in the selection of the most beneficial variants. The efficiency of a facilities management variant is often determined by taking into account many factors. These factors include an account of the economic, aesthetic, technical, technological, management, space, comfort, legal, social and other factors. The model-base of a decision support system includes models that enable a decision-maker to do a comprehensive analysis of the available variants and to make a proper choice.

Below is a list of typical facilities management problems that were solved by users: multiple criteria analysis of space management, administrative management, technical management and management of other services alternatives; analysis of complex facilities management alternatives; analysis of interested parties (competitors, suppliers, contractors, etc.); determination of efficient loans; analysis and selection of rational refurbishment versions (e.g. roof, walls, windows, etc.); multiple criteria analysis and determination of the market value of a real estate (e.g. residential houses, commercial, office, warehousing, manufacturing and agricultural buildings, etc.), analysis and selection of a rational market, determination of efficient investment versions, etc.

3. Conclusions

The analysis of information systems used in construction, real estate and facility management that were developed by researchers from various countries assisted the authors to create of their own Web-based decision support systems. These systems differ from others in the use of new multiple criteria analysis methods as were developed by the authors. The databases were developed providing a comprehensive assessment of alternative versions from the economic, technical, technological, infrastructure, qualitative, technological, legislative and other perspectives. Based on the above complex databases, the developed systems enable the users to analyse alternatives quantitatively (i.e. a system and subsystems of criteria, units of measure, values and weights) and conceptually (i.e. the text, formula, schemes, graphs, diagrams and videotapes). The efficiency of information systems may be increased by applying above Web-based decision support systems.

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A NEW CONTINUAL COMPUTATIONS STRATEGY FOR HETEROGENEOUS NETWORKS

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1. Introduction

The purpose of this paper is to introduce, analyze and optimize the new continual computations strategy. The problem of decision-making related to a specific process produced in any network entity with a specific precision is very important nowadays. The decisions are related to the absolutely different processes performed in any switch, source or destination entities. For example, the characteristics under research are as follows:

- Determination of the optimal/near-optimal route in IP or Multi-protocol Label Switching (MPLS) network [1, 2] considering the constraints imposed on the optimal route searching time;
- Near-optimal implementation of the error recovery procedure;
- Resource reservation in the optimal/near-optimal manner.

The possible metrics for the computation strategy implementation are:

- Direct connection between source and destination;
- Any data link between consecutive switches;
- Asynchronous Transfer Mode (ATM) Virtual Path [3] or MPLS Label Switching Path [2].

The presented strategy takes into consideration the transmission conditions, traffic type and, of course, limitations imposed on the routing, error recovery, or resource reservation time.

The urgency of the problem stated above is caused by the following reasons:

- The modern IP networks are, in essence, heterogeneous and transmit a great amount of different traffic. Therefore, the problem of optimal routing or optimal resource reservation should be solved as fast as possible in the most precise fashion.
- Different users' applications have different demands on the large range of the Quality of Service (QoS) characteristics. Therefore, the continual computation decision must be performed in the differentiated manner considering the different traffic types and users' QoS requirements.

A time-critical computation problem has arisen in recent years in different sciences such as Computer Communications, medicine and management, among others. Due to the problem importance a number of papers have been published. Authors in [4] discuss Continual Computation policies that dictate strategies for pre-fetching into cache portions of documents that a user may wish to review. The probabilistic models were constructed in order to predict a user's interests and access behaviour. A flexible procedure for a resource-bounded agent to allocate limited computational resources to on-line solving is presented in [5]. A new approximate and compensate methodology is defined in this paper. The introduced risk-management factor represents a mean-variance trade-off that may be derived optimally off-line using any available information [5]. The use of decision theory to optimize the value of computation under uncertain and varying resource limitations is described in [6]. Work on simple algorithms and on the control of decision-theoretic inference itself is described in this paper. A new and very attractive approach for time-limited computations and decision making is presented in [7]. The authors discuss how the new LL logic might be used in areas where decision making is critical, such as management and medical diagnosis, and conclude by using LL to give a formal proof of correctness of a protocol for exchanging secrets. Optimizing the performance of computational systems given resource constraints by probabilistic and Bayessian approach accurately described in [8, 9].

2. Mathematical Model

2.1. PROBLEM STATEMENT

The task set is to provide computations considering the strictly limited computing time and satisfying a given computation maximum error. In virtue, computation execution within a strictly defined time interval is more important than satisfying a given acceptable error. We mean that in an extreme case, the suggested strategy provides computation decision within the maximum permitted time; nevertheless, the computing precision may be neglected.

The objective function *F(x)* that determines the studied network resources is introduced. Generally speaking, *F(x)* may be defined in such a way that the optimal/near-optimal resources are computed by determining the *F(x)* root.

2.2. FORMAL MODEL

The problem stated above is to determine a zero (root) of a given objective function *F(x)* in the most precise fashion considering the limitations imposed on the searching time, or on the computation operations number, as well. The following assumptions are made:

- *F(x)* is a "well" defined and clearly described function:
- *F(x)* is defined in any specific range, may be for all real numbers.
- The root of the equation *F(x)=0* satisfies the condition: $\xi \in [a, b]$.
- $f'(x), f''(x)$ are continuous and do not change sign in the *[a, b]* range.

Then the *F(x)* root evaluation is executed by the Newton method which is based on the Tailor expansion. It is implemented in the simplest way as compared to other evaluation methods that satisfy the square time convergence [10].

The root search time T is limited by the maximal permitted time $T_{\max}$. Therefore, the root evaluation has to be executed in the maximum precise fashion considering the constraints imposed on the computation time. In case when the strong limitations are imposed on $T_{\max}$, on the one hand, the optimal root evaluation has to be as fast as possible, while, on the other hand, the final solution may be determined in the non-precise manner.

Let's define the computation algorithm by the following block structure (Figure 1):

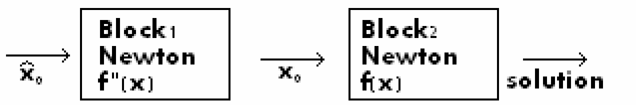

Figure 1. The Computation Algorithm Structure

1) The equation *F"(x)=0* is solved in *Block1* in the maximally precise manner within permitted time T_1. The equation root determines maximum *F'*(x) value. A better choice of the iterations basis for the Newton method in Block2 considering the basic equation *F(x)=0* is, in essence, the argument *x* value that maximizes *F'(x)* [10].
2) The transition to *Block2* is provided when the near-maximum *F"(x)* value is calculated. The basic equation *F(x)=0* has to be solved in the maximally precise manner within T_2 time interval.
3) The initial *Block 0* is introduced when the basic equation *F(x)*=0 solution by Newton method is complicated. In this case it is worth substituting the objective function *F(x)* by the approximation Tailor polynomial *P(x)* whereas the approximation time is limited by T_0.

In a general case, blocks number *N* may be arbitrary on condition:

$$\sum_{i=0}^{N} T = T_{\max}. \tag{1}$$

In the extreme time-limited case the *F(x)* root estimation, in essence, may not be closed to the near-optimal/optimal solution, whereas the computation time must be strictly limited by $T_{\max}$.

The problem stated, discussed and solved in this paper is to define the near-optimal strategy for the $\{T_i\}$ set determining considering the following parameters:

1) Basic function *F(x)* structure;
2) A given $T_{\max}$ value;
3) A given computation precision ε.

In the next section we show that the defined strategy may be perfectly applied not only in the deterministic, but also in the probabilistic manner.

Remark:

The *F(x)* structure determines the relative time occurrences in each block. For example, the smoother is *F(x)*, T_1 is smaller because the additional iterations performed by Newton method are not relevant. Moreover, if $F'(x) \approx constant$ in the *[a, b]* range then *Block1* execution should be neglected. In such a case the sequential iterative x_n values might be calculated in *Block2* in the simplest way by substituting $F'(x_n)$ values by $F'(x_0)$ for any x_n:

$$x_{n+1} = x_n - \frac{F(x_n)}{F''(x_0)}, n \in N. \tag{2}$$

2.3. DETERMINING OF THE COMPUTATION ALGORITHM

On the first cut, let's define the computation algorithm as a multi-dimensional functional

$$G = G(par_1, \dots, par_n). \tag{3}$$

The parameters set $\{par_i\}_{i=1}^{n}$ are defined as follows:

1) The first parameter is the defined above objective function *F(x)*. The computation strategy dependence on the *F(x)* structure will be particularized hereafter. In essence, the computation algorithm is defined in the iterative manner and it is described for each iteration.
2) The computation error precision ε influences the iteration mechanism, as well.
3) Maximum available computation time is T_{max}. The larger is T_{max}, the permitted T_1 time is larger.
4) The relative weigh of the time occurrence in *Block1* is defined as a rate:

$$\rho = \frac{T_1}{T_1 + T_2}. \tag{4}$$

Without loss of generality, the ρ parameter can be generalized on the arbitrary number of the sequential blocks. The measure

$$\theta = \frac{F(x)}{F'(x)} \cdot \frac{F^{(3)}(x)}{F^{(2)}(x)} \tag{5}$$

is defined as the computation algorithm parameter, as well. The first item in (5) determines the distance of the evaluated in *Block2* root value *x* from the real *F(x)* root *a* (it means that *F(a)=0*), with the error of $(x-a)^2$, while the second item denotes the distance of the evaluated in *Block1* the maximum *F'(x)* value from the exact maximum *b* with the error of $(x-b)^2$. According to the definition, if $\theta \succ\succ 1$, then the approximation is close to *b* and it is worth continuing computations in *Block1*. Otherwise, if $\theta \prec\prec 1$, then it is reasonable to apply the regular Newton method for *F(x)* in *Block2*. The problem under consideration is to determine the compromise between two or more defined blocks in the most interesting and problematic case when θ is close to 1: $|\theta - 1| \prec \Delta$ for sufficiently small Δ.

We assume that the computation time for any iteration is the same in any block. This simplified approach will be generalized later considering random time required for any iteration.

2.4. DETERMINING OF THE COMPUTATION STRATEGY

In a general case, the computation algorithm should have all the necessary ingredients to be extended to the computation strategy. The following considerations are made considering the defined above functional $G = G(par_1 ,\ldots ,par_n)$:

- The *G* parameters $T_{max,}\ \rho,\theta$ are not independent values. ρ depends on T_{max} considering the computation peculiarity ε as a parameter, in the following manner:
- Function $\rho = f(T)$ is defined in the range $T \in [T_{min}, T_{max}]$. Here T_{min} is the minimal computation time that is necessary for a single iteration. Clearly, $\rho_1 = f(T_{min}) = 1$ because a single iteration must be performed in *Block2* according to the regular Newton method. $\rho = f(T)$ is the strongly decreasing function. The smaller is ε, the faster is the function decrease (Figure 2). The graph beyond is appropriate to parameter ε_2 and the graph below is appropriate to parameter $\varepsilon_1 \prec \varepsilon_2$.

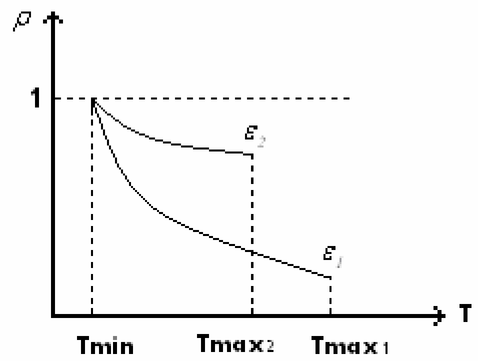

Figure 2. Dependence ρ on T

- The permitted T_{max} values are significantly different for the various range of ε values. As a matter of fact, ρ should be defined as a two-dimensional functional:

$$\rho_i = f(\varepsilon_{ij}, T_{\max_i j}). \tag{6}$$

In order to provide the functional dependence in (6), let's consider ρ as a simple fraction. In a general case, the ρ functional has to be extended by the third dimension T_1 or T_2. Then ρ will include not only the relative weighs of each time occurrence, but also the absolute time occurrence in each block.

- The dependence ρ on θ should be also considered. The reason is that the relative rate θ determines the relation of the two main distances (5). Therefore, $\rho = f(T,\theta)$ is a two-dimensional functional.
- T_{max}, in essence, is not a constant value. Moreover, it is the decreasing function of the ε argument. The reason is that the smaller is ε, the more particular root approximation has to be performed and, therefore, the more computation time should be applied. The dependence $T_{max} = h(\varepsilon)$ is of the square fashion because the Newton method satisfies the square convergence [10].

Let us pass to the $\rho = f(T,\theta)$ analytical description. Assume that θ is a given parameter. The $\rho = f(T,\theta)$ analysis should be performed considering different range of θ values.

2.4. COMPUTATION STRATEGY IMPLEMENTATION

Let's study the $\rho = f(T,\theta)$ functional assuming that *T=constant* and $\varepsilon = constant$. Clearly, the more is θ, the estimation error in *Block2* is larger and it is reasonable to execute operations in *Block1*. Actually, the opposite prediction is also correct. Therefore, $\rho = f(\theta)$ satisfies the following features:

- $\rho = f(\theta)$ is the decreasing function in the range $\theta \prec 1$ and is the increasing function in the range $\theta \succ 1$.
- In the point $\theta = 1$ the function has minimum value. Evidently, $\rho = f(\theta)$ changes extremely slow in the range $|\theta - 1| \prec \Delta$ for sufficiently small Δ. Therefore, $f'(\theta) \approx 0$ in the mentioned above range (Figure 3).

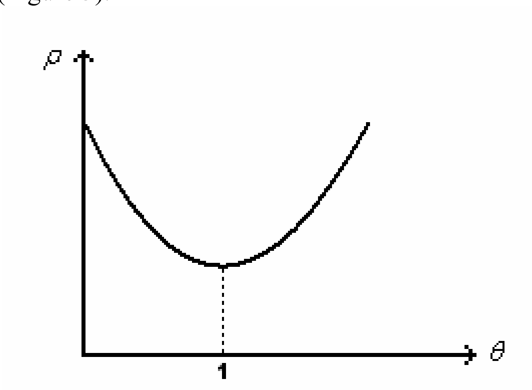

Figure 3. Dependence ρ on θ

Let us define the functional dependence $\rho = f(T)$ in the following manner considering the different θ values:

- In the extreme case when $\theta \approx 1$, $\rho = f(T)$ is a linear decreasing function. Such assumption is reasonable because any other rational or exponential decreasing function decreases faster than a linear function. By defining $\rho = \theta \cdot T + b$ and taking into consideration that $f(T_{min}) = 1$ and $b \prec 0$, it is evident that $b = \frac{-1}{\theta} \cdot T_{min}$. As a result, $\rho = f(T)$ is defined as

$$\rho = \theta \cdot T - \frac{1}{\theta} \cdot T_{min}. \tag{7}$$

- In case when $\theta \prec\prec 1$ ρ is a drastically decreasing function of the *T* argument. It is correctly defined in the exponential form as

$$\rho = e^{-T\theta}. \tag{8}$$

The other relevant $\rho = f(T)$ definition is by rational function:

$$\rho = \frac{1}{a_n T^n + a_{n-1} T^{n-1} + \ldots a_0}. \tag{9}$$

Here the polynomial degree *n* must be sufficiently large.

- In case when $\theta \succ\succ 1$, the dependence ρ on *T* is described by the increasing function that may be symmetric to the function defined for $\theta \prec\prec 1$.
- The more interesting case is, in essence, when $|\theta - 1| \prec \Delta$ for a given confidence level Δ. It is reasonable to define $\rho = f(T)$ as the rational function, whereas the polynomial degree $m \in N$ depends on θ:

$$\rho = \frac{1}{T^m + \alpha}. \tag{10}$$

The smaller is $|\theta - 1|$, the faster decreases $\rho = \dfrac{1}{T^m + \alpha}$ and the appropriate *m* value is larger.
Generally speaking, function $\rho = f(T)$ may be defined in the following manner:

$$\begin{aligned} &\rho = \frac{1}{T^{h(\theta)} + \alpha}, \theta \succ 1; \\ &\rho = \frac{1}{T^{-h(\theta)} + \alpha}, \theta \prec 1; \\ &\rho = constant, \theta = 1. \end{aligned} \tag{11}$$

Here $h(\theta)$ is a non-decreasing function of the θ argument.

The presented analysis will be very useful considering the introduction of the randomized strategy that will be introduced later.

2.5. OBJECTIVE FUNCTION F(X) AS THE COMPUTATION FUNCTIONAL PARAMETER

By virtue, the objective function's *F(x)* nature influences the computation strategy near-optimal determination. Let us determine the functions' classes according to the following guiding principles:

1) Any researched *F(x)* function pertains to a certain class. The computation strategy implementation is different in each class.

2) The class is defined via several parameters. These parameters under consideration are:

- Description and prediction of the *F(x)* behaviour.
- Numerical values of $F'(x_n)$ and of $F^{(2k)}(x_n)$ at each *n-th* iteration in *Block2* and *Block1* respectively.

The following classes' definition is relevant:

- *Class1*={exponential functions}
- *Class2*={polynomial functions}
- *Class3*={rational functions}
- *Class4*={orthogonal functions}
- *Class5*={non-clearly defined functions}.

A correspondence between the class and the appropriate computation algorithm is determined by the Strategic Mapping Matrix (Figure 4).

Algorithm*Class*	1	2	. . . .	M
1				
2				
3				
. . . .				
A				

Figure 4. Strategic Mapping Matrix

As a result, the limited resource computation is performed by the differentiated approach considering the objective function *F(x)* type. For example, if a given function $y(x) \in Class5$ then it is worth to approximate it previously by the corresponding Tailor polynomial with a certain approximation error, whereas the polynomial root calculation will be done at the next stage.

3) As a matter of fact, a number of computation algorithms may be implemented within a given class. The appropriate algorithm choice is determined by the other above mentioned computation functional parameters (8)-(11).

2.6. DETERMINING OF THE RANDOMIZED COMPUTATION STRATEGY

Let us pass to the probabilistic approach in Computation Strategy determining. The randomized strategy is defined in the following manner.

1) The iterations number choice (which is equivalent to the time occurrence in any block) is determined randomly considering a certain confidence level $\tilde{\varepsilon} = max\left((x_i - \alpha)^2, (\tilde{x}_i - \beta)^2\right)$. Here α and β are the exact roots in the consequent blocks and x_i and $\tilde{x}_i$ are the corresponding roots approximations on the *i-th* iteration. The conditional probability $prob\{\theta \prec\prec 1 \; in\, i-th \; iteration | \theta \approx 1 \; in \; the \; (i-1) \; iteration\}$ should be computed at each step.

The following two types of statistics should be applied:

- Long Term Statistics that describes the system's history;
- Short Term Statistics that is sensitive to the current computation conditions [11].

The Markov chain approach describes the Computation Strategy history, as well [12]. For this purpose, the set $\{A_i\}$ of the Markov chain discrete states is defined. Each discrete state corresponds to a specific θ value. The rate θ denotes the distances relation from the estimated root to the exact one in each block (5). The corresponding Markov chain is defined by the Probability Transition Matrix *M* (PTM) and the initial probabilistic vector $\pi(0)$ that determines the initial distribution of each discrete state. The Markov chain step is appropriate to single iteration. As a matter of fact, the Markov chain approach should be completed by the semi-Markovian in the more general case when the computation time for a specific iteration in any block is, in essence, random. Iterations time distribution functions should be added to the basic Markov chain [13].

2) The blocks' number is, in essence, random. For example, if the function differentiation is complicated and requires a lot of time, then it is reasonable to approximate it by a corresponding Tailor polynomial, whereas the approximation time should be taken into account, considering limitations imposed on T_{max}.

3) The randomized strategy is very attractive considering operations in any block. Evidently, not a single Newton method may be applied in the function *F(x)* root computation. Assume that *F'(x)* is close to zero, or $F'(x_n) \prec \varepsilon_2$ for a given confidence level ε_2 in *Block2* on *n-th* iteration. The following question arises.

- Is it worth to execute the next *n+1* iteration by another computation method that gives the more exact root approximation? For example, the chord root approximation method may be applied perfectly. The risk of such a decision is that the chord method satisfies a more hardened first-order convergence comparing with the basic Newton method [10]. Therefore, the following *n+2* iteration performed by the chord method, probably may not be optimal and, moreover, non-precise. In order to construct the corresponding probabilistic model, the Markov chain or the semi-Markovian approaches are suggested. The following probabilities should be analyzes at any *i-th* iteration:
  $prob\left\{(x_i - \beta)^2 \geq \varepsilon_2 | (\hat{x}_{i+1} - \beta) \prec \hat{\varepsilon}_2\right\}$. Here $\hat{x}_{i+1}$ and $\hat{\varepsilon}_2$ denote the root approximation and the approximation error in the *(i+1)-th* iteration applying the chord method.
  The decision scheme choice is based on the confidence level $\mu = max(\varepsilon_2, \hat{\varepsilon}_2)$.

Remark:

1) Considering the randomized strategy approach, the argument *T* in the exponential dependence $\rho = e^{-T\theta}$ may be treated as the mean iteration time. In virtue, function $\rho = e^{-T\theta}$ determines the ρ Poisson distribution [11] with the intensity $\lambda = \frac{1}{T}$.

2) In a general case when the blocks number $n \geq 2,$ the use of the χ^2 – distribution with *n* degrees of freedom or *n*-order Erlang distribution [13] is reasonable in the ρ determining.

3) If $|\theta_n - \theta_{n-1}|$ is sufficiently small for a certain value *n*, then the transition to the chord method should be done in any block.

4) Not a single chord method can substitute the basic Newton method. The approximation by means of Chebyshev polynomials or iterative method which is, essentially, the composition of the chord and the Newton method, may be applied among others [10].

3. Conclusions

The current Continual Computations framework presents an advanced strategy that improves the overall performance of the time-critical issues such as packet error recovery, resource reservation, and routing among others.

The strategy enjoys the following features:

- In general, it matches any Packet Switching Network technology considering the various ranges of the above-mentioned computing characteristics.
- The suggested strategy is adaptive. This feature implies that any technique derived from this practice, will optimally determine the appropriate computation algorithm.
- The additional feature of the presented method is that it may be applied in the most near-optimal way considering different applications and users QoS requirements.
- The strategy may be generalizes by introducing the different approximation and computation methods.

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ESTIMATING OF PARAMETERS OF FATIGUE CURVE OF COMPOSITE MATERIAL

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1. Introduction

Every year the use of composite material in aircraft structure increases. To provide reliability of flight we should study the fatigue phenomenon of this material. One of the main quantitative characteristics of this phenomenon is fatigue curve. There are many offers for its description. Short review of this problem is given in papers [1,2]. We will not repeat it because this paper is in some way a development of [1]. This paper is devoted mainly to processing of dataset of fatigue tests of 125 carbon-fibre laminate specimens. The purpose is to get estimates of parameters of fatigue damage accumulation model, based on the Markov chain theory [1]. It was shown, that, although the model is too simple and does not provide numerical coincidence with experimental fatigue test data, nevertheless it allows to get fatigue curve equation by the use of static strength distribution parameters and some additional parameters, which have some 'physical' interpretation. In this paper we consider inverse problem: by the use of this model, which can be considered now as nonlinear regression analysis model, we'll try to get estimates of local static strength distribution parameters. But it is not the end in itself. The likelihood of these estimates and the likelihood of "theoretical" and experimental fatigue curves can be considered as proof of likelihood of the studied model.

The model discussed in [1] has as 6 parameters all together. For experimental data processing 5 of them is used. In this paper approximately for the same level of precision of fatigue curve description we have used only 4 parameters. It is significant decreasing of the difficulty of statistical analysis. The method of approximate estimation of unknown parameters is offered. Numerical example is given.

Overview. The reminder of the discussed model ideas is given in Section 2. Method for model parameters estimation is discussed in Section 3. Numerical example is given in Section 4.

2. Cumulative damage model based on the Markov chains theory

We consider the process of fatigue damage accumulation as Markov chain with the following matrix of transition probabilities:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{23} & \cdots & p_{1r} & p_{1(r+1)} \\ 0 & p_{22} & p_{23} & p_{24} & \cdots & p_{2r} & p_{2(r+1)} \\ 0 & 0 & p_{33} & p_{34} & \cdots & p_{3r} & p_{3(r+1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & p_{rr} & p_{r(r+1)} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix},$$

corresponding to Markov Chain (MC) with one (r+1)th absorbing state and r non-recurrent states. Cumulative distribution function (CDF) of time to absorption for this process $F_T(t) = p_{1\ r+1}(t)$, $t = 1,2,3,\ldots$, where $p_{1\ r+1}(t)$ is the element of first row and (r+1)th column of matrix $P(t) = P^t$. It can be defined also in this way

$$F_T(t) = aP^t b, \qquad (1)$$

where $a = (100...0)$ is the row vector, $b = (00...01)'$ is the column vector. It is worth to notice, that product $P^t b$ gives a column vector of cumulative times distribution functions for absorption, which components correspond to different start states of MC : $(F_T^{(1)}(t), F_T^{(2)}(t),..., F_T^{(r)}(t))'$. In general case it can be used to get CDF, when the probability distribution on start states of MC, $\pi = (\pi_1, \pi_2,..., \pi_r, \pi_{(r+1)})$, is known $F_T(t) = \pi P^t b = \pi\ (F_T^{(1)}(t), F_T^{(2)}(t),..., F_T^{(r)}(t))'$. This possibility should be considered as some reserve, which can be used to take into account some specific features of specific composite structure, induced by some specific technology. Just now we do not use this possibility, because we deliberately try to decrease the number of parameters of the considered model. Moreover, in this paper we consider S*imple Markov Chain Model of Fatigue Life* (SMCMFL) of composite material. It is a model, for which only transitions to the nearest previous states are allowed and

$$P = \begin{bmatrix} q_1 & p_1 & 0 & & & \cdots & 0 \\ 0 & q_2 & p_2 & 0 & & \cdots & 0 \\ 0 & 0 & q_3 & p_3 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & & & \cdots & 0 & q_r & p_r \\ 0 & & & \cdots & 0 & 0 & 1 \end{bmatrix}$$

, $q_i=1-p_i$, $i=1,...,r$. The main characteristics of this type MC are well known. A time to failure (time to absorption) $T=X_1+X_2+...+X_r$, where X_i (time the process spends in i-th state), $i = 1,...,r$, are independent random variables. A random variable X_i has geometric distribution with a probability mass function (PMF) $P(X_i = n) = (1-p_i)^{n-1} p_i$. Expectation value $E(X_i) = 1/p_i$ and variance $V(X_i) = (1-p_i)/p_i^2$. So probability generating function for random variable T (which can be used to obtain PMF of T) $\boldsymbol{G_T(z) = \sum_{i=0}^{\infty} p_T(i) \cdot z^i = \prod_{i=1}^{r} \frac{z\, p_i}{1 - z(1-p_i)}}$.

The PMF can be calculated also by the use of formula $p_T(t) = F_T(t) - F_T(t-1)$. If we assume, that one step in MC corresponds to k_M cycles in fatigue test, then for calculation of expectation value and variance of T we should use formulae:

$$E(T) = k_M \sum_{i=1}^{r} 1/p_i\ , \tag{2}$$

$$V(T) = k_M{}^2 \sum_{i=1}^{r} (1-p_i)/p_i^2\ . \tag{3}$$

But the main and most difficult problem is to connect these probabilities, p_i, i=1,...,r, with parameter of composite material component strength distribution and applied stress level in such a way that we'll can get the fatigue curve equation. Our offer is to assume that in one step of Markov process (1 cycle or may be 1000 cycles) only one parallel structural item (for example strand) can be failed. If we have *(R-i)* still alive parallel structural items and the same cdf $F(s)$ for every item, then the fracture probability of at least one item is equal $p_i = 1-(1-F(s_i))^{R-i}$, where R is initial number of items, i is the number of items, which are failed already, s_i is the corresponding stress applied uniformly to all *(R-i)* items. We suppose also that $s_i = \dfrac{SR - S_f i}{R-i} = \dfrac{S(1 - S_f\, i/SR))}{1-i/R}$, where S is initial stress (force) in every item (at the start of the test), S_f is stress (force), which already failed item can carry yet (because at least at the beginning of damage accumulation the rupture of fibres can be in different cross sections).

Let us consider the case when cdf $F(s)$ has location and scale parameters:

$F(s)=F_0((g(s)-\theta_0)/\theta_1)$, (4)

where g(s) is some known function, $F_0(.)$ is some known CDF. For example, later on we'll use normal CDF and $g(s)=\log(s)$. Now the considered model has 6 parameters: θ_0, θ_1, r, R, k_M, S_f. They have the following interpretation:

θ_0, θ_1 are parameters of CDF of strength of composite item (strand or fiber); for example, if $g(s)=s$ (normal distribution of strength) then θ_0 is expectation value and θ_1 is standard deviation of item strength;

R is the number of composite items in critical volume, failure of which corresponds to total failure of specimen;

r is a critical number of failed elements inside of this critical volume, corresponding to failure of this volume; the ratio r/R defines the part of the cross section area, the destruction of which we consider as failure of specimen; the value r defines mainly the variance and coefficient of variation of fatigue life;

k_M is number of cycles corresponding to one step in MC;

S_f is residual strength of failed item (it depends on the orientation and number of layers, the characteristics of matrix,...).

So now on we'll use $F_T(t;S,\eta)$ as specific notation of CDF of random variable T instead of more general notation, $F_T(t)$.

3. Estimation of model parameters

Formulae (1), (2) can be used in both direction: for calculation of mean and p-quantile fatigue curves, if parameters are known, or for nonlinear regression analysis for model parameters estimation, if fatigue life dataset is known. Mean and p-quantile fatigue curves are defined by formulae

$E(T(S_j))=k_M\sum_{i=1}^{r}1/p_i(S_j,\eta)$, $\quad t_p(S_j)=F_T^{-1}(p;S_j,\eta)$, where $E(T(S_j))$, $t_p(S_j)$ are mean value and p-quantile of fatigue life for stress S_j.

The parameters of the model can be estimated by the use of Method of Moments (MM), Least Square Method (LSM) and Maximum Likelihood Method (MLM), which is more preferable. For the profound investigation of this model can be recommended nonlinear regression procedure of SAS system. But in any case to find 6 unknown parameters is a very difficult problem. So we limit ourselves to only approximate solution of this problem. First of all we put: $k_M=1$, $S_f=0$, then we'll get approximate estimation of the remaining parameters: r, R, θ_0 and θ_1, and, finally, for fixed approximate estimates of parameters r, R we can find estimates of θ_0 and θ_1 by the use of MLM. Approximate estimate of parameter r can be found, if we assume, that approximately $p_1=p_2=...=p_r=p$.

Then $E(T)\cong\dfrac{r}{p}$; $V(T)\cong\dfrac{r}{p^2}$; a coefficient of variation $C_V=\sqrt{V(T)}/E(T)\cong 1/\sqrt{r}$.

And approximate estimate of parameter r is defined by formula $\hat{r}\cong]1/(\hat{C}_V)^2[+1$, where] x [is the nearest integer towards minus infinity.

The value $E(T)\cong\dfrac{r}{p}$ is very large (10^5-10^7!!!), r is small enough (see section 4), so the value of p is very small and $F(s)$ is very small too. All this gives us an idea to make the following serious enough assumption (not too bad final result is the only justification of it!):

$p_i = 1-(1-F(s_i))^{R-i} \cong p \cong (R-r)F_0((g(S)-\theta_0)/\theta_1)$ for all i=1,2,…,r.

Then, we have the following approximate formula

$$E(T(S)) = \frac{D_f}{F_0((g(S)-\theta_0)/\theta_1)}, \tag{5}$$

where $D_f = r/(R-r)$.

Then at the fixed D_f we can get the following linear regression model

$$y_i = F_0^{-1}(D_f / E(T(S_i))) = -\theta_0/\theta_1 + (1/\theta_1)g(S_i) = \beta_0 + \beta_1 x_i,\ i = 1,2,...,n. \tag{6}$$

Parameters β_0 and β_1 of this model can be estimated by the use of some statistical program of linear regression analysis at every fixed value of parameter D_f. And it is not too serious problem to find only one nonlinear parameter D_f. Then we have the following estimates for θ_0 and θ_1: $\hat{\theta}_1 = 1/\hat{\beta}_1$, $\hat{\theta}_0 = -\hat{\beta}_0/\hat{\beta}_1$.

Estimate of parameter *R* can be made after estimation of ratio $\rho = r/R$. Remind, that this ratio defines the part of the cross section area, the destruction of which we consider as total failure of specimens. In the Daniels's model of static strength [4] this value corresponds to the value of $F(x^*)$, where x^* is such, that $x^*(1-F(x^*)) = \max_x x(1-F(x))$.

We can estimate this value, using estimates of θ_0 and θ_1. So we have $\hat{\rho} = F(x^*)$,

$$\hat{R} =]1/((C_V)^2\hat{\rho})[+1. \tag{7}$$

Now we have approximate estimates of all four parameters θ_0 and θ_1, *r* and *R*. At the fixed estimates of *r* and *R* the more precise estimates of θ_0 and θ_1 can be found by the use of MLM. For the probability mass function now we have following formula $p_T(t;S,\eta) = F_T(t;S,\eta) - F_T(t-1;S,\eta)$. But for calculation of CDF we should get P^t. It needs too much time. So we try to find some approximation for CDF. By comparison of normal and lognormal approximation it appears, that lognormal approximation is more appropriate:

$F_T(t;S,\eta) \cong \Phi(\frac{\log(t)-\theta_{0LT}}{\theta_{1LT}})$, where $\theta_{0LT}, \theta_{1LT}$ are such, that we have the same expectation value and standard deviation $\theta_{0LT} = \log(E(T)) - (\log(C_V^2+1))/2$, $\theta_{1LT} = (\log(C_V^2+1))^{1/2}$.

Now the maximum likelihood function in logarithm scale $l(\eta) = \ln(L(\eta))$, where $L(\eta) = \prod_{i=1}^{n} f_i^{A_i}(1-F_i)^{1-A_i}$, f_i, F_i are the probability density function and cumulative distribution function of random variable T (for fixed η and S); A_i is equal to 1, if a fatigue test is finished by the failure of specimens, and , A_i is equal to 0, if the time of test is limited (right censored observation).

Until now we considered mainly uniformly load-shearing system of isolated parallel items loaded by tension. But we suppose to apply this model to the more complex structure, for which fracture of longitudinal items means the failure of specimen as a whole.

4. Numerical example

For numerical example we consider the problem to fit the experimental data of fatigue test of laminate panel. These data was kindly given to the authors by Professor W.Q. Meeker, who studied them in paper [5] and gives the following description of these data: "the data come from 125 specimens in four-point out-of-plane bending tests of carbon eight-harness-satin/ epoxy laminate. Fiber fracture and final specimen fracture occurred simultaneously. Thus, a fatigue life is defined to be the number of cycles until specimen fracture. The dataset includes 10 right censored observations (known as "run outs" in the fatigue literature)". In [1,2,3] we have considered already extreme values of fatigue lives for 5 stress levels, which we have got from the Figure 1 of paper [4]. But now we have original information, the same on the base of which the fatigue curve of this figure was made. And, as it was told already, this time we decrease the numbers of model parameters in order to increase the stability of others parameters. We put k=1, S_f= 0. Then four main steps were made for estimation of parameters θ_0, θ_1, r and R.

First step. We can make additional assumption, that static strength of items has lognormal distribution [2]: $F_0(.)$ is CDF of standard normal distribution, $g(s) = \log(s)$. By the use of regression analysis (and by sequence of calculation for different D_f) it was found approximate parameter estimates: $\hat{\theta}_0$=7.6906, $\hat{\theta}_1$=0.3541 and $\hat{D}_f$ = 0.0229.

Second step. Estimation of r. For this purposes the calculation of coefficient of variation C_V =0.5839 for some middle stress, at which there was not censuring, was calculated (for S=340 MPa). So $\hat{r}$ =3.

Third step. We have got estimate $\hat{R} = 15$, because at obtained approximate estimates $\hat{\theta}_0$, $\hat{\theta}_1$, the value of $\hat{\rho} = F(x^*)$ appears to be equal to 0.2072 (later on for final MLM the same estimate appears to be equal to 0.2022 and estimate $\hat{R} = 15$ did not change). Remind, that x^* is such, that: $x^*(1 - F(x^*)) = \max_x x(1 - F(x))$).

Fourth step. The estimation of parameters θ_0 and θ_1 by the use of MLM appears to be very difficult problem. The general view of this function is as we can see in Fig. 1. The counter lines of equal levels are shown in Fig. 2. After detail analysis (for the fixed already $\hat{r}$ =3, $\hat{R} = 15$) it was found: $\hat{\theta}_0$ = 7.6461, $\hat{\theta}_1$ = 0.34471.

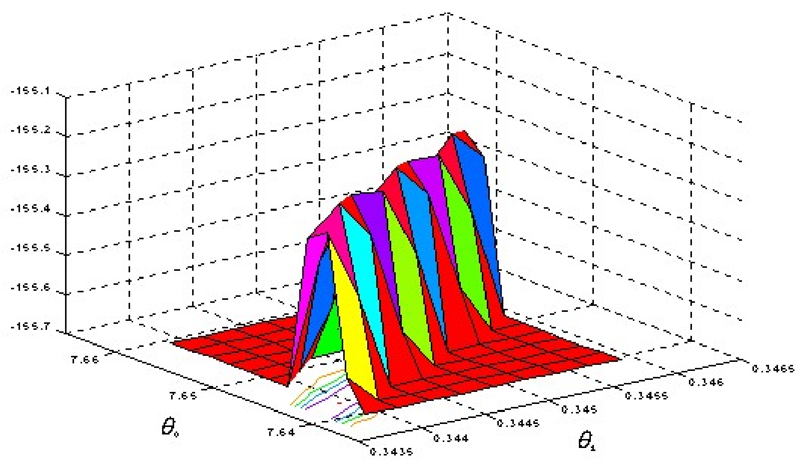

Figure 1. Likelihood function

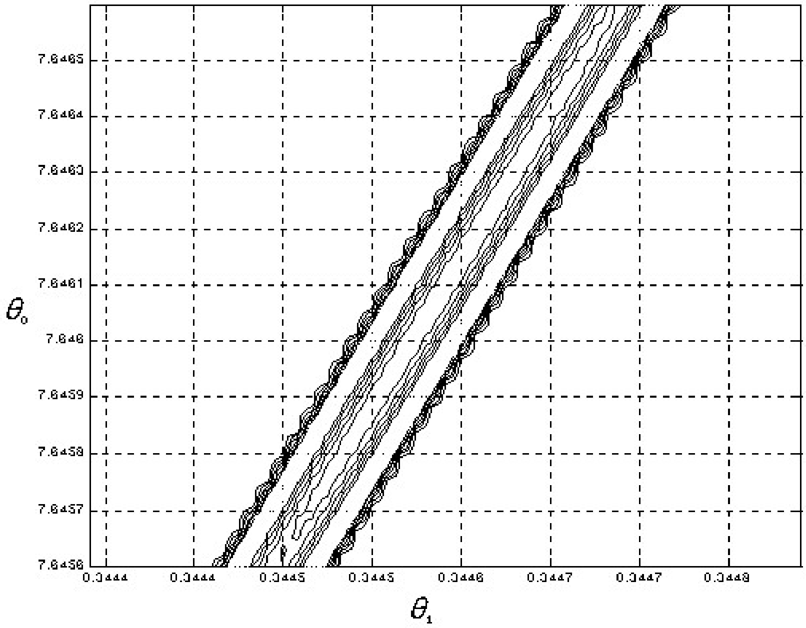

Figure 2. Contour lines of equal levels of likelihood function.
It was found: $\hat{\theta}_0$ = 7.6461 , $\hat{\theta}_1$ =0.34471.

Now we can check the validation of lognormal approximation of $F_T(t;S,\eta)$. We consider two hypotheses
$F_T(t;S,\eta) \cong \Phi(\frac{\log(t)-\theta_{0LT}}{\theta_{1LT}})$ and $F_T(t;S,\eta) \cong \Phi(\frac{t-\theta_{0T}}{\theta_{1T}})$. In the first case we should get straight line

$$\log(t) = \theta_{0LT} + \theta_{1LT}\Phi^{-1}(F_T(t;S,\eta)), \tag{6}$$

In the second case

$$t = \theta_{0T} + \theta_{1T}\Phi^{-1}(F_T(t,S,\eta)). \tag{7}$$

For this purpose we calculate the CDF and PDF for stress level S = 290.1 MPa. The result of calculation is shown in Fig. 3. We see, that formula (6) gives nearly straight line, so the lognormal approximation of $F_T(t;S,\eta)$ is more appropriate than the normal approximation.

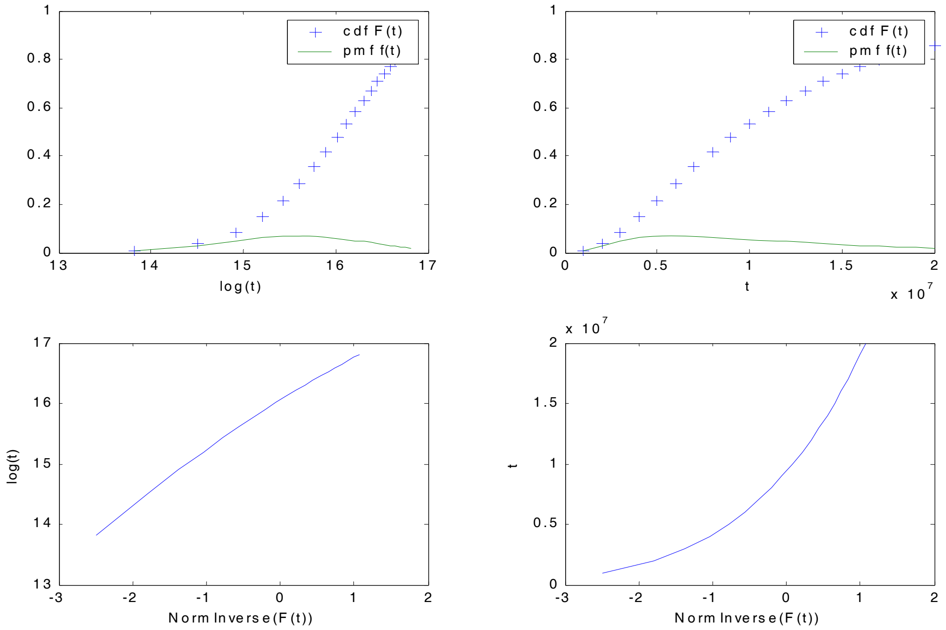

Figure 3. Cumulative distribution (×) and probability mass functions (-) in the upper part . Functions $\log(t) = \theta_{0LT} + \theta_{1LT}\Phi^{-1}(F_T(t;S,\eta))$ and $t = \theta_{0T} + \theta_{1T}\Phi^{-1}(F_T(t,S,\eta))$ in the lower part

Finally we have got the fatigue curve. The experimental data (×) and results of calculations of the expectation values of extreme order statistics (o) (minimum and maximum) are shown in Figure 4.

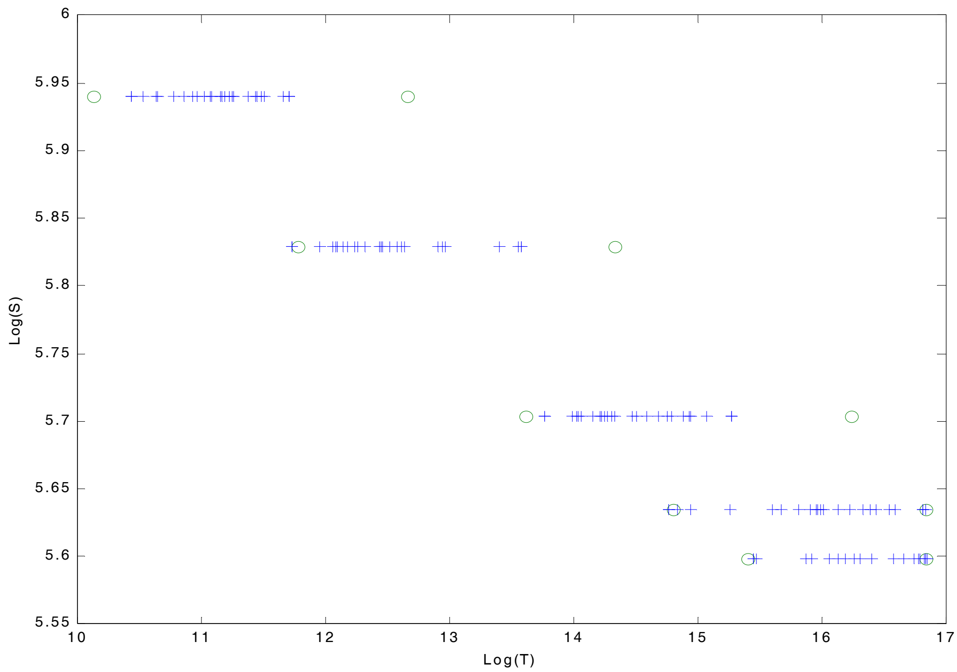

Figure 4. The experimental data (×) and results of calculations of the expectation values of extreme (minimum and maximum) orders statistics (o)

Conclusions

- S*imple Markov Chain Model of Fatigue Life* (SMCMFL) of composite material can be used as nonlinear regression model for fatigue curve approximation. Processing of dataset of fatigue test of carbon-fibre laminate specimens we have got estimates of parameters, which can be interpreted as *equivalent local static strength distribution parameter estimates.* We think, that, probably, there is some discrepancy between these estimations ($\hat{\theta}_0$ = 7.6461, $\hat{\theta}_1$ =0.34471) and real parameters of static strength distribution of carbon fibres, which are unknown for us. For example, in [4] the following estimations of carbon fibre static strength distribution are given: $\theta_0 = 7.198$, $\theta_1 = 0.467$. This discrepancy can be explained by the difference of original material, difference of load types (bending instead of tension), difference of "effective" length of fibres and so on. If we take into account all these circumstances, it seems that it is not too bad result for estimation of static strength distribution function parameters on the base of fatigue data.
- This discrepancy can be used for description of specific features of the specific structure.
- Really we do not need to estimate the static strength distribution parameters on the base of fatigue data. We consider the likelihood of these estimates only as a proof, that considered SMCMFL of composite material has right to exist and it can be used, for example, for forecasting of fatigue curve changes, when there are some changes of real static strength distribution parameters.

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PREDICTION INTERVALS FOR FUTURE OUTCOMES WITH A MINIMUM LENGTH PROPERTY

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1. Introduction

Practical problems often require the computation of predictions and prediction intervals for future values of random quantities. Consider the following examples.
1. A consumer purchasing a refrigerator would like to have a lower bound for the failure time of the unit to be purchased (with less interest in distribution of the population of units purchased by other consumers).
2. Financial managers in manufacturing companies need upper prediction bounds on future warranty costs.
3. When planning life tests, engineers may need to predict the number of failures that will occur by the end of the test to predict the amount of time that it will be taken for a specified number of units to fail.

Some applications require a two-sided prediction interval (z_L,z_U) that will, with a specified high degree of confidence, contain the future random variable of interest, say Z. In many applications, however, interest is focused on either an upper prediction bound or a lower prediction bound (e.g., the maximum warranty cost is more important than the minimum, and the time of the early failures in a product population is more important than the last ones).

Conceptually, it is useful to distinguish between “new-sample’ prediction and “within-sample’ prediction. For new-sample prediction, data from a past sample are used to make predictions on a future unit or sample of units from the same process or population. For example, based on previous (possibly censored) life test data, one could be interested in predicting the time to failure of a new unit, time until k failures in a future sample of m units, or number of failures by time $t^{\bullet}$ in a future sample of m units.

For within-sample prediction, the problem is to predict future events in a sample or process based on early data from that sample or process. For example, if n units are followed until $t_{\bullet}$ and there are r observable failures, $X_{(1)} \le X_{(2)} \le \cdots \le X_{(r)}$, one could be interested in predicting the time of the next failure, $X_{(r+1)}$; time until k additional failures, $X_{(r+k)}$; number of additional failures in a future interval $(t_{\bullet},t^{\bullet})$.

In general, to predict a future realization of a random quantity one needs the following.
1. A statistical model to describe the population or process of interest. This model usually consists of a distribution depending on a vector of parameters $\boldsymbol{\theta}$. In this paper, attention is restricted to invariant families of distributions. In particular, the case is considered where a previously available complete or type II censored sample is from a continuous distribution with CDF $F((x-\mu)/\sigma)$, where $F(\cdot)$ is known but both the location (μ) and scale (σ) parameters are unknown. For such family of distributions the decision problem remains invariant under a group of transformations (a subgroup of the full affine group) which takes μ (the location parameter) and σ (the scale) into $c\mu + b$ and $c\sigma$, respectively, where b lies in the range of μ, $c > 0$. This group acts transitively on the parameter space. The technique used here emphasizes pivotal quantities relevant for obtaining invariant statistics. It is a special case of the method of invariant embedding of sample statistics into a performance index (see, e.g., Nechval et al. (1999;

2000b; 2002)) applicable whenever the statistical problem is invariant under a group of transformations, which acts transitively on the parameter space. The analysis of problem considered here is easily seen to be invariant under location and scale changes.
2. Information on the values of components of the parametric vector $\boldsymbol{\theta}$. It is assumed that only the functional form of the distribution is specified, but some or all of its parameters are unspecified. In such cases an invariant statistic, whose distribution does not depend on the unknown parameters, is used.

2. Preliminaries

Suppose that we observe a sample $\mathbf{X}=(X_1, \ldots, X_n)$ of independent random variables whose distribution depends on the unknown parameter $\boldsymbol{\theta}=(\theta_1, \ldots, \theta_k)\in\Theta$ (a parameter space). On the basis of the outcomes $X_1, \ldots, X_n$ we wish to make a prediction about some function $z(\mathbf{Y})$ (statistic) in a future sample $\mathbf{Y}=(Y_1, \ldots, Y_m)$ of m independent observations for the same distribution, usually in the form of an interval or region that we reasonably confident will contain $z(\mathbf{Y})$. Thus, we want to construct a $(1-\alpha)$ prediction interval $C(S)$ for some function $z(\mathbf{Y})$, where $S\equiv s(\mathbf{X})$ is some statistic. One method that is often used to construct such a prediction interval is to find an invariant statistic, a function $V(S,z(\mathbf{Y}))$ whose distribution does not depend on any unknown parameter. For V we use a random variable $V(S,z(\mathbf{Y}))$, which is a function of S and $\mathbf{Y}$, and whose distribution is independent of $\boldsymbol{\theta}$. This function is called an invariant statistic. The simplest invariant statistic represents a function of a sufficient statistic S and $Z\equiv z(\mathbf{Y})$.

In this paper we are interested in the problem of finding a family of random sets $C(S)$ for a function $z(\mathbf{Y})$ such that, for a given α, $0<\alpha<1$ (usually small),

$$\Pr\{C(S)\ni z(\mathbf{Y})\}\geq 1-\alpha \quad \text{for all } \boldsymbol{\theta}\in\Theta. \tag{1}$$

A family of subsets $C(s)$ for $z(\mathbf{y})$, where $\boldsymbol{\theta}\in\Theta\subseteq\mathscr{R}^k$, is said to constitute a family of prediction sets at confidence level $1-\alpha$ if the random set $C(S)$ covers the true value of $z(\mathbf{Y})$ with probability $\geq 1-\alpha$. A lower prediction bound $z_L(s)$ corresponds to the case where

$$C(s)=\{z(\mathbf{y}):\ z_L(s) < z(\mathbf{y})\}; \tag{2}$$

and an upper prediction bound $z_U(s)$ to the case where

$$C(s)=\{z(\mathbf{y}):\ z_U(s) > z(\mathbf{y})\}. \tag{3}$$

If $C(s)$ is of the form

$$C(s)=(z_L(s),\ z_U(s)) \tag{4}$$

we will call it a prediction interval at confidence level $1-\alpha$, provided that

$$\Pr\{(z_L(S)<z(\mathbf{Y})<z_U(S)\} \geq 1-\alpha \quad \text{for all } \boldsymbol{\theta}\in\Theta, \tag{5}$$

and the quantity $\inf_{\boldsymbol{\theta}} \Pr\{(z_L(S)<z(\mathbf{Y})<z_U(S)\}$ will be referred to as the confidence coefficient associated with the random interval.

3. Main theorem

The following result provides a general method of finding prediction intervals and covers most cases in practice.
Theorem 1. Let $\mathbf{X}=(X_1, \ldots, X_n)$ and $\mathbf{Y}=(Y_1, \ldots, Y_m)$ be independent random samples of n and m observations, respectively, with the same distribution. Let $S\equiv s(\mathbf{X})$ be a statistic, based on a random sample $\mathbf{X}=(X_1, \ldots, X_n)$, whose range is region of the real line, with a distribution function $F_{\boldsymbol{\theta}}$, $\boldsymbol{\theta}\in\Theta$, where Θ is a region of $\mathscr{R}^k$. Let $V(S,z(\mathbf{Y}))$ be a real-valued function defined on $\mathscr{S}\times\mathscr{Z}$, such that, for each

$\mathbf{Y}$, $V(S,z(\mathbf{Y}))$ is a statistic, and as a function of $z(\mathbf{y})$, V is strictly increasing or decreasing at every $s\in\mathcal{S}$, where $\mathcal{S}\subseteq\mathcal{R}$ and $\mathcal{Z}\subseteq\mathcal{R}$ are the ranges of S and $Z\equiv z(\mathbf{Y})$, respectively. Let $\mathcal{V}\subseteq\mathcal{R}$ be the range of V, and for every $v\in\mathcal{V}$ and $s\in\mathcal{S}$ let the equation $v=V(s,z(\mathbf{y}))$ be solvable. If the distribution of $V(S,z(\mathbf{Y}))$ is independent of $\boldsymbol{\theta}$, one can construct a confidence interval for $z(\mathbf{Y})$ at any level.

Proof. Let $0 < \alpha < 1$. Then we can choose a pair of numbers $v_1(\alpha)$ and $v_2(\alpha)$ in $\mathcal{V}$ not necessarily unique, such that

$$\Pr\{v_1(\alpha)<V(S,z(\mathbf{Y}))<v_2(\alpha)\}\geq 1-\alpha \quad \text{for all } \boldsymbol{\theta}\in\Theta. \tag{6}$$

Since the distribution of V is independent of $\boldsymbol{\theta}$, it is clear that v_1 and v_2 are independent of $\boldsymbol{\theta}$. Since, moreover, V is monotone in $z(\mathbf{y})$, we can solve the equations

$$V(s,z(\mathbf{y}))=v_1(\alpha) \quad \text{and} \quad V(s,z(\mathbf{y}))=v_2(\alpha) \tag{7}$$

for every s, uniquely for $z(\mathbf{y})$. We have

$$\Pr\{(z_L(S)<z(\mathbf{Y})<z_U(S)\}\geq 1-\alpha \quad \text{for all } \boldsymbol{\theta}\in\Theta, \tag{8}$$

where $z_L(S)<z_U(S)$ are rv's. This completes the proof. □

4. Problem statement

It follows from the above that, for a given confidence level $1-\alpha$, a wide choice of prediction intervals is available. Clearly, the larger the interval, the better is the chance of trapping a true value of $z(\mathbf{Y})$. Thus the interval $(-\infty, +\infty)$, which ignores the data completely, will include the real-valued parameter function $z(\mathbf{Y})$ with confidence level 1. In practice, one would like to set the level at a given fixed number $1-\alpha$ ($0 < \alpha < 1$) and, if possible, construct an interval as short as possible among all prediction intervals with the same level. Such an interval is desirable since it is more informative. Unfortunately, shortest-length prediction intervals do not always exist. In this paper we will investigate the possibility of constructing shortest-length prediction intervals based on some simple rv's. Theorem 1 is really the key to the following discussion.

Let $V(S,z(\mathbf{Y}))\equiv V$ be a random variable with distribution independent on $\boldsymbol{\theta}$. Also, let $v_1=v_1(\alpha)$, $v_2=v_2(\alpha)$ be chosen so that

$$\Pr\{v_1<V<v_2\} = 1-\alpha, \tag{9}$$

and suppose that (9) can be rewritten as

$$\Pr\{(z_L(S)<z(\mathbf{Y})<z_U(S)\}=1-\alpha \tag{10}$$

(see Theorem 1 for a set of sufficient conditions). For every V, v_1 and v_2 can be chosen in many ways. We would like to choose v_1 and v_2 so that z_U-z_L is minimum. Such an interval is a $1-\alpha$ level shortest-length prediction interval based on V. It may be possible, however, to find another rv V* that may yield an even shorter interval. Therefore we are not asserting that the procedure, if it succeeds, will lead to a $1-\alpha$ level confidence interval that has shortest length among all intervals of this level. Let F be the distribution function of the invariant statistic $V(S,z(\mathbf{Y}))$ and let v_1, v_2 be such that

$$F(v_2)-F(v_1) = \Pr\{v_1<V<v_2\}=1-\alpha. \tag{11}$$

$100(1-\alpha)\%$ prediction interval of $z(\mathbf{Y})$ is $(z_L(S),z_U(S))$ and the length of this interval is $\Delta_z(S, v_1, v_2) = z_U - z_L$. We want to choose v_1, v_2, minimizing $z_U - z_L$ and satisfying (11). Thus, we consider the problem:
Minimize:

$$\Delta_z(S,v_1,v_2) = z_U - z_L, \tag{12}$$

Subject to:

$$F(v_2) - F(v_1) = 1-\alpha. \tag{13}$$

The search for the prediction interval with the shortest length $\Delta_z=z_U - z_L$ is greatly facilitated by the use of the result of the following section.

5. Finding the shortest-length prediction intervals

Theorem 2. Under appropriate derivative conditions, there will be a pair (v_1,v_2) giving rise to the prediction interval with the shortest length $\Delta_z(S,v_1,v_2)=z_U-z_L$ for $z(\mathbf{Y})$ as a solution to the simultaneous equations:

$$\frac{\partial \Delta_z}{\partial v_1} + \frac{\partial \Delta_z}{\partial v_2}\frac{F'(v_1)}{F'(v_2)} = 0, \tag{14}$$

$$F(v_2) - F(v_1) = 1-\alpha. \tag{15}$$

Proof. Note that (15) forces v_2 to be a function of v_1 (or visa-versa). Take $\Delta_z(S,v_1,v_2)$ as a function of v_1, say $\Delta_z(S,v_1,v_2(v_1))$. Then, by using the method of Lagrange multipliers, the proof follows immediately. □

6. Examples

Example 1. Suppose that the times-to-failure for some product follow an exponential distribution. The given data $\mathbf{X}=(X_1, \ldots, X_n)$ consist of the failure times of n randomly selected units. It is desired to construct, using the results of a previous sample, the shortest-length (1-α) prediction interval on the mean time-to-failure of future sample $\mathbf{Y}=(Y_1, \ldots, Y_m)$ consisting of m units. It is assumed that X and Y are stochastically independent random variables from an exponential distribution with probability density functions

$$f(x;\theta) = \frac{1}{\sigma}e^{-x/\sigma}, \quad x \in (0,\infty), \quad \sigma > 0, \tag{16}$$

and

$$f(y;\sigma) = \frac{1}{\sigma}e^{-y/\sigma}, \quad y \in (0,\infty), \quad \sigma > 0, \tag{17}$$

respectively. Also let

$$\overline{X} = \sum_{i=1}^{n} X_i / n \quad \text{and} \quad \overline{Y} = \sum_{j=1}^{m} Y_j / m \tag{18}$$

denote the mean times to failure for the above units. It is well known that $2n\overline{X}/\sigma$ and $2m\overline{Y}/\sigma$ are chi-square distributed variables with 2n and 2m degrees of freedom, respectively. Since they are independent due to the independence of the variables X and Y, the obvious choice for an invariant statistic $V(S,z(\mathbf{Y}))$ is given by

$$V(S,z(\mathbf{Y})) = \left(\frac{2n\overline{X}/\sigma}{2n}\right)\Bigg/\left(\frac{2m\overline{Y}/\sigma}{2m}\right) = \frac{\overline{X}}{\overline{Y}}, \tag{19}$$

which is F-distributed with (2n,2m) degrees of freedom, $S = \overline{X}$, and $z(\mathbf{Y}) = \overline{Y}$. Now

$$\Pr\{v_1 < V < v_2\} = 1 - \alpha, \tag{20}$$

so that

$$\Pr\{z_L < \overline{Y} < z_U\} = 1 - \alpha, \tag{21}$$

where

$$z_L = \overline{X} / v_2, \quad z_U = \overline{X} / v_1. \tag{22}$$

Let us assume that we wish to minimize

$$\Delta_z = z_U - z_L = \left(\frac{1}{v_1} - \frac{1}{v_2}\right)\overline{X} \tag{23}$$

subject to

$$\Pr\{v_1 < V < v_2\} = \int_{v_1}^{v_2} f_{2n,2m}(v)dv = 1 - \alpha, \tag{24}$$

where $f_{2n,2m}(\cdot)$ is the PDF of an F-distributed rv with (2n,2m) d.f. It follows from (14) and (15) that v_1 and v_2 are a solution of

$$v_1^2 f_{2n,2m}(v_1) = v_2^2 f_{2n,2m}(v_2) \tag{25}$$

and

$$\Pr\{v_1 < F_{2n,2m} < v_2\} = 1 - \alpha, \tag{26}$$

where $F_{2n,2m}$ is an F random variable, with (2n,2m) degrees of freedom. This result can be used to obtain the shortest-length prediction interval at confidence level $1-\alpha$,

$$C_{\overline{Y}}(\overline{X}) = \left(\frac{\overline{X}}{v_2}, \frac{\overline{X}}{v_1}\right), \tag{27}$$

for $\overline{Y}$.

Consider the following data. A total of 400 hours of operating time with n=2 times-to-failure X_1=97 hours and X_2=303 hours were observed for aircraft air-conditioning equipment. What is the shortest-length prediction interval for the mean time-to-failure of future sample **Y**=(Y_1, Y_2) of size m=2 of times-to-failure of this equipment at the 90% confidence level? From (27), the shortest-length prediction interval is

$$C_{\overline{Y}}^*(\overline{X}) = \left(\frac{\overline{X}}{v_2^*}, \frac{\overline{X}}{v_1^*}\right) = (1.235,\ 823.045), \tag{28}$$

where $\overline{X} = 200$; v_1^* =0.243 and v_2^* =161.89 are a solution of (25) and (26). Thus, the length of this interval is Δ_z^* = 823.045 − 1.235 = 821.81.

It will be noted that the simpler equal tails prediction interval at the 90% confidence level, which is used, as a rule, in practice, is given by

$$C^{\circ}_{\bar{Y}}(\bar{X}) = \left(\frac{\bar{X}}{v_2}, \frac{\bar{X}}{v_1}\right) = (31.299,\ 1282.051), \tag{29}$$

where $v_1=F_{n,m;\alpha/2}=F_{2,2;0.05}=0.156$, $v_2=F_{n,m;1-\alpha/2}=F_{2,2;0.95}=6.39$,

$$\Pr\{F_{n,m} \le F_{n,m;\alpha}\} = \alpha. \tag{30}$$

The length of this interval is $\Delta^{\circ}_z = 1282.051 - 31.299 = 1250.752$. The relative efficiency of $C^{\circ}_{\bar{Y}}(\bar{X})$ relative to $C^{*}_{\bar{Y}}(\bar{X})$, taking into account $E_{\theta}\{\Delta_z\}$, is given by

$$\text{rel.eff.}_{E_{\theta}\{\Delta_z\}}\left(C^{\circ}_{\bar{Y}}(\bar{X}), C^{*}_{\bar{Y}}(\bar{X})\right) = \frac{E_{\theta}\{\Delta^{*}_z\}}{E_{\theta}\{\Delta^{\circ}_z\}} = \frac{\left(\dfrac{1}{v^{*}_1} - \dfrac{1}{v^{*}_2}\right)}{\left(\dfrac{1}{F_{2,2;0.05}} - \dfrac{1}{F_{2,2;0.95}}\right)} = 0.657. \tag{31}$$

Example 2. Let $X_{(1)} \le X_{(2)} \le \cdots \le X_{(k)}$ be the k smallest observations in a sample of size n from the two-parameter exponential distribution, with density

$$f(x;\theta) = \frac{1}{\sigma}\exp\left(-\frac{x-\mu}{\sigma}\right), \quad x\ge\mu, \tag{32}$$

where $\sigma>0$ and μ are unknown parameters, $\theta=(\mu,\sigma)$. Let $Y_{(r)}$ be the rth smallest observation in a future sample of size m from the same distribution. We wish, on the basis of observed $X_{(1)}, \ldots, X_{(k)}$ to construct prediction intervals for $Y_{(r)}$.

Let $S_r=(Y_{(r)}-\mu)/\sigma$, $S_1=(X_{(1)}-\mu)/\sigma$ and $T_1=T/\sigma$, where

$$T = \sum_{i=1}^{k}(X_{(i)} - X_{(1)}) + (n-k)(X_{(k)} - X_{(1)}). \tag{33}$$

To construct prediction intervals for $Y_{(r)}$, consider the quantity (invariant statistic)

$$V = n(S_r-S_1)/T_1 = n(Y_{(r)}-X_{(1)})/T. \tag{34}$$

It is well known (Epstein and Sobel, 1954) that nS_1 has a standard exponential distribution, that $2T_1\sim\chi^2_{2k-2}$ and that S_1 and T_1 are independent. Also, S_r is the rth order statistic from a sample of size m from the standard exponential distribution and thus has probability density function (Kendall and Stuart, 1969),

$$f(s_r) = r\binom{m}{r}(1-e^{-s_r})^{r-1}e^{-s_r(m-r+1)}, \tag{35}$$

if $s_r>0$, and $f(s_r)=0$ for $s_r\le 0$. Using the technique of invariant embedding (Nechval et al. (2000a; 2001a; 2001b)), we find after some algebra that

$$F(v)=\Pr\{V\le v\} = \begin{cases} 1-nr\binom{m}{r}\sum_{j=0}^{r-1}\dfrac{\binom{r-1}{j}(-1)^j[1+v(m-r+j+1)/n]^{-k+1}}{(m+n-r+j+1)(m-r+j+1)}, & v>0, \\ m^{(r)}(1-v)^{-k+1}/(m+n)^{(r)}, & v\le 0, \end{cases} \tag{36}$$

where $m^{(r)}=m(m-1)\cdots(m-r+1)$.

The special case in which r=1 is worth mentioning, since in this case (36) simplifies somewhat. We find here that we can write

$$F(v)=\Pr\{V\le v\} = \begin{cases} 1-\dfrac{\vartheta}{\vartheta+1}\left(\dfrac{\vartheta}{\vartheta+v}\right)^{k-1}, & v>0, \\ (\vartheta+1)^{-1}(1-v)^{-k+1}, & v\le 0, \end{cases} \tag{37}$$

where $\vartheta=n/m$.

Consider the ordered data given by Grubbs (1971) on the mileages at which nineteen military carriers failed. These were 162, 200, 271, 302, 393, 508, 539, 629, 706, 777, 884, 1008, 1101, 1182, 1463, 1603, 1984, 2355, 2880, and thus constitute a complete sample with k=n=19. We find

$$T=\sum_{i=1}^{19}(X_{(i)}-X_{(1)})=15869 \tag{38}$$

and of course $X_{(1)}=162$. Suppose we wish to set up the shortest-length ($1-\alpha=0.95$) prediction interval for the smallest observation $Y_{(1)}$ in a future sample of size m=5. Consider the invariant statistic

$$V=n(Y_{(1)}-X_{(1)})/T. \tag{39}$$

Then

$$1-\alpha=\Pr\left\{v_1<\frac{n(Y_{(1)}-X_{(1)})}{T}<v_2\right\}=\Pr\left\{X_{(1)}+v_1\frac{T}{n}<Y_{(1)}<X_{(1)}+v_2\frac{T}{n}\right\}$$

$$=\Pr\{z_L<Y_{(1)}<z_U\}, \tag{40}$$

where $z_L=X_{(1)}+v_1T/n$ and $z_U=X_{(1)}+v_2T/n$. The length of the prediction interval is $\Delta_z=z_U-z_L=(T/n)(v_2-v_1)$. We wish to minimize Δ_z subject to

$$F(v_2)-F(v_1)=1-\alpha. \tag{41}$$

It follows from (14) and (15) that the minimum occurs when $f(v_1)=f(v_2)$, where v_1 and v_2 satisfy (41). The shortest-length prediction interval is given by

$$C^{*}_{Y_{(1)}}(X_{(1)},T)=\left(X_{(1)}+v_1^{*}\frac{T}{n},X_{(1)}+v_2^{*}\frac{T}{n}\right)=(10.78,\ 736.62), \tag{42}$$

where $v_1^{*}=-0.18105$ and $v_2^{*}=0.688$. Thus, the length of this interval is $\Delta^{*}_{z}=736.62-10.78=725.84$. The equal tails prediction interval at the $1-\alpha=0.95$ confidence level is given by

$$C^{\circ}_{Y_{(1)}}(X_{(1)},T)=\left(X_{(1)}+v_{\alpha/2}\frac{T}{n},X_{(1)}+v_{1-\alpha/2}\frac{T}{n}\right)=(57.6,\ 834.34), \tag{43}$$

where $F(v_\alpha)=\alpha$, $v_{\alpha/2}=-0.125$ and $v_{1-\alpha/2}=0.805$. The length of this interval is $\Delta^{\circ}_{z}=834.34-57.6=776.74$. The relative efficiency of $C^{\circ}_{Y_{(1)}}(X_{(1)},T)$ relative to $C^{*}_{Y_{(1)}}(X_{(1)},T)$, taking into account $E_{\theta}\{\Delta_z\}$, where $\theta=(\mu,\sigma)$, is given by

$$\text{rel.eff.}_{E_{\theta}\{\Delta_z\}}\left(C^{\circ}_{Y_{(1)}}(X_{(1)},T),C^{*}_{Y_{(1)}}(X_{(1)},T)\right)=\frac{E_{\theta}\{\Delta^{*}_{z}\}}{E_{\theta}\{\Delta^{\circ}_{z}\}}=\frac{v_2^{*}-v_1^{*}}{v_{1-\alpha/2}-v_{\alpha/2}}=0.934. \tag{44}$$

One may also be interested in predicting the mean $\overline{Y}=\sum_{j=1}^{m}Y_j/m$ or total lifetime in a future sample. Consider the quantity:

$$V=n(\overline{Y}-X_{(1)})/T. \tag{45}$$

Using the invariant embedding technique, we find after some algebra that

$$F(v)=\Pr\{V\le v\}=\begin{cases}1-\displaystyle\sum_{j=0}^{m-1}\binom{k+j-2}{j}\frac{(v/\vartheta)^{j}[1-(1+\vartheta)^{-m+j}]}{(1+v/\vartheta)^{k+j-1}}, & v>0,\\[2ex] (1+\vartheta)^{-m}(1-v)^{-k+1}, & v\le 0.\end{cases} \tag{46}$$

Probability statements about V lead to prediction intervals for $\overline{Y}$ or $\sum_{j=1}^{m}Y_j=m\overline{Y}$.

Example 3. Consider the system consisting of m independent components, which have a common life-distribution. Let $R(\cdot)$ be a survivor function of the life of each component in the system and Y_s be a lifetime of the system of components, all of which have the same life distribution. Let $F_s(\cdot)$ be a cumulative distribution function of Y_s. In this case, the reliability function of the system is a polynomial in component reliability. In other words, the relationship between $F_s(y)$ and $R(y)$ is described as follows (Barlow and Proschan, 1981; Hoyland and Rausand, 1994).

For a given system, there exists a set of integers $\{a_0, a_1, \ldots, a_m\}$ such that

$$F_s(y)=\sum_{j=1}^{m}a_j[R(y)]^{j},\quad a_0=1,\quad \sum_{j=1}^{m}a_j=-1. \tag{47}$$

The set $\{a_0, a_1, \ldots, a_m\}$ is determined by the reliability function of the system.

Let us assume that a sample of n components with the same life distribution is put on life test simultaneously and the lifetimes of some components in the sample are observed. Let $\mathbf{X}=(X_{(h)}, X_{(h+1)}, \ldots, X_{(k)})$, $1\le h\le k\le m$, be a sample of some or all observed ordered lifetimes on life test of components. The probability density function for the lifetimes of all components is the exponential (see (32)). Consider 2 cases: (i) $\mu=0$ and σ is unknown (the one-parameter exponential distribution), (ii) μ and σ are unknown (the two-parameter exponential distribution).

To construct the exact shortest-length prediction interval for Y_s based on all or some (in the case of missing values) of the observations **X** from a doubly-censored sample in the (i)th case, consider an invariant statistic

$$V = \frac{Y_s}{T_\omega} \tag{48}$$

that has a distribution, not involving σ, with ω degrees of freedom, where

$$T_\omega = \sum_{i=h}^{k} \omega_i (n-i+1)(X_{(i)} - X_{(i-1)}), \quad X_{(0)} = 0, \tag{49}$$

and

$$\omega = \sum_{i=h}^{k} \omega_i, \quad \omega_i = 0,1. \tag{50}$$

In the case of missing values ω_i=0. Using the technique of invariant embedding (Nechval et al. (2000a; 2001a; 2001b)), it can be shown that the cumulative distribution function of V is given by

$$F^{(1)}(v) = \sum_{j=0}^{m} \frac{a_j}{(1+jv)^\omega}, \quad \text{for } v > 0. \tag{51}$$

The statement:

$$\Pr\{v_{\alpha/2} < V < v_{1-\alpha/2}\} = 1-\alpha, \tag{52}$$

gives

$$C^{\circ}_{Y_s}(T_\omega) = \left(v_{\alpha/2}T_\omega, v_{1-\alpha/2}T_\omega\right) \tag{53}$$

as the equal tails (1−α) prediction interval for Y_s, where $F^{(1)}(v_\alpha)$=α. For given values of a_1, a_2, …, a_m, ω, and v, the $F^{(1)}(v)$ in (51) is easily evaluated on a computer. If, for specified a_1, a_2, …, a_m, and ω, the values of v are desired which make F(v) equal to some specified value, such as 0.05, these values can be found easily on a computer using a simple iteration scheme.
The statement:
Minimize

$$\Delta_z = z_U - z_L = \left(v_2 - v_1\right)T_\omega \tag{54}$$

Subject to

$$\Pr\{v_1 < V < v_2\} = F^{(1)}(v_2) - F^{(1)}(v_1) = 1-\alpha \tag{55}$$

gives

$$C^{*}_{Y_s}(T_\omega) = \left(v_1^* T_\omega, v_2^* T_\omega\right) \tag{56}$$

as the shortest-length (1−α) prediction interval for Y_s.

To construct the exact shortest-length prediction interval for Y_s based on all or some (in the case of missing values) of the observations **X** from a doubly-censored sample in the (ii)th case, consider an invariant statistic

$$V = \frac{Y_s - X_{(h)}}{T_\omega} \tag{57}$$

that has a distribution, not involving μ and σ, with ω degrees of freedom.

Using the technique of invariant embedding (Nechval et al. (2000a; 2001a; 2001b)), it can be shown that the cumulative distribution function of V is given by

$$F^{(2)}(v) = \begin{cases} \displaystyle\sum_{j=0}^{m} \sum_{i=0}^{h-1} \frac{a_j(-1)^i h \binom{n}{h} \binom{h-1}{i}}{(1+jv]^\omega (j+n-h+1+i)}, & \text{for } v > 0, \\ \\ \displaystyle\sum_{j=0}^{m} \sum_{i=0}^{h-1} \frac{a_j(-1)^i h \binom{n}{h} \binom{h-1}{i}}{[1-(n-h+1+i)v]^\omega (j+n-h+1+i)}, & \text{for } v \le 0, \end{cases} \tag{58}$$

The statement:

$$\Pr\{v_{\alpha/2} < V < v_{1-\alpha/2}\} = 1-\alpha, \tag{59}$$

gives

$$C^{\circ}_{Y_s}(X_{(h)}, T_\omega) = \left(X_{(h)} + v_{\alpha/2} T_\omega, X_{(h)} + v_{1-\alpha/2} T_\omega\right) \tag{60}$$

as the equal tails $(1-\alpha)$ prediction interval for Y_s.
The statement:

Minimize

$$\Delta_z = z_U - z_L = (v_2 - v_1) T_\omega \tag{61}$$

Subject to

$$\Pr\{v_1 < V < v_2\} = F^{(2)}(v_2) - F^{(2)}(v_1) = 1-\alpha \tag{62}$$

gives

$$C^{*}_{Y_s}(X_{(h)}, T_\varpi) = \left(X_{(h)} + v_1^{*} T_\omega, X_{(h)} + v_2^{*} T_\omega\right) \tag{63}$$

as the shortest-length $(1-\alpha)$ prediction interval for Y_s.

Let n=20 components, with the two-parameter exponential distribution, be put on test simultaneously (Futatsuya, 2000). The test is terminated after the first 15 failures, but failures #1, #2, #7 were unobserved. The failure times are: ?, ?, 59, 62, 67, 79, ?, 96, 98, 114, 119, 151, 156, 172, 208. Thus, n=20, h=3, ω_i=1 for i=4, 5, 6, 9, 10, …, 15, and ω_i=0, otherwise; ω=10, $X_{(h)}$=59, T_ω=1217. Consider a bridge system with 5 similar components (Hoyland and Rausand, 1994), for which

$$F_s(y)=1-2[R(y)]^2-2[R(y)]^3+5[R(y)]^4-2[R(y)]^5. \tag{64}$$

The equal tails 0.95-prediction interval for Y_s is (Futatsuya, 2000)

$$C^{\circ}_{Y_s}(X_{(h)}, T_\omega) = \left(X_{(h)} + v_{\alpha/2}T_\omega, X_{(h)} + v_{1-\alpha/2}T_\omega\right) = (48.87, 373.59), \tag{65}$$

where $v_{\alpha/2} = v_{0.025} = -0.00832$, $v_{1-\alpha/2} = v_{0.975} = 0.25849$. The length of this interval is $\Delta^{\circ}_z = 373.59 - 48.87 = 324.72$.

The shortest-length 0.95 prediction interval for Y_s is

$$C^{*}_{Y_s}(T_\omega) = \left(X_{(h)} + v^*_1 T_\omega, X_{(h)} + v^*_2 T_\omega\right) = (32.23, 323.03), \tag{66}$$

where $v^*_1 = -0.022$, $v^*_2 = 0.21695$. The length of this interval is $\Delta^*_z = 323.03 - 32.23 = 290.8$.

The relative efficiency of $C^{\circ}_{Y_s}(X_{(h)}, T_\omega)$ relative to $C^{*}_{Y_s}(X_{(h)}, T_\omega)$, taking into account $E_\theta\{\Delta_z\}$, where $\theta=(\mu,\sigma)$, is given by

$$\text{rel.eff.}_{E_\theta\{\Delta_z\}}\left(C^{\circ}_{Y_s}(X_{(h)}, T_\omega), C^{*}_{Y_s}(X_{(h)}, T_\omega)\right) = \frac{E_\theta\{\Delta^*_z\}}{E_\theta\{\Delta^{\circ}_z\}} = \frac{v^*_2 - v^*_1}{v_{1-\alpha/2} - v_{\alpha/2}} = 0.8955. \tag{67}$$

Example 4. Suppose that $Y_{(1)} \le Y_{(2)} \le \dots \le Y_{(n)}$ are ordered observations in a sample of size n from the two-parameter exponential distribution with parameters μ and σ, having density

$$f(y;\theta) = (1/\sigma)\exp[-(y-\mu)/\sigma], \quad \sigma > 0, \quad y \ge \mu, \tag{68}$$

where $\theta=(\mu,\sigma)$.
At first, we consider the case when the parameter μ is known, but the parameter σ is unknown. Let

$$S_k = \sum_{i=1}^{k}(Y_{(i)} - \mu) + (n-k)(Y_{(k)} - \mu), \quad 1 \le k < r \le n, \tag{69}$$

and consider the invariant statistic

$$V_r = (Y_{(r)} - Y_{(k)})/S_k. \tag{70}$$

In deriving the density function of V_r we note two well-known results (Nechval (1982; 1984)) concerning ordered observations from an exponential distribution: (i) the random variables $Z_1 = n(Y_{(1)} - \mu)$, $Z_i = (n-i+1)(Y_{(i)} - Y_{(i-1)})$, $i=2(1)n$, are independently distributed with density (22) for $\mu=0$, and (ii) $2S_k/\sigma = 2(Z_1 + \dots + Z_k)/\sigma$ is hence a χ^2 variable with 2k degrees of freedom. It then follows rather easily that $(Y_{(r)} - Y_{(k)})/\sigma$ and S_k/σ are pivotal quantities, which are independently distributed (both independent of σ), and that V_r has a distribution not involving σ. Then the probability density function of the invariant (ancillary) statistic V_r is found as follows.

It follows readily from standard theory of order statistics (see, for example, Kendall and Stuart (1969)) that the joint density function of $Y_{(r)}$ and $Y_{(k)}$ is

$$f(y_{(k)}, y_{(r)}; \theta)$$

$$= \frac{[F(y_{(k)};\theta)]^{k-1}[F(y_{(r)};\theta) - F(y_{(k)};\theta)]^{r-k-1}[1 - F(y_{(r)});\theta]^{n-r}}{B(k, r-k)B(r, n-r+1)} \, dF(y_{(k)};\theta)dF(y_{(r)};\theta), \tag{71}$$

where

$$F(y;\theta) = 1 - \exp[-(y-\mu)/\sigma]. \tag{72}$$

Making the transformation

$$X_r = (Y_{(r)} - \mu) - (Y_{(k)} - \mu), \quad Y_{(k)} - \mu = Y_{(k)} - \mu, \tag{73}$$

and integrating out $y_{(k)}-\mu$, we find the density of X_r as the beta density

$$f(x_r;\sigma) = \frac{[e^{-x_r/\sigma}]^{n-r+1}[1-e^{-x_r/\sigma})^{r-k-1}}{\sigma B(r-k, n-r+1)}, \quad x_r \in (0,\infty). \tag{74}$$

The density of S_k is

$$f(s_k;\sigma) = \frac{1}{\Gamma(k)\sigma^k}\, s_k^{k-1}\, e^{-s_k/\sigma}, \quad s_k \in (0,\infty), \tag{75}$$

and since X_r, S_k are independent, we have the joint density of X_k and S_k as

$$f(x_r, s_k;\sigma) = \frac{[e^{-x_r/\sigma}]^{n-r+1}[1-e^{-x_r/\sigma}]^{r-k-1} s_k^{k-1} e^{-s_k/\sigma}}{\Gamma(k)B(r-k, n-r+1)\sigma^{k+1}}. \tag{76}$$

Making the transformation $V_r=X_r/S$, $S=S_k$, we find the joint density of V_r and S as

$$f^\circ(v_r, s;\sigma) = \frac{[e^{-v_r s/\sigma}]^{n-r+1}[1-e^{-v_r s/\sigma})^{r-k-1}\, s^k\, e^{-s/\sigma}}{\Gamma(k)B(r-k, n-r+1)\sigma^{k+1}}$$

$$= \frac{e^{-(n-r+1)v_r s/\sigma} s^k e^{-s/\sigma}}{\Gamma(k)B(r-k, n-r+1)\sigma^{k+1}} \sum_{j=0}^{r-k-1} \binom{r-k-1}{j} (-1)^j e^{-jv_r s/\sigma}$$

$$= \frac{1}{\Gamma(k)B(r-k, n-r+1)\sigma^{k+1}} \sum_{j=0}^{r-k-1} \binom{r-k-1}{j} (-1)^j\, s^k\, e^{-s[1+(n-r+j+1)v_r]/\sigma}. \tag{77}$$

It is then straightforward to integrate out s, leaving the density of V_r as

$$f(v_r) = \frac{k}{B(r-k, n-r+1)} \sum_{j=0}^{r-k-1} \frac{\binom{r-k-1}{j}(-1)^j}{[1+v_r(n-r+j+1)]^{k+1}}, \quad v_r \in (0,\infty). \tag{78}$$

The cumulative distribution function of V_r is given by

$$F(v_r) = 1 - \frac{1}{B(r-k, n-r+1)} \sum_{j=0}^{r-k-1} \frac{\binom{r-k-1}{j}(-1)^j}{[n-r+j+1][1+v_r(n-r+j+1)]^k}. \tag{79}$$

In the important case where r=n (that is, we wish to predict the largest observation on the basis of the k smallest), expression (79) simplifies to

$$F(v_n) = \sum_{j=0}^{n-k} \frac{\binom{n-k}{j}(-1)^j}{[1+jv_n]^k}. \tag{80}$$

When the parameter μ is unknown, it can be shown in the same manner as above that

$$f(v_r) = \frac{k-1}{B(r-k, n-r+1)} \sum_{j=0}^{r-k-1} \frac{\binom{r-k-1}{j}(-1)^j}{[1+v_r(n-r+j+1)]^k}, \quad v_r \in (0,\infty), \tag{81}$$

and

$$F(v_r) = 1 - \frac{1}{B(r-k, n-r+1)} \sum_{j=0}^{r-k-1} \frac{\binom{r-k-1}{j}(-1)^j}{[n-r+j+1][1+v_r(n-r+j+1)]^{k-1}}, \tag{82}$$

where $V_r=(Y_{(r)}-Y_{(k)})/S_k$ with

$$S_k = \sum_{i=2}^{k}(Y_{(i)} - Y_{(1)}) + (n-k)(Y_{(k)} - Y_{(1)}), \quad 2 \le k < r \le n. \tag{83}$$

In the case where r=n, expression (82) simplifies to

$$F(v_n) = \sum_{j=0}^{n-k} \frac{\binom{n-k}{j}(-1)^j}{[1+jv_n]^{k-1}}. \tag{84}$$

Consider the following data, which represent failure times, in minutes, for a specific type of electrical insulation in an experiment in which the insulation was subjected to a continuously increasing voltage stress (Lawless, 1982): 21.8, 70.7, 24.4, 138.6, 151.9, 75.3, 12.3, 95.5, 98.1, 43.2, 28.6, 46.9. An exponential model is appropriate for these data, where the parameter μ=0 and the parameter σ is unknown.

Assume that only the first ordered failure times are available and that the last 4 failure times are censored–since the experimenter failed to observe these failure times. Hence: n=12, k=8,

$$S_k = \sum_{i=1}^{k} Y_{(i)} + (n-k)Y_{(k)} = 624.4. \tag{85}$$

The equal tails 0.95-prediction interval for $Y_{(r)}$ (where r=n=12) is

$$C^{\circ}_{Y_{(n)}}(X_{(k)}, S_k) = \left(X_{(k)} + v_{\alpha/2}S_k, X_{(k)} + v_{1-\alpha/2}S_k\right) = (111.14,\ 618.528), \tag{86}$$

where $v_{\alpha/2}=v_{0.025}=0.0574$, $v_{1-\alpha/2} = v_{0.975} = 0.87$. The length of this interval is Δ°_z = 618.528 – 111.14 = 507.388. The shortest-length 0.95 prediction interval for $Y_{(n)}$ is

$$C^{*}_{Y_{(n)}}(X_{(k)}, S_k) = \left(X_{(k)} + v_1^{*}S_k, X_{(k)} + v_2^{*}S_k\right) = (90.2856,\ 522.9948), \tag{87}$$

where $v_1^* = 0.024$, $v_2^* = 0.717$. The length of this interval is Δ^*_z = 522.9948 – 90.2856 = 432.7092.

The relative efficiency of $C^{\circ}_{Y_{(n)}}(X_{(k)}, S_k)$ relative to $C^{*}_{Y_{(n)}}(X_{(k)}, S_k)$, taking into account $E_{\boldsymbol{\theta}}\{\Delta_z\}$, where $\boldsymbol{\theta}=(\mu,\sigma)$, is given by

$$\text{rel.eff.}_{E_{\boldsymbol{\theta}}\{\Delta_z\}}\left(C^{\circ}_{Y_{(n)}}(X_{(k)}, S_k), C^{*}_{Y_{(n)}}(X_{(k)}, S_k)\right) = \frac{E_{\boldsymbol{\theta}}\{\Delta^*_z\}}{E_{\boldsymbol{\theta}}\{\Delta^{\circ}_z\}} = \frac{v_2^* - v_1^*}{v_{1-\alpha/2} - v_{\alpha/2}} = 0.8528. \tag{88}$$

Conclusions

The technique given in this paper for constructing the shortest-length prediction intervals on functions of future observations is easy to apply. This technique may be useful for solving the similar problems in the area of intellectual information systems.

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CUMULATIVE INDEX

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(Abstracts)

M. Gulyaev, V. Boicov. Productivity Estimation of *Unix* Operating System, *Computer Modelling and New Technologies,* vol. 8, No 2, 2004, pp. 7–13.

The research and analysis of the problems in productivity of Internet server on UNIX operating system are described.

Keywords: *WWW- World Wide Web, CPU - Central Processing Unit, HTTP – Hypertext Transfer Protocol, TCP – Transmission Control Protocol, UDP – User Datagram Protocol, IP – Internet Protocol*

D. Golenko-Ginzburg, Z. Laslo. Chance Constrained Oriented Dispatching Rules for Flexible Job-Shop Scheduling, *Computer Modelling and New Technologies,* vol. 8, No 2, 2004, pp. 14–18.

We search for a machines' delivery schedule in order to minimize the average scheduling expenses within the time period of processing all the jobs subject to their chance constraints. A newly developed decision-making model for controlling a flexible job-shop manufacturing cell with cost objectives by means of pair-wise comparison is presented. Decision-making is based on analyzing long-term forecasting results with modified average values of total cost expenses within the job-shop make span. The modified heuristic for choosing jobs for the machines has been implemented into a generalized simulation model including a broad variety of cost expenses.

Keywords: *general flexible job-shop; scheduling problem; cost-optimization model; random operation; coordinate descent search algorithm; machines' delivery schedule.*

S. Guseinov. The Total Error Estimate of One Regularization Iterative Method for the Solution of the First Kind Operator Equations with Inexact Both Operators and Right-Hand Sides, *Computer Modelling and New Technologies,* vol. 8, No 2, 2004, pp. 19–25.

Modelling many problems of mathematical physics, economy, statistics, actuary mathematics and etc., frequently we obtain operator equations of the first kind with inexact both operators and right-hand sides. As a rule, these equations concern to ill-posed problems. The paper deals with some iterative methods for solution of such problems.

In the present work we consider the concrete iterative method for the solution of such first kind operator equations in the abstract separable Hilbert Space. We prove strong convergence of concerned iterative process in the initial separable Hilbert Space and we estimate its total degree of convergence under enough weak and reasonable additional conditions.

Keywords: *the first kind operator equations, Hilbert Space*

A. Kaklauskas, E. K. Zavadskas, S. Raslanas. Decision Support Systems in Lithuania, *Computer Modelling and New Technologies,* vol. 8, No 2, 2004, pp. 26–31.

Decision support systems (construction, real estate, facilities management, etc.) created in Lithuania are described in this paper. The above decision support systems comprise of the following constituent parts: data (database and its management system), models (model base and its management system) and a user interface. Presentation of information in databases may be in conceptual (digital, textual, graphical, photographic, video) and quantitative forms. Quantitative information presented involves criteria systems and subsystems, units of measurement, values and initial weight fully defining the variants provided. The databases were developed providing a multiple criteria analysis of alternatives from economical, legislative, infrastructure, social, qualitative, technical, technological and other perspectives. This information is provided in a user-oriented way. Since the analysis of alternatives is usually performed by taking into account economic, quality, technical, legal, social and other factors, a model-base include models which enable a decision maker to carry out a comprehensive analysis of the variants available and make a proper choice. These systems and the related questions were analysed in the paper.

Keywords: *construction, real estate, Lithuania, decision support systems*

M. Kopeetsky, Avi Lin. A New Continual Computations Strategy for Heterogeneous Networks, *Computer Modelling and New Technologies,* vol. 8, No 2, 2004, pp. 32–39.

The paper is devoted to a new continual-computation strategy for heterogeneous networks. This strategy presents the near-optimal solution of the basic equation $F(x)=0$ by Newton or any other approximation method under constraints imposed on the computation time. The differentiated approach considering current transmission conditions, objective function $F(x)$ type, and limitations imposed on computation time and on required computation precision is provided. The objective function $F(x)$ may be treated in different ways depending on the decision type that has to be taken. The presented analysis shows that the strategy implementation in the statistical and probabilistic manner is more flexible and dynamic as compared to the deterministic approach.

Keywords: *Continual Computation Strategy, limited computation time, Newton method*

Yu. Paramonova, M. A. Kleinhof, Yu. M. Paramonov. Estimating of Parameters of Fatigue Curve of Composite Material, *Computer Modelling and New Technologies,* vol. 8, No 2, 2004, pp. 40–47.

Maximum likelihood estimation of parameters of mathematical model of fatigue damage accumulation on the base of laminate fatigue life data processing is considered. The model, which is founded on the use of Markov chain theory, allows seeing the connection between static strength distribution parameters and S-N fatigue curve. It was shown already that, although the model is too simple and does not provide numerical coincidence with experimental fatigue test data, nevertheless it can be used as nonlinear regression model of S-N fatigue curve. Simple method of approximate estimation of model parameters is offered. Numerical example is given. Some parameters of this fatigue curve model can be considered as local static strength distribution function parameters. By the use of this model we can predict fatigue curve changes as consequence of static strength parameter changes.

Keywords: *strength, fatigue life, composite*

N. A. Nechval, K. N. Nechval, E. K. Vasermanis. Prediction Intervals for Future Outcomes with a Minimum Length Property, *Computer Modelling and New Technologies,* vol. 8, No 2, 2004, pp. 48–61.

In this paper, a technique is proposed for constructing shortest-length prediction intervals for future outcomes. The prediction intervals depend upon a previously available complete or type II censored sample from the same distribution belonging to invariant family. Both new-sample prediction (e.g., using data from a previous sample to make predictions, say, on the future failure time of a new unit) and within-sample prediction (e.g., predicting the number of future failures from a sample, based on early data from that sample) problems are considered. Prediction intervals are required as specifications on future life for components, as warranty limits for the future performance of a specified number of systems with standby units, and in various other applications.

The purpose of this paper is to give a simple technique for deriving prediction intervals with a minimum length property and providing exactly the nominal coverage probability.

Keywords: *future outcomes, prediction intervals, shortest length, technique for constructing*

COMPUTER MODELLING and NEW TECHNOLOGIES, 8.sējums, Nr.2, 2004 (Anotācijas)

M. Gulyaev, V. Boicov. *UNIX* operētājsistēmas produktivitātes novērtējums, *Computer Modelling and New Technologies,* 8.sēj., Nr.2, 2004, 7.–13. lpp.

Šajā rakstā. tiek apskatīta Interneta servera produktivitātes problēmas UNIX operatora sistēmā izpēte un analīze.

Atslēgvārdi: *WWW – World Wide Web, CPU – Central Processing Unit, HTTP – Hypertext Transfer Protocol, TCP – Transmission Control Protocol, UDP – User Datagram Protocol, IP – Internet Protocol*

D. Golenko-Ginzburg, Z. Laslo. Iespēju ierobežojoši orientējoši izsūtāmie noteikumi elastīgajai "job-shop" plānošanai, *Computer Modelling and New Technologies,* 8.sēj., Nr.2, 2004, 14.–18. lpp.

Rakstā tiek izskatītas iespējas darbgaldu piegādes termiņiem, lai samazinātu vidējās plānojamās izmaksas laika periodā, kad visi darbi, kas pakļauti iespēju ierobežojumiem, tiek veikti. Tiek izstrādāts jauns lēmumu pieņemšanas modelis, lai kontrolētu elastīgo apstrādājamo „job-shop" elementu ar pāru veida salīdzinājuma līdzekļiem cenu noteikšanā.

Atslēgvārdi: *vispārīgi elastīgs „job-shop", plānojamā problēma, cenu optimizācijas modelis, nejauša darbība, darbgaldu piegādes plāns*

S. Guseinov. Vienas regulējuma iteratīvās metodes kopējās kļūdas noteikšana pirmās kārtas vienādojuma risināšanai ar nepilnīgu operatora un labējo pusi, *Computer Modelling and New Technologies,* 8.sēj., Nr.2, 2004, 19.–25. lpp.

Modelējot matemātiskās fizikas, ekonomikas, statistikas, aktuārās matemātikas u.c. problēmas, mēs bieži vien iegūstam pirmās kārtas operatora vienādojumus ar neprecizitātēm, kā operatora, tā arī labajā pusē. Kā likums, šie vienādojumi pieder pie sāpīgākām problēmām. Pastāv dažas iteratīvas metodes šādu vienādojumu risināšanai.

Šajā darbā tiek izskatīta iteratīvā metode pirmās kārtas operatora vienādojumu risināšanā abstraktā sadalāmā Hilberta telpā.

Atslēgvārdi: *pirmās kārtas operatora vienādojumi, Hilberta telpa*

A. Kaklauskas, E. K. Zavadskas, S. Raslanas. Lēmumu atbalsta sistēmas Lietuvā, *Computer Modelling and New Technologies,* 8.sēj., Nr.2, 2004, 26.–31. lpp.

Lēmumu atbalsta sistēmas (celtniecība, nekustamais īpašums, vadības spējas u.c.), kas izstrādātas Lietuvā, tiek izskatītas šajā rakstā.. Iepriekšminētās lēmumu atbalsta sistēmas ietver šādas sastāvdaļas: datus (datu bāze un tās vadības sistēma), modeļi (modeļu bāze un tās vadības sistēma) un lietotāja interfeiss. Informācijas iekļaušana datu bāzēs var tikt veikta divējādi: konceptuāli (digitāli, tekstuāli, grafiski, foto veidā, video veidā) un kvantitatīvi. Visi iepriekšminētie jautājumi tiek iztirzāti rakstā.

Atslēgvārdi: *celtniecība, nekustamais īpašums, Lietuva, atbalsta sistēmas*

M. Kopeetsky, Avi Lin. Jauna nepārtrauktās skaitļošanas stratēģija datortīkliem, *Computer Modelling and New Technologies,* 8.sēj., Nr.2, 2004, 32.–39. lpp.

Raksts tiek veltīts jaunajai nepārtrauktās skaitļošanas stratēģijai, kas paredzēta heterogēnajiem tīkliem. Veiktā analīze parāda, ka stratēģijas ieviešana statistiskā un varbūtības veidā ir daudz elastīgāka un dinamiskāka nekā determinētā pieeja.

Atslēgvārdi: *nepārtrauktās skaitļošanas stratēģija, ierobežotais skaitļošanas laiks, Ņūtona metode*

A. Yu. Paramonova, M. A. Kleinhof, Yu. M. Paramonov. Kompozītmateriālu noguruma līknes parametru novērtējums, *Computer Modelling and New Technologies,* 8.sēj., Nr.2, 2004, 40.–47. lpp.

Rakstā tiek izskatīts noguruma bojājumu uzkrāšanās matemātisko modeļu parametru maksimālais varbūtējais novērtējums, pamatojoties uz lamināta noguruma dzīves datu ievadīšanu. Modelis, kurš ir izveidots, lietojot Markova ķēdes teoriju, atļauj saskatīt saikni starp sadalījuma parametru statisko spēku un S-N noguruma līkni. Tiek piedāvāti skaitliskie piemēri.

Atslēgvārdi: *spēks, noguruma dzīve, kompozīts*

N. A. Nechval, K. N. Nechval, E. K. Vasermanis. Prognožu intervāli paredzējumiem ar minimālā garuma īpašību, *Computer Modelling and New Technologies,* 8.sēj., Nr.2, 2004, 48.–61. lpp.

Autori šajā rakstā piedāvā tehnoloģiju, lai veidotu īsākā garuma prognožu intervālus paredzējumiem. Prognožu intervāli ir atkarīgi no iepriekš pieejamā pilnīga vai II tipa cenzēta parauga, kurš ir no tā paša sadalījuma, kas pieder invariantu saimei. Abas problēmas tiek izskatītas – gan jaunā parauga prognoze, gan prognoze parauga robežās.

Šī raksta mērķis ir sniegt vienkārši tehnoloģiju, lai iegūtu prognožu intervālus ar minimālā garuma īpašību un nodrošinātu tieši nominālās pārklāšanas varbūtību.

Atslēgvārdi: *paredzējumi, prognožu intervāli, īsākais garums, tehnoloģija celtniecībai*

COMPUTER MODELLING & NEW TECHNOLOGIES

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PREPARATION OF CAMERA-READY TYPESCRIPT: COMPUTER MODELLING AND NEW TECHNOLOGIES

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Abstract reviews the main results and peculiarities of a contribution. Abstract is presented always in English or in English and the second (presentation) language both.
Keywords: main terms, concepts

1. Introduction

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If you produce your camera-ready copy using a laser printer, use a 15 x 23 cm typing area (in A4 format: 30 mm - left, 30 mm- right, 30 mm- top, 30 - bottom, line spacing - single), as in these instructions, in combination with the **10 points Times** font. The pages will then be reproduced one to one in printing.
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The text should be in clear, concise English (or other declared language). Please be consistent in punctuation, abbreviations, spelling (***British English***), headings and the style of referencing.

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In general, if you prepare your typescript on a computer using a word processing package, use styles for the font(s), margin settings, headings, etc., rather than inserting these layout codes every time they are needed. This way, you will obtain maximum consistency in layout. Changes in the layout can be made by changing relevant style(s).

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A sample for the opening page of a contribution is shown in Figure 1 on page 3.
Requirements for the opening page of a contribution are (see also Figure 1) : the titles should always be a centred page and should consist of: the title in capital letters, bold font, flush centre, on the fourth text line; followed by the subtitle (if present) in italics, flush centre, with one line of white above. The author's name(s) in capitals and the affiliation in italics should be centred and should have two lines of white space above and three below, followed by the opening text, the first heading or the abstract.

5. Headings

Please distinguish the following four levels of headings:

1. First-order Heading

This heading is in bold, upper and lowercase letters, numbered in Arabic figures, and has two lines of space above and one line below. The text begins full out at the left margin.

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This heading is in roman capitals, numbered in Arabic figures and has one line of space above and below. The text begins full out at the left margin.

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Fourth-order Heading in Italics. This heading is in italics, upper and lowercase letters, with one line of space above the heading. The heading has a full stop at the end and the text runs on the same line.

3 blank lines

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Abstract

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Figure l. Example of an opening part of contribution to a Volume of CM&NT

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- *Line drawings* must be submitted in original form, on good quality tracing paper, or as a glossy photographic print.
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Mount all illustrations directly into the text at the appropriate places. Alternatively, it is acceptable to leave the appropriate space blank in the text, and submit the illustrations separately. In this case you must put the figure numbers in pencil in the open spaces in the text and on the back of the figures. Also indicate the top of the illustration.

For computer plotting the ORIGIN Software is preferable.

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Displayed equations should be in the left side of the page, with the equation number in parentheses, flush right.

$$E_{int} = \iint \psi^{+}(\mathbf{x})\psi(\mathbf{x})K(\mathbf{x}-\mathbf{x}')(-div\mathbf{P}(\mathbf{x}'))d^{3}xd^{3}x' , \qquad (1)$$

$$K(\mathbf{x}-\mathbf{x}')=C_0\,\frac{exp(-\lambda(|\mathbf{x}-\mathbf{x}'|)}{|\mathbf{x}-\mathbf{x}'|}\ . \qquad (2)$$

Font sizes for equations are as follows: 12pt -full, 7pt - subscripts/superscripts, 5pt - sub-subscripts/superscripts, 18pt - symbols, 12pt - sub symbols.

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Please centre tables on the page, unless it is necessary to use the full page width. Exceptionally large tables may be placed landscape (90^0 rotated) on the page, with the top of the table at the left-hand margin. An example of a table is given below:

TABLE 1. National programs of fusion research [1]

Experiment	Type	Laboratory	Task	Begin of operation
JET	tokamak	Joint European Torus, Culham, UK	Plasma physics studies in the region close to ignition	1983
TEXTOR	tokamak	FA, Jülich. Germany	Studies of plasma-wall interaction	1982
TORE SUPRA	tokamak	CEA, Cadarache, France	Testing of super-conducting coils, stationary operation	1988
ASDEX Upgrade	tokamak	IPP, Garching, Germany	Plasma boundary studies in diverter plasmas	1990
WENDELSTEIN 7-AS	stellarator	IPP, Garching, Germany	Testing the principles of ”advanced stellarator”	1988
WENDELSTEIN 7-X	stellarator	IPP, Greifswald, Germany	Testing feasibility of ”advanced stellarator” for power station	2004

9. References

The References should be typeset in a separate section as a numbered list at the end of your contribution in the following style:

Journal articles should consist of: author's name, initials, year, title of article, journal title, volume number, inclusive page numbers, e.g.:

[1] Dumbrajs O. (1998) Nuclear Fusion. *RAU Scientific Reports &Computer Modeling & New Technologies* **2**, aa-zz
[2] Kiv A.E. , Polozovskaya I.A., Tavalika L.D. and Holmes S. (1998) Some problems of operator-machine interaction.. *RAU Scientific Reports &Computer Modelling & New Technologies* **2**, aa-zz
[3] Shunin Yu.N. (1996) Elementary excitations and radiation defects in solids induced by swift heavy ions. *RAU Scientific Reports & Solid State Electronics & Technologies* **1**, 15-35
[4] Schwartz K. (1996) Excitons and radiation damage in alkali halides. *RAU Scientific Reports & Solid State & Electronics & Technologies* **1**, 3-14

Book references should consist of: author's name, initials, year, title of book, publisher, place of publication, e.g.:

[5] Schwartz K. (1993) The Physics of Optical Recording. Springer-Verlag, Berlin Heidelberg New York
[6] Shunin Yu.N. and Schwartz K.K. (1997) Correlation between electronic structure and atomic configurations in disordered solids. In: R.C. Tennyson and A.E. Kiv (eds.). Computer Modelling of Electronic and Atomic Processes in Solids. Kluwer Academic Publishers, Dordrecht, pp. 241-257.

Unpublished papers should consist of: author's name, initials, year (or: in press), title of paper, report, thesis, etc., other relevant details, e.g.:

[7] Shunin Yu.N. (1995) Elementary excitations in amorphous solids accompanying the swift heavy ions passages. Private communication. GSI Seminar. Darmstadt

The references above should be cross-referenced by numbers within square brackets: ...as cited in [1], or Kiv *et al.* [2]... The use of author initials for cross-references is not encouraged.

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Acknowledgements

Acknowledgements (if present) mention some specialists, grants and foundations connected with the presented paper. The first page of the contribution should start on page 1 (right-hand, upper, without computer page numbering). Please paginate the contributions, in the order in which they are to be published. Use simple pencil only.

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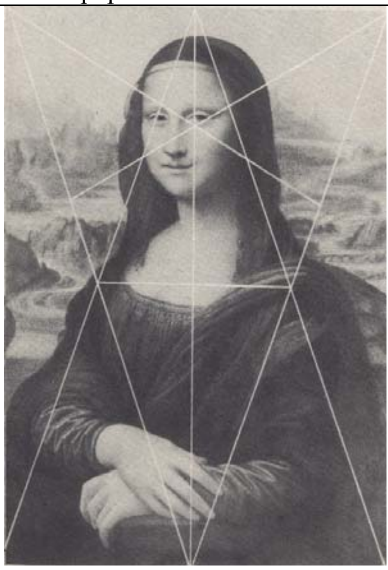

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