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Endless Time

by Rabindranath Tagore

Time is endless in thy hands, my lord.
There is none to count thy minutes.

Days and nights pass and ages bloom and fade like flowers.
Thou knowest how to wait.

Thy centuries follow each other perfecting a small wild flower.

We have no time to lose,
and having no time we must scramble for a chance.
We are too poor to be late.

And thus it is that time goes by
while I give it to every querulous man who claims it,
and thine altar is empty of all offerings to the last.

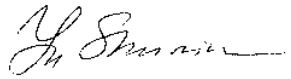
At the end of the day I hasten in fear lest thy gate be shut;
but I find that yet there is time.

****

Rabindranath Tagore (1861–1941) ♣

This 17th volume no. 1 presents actual papers on two main topics of Journal specialization, namely, Operation Research, Computer Simulation and Mathematical Modelling. Contributors of this issue represent scientific institutions of Belarus, Poland, Australia, USA, India, Latvia, and Israel. Our journal policy is directed on the fundamental and applied sciences researches, which are the basement of a full-scale modelling in practice. This edition is the continuation of our publishing activities. We hope our journal will be interesting for research community, and we are open for collaboration both in research and publishing. We hope that Journal's contributors will consider the collaboration with the Editorial Board as useful and constructive.

EDITORS



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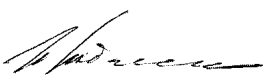

I.V. Kabashkin

♣ **Rabindranath Tagore** (1861–1941) – Greatest writer in modern Indian literature, Bengali poet, novelist, educator, who won the Nobel Prize for Literature in 1913. Tagore was awarded the knighthood in 1915, but he surrendered it in 1919 as a protest against the Massacre of Amritsar, where British troops killed some 400 Indian demonstrators protesting colonial laws. Tagore's reputation in the West as a mystic has perhaps misled his Western readers to ignore his role as a reformer and critic of colonialism.

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OPTIMIZING THE MANAGEMENT OF TRAFFIC LIGHT OBJECT BASED ON NATURAL ALGORITHMS

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The article proposes an adaptive control method for traffic lights, which operates at the strategic level of management. The algorithm uses data on changes in the intensity during the day, which provides forecasting module (for the experiments used neural network prediction). An adaptive algorithm is based on finding the minimum of delay at the crossroad, based on genetic algorithm and the method of 'swarm of bees'.

Keywords*:* traffic light, adaptive control, genetic algorithm, artificial bee colony algorithm

1. Introduction

Crossroads are the most important nodes of the road network of the city. The largest losses are observed by the use of the roadway. The main parameter that characterizes the management of traffic lights is the average delay of transport at crossroads. The minimizing of this parameter leads to an improvement quality of service at the crossroads.

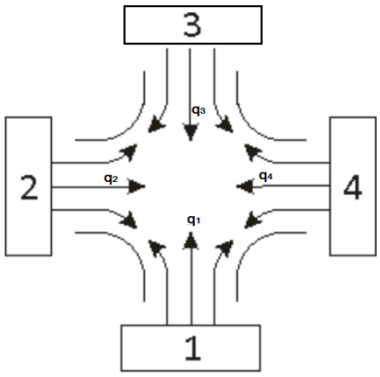

Figure 1. The scheme of traffic light object

Scheme of a typical traffic light object is shown on Figure 1.

The optimization problem consists in the fact that to choose a number and duration of cycles in a controlled cycle in which the delay for traffic would be minimal. To calculate the delay formula can be used [1]:

$$e_{ti}^{'} = 0.45 \cdot \left(\frac{C \cdot (1-\lambda^2)}{1-\lambda x} + \frac{x^2}{q_i \cdot (1-x)} \right), \text{ s/vehicle,} \tag{1}$$

where C – duration of the traffic light cycle, s;
λ – proportion of the green signal for the direction in the cycle;
q_i – traffic volume in this direction, vehicle/s;
x – coefficient of loading.

Then the target function for the optimization task is as follows:

$$f = \sum_{i=1}^{m} w_i e_{ti}^{'} \rightarrow \min, \tag{2}$$

where w_i – weighting factor of importance (priority) direction of optimization.

The coefficients w_i are determined at the design stage. It is also possible that at which these values will change during the day. If all values are equal to one, then the intersection is considered to be equivalent for each of the directions of optimization.

Thus, a complex target function f depends on many parameters t_{zi}. It is required to find such values t_{zi}, which satisfy (2).

To solve the problem of optimization methods were chosen:

1) the genetic algorithm,
2) the method of 'swarm of bees'.

2. Application of Genetic Algorithm for Adaptive Management at the Crossroads

Genetic Algorithms – adaptive search methods, which in recent times are often used for solving a functional optimization. They are based on the genetic processes of biological organisms: biological populations evolve over several generations, subject to the laws of natural selection and the principle of "survival of the fittest", an open Charles Darwin. Genetic algorithm is a simple model of evolution in nature, implemented by the algorithm. It is used as an analogue of the mechanism of genetic inheritance, as well as an analogue of natural selection. This preserves the biological terminology in a simplified form.

To simulate the evolutionary process, initially generated by a random population i.e. some individuals with a random set of chromosomes (numeric vectors). A genetic algorithm simulates the evolution of this population as a cyclic process of crossing individuals and change of generations.

The life cycle of the population – a few random crossings and mutation, which resulted in a population, is added to a number of new individuals. The selection of a genetic algorithm – it is the process of forming a new population from old one, after which the old population is removed. Following the selection of a new population again used the operation of crossover and mutation, then again there is a selection, and so on (Figure 2).

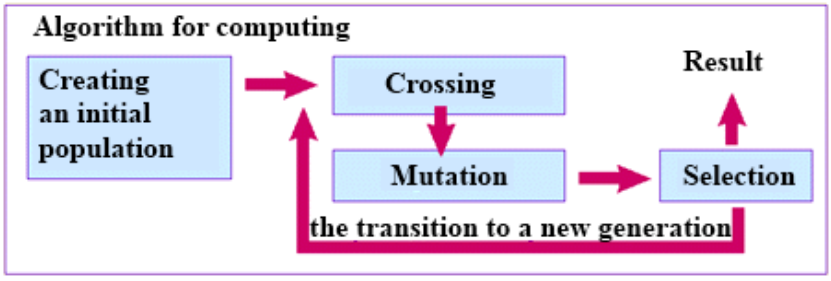

Figure 2. Steps of genetic algorithm

Step 1 – Create the initial population.

Each member of the population consists of a set of chromosomes t_{zi}, $i = \overline{1,m}$.

Figure 3. A set of chromosomes of an individual with a fitness function

I member of the population of chromosomes contains a value t_{zi}, which is equal to the duration of the *i*-th phase of the traffic light cycle, if the value is zero, then this phase is not in the cycle of regulation. The value of the genome for each chromosome is in the interval (t_{min},t_{max}). Number of phase's m and variations of flow are defined at the design stage of the system. At this stage, as well as during crossing check the admissibility condition of existence of this individual. Testing is to analyse all phases and check on the condition that the entire cycle of time allocated for each direction of motion. If this condition is not satisfied, the fitness function is calculated with an error, so not to waste time on the calculation of this set of chromosomes varies.

Step 2 – Crossing.

The selection of a genetic algorithm is closely related to the principles of natural selection in nature as follows:

- The fitness of the individual – the value target function (fitness function) for this individual.
- Survival the most adapted – the population of the next generation is formed in accordance with the target function. The more adapted an individual, the greater chance of his participation in the crossover.

Model selection defines how to build the next generation population. As a rule, the probability of an individual's participation in crossing is taken proportional to its fitness. Thus, each successive generation will be on average better than the last. The probability of involvement of *i*-th member of the population as a crossing is determined by:

$$P_i = \frac{1}{m-1}\left(1 - \frac{f_i}{\sum_{j=1}^{m} f_j}\right). \tag{3}$$

After determination of individuals i.e. members crossing, the operation itself is executed crossover.

Single-point crossover works as follows. First, randomly select a point of discontinuity (break point – the area between adjacent values in a row.). Both the parent structure is broken into two segments at this point. Then, corresponding to different segments of the parents stick together and produce two offspring genotypes; can be applied to multi-point crossover or uniform crossover. In uniform crossover, each gene is inherited by the first parent first child with a given probability; otherwise it is passed to the second child.

Step 3 – Mutation.

After the end stage of the crossover, mutation operators are performed. For each of the individual is subjected to mutation of each gene with probability *Pm*. The population obtained after mutation overwrites the old one. The mutation changes the value for the individual genome t_{zi} to some value in the range [–5,5].

Step 4 – Selection.

At this stage there is the sorting of all genotypes of the objective function; and the inclusion of the next generation of individuals with the best target function values (Eq. (2)). Just switch on parental genotypes are there with the best values of f, in accordance with the principles of "elitism". The use of "elitism" cannot lose a good interim solution.

Then the steps of the algorithm are performed again starting from the second. So occurs the limited number of epochs (an acceptable calculation time), resulting in the best option solving the optimisation problem is chosen. Genomes of the options will be used to specify the structure and duration of phases for the traffic light cycle.

Genetic algorithm is a combined method of iterate and gradient descent. The mechanisms of crossover and mutation, in a sense part of the implement then iterative method, and the selection of the best solutions – gradient descent.

3. The Method of "Swarm of Bees" for the Solution of the Optimisation of Management at the Crossroads

One of the newest varieties of genetic algorithms is the search algorithm for 'swarm of bees'. The algorithm for finding the global extrema of functions of complex multidimensional emerged relatively recently. In [2] first described the basis of the method Particle Swarm Optimization.

Every bee in the swarm is considered as a particle or agent. All swarm particles act individually in accordance with one control principle: to move towards the best personal and global best position, constantly checking the value of the current position.

The position coordinates of the bees in the study is *m*-dimensional space.

The personal best position (BPP) is a position with the best value of the target function, discovered by a bee. Each bee has its own BPP. At each point along the path of the bee compares the value of

the target function at the current position with a value of BPP. If the current position is set to the suitability of the above, the importance of BPP is replaced by the value of the current position. The global best position (BGP) is defined as the position with the best value of the target function, found all the swarm. Information about the value of BGP is available for each individual bee. If in the process of moving from a bee finds a position with the best target function replaced by the current position of the BGP of the bees.

Description of the algorithm for finding the optimal solution using the 'swarm of bees' method is given below.

Step 1.

Similarly, the genetic algorithm creates a population of bees, each of which contains *m* coordinates and the current value of the optimality of *f* (which is determined by the formula (2)). Also, there is given a random initial speed of movement. Each coordinate corresponds to the duration of some phases in the traffic light cycle t_{zi}.

Step 2.

Each bee in the swarm moves in a new direction in accordance with its position and speed. Checks exit bee's solutions of bounds, and limit the actions required are performed.

Step 3.

For each bee we calculate the value of the objective function in its new position. Comparing this value with the value of the BPP bees, and if necessary, replace the BPP's current position. Comparing this value with the value of BGP swarm, and if necessary, replace the current position of the BGP.

Step 4.

For each bee calculate new speed of movement in accordance with the equation:

$$v_i^{j+1} = w \cdot v_i^j + c_1\psi_1(p_i - t_{zi}) + c_2\psi_2(g_i - t_{zi}), \tag{4}$$

where v_i^j – speed of the bees in the measurement of *i* to *j* iteration;

w – inertia weight, the number (located in the interval [0,1]) reflects the extent to which the particle retains its original speed;

p_i, g_i – value of the *i* position, respectively, for the BPP and the bees swarm of BGP;

ψ_1, ψ_2 – random value in the range [–1, 1];

c_1, c_2 – constant weighting factors determining the attraction of its own BPP and BGP of swarm.

The parameter c_1 determines what effect on the particle has its memories of BPP, and c_2 determines what effect on the particle has the rest of the swarm. These factors are sometimes considered as a cognitive and social factor.

Step 5.

Checking the termination condition of the algorithm (5) if the search is completed, executed the transition to Step 3.

As an estimate of the current state of the search process is proposed to use the average value for the swarm of Euclidean distance ε from each bee to the centre of gravity of the cluster:

$$\varepsilon = \frac{1}{k}\sum_{j=1}^{k}\sqrt{\sum_{i=1}^{m}\left(t_{zi}^j - \widetilde{t}_{zi}\right)^2}, \tag{5}$$

where *k* – population size;

$\widetilde{t}_{zi}$ – centre of gravity of the swarm in the coordinate *i*:

$$\widetilde{t}_{zi} = \frac{1}{k}\sum_{j=1}^{k} t_{zi}^j. \tag{6}$$

The search result is a swarm of BGP. The value of the response function at this point in relation to the known value of global optimisation determines the accuracy of search results. An important advantage of this method for finding the optimal solution is its robustness, i.e. it keeps performance at rather complex response surfaces, as well as the presence of the stochastic component in the measured value of the response function.

4. Results of Testing Algorithms

For the beginning let's generate the intensity of the 8 tests of 15 minutes, equivalent to 2 hours. Table 1 shows the intensity of the input streams at a crossroads.

Table 1. Initial intensity for adaptive algorithms

№	**The intensity of the *i* phase, 15 min**			
	1	2	3	4
1	157	86	162	124
2	142	153	165	112
3	145	157	167	114
4	157	135	159	126
5	156	138	166	130
6	144	116	174	120
7	140	116	161	114
8	166	122	145	117

At received intensities using adaptive algorithms obtain the duration of the phase traffic light cycle. These data are summarized in Table 2 for the genetic algorithm in Table 3 for the algorithm 'swarm of bees'.

Table 2. Duration of the phases of the genetic algorithm

№	**Duration of the *i* phase of the traffic light cycle**					
	1	2	3	4	1+3	2+4
1	32	0	23	0	0	14
2	21	29	0	0	14	14
3	18	0	32	0	0	28
4	27	0	0	0	14	14
5	19	0	0	21	14	24
6	39	27	0	0	26	14
7	16	0	35	0	22	14
8	38	32	0	0	23	14

Table 3. Duration of the phases of the algorithm of bee swarm

№	**Duration of the *i* phase of the traffic light cycle**					
	1	2	3	4	1+3	2+4
1	18	0	0	0	14	14
2	27	0	32	0	0	14
3	0	23	30	0	14	14
4	0	0	0	0	39	19
5	0	0	0	0	31	24
6	28	0	38	14	19	14
7	0	0	31	0	14	17
8	0	0	0	0	17	14

The inputs to the algorithm are the intensity, where for five phase equals the amount of the intensities of the first and third, and 6 phase – the second and fourth.

Figure 4 shows the dynamics of the queue number 2 for the rigid control and adaptive, based on genetic algorithm.

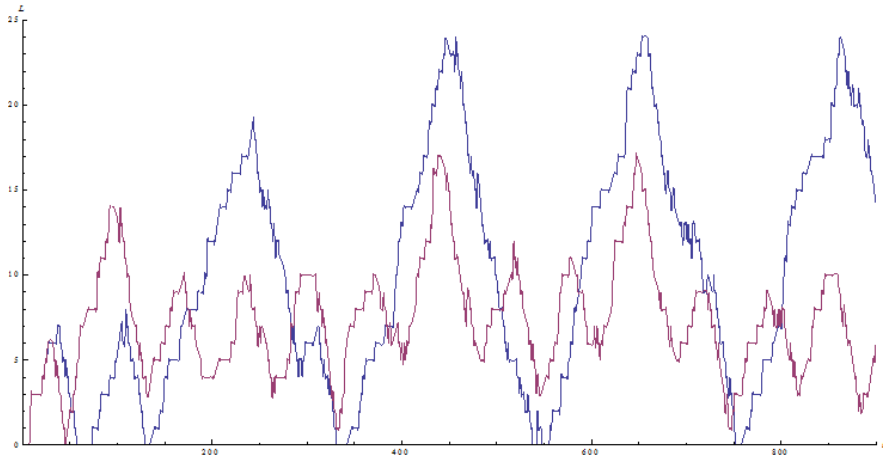

Figure 4. The dynamics of the queue for the genetic algorithm

Figure 5 shows the dynamics of the queue number 2 for the rigid control and an adaptive algorithm based on 'swarm of bees'.

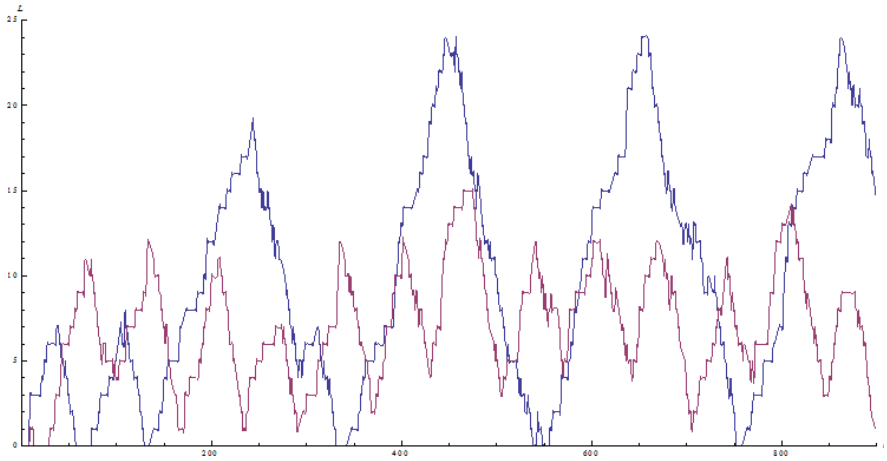

Figure 5. The dynamics of the queue for the algorithm bee swarm

Comparison of changes in the length of the queue of two adaptive algorithms is shown on Figure 6.

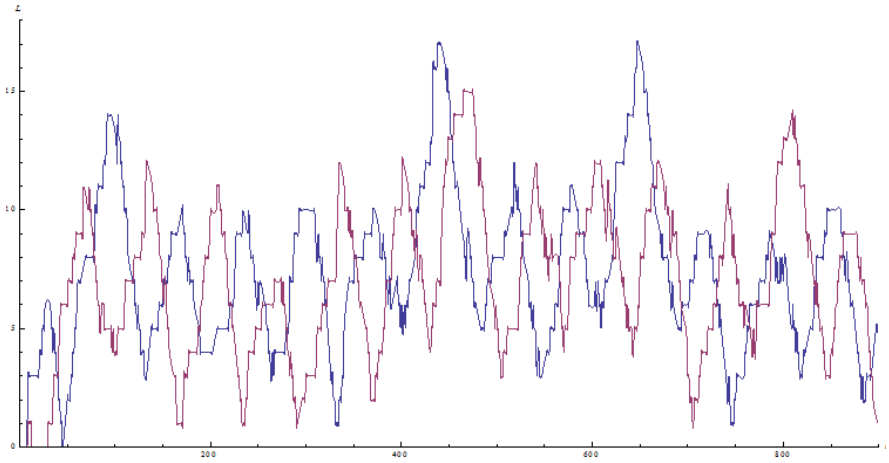

Figure 6. The dynamics of the queue adaptive algorithms

Statistics on the queue for the first 15 minutes is shown in Table 4.

Table 4. Statistics queues for the first 15 minutes

№	Number of queue	The maximum value of queue	The average value of queue	Average time in queue
Rigid algorithm				
1	Queue1	22	7,726	45,298
	Queue 2	10	1,598	34,097
	Queue 3	23	8,088	44,732
	Queue 4	19	5,8	40,85
The genetic algorithm				
2	Queue1	15	4,14	24,014
	Queue2	9	1,482	30,507
	Queue3	16	5,35	30,205
	Queue4	16	5,285	39,44
The algorithm of 'swarm of bees'				
3	Queue1	9	1,108	6,45
	Queue 2	6	0,902	18,259
	Queue 3	12	4,711	25,363
	Queue 4	11	3,276	23,411

5. Conclusions

The article considers the option to optimise the management of traffic lights at the level of strategic management.

The proposed methods: genetic algorithm and the method of 'swarm of bees' are methods for solving the optimisation problem in traffic management, its effectiveness in comparison with a rigid management mode is shown. Since the information for calculating the parameters of traffic lights cycle continuously enters to the adaptive control module, it allows responding in a quicker way to the current transport situation at the crossroads. Method of 'swarm of bees' shows better performance in comparison with the genetic algorithm.

These methods belong to a class of strategic management and are a synthesis of control algorithms for a sequence of phases and the calculated algorithm for determining the duration of the cycle and phases.

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COMPUTATIONAL METHODS FOR ADAPTATION OF MARKOV MODELS TO REQUESTED MAINTENANCE POLICIES

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State-transition models are often used in reliability analysis and one specific approach of this kind is the subject of this paper. By incorporating elements of both the deterioration process and the maintenance activities (inspections and repairs) in a semi-Markov model, a common computational platform has been created which serves as a foundation for various dependability studies that can investigate different maintenance scenarios. Having available some basic model it is possible to adjust its parameters so that it represents some hypothetical new maintenance policy and then to examine an impact which changing to the new policy has on various reliability characteristics of the system. Particularly, this paper discusses an extension of the generic adjustment algorithm to specific situations of the so-called model saturation when, as a result of tweaking the model towards higher repair frequencies, sum of repairs probabilities in the states reach the maximum value and there is no room for further increase. The general idea is to modify the model in such cases by forcing some non-zero value of a repair probability in those states where it is zero initially but in a manner that will not destroy the overall model behaviour.

After theoretical presentation of the modified method its effectiveness is illustrated on practical examples. It is shown that the proposed extension allows to successfully evaluating a class of cases that has not been properly handled by the generic method and thus broadens the range of dependability studies that can be effectively evaluated.

Keywords*:* state-transition deterioration model, semi-Markov process, model adaptation, maintenance analysis, model adjustment

1. Introduction

Cost-effective maintenance is the crucial point in management of any complex contemporary technical system. Selecting the optimal maintenance strategy is not an easy task and often requires extensive analysis of reliability, security, safety and economic aspects. Finding a reasonable balance between extensive and frequent maintenance actions on one side and redundant and excessive maintenance expenses on the other is the key point in reliable yet cost-effective system operation.

The subject of this paper is connected with original methodology that assists a person who decides about maintenance activities by evaluating risks and costs associated with choosing different maintenance strategies. Instead of searching for a globally optimal solution to a problem: "what maintenance strategy would lead to the best dependability parameters of system operation incurring the minimal cost", in this approach different maintenance scenarios can be examined in "what-if" studies and their reliability and economic effects can be compared so that a person managing the maintenance is assisted in making informed decisions ([1–3]).

Our method of deterioration representation based on Markov models has been presented originally in [1] and its specific extensions are further described in [4–8]. Additionally, presentations in [9] and [10] concentrated on one important aspect of the methodology: fully automatic adjustment of the model to possible modifications of the maintenance policy that are often required in studies requested by the user. In this work we extend the research in this context with additional studies related to the problem of so-called *model saturation* that may occur when increased repair frequencies are requested and, furthermore, we propose an automatic modification mechanism that remedies the saturation limits.

The main content of the paper is divided in three parts. The first one (section 2) briefly summarizes the method of automatic model adjustment to the requested repair frequencies, which is the core task in maintenance studies, the second part (section 3) discusses specific issues of the adjustment when probability saturation occurs and proposes an extension that circumvents these kinds of problems, while the third part (section 4) is devoted to practical verification of the extension and includes real-world examples of reliability analyses which are based on this approach.

2. Modelling the Ageing Process

In this chapter we will briefly resume basic information about Markov models and the adjustment procedure that has been included in [9]. Further, in Chapter 3, this information is used for discussion of model saturation phenomenon and its proposed remedies.

2.1. Construction of the Model

Deterioration is a complex process and its modelling is not an easy task. In the literature there are numerous approaches proposed that, in general, try to encompass the three major factors that affect equipment wear: physical characteristics of the object under consideration, operating practices, and the maintenance policy. In the proposed solution especially the maintenance activities relate to the events and actions that should be properly incorporated in the representation and should be described with a distinctive set of parameters that would later be used in maintenance studies.

One of the approaches that can properly incorporate all the above suppositions about the aging process and maintenance activities is based on state-space (Markov) model ([12–17]). A Markov model consists of the states the equipment can assume in the process, and the possible transitions between them. The method described in this paper is based on a model of the Asset Maintenance Planner (AMP) that has been initially developed and implemented by George J. Anders and Henryk Maciejewski ([18–19]).

For structure of a typical AMP model see Figure 1. In the model, the deterioration progress is represented by a chain of deterioration states $D1 \ldots DK$, which then leads to the failure state F. In most situations, it is sufficient to represent deterioration by three stages: an initial ($D1$), a minor ($D2$), and a major ($D3$) stage of deterioration ($K = 3$). This last is followed, in due time, by equipment failure (F). Other states are related to the maintenance activities: regular inspections (Is states) result in decisions to continue with, e.g., minor ($Ms1$) or major ($Ms2$) repair or to return to the deterioration state Ds without any repair. The expected result of all repair actions is a single-step improvement in the deterioration chain.

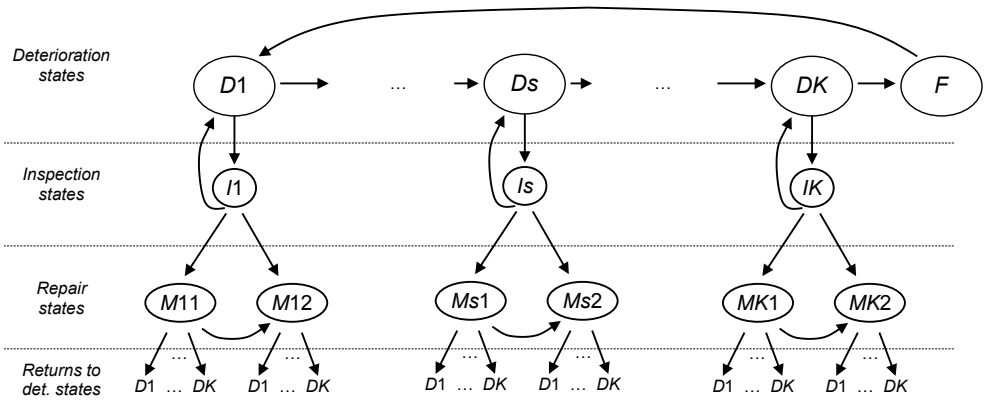

Figure 1. The state-transition model representing the deterioration chain with inspection and repair states (an example with two types of repairs is shown)

2.2. Adjusting the Model to Modified Repair Frequencies

Preparing the Markov model for some specific equipment is not an easy task and requires expert intervention. The goal is to create the model representing closely the real-life deterioration process known from the records that usually describe equipment operation under a regular maintenance policy with some specific frequencies of inspections and repairs. The model itself permits calculation of the repair frequencies and compliance of the computed and recorded frequencies is a very desirable feature that verifies trustworthiness of the model. In this sub-section, we will summarize the method of model adjustment proposed in [9] and [10] that aims at reaching such compliance. It can be used also for a different task of a fully automatic generation of a model for some hypothetical new maintenance policy with modified frequencies of repairs. Such a task is typical during evaluation of various maintenance scenarios.

In our analysis let the deterioration model under consideration consists of K deterioration states and R repairs. Also, let P^{sr} denotes probability of selecting maintenance r in state s (assigned to the decision after inspection state Is) and P^{s0} represents probability of returning to state Ds from inspection Is which corresponds to a situation when no maintenance is scheduled as a result of the inspection. The foremost condition that must be met at all times is that in all deterioration states $s = 1 \ldots K$:

$$\mathrm{P}^{s0} + \sum_r \mathrm{P}^{sr} = 1. \tag{1}$$

Let F^r represents the frequency of some repair r as it is generated by the model. The problem of model adjustment can be formulated with various assumptions and with different goals in mind but in this approach it is defined as follows:

> Given an initial semi-Markov model M_0, with internal structure representing deterioration, inspection and repair states as described above and producing the initial vector of repair frequencies $\mathbf{F}_0 = [\,\mathrm{F}_0^1,\ \mathrm{F}_0^2\ \ldots\ \mathrm{F}_0^R\,]$, modify the probabilities P^{sr} assigned to transitions from inspections states Is so that the resulting model generates some requested vector of goal frequencies $\mathbf{F}_\mathrm{G}$.

There are various approaches that can be used in order to accomplish such model modification. In the proposed solution, a method of iterative approximations has been chosen in order to preserve an original construction of the model M_0 as mush as possible. In this method a sequence of *tuned* models $M_0, M_1, M_2, \ldots M_N$ is evaluated with each consecutive model approximating desired goal with a better accuracy. Starting with $i = 0$ and the initial model M_0, the procedure consists in the following steps:

1° for the current model M_i compute its vector of repair frequencies $\mathbf{F}_i$;

2° evaluate an error of M_i as a distance between vectors $\mathbf{F}_\mathrm{G}$ and $\mathbf{F}_i$;

3° if the error is within the user-defined accuracy ε, consider M_i as the final model and stop the procedure ($N = i$); otherwise proceed to the next step;

4° create a new model M_{i+1} by *tweaking* values of P_i^{sr}, then correct P_i^{s0} according to condition (1);

5° return to the step 1° for the next iteration.

Of all the steps 1–5, it is clear that adjusting probabilities P_i^{sr} in step 4 is the heart of the whole method. This is accomplished with the following two assumptions.

The first assumption is a restrictive condition: if the probability of some particular repair must be modified, it is modified *proportionally* in *all* deterioration states, so that during the adjustment the proportion between this repair probabilities over all states remains unchanged and is the same as in M_0:

$$\forall\ i, r \quad \mathrm{P}_0^{1r} : \mathrm{P}_0^{2r} : \ldots : \mathrm{P}_0^{Kr} \ \sim\ \mathrm{P}_i^{1r} : \mathrm{P}_i^{2r} : \ldots : \mathrm{P}_i^{Kr}. \tag{2}$$

Now only R scaling factors, denoted as $\mathbf{X}_{i+1} = [\,\mathrm{X}_{i+1}^1,\ \mathrm{X}_{i+1}^2,\ \ldots\ \mathrm{X}_{i+1}^R\,]$, must be found to compute all new probabilities and to create the next model M_{i+1}:

$$\mathrm{P}_{i+1}^{sr} = \mathrm{X}_{i+1}^r \cdot \mathrm{P}_0^{sr}, \quad r = 1 \ldots R,\ s = 1 \ldots K. \tag{3}$$

Moreover, and this observation leads to the second assumption, although the frequency of a repair r depends on the probabilities of all repairs (modifying probability of one repair changes, among others, state durations in the whole model; thus, it changes the frequency of all states) it can be assumed that, in a case of a single-step small adjustment, its dependence on repairs other than r can be considered negligible and

$$\mathrm{F}_i^r = \mathrm{F}_i^r\left(\mathrm{X}_i^1, \mathrm{X}_i^2 \ldots \mathrm{X}_i^R\right) \approx \mathrm{F}_i^r\left(\mathrm{X}_i^r\right). \tag{4}$$

Now, generation of a tweaked model in step 4 is reduced to of solving a set of R equations in the form:

$$\mathrm{F}_i^r\left(\mathrm{X}_i^r\right) = \mathrm{F}_G^r \tag{5}$$

and this can be accomplished with one of the standard root-finding numerical algorithms.

Additionally it should be pointed out that applying equation (3) with $X_{i+1} > 1$ may, in some state s, violate the condition

$$\sum_r \mathrm{P}_{i+1}^{sr} \le 1 . \tag{6}$$

Such situation needs special tests and, if detected, execution of *a scale-down transformation*:

$$\mathrm{P}^{sr} = \mathrm{P}^{sr} \big/ S_{Ds} , \quad S_{Ds} = \sum_{r=1}^{R} \mathrm{P}^{sr} . \tag{7}$$

Efficient numerical methods for approximate solving the equation (5) and, consequently, implementing the whole adjustment procedure of the steps 1–5, were presented in [9] and [10].

3. Modifying the Model in Case of Probability Saturation

As it was discussed in [9] and [10], the above defined procedure can successfully adjust repair probabilities P^{sr} when the goal (the $\mathbf{F}_G$ vector) include the frequencies that are lower than the ones for which the model was created (the $\mathbf{F}_0$ vector). That is, the method does not have problems if the model is transformed for studies like "what if some or all repairs are performed with lower frequency and deterioration rate is increased". In these kinds of situations the P^{sr} values need to be scaled down and the numerical algorithms are capable of their precise tuning so that the goal is reached in a reasonably limited number of iterations. On the other hand, adjusting the model to the repair frequencies that are substantially higher than the original ones may lead to so called *model saturation* – a condition in which repair probabilities reach the limit (6) and there is no room for further increase if the adjustment procedure is limited only to the simple scaling expressed by equation (3). In this situation bringing together two requirements such as tuning the model towards high repair frequencies, and at the same time keeping the modifications of the internal structure within a safe range, which does not break the proper relation with the original, it is a particular task that needs a devoted new development.

3.1. The Problem of Model Saturation

Discussion included in [6] investigated main challenges that are brought when the goal vector $\mathbf{F}_G$ contains increased values of repair frequencies. The two main factors that were recognized were as follows: (1) although it may seem that in the initial (minor) deterioration state no repairs are performed after inspections, still some non-zero probabilities are required in $D1$ if purely hypothetical questions like "What if I start some repair twice as often as previously?" shall be allowed; (2) including an option of not doing any repair after inspection in the later deterioration states, albeit with small probability, is also desirable because it increases ability of the model to represent diverse maintenance configurations found in the studies.

For the purpose of this presentation, we will illustrate these issues with an example of two models with three deterioration states and two repairs: minor (index 1) and major (index 2). Thus, there are a total of 6 repair probabilities in the model that will be fine-tuned by the procedure: P^{11} and P^{12} in deterioration state $D1$, P^{21} and P^{22} in $D2$, P^{31} and P^{32} in $D3$. Initial distribution of the probabilities is presented in Table 1. The model 1 has been created with assumption that although there are no repairs in the first state $D1$, when the equipment is in subsequent states $D2$ and $D3$ every inspection leads to some sort of repair and in these states the totals $S_{D2} = S_{D3} = 1$ ($\mathrm{P}^{20} = \mathrm{P}^{30} = 0$). Looking at the probability distribution in each state it can be seen that in the medium deterioration state $D2$ the minor repair evidently prevails ($\mathrm{P}^{21} = 0.8$) while in the major deterioration state $D3$ the distribution is in favour of the major repair with only a little more balanced allocation of probabilities ($\mathrm{P}^{32} = 0.7$). The model 2 is a sibling of 1 with one important difference: repair probabilities in $D2$ and $D3$ are lowered to, respectively, 80% and 95% of the values taken from 1, which means that after inspections in these states it is possible to return to Ds without undertaking any repair: $\mathrm{P}^{20} = 0.2$ and $\mathrm{P}^{30} = 0.05$.

Table 1. Repair probabilities in the sample models used as examples in this work

Deterioration state	$D1$		$D2$		$D3$	
Repair probability	P^{11}	P^{12}	P^{21}	P^{22}	P^{31}	P^{32}
Model 1	0	0	0.80	0.20	0.30	0.70
Model 2	0	0	0.64	0.16	0.28	0.67

The studies which will be analysed in the further part of this work will consist in generation of a sequence of models adjusted to goal frequencies created by modifying frequency of the major repair (index 2):

$$\mathbf{F}_G = \left[\mathrm{F}_0^1,\ \alpha \cdot \mathrm{F}_0^2\right], \tag{8}$$

where the factor α will increase from 0.5 (frequency of the major repair reduced by half) to 2.0 (the repair performed twice as often), in steps of 0.1. Values of α in the figures and in the following discussion will be expressed as %. To create a field of discussion the frequency of the major repair was selected as the varying parameter of $\mathbf{F}_\mathrm{G}$ just for an example, but equivalent results could be demonstrated with changing frequencies of the minor repair. The figures will include graphs presenting variations of repair probabilities P^{sr} and their sums S_{Ds} in particular deterioration states of the final adjusted models as functions of the α factor.

The problem of probability saturation is illustrated in Figure 2 which shows major repair probabilities in all states for models 1 and 2 tuned with the standard procedure described in the previous section (upper graphs) along with sums of probabilities over all states (lower graphs). Probability of the minor repair is not included to preserve space. Although it does not remain constant (see, for example, discussion in [6]) its variation does not demonstrate any significant aspects of saturation mechanism and just follow the general rules of repair inter-dependence.

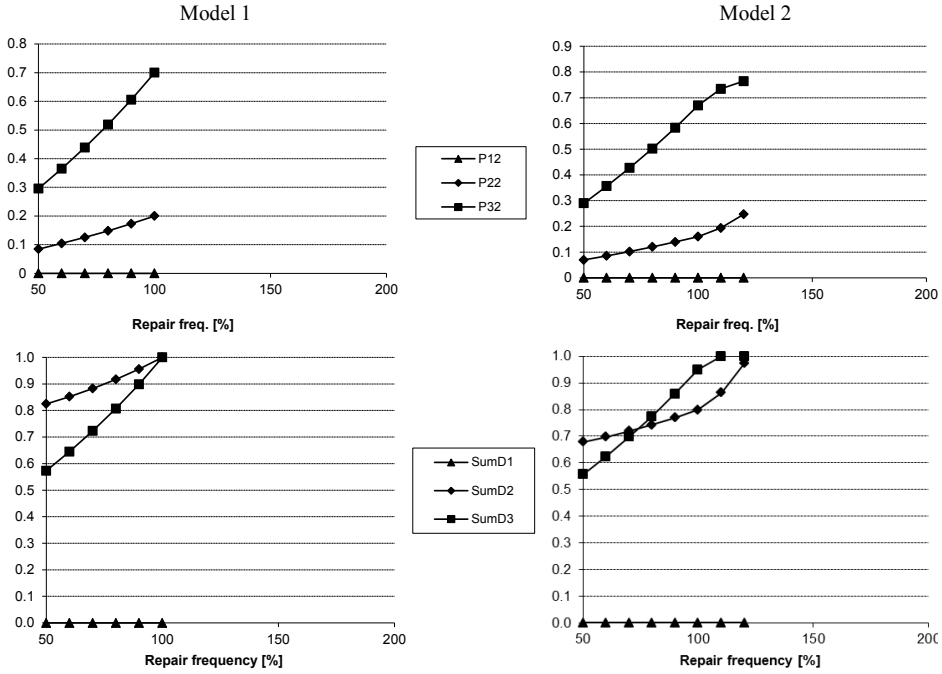

Figure 2. Adjusting the models 1 (left) and 2 (right) to the modified frequency of the major repair with the standard procedure (in case of model saturation the iteration stops reporting a fatal error)

The graphs show that both models can be successfully adjusted only up to the point of saturation which is reached for α = 100% for the model 1 (i.e. the initial model is already saturated) and 120% for model 2. This shows that in this particular case of model 2 probabilities $\mathrm{P}^{20} = 0.2$ and $\mathrm{P}^{30} = 0.05$ leave potential which is enough for 20% increase in frequency of the major repair. In both cases in the saturation points (α = 100% and 120%) probabilities in states $D2$ and $D3$ sum up to unity and cannot be further increased, while in $D1$ the P^{12} is zero and applying the scaling factor as in equation (3) cannot produce any growth. On the other hand, the procedure has no problems with adjustment towards lower frequencies and in such cases the probabilities are scaled accordingly which only confirms discussions presented in [9] and [10].

3.2. Automatic Modification of the Model in Case of Saturation

The above example of unsuccessful tuning can be used also for illustration of the main idea of the proposed extension to the algorithm: if the model gets saturated during the adjustment iteration but there is still some state with null repair probability, the process can be continued in the same iterative way after some non-zero probability is introduced into this state. Such modification, though, goes far beyond the restrictive assumption expressed by equation (2) and, being a more serious invasion into the structure of the model, must be applied in a cautious and thoughtful manner.

As it was investigated in [7], the following two particular issues must be taken into account: (1) forcing a non-zero probability in some state before it is not absolutely necessary, i.e. prior to model saturation, instantly changes reaction to the adjustment iterations, hence may change the final result of the tuning also in cases when the standard procedure applied to the unmodified model would be able to produce the correct result; (2) replacing the null value of P^{sr}, even if delayed up to the moment of saturation, but with probability which is too high for the needs of the situation also may affect the final result in a way that is against the general idea of the conservative tuning which tries to preserve the structure of the original model with minimal possible modifications. Consequently, it is advantageous to minimize the value of the newly introduced probability even below the anticipated level: the standard adjustment procedure that continues afterwards would increase it accordingly on the cost of extra iterations anyway, whereas the overestimated value would adversely corrupt the adjustment process and harm the result – the goal frequencies would be reached but with unwanted changes of the model structure ([7]).

Taking all these aspects into account, the following approach has been selected as the optimal and robust solution. In order to limit the changes of the already implemented method, the overall process of the 5 steps outlined in section 2.2 remains unchanged while only the internal procedure of *probability tweaking* in step 4 becomes extended. The procedure receives as the input the current M_i model and produces as the output the next M_{i+1} model in a sequence of operations that can be illustrated by the pseudo-code from Listing 1.

The operation begins as the previous unmodified method: initially, in the line no. 2 the scaling factors X^r are computed using any of the numerical approximation methods (NOLA, secant or *falsi*), as it was described in detailed discussions of [9] and [10]. Afterwards, the conditional instruction in the line no. 3 divides the rest of the code into two parts which are executed either for non-saturated (lines 4 ÷ 10) or for saturated (lines 12 ÷ 26) models. The model is considered saturated (which is tested as the condition of the line 3) if in all states $S_{Ds} = 1$ (probabilities reached their maximum) or $S_{Ds} = 0$ (probabilities are zero and cannot be increased by simple multiplication) *and* any of the X^r factor is greater than 1 (if all probabilities are to be decreased during the adjustment the saturation is not an obstacle and the standard method should not be altered). If the model is not saturated, the standard procedure just multiplies the probabilities by the tweaking factors (second part of the line 5 actually implements the equation (3)) and, per every state, the new values are scaled down if their sum exceeds the limit (lines 6 ÷ 9 implement the equation (7)).

The actual extension to the algorithm is included in the special processing path included in the lines 12 ÷ 26 which is executed if the model has reached the saturation state. In this case, in the beginning values of predicted average increases of the probabilities are computed in line 13 and stored in auxiliary `dAvgInc[]` array. Details of this computation has been omitted for brevity, but the code actually repeats the standard path similar to that of lines 4 ÷ 10 with the only difference that the new values are not assigned but they are used for storing the P^{sr} changes in elements of the `dAvgInc[]` array. If, afterwards, for any repair *r* the accumulated change is negative (a common case when this particular repair frequency should be lowered) it is replaced with zero while positive values are later used in forcing non-zero P^{sr} values. The rest of the processing is executed on state-by-state basis and is expressed with a single `for...each` instruction spanning lines 14 ÷ 26. In every state, each probability is either multiplied by the scaling factor if it is positive and such multiplication does lead to any increase (line 17 analogous to the equation (3)), or it is replaced with its pre-computed average increase in other states `dAvgInc[`*r*`]`. The line no. 19 implements the actual operation of replacing a zero value of P^{sr} with a positive one, which is the essence of the method. Afterwards, in lines 22 ÷ 25 the probabilities are scaled down if their sum exceeds the limit, although this conditional operation usually will not be executed in states where the probability increase was done as the condition in the line no. 23 will not be fulfilled.

Usually, once a non-zero probability has been forced into the model, the further iterations operate on a non-saturated model and they can proceed only with the regular scaling operations of the lines 4 ÷ 10.

4. Practical Verification of the Proposed Extension

4.1. Modification of Repair Probabilities in the Case Study Model

To illustrate operation of the modification method we will analyse adjustment progress in the case of model 2 adjusted for $\alpha = 200\%$, i.e. for a maintenance policy with the major repair performed twice as often as in the initial one. In this case the goal frequency vector equals to $\mathbf{F}_G = \left[\mathrm{F}_0^1,\ 2 \cdot \mathrm{F}_0^2\right]$ and this was the topmost value taken into account in the discussion of point 3.1. As it was then demonstrated, without modification this model can be tuned only up to the factor α = 120% and any further increase in frequency F^2 leads to model saturation (see Figure 2).

Figure 3 can be used to analyse how the modification mechanism has worked on the model in this case. In the graphs we can see how the probabilities (P^{sr} and their sums S_{Ds}) changed when in each iteration the *TweakProbs*() procedure was called. In total, 28 iterations were needed to reach the goal for imposed accuracy of 1% (in practical studies smaller precision is usually acceptable).

```
1.  procedure TweakProbs( MarkovModel M )
2.     EstimateTweakFactors( M, X );      // ...with numerical method NOLA, secant or falsi
3.     if not ModelSaturated( M ) then
4.        // The model IS NOT saturated, use the standard procedure
          for each s in M.DeteriorationSates do
5.           for each r in M.Repairs do   P^sr := P^sr · X^r;
6.           S_Ds = Σ_r P^sr;
7.           if S_Ds > 1 then
8.              for each r in M.Repairs do   P^sr := P^sr / S_Ds;
9.           end if;
10.       next s;
11.    else
12.       // Model IS saturated
          double dAvgInc[ R ]; // an array for predicted average increases of P^sr
13.       EstimateAverageProbIncrease( M, X, dAvgInc );
14.       for each s in M.DeteriorationSates do
15.          for each r in M.Repairs do
16.             if P^sr > 0 then
17.                // The probability is above zero, apply the regular scaling
                   P^sr := P^sr · X^r;
18.             else
19.                // The probability is zero, force the positive value
                   P^sr := dAvgInc[ r ];
20.             end if;
21.          next r;
22.          S_Ds = Σ_r P^sr;
23.          if S_Ds > 1 then
24.                for each r in M.Repairs do   P^sr := P^sr / S_Ds;
25.          end if;
26.       next s;
27.    end if;
28. end procedure;
```

Listing 1. Pseudo-code of the procedure used for detecting saturation of the model and its further modification

Initially, the sums equalled to, respectively, $S_{D1} = 0$, $S_{D2} = 0.80$, $S_{D3} = 0.95$ and this left some room for increases in the first three iterations after which both S_{D2} reached S_{D3} the maximum (1.0). With $P^{12} = 0$ this was identified as the situation of model saturation (test *ModelSaturated*() in line 3.) and, making the actual modification, P^{12} was replaced with and average predicted increase of P^{s2} as it was evaluated by *EstimateTweakFactors*() subroutine in line 2. Additionally, one can note that during the first 3 iterations before model modification increases of P^{22} and P^{32} values were accompanied with slight decreases of P^{21} and P^{31} values – this was caused by the scaling operation (equation 7) which, in case of nearly-saturated model, can visibly change probabilities other than the one being adjusted.

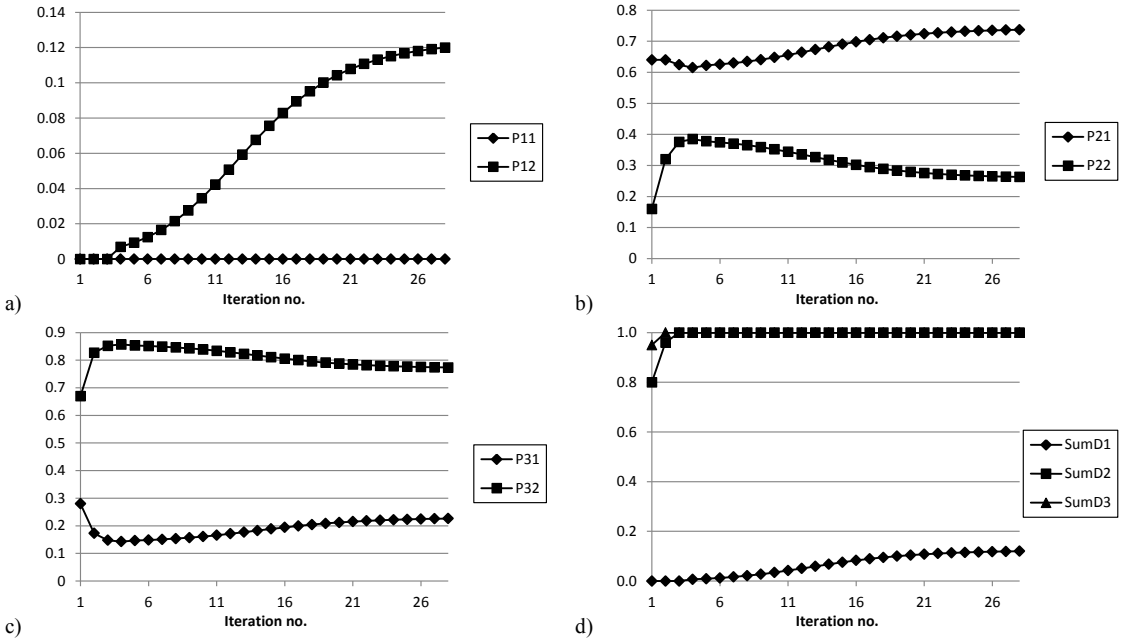

Figure 3. Iterations of the modified adjustment algorithm for the model 2 and the goal $\mathbf{F}_G = \left[\mathrm{F}_0^1,\ 2 \cdot \mathrm{F}_0^2\right]$: variation of the repair probabilities in the deterioration states $D1 \div D3$ (a ÷ c) and sums of repair probabilities in each state (d)

After the P^{12} probability got a non-zero value, the increase in the further iterations concentrated on this parameter. Starting with $P^{12} = 0.09$ in iteration no. 4 it quickly raised to 0.1 and then stabilized at the final level of 0.12. Simultaneously, the growth of P^{12} was accompanied by related fluctuations of probabilities of the major repair in the other two states. In the medium deterioration state $D2$ the P^{22} was initially raised from 0.16 to 0.38 but after modification of the model in iteration no. 4 it was somewhat lowered back to the final value of 0.28. In the major deterioration state $D3$ the P^{32} probability followed the same path: before model modification (i.e. before model saturation) it was enlarged from 0.67 to 0.86 but then, when P^{12} took over the main burden of increasing the major repair frequency, it reverted to slightly lower range settling finally at the value of 0.78.

In cases of both probabilities we can see the same behaviour: after initial growth in the first iterations before model saturation, when P^{12} was zero and did not take part in the tuning process, after modification of P^{12} both probabilities were decreased returning, to some extent, towards their initial values. This shows that the proposed modification of the model not only expanded tuning capabilities to the level needed for reaching the adjustment goal, but also released the stress from the states that remain unmodified: probabilities in these states to some extent return towards the initial values and within this aspect the final model is closer to the original one.

Figure 4 demonstrates the final results of the tuning of both case models over the whole range of α parameter from 50 to 200%, as it was analysed in point 3.1. By the virtue of the method, the results are identical to those included in Figure 2 up to the point of saturation: $\alpha = 100\%$ for model 1 and $\alpha = 120\%$ for model 2. After those points it can be seen that the results do not exhibit any unexpected fluctuations or erroneous instabilities: the P^{12} value begins to raise with P^{22} and P^{32} remaining at the saturation levels but the overall growth runs as anticipated.

4.2. Application of the Modified Models in Exemplary Reliability Analyses

Ability to create a Markov model modified for a specific maintenance policy is the starting point to various reliability analyses.

A convenient way to represent the deterioration process is by the life curve of the equipment ([5]). Such a curve shows the relationship between asset condition, expressed in either engineering or financial terms or simply as percentage of "as new" condition, and time. Starting with 100% value shape of the curve illustrates advancements of the deterioration process for the equipment operating in some application environment and undergoing regular inspections, while the overall length of the curve represents the average expected life of the equipment up to the point of its failure. To create a life curve from a Markov model representing some equipment and its maintenance activities, first its overall length is calculated as the average first passage time (FPT) from the first deterioration state ($D1$) to the failure state (F) in the model. Afterwards, solving the model for state probabilities of all consecutive deterioration states makes possible computing state durations, which in turns determine shape of the curve.

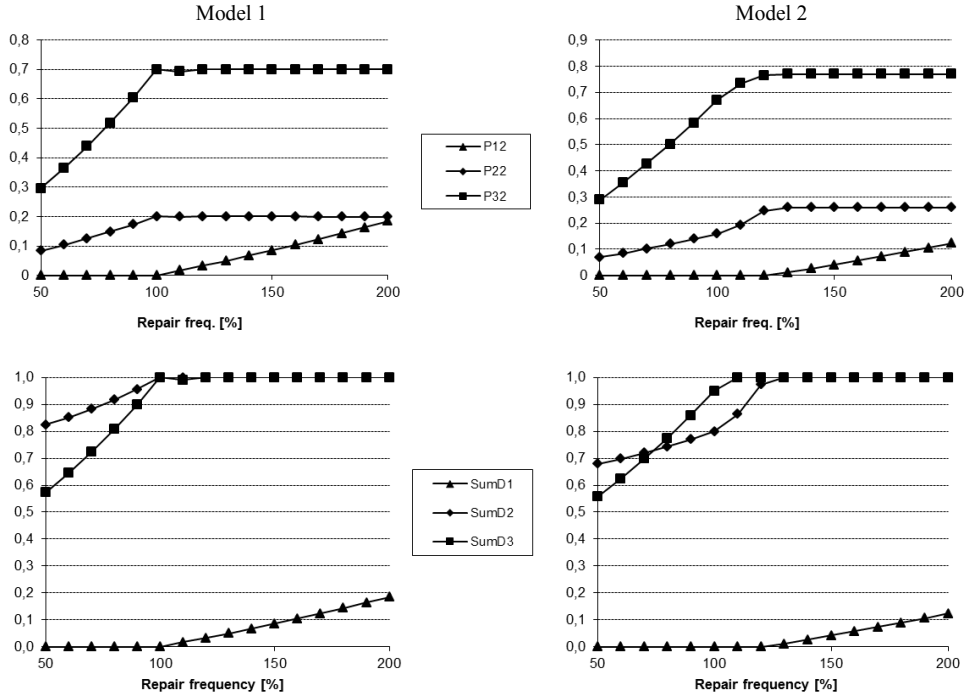

Figure 4. Adjusting the models 1 (left) and 2 (right) to the modified frequency of the major repair with the extended procedure proposed in this paper

Figure 5 shows the life curves computed from the models that all originated from the case study model 2 and were then adjusted to various frequencies of the major repair using the method presented in this paper. The goal frequencies were derived from the initial frequency F_0^2: $F_0^2 \times 0.8$, $F_0^2 \times 1.2$, and $F_0^2 \times 1.4$ (the case $F_0^2 \times 1.0$ represents the unchanged frequency so it corresponds to the original unmodified model). Comparing shapes and, first of all, lengths of the curves one can judge if the savings or the extra costs caused by reduced or extended maintenance are justified by the changes in equipment lifetime. In particular, in this specific case it can be seen that although increasing the major repair frequency by 20% ($F_0^2 \times 1.2$) extends expected equipment lifetime by 17% (from 16.9 to 19.7 years) further increase to $F_0^2 \times 1.4$ extends the lifetime only by extra 2% to 20.1 years.

5. Conclusions

In this work we were dealing with a deterioration modelling methodology which was designed to help in choosing effective yet cost-efficient maintenance policy. Incorporating in the state-transition semi-Markov model elements of both deterioration characteristics of the object and the maintenance activities (inspections and repairs), a common computational foundation has been created for various dependability

studies on possible maintenance scenarios. Specifically, this paper presented an extension of the model adjustment algorithm which was proposed and refined in our previous works. The general idea is to modify the model during the iteration by forcing a value greater than zero for a repair probability in situation when this probability reach the limit in other states, i.e. the model saturates. This extension allows to successfully evaluating a class of cases that was not properly handled by the original method and broadens the range of dependability studies that can be effectively evaluated.

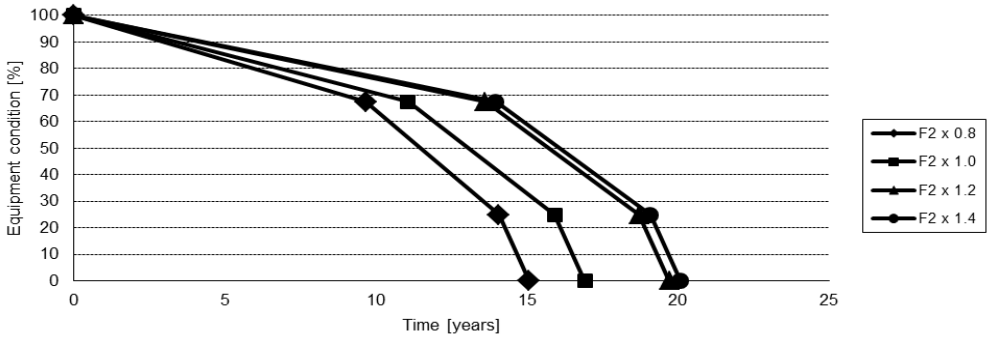

Figure 5. Life curves computed from Markov models adjusted to different frequencies of the major repair

The proposed approach strives to be as conservative as possible with regard to the amount of alterations introduced to the existing model. While the original method constrains the adjustment operations so that the distribution of the repair probabilities over all deterioration states is altered to the least possible degree, the modification introduced by this extension is a far more significant one and must be applied in a very cautious manner in order to avoid deformation of the model and corruption of the produced results. Consequently, there is a growing need for methods that would evaluate trustworthiness of the generated results, for example definition of new metrics of the model that would quantitatively assess extensions of its modification and would allow estimating the range of its valid use.

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PREDICTIVE INFERENCES FOR FUTURE ORDER STATISTICS UNDER PARAMETRIC UNCERTAINTY

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Prediction intervals for order statistics are widely used for reliability problems and other related problems. The determination of these intervals has been extensively investigated. But the optimality property of these intervals has not been fully explored. In this paper, in order to discuss this problem, a risk function is introduced to compare prediction intervals. In particular, new-sample prediction based on a previous sample (i.e., when for predicting the future observation in a new sample there are available the data only from a previous sample), and within-sample prediction based on the early observed data from a current experiment (i.e., when for predicting the future observation in a sample there are available the early observed data only from that sample). We restrict attention to families of distributions invariant under location and/or scale changes. The technique used here for optimization of prediction intervals based on censored data emphasizes pivotal quantities relevant for obtaining ancillary statistics. It allows one to solve the optimization problems in a simple way. An illustrative example is given.

Keywords: order statistic, prediction interval, risk function, optimisation

1. Introduction

Prediction of an unobserved random variable is a fundamental problem in statistics. Patel [1] provides an extensive survey of literature on this topic. In the areas of reliability and life-testing, lifetime data are often modeled via the Exponential and the Weibull in order to make predictions about future observations. Prediction intervals are constructed to have a reasonably high probability of containing a specified number of such future observations. These limits may be helpful in establishing warranty policy, determining maintenance schedules, etc. For a very readable discussion of prediction limits and related intervals, see Hahn and Meeker [2]. Many authors have reported their efforts for constructing prediction limits for the Weibull and for the related extreme value distributions (see Patel [1]). Mann and Saunders [3] proposed prediction limits for the Weibull which make use of only two or three order statistics (see also Mann [4]). Antle and Rademaker [5] used simulation to produce a table of factors to use with ML estimates to obtain prediction limits. Lawless [6] proposed prediction limits based on a conditional confidence approach; his limits require both determination of the ML estimates and numerical integration. Engelhardt and Bain [7–8] and Fertig, Meyer and Mann [9] have proposed various approximate prediction limits for the Weibull. Mee and Kushary [10] provided a simulation based procedure for constructing prediction intervals for Weibull populations for Type II censored case. This procedure is based on maximum likelihood estimation and requires an iterative process to determine the percentile points. Bhaumik and Gibbons [11] and Krishnamoorthy et al. [12] proposed approximate methods for constructing upper prediction limits for a gamma distribution. Consider the following examples of practical problems which often require the computation of prediction bounds and prediction intervals for future values of random quantities: (i) a consumer purchasing a refrigerator would like to have a lower bound for the failure time of the unit to be purchased (with less interest in distribution of

the population of units purchased by other consumers); (ii) financial managers in manufacturing companies need upper prediction bounds on future warranty costs; (iii) when planning life tests, engineers may need to predict the number of failures that will occur by the end of the test to predict the amount of time that it will be take for a specified number of units to fail. Some applications require a two-sided prediction interval that will, with a specified high degree of confidence, contain the future random variable of interest. In many applications, however, interest is focused on either an upper prediction bound or a lower prediction bound (e.g., the maximum warranty cost is more important than the minimum, and the time of the early failures in a product population is more important than the last ones). Conceptually, it is useful to distinguish between 'new-sample' prediction and 'within-sample' prediction. For new-sample prediction, data from a past sample are used to make predictions on a future unit or sample of units from the same process or population. For example, based on previous (possibly censored) life test data, one could be interested in predicting the time to failure of a new unit, time until r failures in a future sample of m units, or number of failures by time $t^{\bullet}$ in a future sample of m units. For within-sample prediction, the problem is to predict future events in a sample or process based on early data from that sample or process. If, for example, n units are followed until $t_{\bullet}$ and there are k observable failures, $X_1 < X_2 < \cdots < X_k$, one could be interested in predicting the time of the next failure, $X_{(k+1)}$; time until l additional failures, $X_{(k+l)}$; number of additional failures in a future interval $(t_{\bullet},t^{\bullet})$. In general, to predict a future realization of a random quantity one needs the following:

1) *A statistical model to describe the population or process of interest*. This model usually consists of a distribution depending on a vector of parameters $\boldsymbol{\theta}$. In this paper, attention is restricted to families of distributions which are invariant under location and/or scale changes. In particular, the case may be considered where a previously available complete or type II censored sample is from a continuous distribution with cdf $F((x-\mu)/\sigma)$, where $F(\cdot)$ is known but both the location (μ) and scale (σ) parameters are unknown. For such family of distributions the decision problem remains invariant under a group of transformations (a subgroup of the full affine group) which takes μ (the location parameter) and σ (the scale) into $c\mu + b$ and $c\sigma$, respectively, where b lies in the range of μ, $c > 0$. This group acts transitively on the parameter space.

2) *Information on the values of components of the parametric vector* $\boldsymbol{\theta}$. It is assumed that only the functional form of the distribution is specified, but some or all of its parameters are unspecified. In such cases ancillary statistics and pivotal quantities, whose distribution does not depend on the unknown parameters, are used.

The technique used here for constructing prediction intervals (or bounds) emphasizes pivotal quantities relevant for obtaining ancillary statistics. It represents a simple procedure that can be utilized by non-statisticians, and which provides easily computable explicit expressions for both prediction bounds and prediction intervals. The technique is a special case of the method of invariant embedding of sample statistics into a performance index (see, e.g., Nechval et al. [13–18]) applicable whenever the statistical problem is invariant under a group of transformations, which acts transitively on the parameter space.

2. Within-Sample Prediction Problem

For within-sample prediction, the problem is to predict future events in a sample or process based on early data from that sample or process. For example, if n units are followed until t_k and there are k observed failures, $t_1, \ldots, t_k$, one could be interested in predicting the time of the next failure t_{k+1}; time until l additional failures, t_{k+l}; number of additional failures in a future interval.

2.1. Location-Scale Family of Distributions

Consider a situation described by a location-scale family of probability distribution functions, indexed by the vector parameter $\boldsymbol{\theta} = (\mu,\sigma)$, where μ and σ (> 0) are respectively parameters of location and scale. For this family, invariant under the group G of positive linear transformations: $x \to ax + b$ with $a > 0$, we shall assume that there is obtainable (from some informative experiment) the first k order statistics $X_1 < X_2 < \cdots < X_k$ from a random sample of size n with cumulative distribution function

$$F(x \mid \mu,\sigma) \equiv F([x-\mu]/\sigma),\ (-\infty)\mu < x < \infty,\ \ \sigma > 0. \tag{1}$$

If Y is an independent future observation from the same sample of size n, then $W = (Y - X_k)/S_k$ (or $W = (Y - X_k)/X_k$) is an invariant statistic, the distribution of which does not depend on (μ,σ); S_k is a sufficient statistic (or a maximum likelihood estimator $\hat{\sigma}_k$) for σ based on $\mathbf{X} = (X_1, X_2, \ldots, X_k)$.

2.2. Piecewise-Linear Loss Function

We shall consider the interval prediction problem for the rth order statistic X_r, $k < r \le n$, in the same sample of size n for the situation where the first k observations $X_1 < X_2 < \cdots < X_k$, $1 \le k < n$, have been observed. Suppose that we assert that an interval $\mathbf{d} = (d_1,d_2)$ contains X_r. If, as is usually the case, the purpose of this interval statement is to convey useful information we incur penalties if d_1 lies above X_r or if d_2 falls below X_r. Suppose that these penalties are $c_1(d_1 - X_r)$ and $c_2(X_r - d_2)$, losses proportional to the amounts by which X_r escapes the interval. Since c_1 and c_2 may be different the possibility of differential losses associated with the interval overshooting and undershooting the true μ is allowed. In addition to these losses there will be a cost attaching to the length of interval used. For example, it will be more difficult and more expensive to design or plan when the interval $\mathbf{d} = (d_1,d_2)$ is wide. Suppose that the cost associated with the interval is proportional to its length, say $c(d_2 - d_1)$. In the specification of the loss function, σ is clearly a 'nuisance parameter' and no alteration to the basic decision problem is caused by multiplying all loss factors by $1/\sigma$. Thus we are led to investigate the piecewise-linear loss function

$$r(\boldsymbol{\theta},\mathbf{d}) = \begin{cases} \dfrac{c_1(d_1 - X_r)}{\sigma} + \dfrac{c(d_2 - d_1)}{\sigma} & (X_r < d_1), \\ \dfrac{c(d_2 - d_1)}{\sigma} & (d_1 \le X_r \le d_2), \\ \dfrac{c(d_2 - d_1)}{\sigma} + \dfrac{c_2(X_r - d_2)}{\sigma} & (X_r > d_2). \end{cases} \tag{2}$$

The decision problem specified by the informative experiment probability distribution function (1) and the loss function (2) is invariant under the group G of transformations. Thus, the problem is to find the best invariant interval predictor of X_r,

$$\mathbf{d}^* = \arg\min_{\mathbf{d}\in\mathfrak{D}} R(\boldsymbol{\theta},\mathbf{d}), \tag{3}$$

where $\mathfrak{D}$ is a set of invariant interval predictors of X_r, $R(\boldsymbol{\theta},\mathbf{d}) = E_{\boldsymbol{\theta}}\{r(\boldsymbol{\theta},\mathbf{d})\}$ is a risk function.

2.3. Transformation of the Loss Function

It follows from (2) that the invariant loss function, $r(\boldsymbol{\theta},\mathbf{d})$, can be transformed as follows:

$$r(\boldsymbol{\theta},\mathbf{d}) = \ddot{r}(\mathbf{V},\boldsymbol{\eta}), \tag{4}$$

where

$$\ddot{r}(\mathbf{V},\boldsymbol{\eta}) = \begin{cases} c_1(-V_1 + \eta_1 V_2) + c(\eta_2 - \eta_1)V_2 & (V_1 < \eta_1 V_2), \\ c(\eta_2 - \eta_1)V_2 & (\eta_1 V_2 \le V_1 \le \eta_2 V_2), \\ c_2(V_1 - \eta_2 V_2) + c(\eta_2 - \eta_1)V_2 & (V_1 > \eta_2 V_2), \end{cases} \tag{5}$$

$$\mathbf{V} = (V_1,V_2),\ V_1 = (X_r - X_k)/\sigma,\ V_2 = S_k/\sigma;\ \boldsymbol{\eta} = (\eta_1,\eta_2),\ \eta_1 = (d_1 - X_k)/S_k,\ \eta_2 = (d_2 - X_k)/S_k. \tag{6}$$

2.4. Risk Function

It follows from (5) that the risk associated with $\mathbf{d}$ and $\boldsymbol{\theta}$ can be expressed as

$$R(\boldsymbol{\theta},\mathbf{d}) = E_{\boldsymbol{\theta}}\{r(\boldsymbol{\theta},\mathbf{d})\} = E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\} = c_1\int_0^{\infty}\int_0^{\eta_1 v_2}(-v_1 + \eta_1 v_2)f(v_1,v_2)dv_1dv_2$$

$$+ c_2\int_0^{\infty}\int_{\eta_2 v_2}^{\infty}(v_1 - \eta_2 v_2)f(v_1,v_2)dv_1dv_2 + c(\eta_2 - \eta_1)\int_0^{\infty}\int_0^{\infty} v_2 f(v_1,v_2)dv_1dv_2 \equiv R(\eta_1,\eta_2), \tag{7}$$

which is constant on orbits when an invariant predictor (decision rule) $\mathbf{d}$ is used, where $f(v_1,v_2)$ is defined by the joint probability density of the first k observations $X_1 < X_2 < \cdots < X_k$ and X_r,

$$f(x_1,x_2,...,x_k,x_r \mid \mu,\sigma)=\frac{n!}{(r-k-1)!(n-r)!}$$

$$\times[F(x_r \mid \mu,\sigma)-F(x_k \mid \mu,\sigma)]^{r-k-1}[1-F(x_r \mid \mu,\sigma)]^{n-r}\prod_{i=1}^{k} f(x_i \mid \mu,\sigma)f(x_r \mid \mu,\sigma). \tag{8}$$

2.5. Risk Minimization and Optimal Predictors

The following theorem gives the central result in this section.

Theorem 1 (*Optimal invariant predictor of X_r based on* **X**). Suppose that (u_1, u_2) is a random vector having density function

$$u_2 f(u_1,u_2)\left[\int_0^{\infty}\int_0^{\infty} u_2 f(u_1,u_2)du_1du_2\right]^{-1} \quad (u_1,u_2>0), \tag{9}$$

where f is defined by $f(v_1,v_2)$, and let Q be the probability distribution function of u_1/u_2.

(i) If $c / c_1 + c / c_2 < 1$ then the optimal invariant linear-loss interval predictor of X_r based on **X** is $\mathbf{d}^* = (X_k + \eta_1 S_k, X_k + \eta_2 S_k)$, where

$$Q(\eta_1)=c/c_1, \quad \mathrm{Q}(\eta_2)=1-c/c_2. \tag{10}$$

(ii) If $c / c_1 + c / c_2 \geq 1$ then the optimal invariant linear-loss interval predictor of X_r based on **X** degenerates into a point predictor $X_k + \eta_\bullet S_k$, where

$$Q(\eta_\bullet)=c_2/(c_1+c_2). \tag{11}$$

Proof. From (7)

$$\frac{\partial E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\}}{\partial \eta_1}=c_1\int_0^{\infty}\int_0^{\eta_1 v_2} v_2 f(v_1,v_2)dv_1dv_2 - c\int_0^{\infty}\int_0^{\infty} v_2 f(v_1,v_2)dv_1dv_2$$

$$=\int_0^{\infty}\int_0^{\infty} v_2 f(v_1,v_2)dv_1dv_2[c_1Q(\eta_1)-c], \tag{12}$$

and

$$\frac{\partial E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\}}{\partial \eta_2}=\int_0^{\infty}\int_0^{\infty} v_2 f(v_1,v_2)dv_1dv_2[-c_2(1-Q(\eta_2))+c], \tag{13}$$

where

$$Q(\eta)=\int_0^{\eta} q(w)dw, \tag{14}$$

$$q(w)=\frac{\int_0^{\infty} v_2^2 f(wv_2,v_2)dv_2}{\int_0^{\infty}\int_0^{\infty} v_2 f(v_1,v_2)dv_1dv_2}, \tag{15}$$

$$W=V_1/V_2. \tag{16}$$

Now $\partial E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\} / \partial \eta_1 = \partial E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\} / \partial \eta_2 = 0$ if and only if (10) hold. Thus, $E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\}$ provided (10) has a solution with $\eta_1 < \eta_2$ and this is so if $1 - c / c_2 > c / c_1$. It is easily confirmed that this $\boldsymbol{\eta} = (\eta_1, \eta_2)$ gives the minimum value of $E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\}$. Thus (i) is established. If $c / c_1 + c / c_2 \geq 1$ then the minimum of $E\{\ddot{r}(\mathrm{v},\eta)\}$ in the region $\eta_2 \geq \eta_1$ occurs where $\eta_1 = \eta_2 = \eta_\bullet$, $\eta_\bullet$ being determined by setting

$$\partial E\{\ddot{r}(V,(\eta_\bullet,\eta_\bullet))\}/\partial \eta_\bullet = 0 \tag{17}$$

and this reduces to

$$c_1 Q(\eta_\bullet) - c_2[1 - Q(\eta_\bullet)] = 0, \tag{18}$$

which establishes (ii).

Corollary 1.1 (*Minimum risk of the optimal invariant predictor of X_r based on* **X**). The minimum risk is given by

$$R(\boldsymbol{\theta},\mathbf{d}^*) = E_{\boldsymbol{\theta}}\{r(\boldsymbol{\theta},\mathbf{d}^*)\} = E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\}$$

$$= -c_1 \int_0^\infty \int_0^{\eta_1 v_2} v_1 f(v_1,v_2)\,dv_1 dv_2 + c_2 \int_0^\infty \int_{\eta_2 v_2}^\infty v_1 f(v_1,v_2)\,dv_1 dv_2 \tag{19}$$

for case (i) with $\boldsymbol{\eta} = (\eta_1, \eta_2)$ as given by (10) and for case (ii) with $\eta_1 = \eta_2 = \eta_\bullet$ as given by (11).

Proof. These results are immediate from (7) when use is made of $\partial E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\}/\partial \eta_1 = \partial E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\}/\partial \eta_2 = 0$ in case (i) and $\partial E\{\ddot{r}(\mathbf{V},(\eta_\bullet,\eta_\bullet))\}/\partial \eta_\bullet = 0$ in case (ii).

The underlying reason why $c / c_1 + c / c_2$ acts as a separator of interval and point prediction is that for $c / c_1 + \mathrm{c} / c_2 \geq 1$ every interval predictor is inadmissible, there existing some point predictor with uniformly smaller risk.

Theorem 2 (*Optimal invariant predictor of X_r based on X_k*). Suppose that $\mu = 0$ and

$$\mathbf{V} = (V_1,V_2),\ V_1 = (X_r - X_k)/\sigma,\ V_2 = X_k/\sigma;\ \boldsymbol{\eta} = (\eta_1,\eta_2),\ \eta_1 = (d_1 - X_k)/X_k,\ \eta_2 = (d_2 - X_k)/X_k. \tag{20}$$

Let us assume that (u_1, u_2) is a random vector having density function

$$u_2 f_0(u_1,u_2)\left[\int_0^\infty \int_0^\infty u_2 f_0(u_1,u_2)\,du_1 du_2\right]^{-1} \quad (u_1, u_2 > 0), \tag{21}$$

where f_0 is defined by $f_0(v_1,v_2)$, and let Q_0 be the probability distribution function of u_1 / u_2.

(i) If $c / c_1 + c / c_2 \quad 1$ then the optimal invariant linear-loss interval predictor of X_r based on X_k is $\mathbf{d}^* = ((1 + \eta_1)X_k, (1 + \eta_2)X_k)$, where

$$Q_0(\eta_1) = c/c_1, \quad \mathrm{Q}_0(\eta_2) = 1 - c/c_2. \tag{22}$$

(ii) If $c / c_1 + c / c_2 \geq 1$ then the optimal invariant linear-loss interval predictor of X_r based on X_k degenerates into a point predictor $(1+\eta_\bullet)X_k$, where

$$Q_0(\eta_\bullet) = c_2/(c_1 + c_2). \tag{23}$$

Proof. For the proof we refer to Theorem 1.

Corollary 2.1 (*Minimum risk of the optimal invariant predictor of X_r based on X_k*). The minimum risk is given by

$$R(\boldsymbol{\theta},\mathbf{d}^*) = E_{\boldsymbol{\theta}}\{r(\boldsymbol{\theta},\mathbf{d}^*)\} = E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\}$$

$$= -c_1\int_0^{\infty}\int_0^{\eta_1 v_2} v_1 f_0(v_1,v_2)dv_1dv_2 + c_2\int_0^{\infty}\int_{\eta_2 v_2}^{\infty} v_1 f_0(v_1,v_2)dv_1dv_2 \tag{24}$$

for case (i) with $\boldsymbol{\eta} = (\eta_1,\eta_2)$ as given by (22) and for case (ii) with $\eta_1 = \eta_2 = \eta_\bullet$ as given by (23).

Proof. For the proof we refer to Corollary 1.1.

2.6. Equivalent Confidence Coefficient

For case (i) when we obtain an interval predictor for X_r we may regard the interval as a confidence interval in the conventional sense and evaluate its confidence coefficient. The general result is contained in the following theorems.

Theorem 3 (*Equivalent confidence coefficient for* $\mathbf{d}^*$ *based on* $\mathbf{X}$). Suppose that $\mathbf{V} = (V_1,V_2)$ is a random vector having density function $f(v_1,v_2)$ $(v_1,v_2 > 0)$ where f is defined by (8) and let H be the distribution function of $W = V_1 / V_2$, i.e., the probability density function of W is given by

$$h(w) = \int_0^{\infty} v_2 f(wv_2,v_2)dv_2. \tag{25}$$

Then the confidence coefficient associated with the optimum prediction interval $\mathbf{d}^* = (d_1,d_2)$, where $d_1 = X_k + \eta_1 S_k$, $d_2 = X_k + \eta_2 S_k$, is

$$\Pr\{\mathbf{d}^* : d_1 < X_r < d_2 \mid \mu,\sigma\} = H[Q^{-1}(1-c/c_2)] - H[Q^{-1}(c/c_1)]. \tag{26}$$

Proof. The confidence coefficient for $\mathbf{d}^*$ corresponding to (μ,σ) is given by

$$\Pr\{(X_k,S_k) : X_k + \eta_1 S_k < X_r < X_k + \eta_2 S_k \mid \mu,\sigma\} = \Pr\{(v_1,v_2) : \eta_1 < v_1/v_2 < \eta_2\}$$

$$= H(\eta_2) - H(\eta_1) = H[Q^{-1}(1-c/c_2)] - H[Q^{-1}(c/c_1)]. \tag{27}$$

This is independent of (μ,σ).

Theorem 4 (*Equivalent confidence coefficient for* $\mathbf{d}^*$ *based on* X_k). Suppose that $\mathbf{V} = (V_1,V_2)$ is a random vector having density function $f_0(v_1,v_2)$ $(v_1, v_2 > 0)$, where f_0 is defined by

$$f(x_k,x_r \mid \mu,\sigma) = \frac{1}{\mathrm{B}(k,r-k)\mathrm{B}(r,n-r+1)}[F(x_k \mid \mu,\sigma)]^{r-1}[F(x_r \mid \mu,\sigma) - F(x_k \mid \mu,\sigma)]^{r-k-1}$$

$$\times[1 - F(x_r \mid \mu,\sigma)]^{n-r} f(x_k \mid \mu,\sigma) f(x_r \mid \mu,\sigma), \tag{28}$$

and let H_0 be the distribution function of $W = V_1 / V_2$, i.e., the probability density function of W is given by

$$h_0(w) = \int_0^{\infty} v_2 f_0(wv_2,v_2)dv_2. \tag{29}$$

Then the confidence coefficient associated with the optimum prediction interval $\mathbf{d}^* = (d_1,d_2)$, where $d_1 = (1 + \eta_1)X_k$, $d_2 = (1 + \eta_2)X_k$, is

$$\Pr\{\mathbf{d}^* : d_1 < X_r < d_2 \mid \mu,\sigma\} = H_0[Q_0^{-1}(1-c/c_2)] - H_0[Q_0^{-1}(c/c_1)]. \tag{30}$$

Proof. For the proof we refer to Theorem 3.

The way in which (26) (or (30)) varies with c, c_1 and c_2, and the fact that c_1 and c_2 are the factors of proportionality associated with losses from overshooting and undershooting relative to loss involved in increasing the length of interval, provides an interesting interpretation of confidence interval prediction.

3. New-Sample Prediction Problem

For new-sample prediction, data from a past sample are used to make predictions on a future unit or sample of units from the same process or population. For example, based on previous (possibly censored) life test data, one could be interested in predicting the time to failure of a new item, time until l failures in a future sample of m units, or number of failures by time $t_\bullet$ in a future sample of m units.

3.1. Location-Scale Family of Density Functions

Consider a situation described by a location-scale family of density functions, indexed by the vector parameter $\boldsymbol{\theta} = (\mu,\sigma)$, where μ and σ (> 0) are respectively parameters of location and scale. For this family, invariant under the group of positive linear transformations: $x \to ax + b$ with $a > 0$, we shall assume that there is obtainable from some informative experiment (the first k order statistics $X_1 < X_2 < \cdots < X_k$ from a random sample of size n) a sufficient statistic (M_k,S_k) (or a maximum likelihood estimator $(\hat{\mu}_k, \hat{\sigma}_k)$) for (μ,σ) based on $\mathbf{X} = (X_1, X_2,\ldots, X_k)$ with density function

$$\begin{aligned} &p(m_k, s_k \mid \mu,\sigma) = \sigma^{-2} p_0[(m_k - \mu)/\sigma, s_k/\sigma], \\ &-\infty < m_k < \infty, \quad 0 < s_k < \infty, \quad -\infty < \mu > \infty, \quad \sigma > 0. \end{aligned} \tag{31}$$

We are thus assuming that for the family of density functions an induced invariance holds under the group G of transformations: $m_k \to am_k + b$, $s_k \to as_k$ or $\hat{\mu}_k \to a\hat{\mu}_k + b$, $\hat{\sigma}_k \to a\hat{\sigma}_k$ (a> 0). The family of density functions satisfying the above conditions is, of course, the limited one of normal, negative exponential, Weibull and gamma (with known index) density functions. The structure of the problem is, however, more clearly seen within the general framework. Let Y be an independent future observation from a new sample. If Y is invariantly predictable then $W = (Y - M_k) / S_k$ (or $W = (Y - \hat{\mu}_k)/\hat{\sigma}_k$) is a maximal invariant pivotal, conditional on $\mathbf{X}$.

3.2. Piecewise-Linear Loss Function

We shall consider the interval prediction problem for the sth order statistic Y_s, $1 \le s \le m$, in a future sample of size m for the situation where the first k observations $X_1 < X_2 < \cdots < X_k$, $1 \le k < n$, from a past sample of size n have been observed. Suppose that we assert that an interval $\mathbf{d} = (d_1,d_2)$ contains Y_s. If, as is usually the case, the purpose of this interval statement is to convey useful information we incur penalties if d_1 lies above Y_s or if d_2 falls below Y_s. Suppose that these penalties are $c_1(d_1 - Y_s)$ and $c_2(Y_s - d_2)$, losses proportional to the amounts by which Y_s escapes the interval. Since c_1 and c_2 may be different the possibility of differential losses associated with the interval overshooting and undershooting the true μ is allowed. In addition to these losses there will be a cost attaching to the length of interval used. For example, it will be more difficult and more expensive to design or plan when the interval $\mathbf{d} = (d_1,d_2)$ is wide. Suppose that the cost associated with the interval is proportional to its length, say $c(d_2 - d_1)$. In the specification of the loss function, σ is clearly a 'nuisance parameter' and no alteration to

the basic decision problem is caused by multiplying all loss factors by $1/\sigma$. Thus we are led to investigate the piecewise-linear loss function

$$r(\boldsymbol{\theta},\mathbf{d})=\begin{cases}\dfrac{c_1(d_1-Y_s)}{\sigma}+\dfrac{c(d_2-d_1)}{\sigma} & (Y_s<d_1),\\ \dfrac{c(d_2-d_1)}{\sigma} & (d_1\le Y_s\le d_2),\\ \dfrac{c(d_2-d_1)}{\sigma}+\dfrac{c_2(Y_s-d_2)}{\sigma} & (Y_s>d_2).\end{cases} \tag{32}$$

The decision problem specified by the informative experiment density function (31) and the loss function (32) is invariant under the group G of transformations. Thus, the problem is to find the optimal interval predictor of Y_s,

$$\mathbf{d}^*=\arg\min_{\mathbf{d}\in\mathbb{D}} R(\boldsymbol{\theta},\mathbf{d}), \tag{33}$$

where $\mathbb{D}$ is a set of invariant interval predictors of Y_s, $R(\boldsymbol{\theta},\mathbf{d})=E_{\boldsymbol{\theta}}\{r(\boldsymbol{\theta},\mathbf{d})\}$ is a risk function.

3.3. Transformation of the Loss Function

It follows from (32) that the invariant loss function, $r(\boldsymbol{\theta},\mathbf{d})$, can be transformed as follows:

$$r(\boldsymbol{\theta},\mathbf{d})=\ddot{r}(\mathbf{V},\boldsymbol{\eta}), \tag{34}$$

where

$$\ddot{r}(\mathbf{V},\boldsymbol{\eta})=\begin{cases}c_1(-V_1+\eta_1V_2)+c(\eta_2-\eta_1)V_2 & (V_1<\eta_1V_2),\\ c(\eta_2-\eta_1)V_2 & (\eta_1V_2\le V_1\le\eta_2V_2),\\ c_2(V_1-\eta_2V_2)+c(\eta_2-\eta_1)V_2 & (V_1>\eta_2V_2),\end{cases} \tag{35}$$

$$\mathbf{V}=(V_1,V_2),\quad V_1=(Y_s-M_k)/\sigma,\quad V_2=S_k/\sigma;\quad \boldsymbol{\eta}=(\eta_1,\eta_2),\quad \eta_1=(d_1-M_k)/S_k,\quad \eta_2=(d_2-M_k)/S_k. \tag{36}$$

3.4. Risk Function

It follows from (35) that the risk associated with $\mathbf{d}$ and $\boldsymbol{\theta}$ can be expressed as

$$R(\boldsymbol{\theta},\mathbf{d})=E_{\boldsymbol{\theta}}\{r(\boldsymbol{\theta},\mathbf{d})\}=E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\}=c_1\int_0^{\infty}\int_{-\infty}^{\eta_1v_2}(-v_1+\eta_1v_2)f(v_1,v_2)dv_1dv_2$$

$$+c_2\int_0^{\infty}\int_{\eta_2v_2}^{\infty}(v_1-\eta_2v_2)f(v_1,v_2)dv_1dv_2+c(\eta_2-\eta_1)\int_0^{\infty}\int_{-\infty}^{\infty}v_2f(v_1,v_2)dv_1dv_2\equiv R(\eta_1,\eta_2), \tag{37}$$

which is constant on orbits when an invariant predictor (decision rule) $\mathbf{d}$ is used, where $f(v_1,v_2)$ is defined by the joint probability density function of the first k ordered observations $X_1<X_2<\cdots<X_k$ from the past random sample of observations of size n and the sth order statistic Y_s in the future sample of size m,

$$f(x_1,x_2,\ldots,x_k,y_s\mid\mu,\sigma)=\frac{n!}{(n-k)!}\frac{m!}{(s-1)!(m-s)!}\prod_{i=1}^{k}f(x_i\mid\mu,\sigma)[1-F(x_k\mid\mu,\sigma)]^{n-k}$$

$$\times[F(y_s\mid\mu,\sigma)]^{s-1}[1-F(y_s\mid\mu,\sigma)]^{m-s}f(y_s\mid\mu,\sigma). \tag{38}$$

3.5. Risk Minimization and Optimal Predictors

The following theorem gives the central result in this section.

Theorem 5 (*Optimal invariant predictor of Y_s based on* **X**). Suppose that (u_1, u_2) is a random vector having density function

$$u_2 f(u_1,u_2)\left[\int_0^{\infty}\int_{-\infty}^{\infty} u_2 f(u_1,u_2)du_1du_2\right]^{-1} \quad (u_1 \text{ real}, u_2>0), \tag{39}$$

where f is defined by $f(v_1,v_2)$, and let Q be the probability distribution function of u_1 / u_2.

(i) If $c / c_1 + c / c_2 < 1$ then the optimal invariant linear-loss interval predictor of Y_s based on **X** is $\mathbf{d}^* = (M_k + \eta_1 S_k, M_k + \eta_2 S_k)$, where

$$Q(\eta_1) = c/c_1, \quad Q(\eta_2) = 1 - c/c_2. \tag{40}$$

(ii) If $c / c_1 + c / c_2 \geq 1$ then the optimal invariant linear-loss interval predictor of Y_s based on **X** degenerates into a point predictor $M_k + \eta_\bullet S_k$, where

$$Q(\eta_\bullet) = c_2/(c_1 + c_2). \tag{41}$$

Proof. For the proof we refer to Theorem 1.

Corollary 5.1 (*Minimum risk of the optimal invariant predictor of Y_s based on* **X**). The minimum risk is given by

$$R(\boldsymbol{\theta},\mathbf{d}^*) = E_{\boldsymbol{\theta}}\{r(\boldsymbol{\theta},\mathbf{d}^*)\} = E\{\ddot{r}(\mathbf{V},\boldsymbol{\eta})\} = -c_1\int_0^{\infty}\int_{-\infty}^{\eta_1 v_2} v_1 f(v_1,v_2)dv_1dv_2 + c_2\int_0^{\infty}\int_{\eta_2 v_2}^{\infty} v_1 f(v_1,v_2)dv_1dv_2 \tag{42}$$

for case (i) with $\boldsymbol{\eta} = (\eta_1, \eta_2)$ as given by (40) and for case (ii) with $\eta_1 = \eta_2 = \eta_\bullet$ as given by (41).

Proof. For the proof we refer to Corollary 1.1.

3.6. Equivalent Confidence Coefficient

Theorem 6 (*Equivalent confidence coefficient for* $\mathbf{d}^*$ *based on* **X**). Suppose that $\mathbf{V} = (V_1, V_2)$ is a random vector having density function $f(v_1,v_2)$ (v_1 real, $v_2 > 0$) where f is defined by (38) and let H be the distribution function of $W = V_1 / V_2$, i.e., the probability density function of W is given by

$$h(w) = \int_0^{\infty} v_2 f(wv_2, v_2)dv_2. \tag{43}$$

Then the confidence coefficient associated with the optimum prediction interval $\mathbf{d}^* = (d_1, d_2)$, where $d_1 = M_k + \eta_1 S_k$, $d_2 = M_k + \eta_2 S_k$, is

$$\Pr\{\mathbf{d}^* : d_1 < X_r < d_2 \mid \mu,\sigma\} = H[Q^{-1}(1 - c/c_2)] - H[Q^{-1}(c/c_1)]. \tag{44}$$

Proof. For the proof we refer to Theorem 3.

4. Example

4.1. Within-Sample Prediction

Exponential distribution. Let $X_1 < X_2 < \cdots < X_n$ be the order statistics of a random sample of size n from the exponential distribution with the density

$$f(x \mid \sigma) = \frac{1}{\sigma}\exp\left(-\frac{x}{\sigma}\right), \quad x > 0, \quad \sigma > 0. \tag{45}$$

We shall consider the prediction problem of X_r for the situation where the first k observations $X_1 < X_2 < \cdots < X_k$, $1 \le k < r \le n$, have been observed. Let G be the group of transformations $x_i = ax_i$ ($i = 1, \ldots, k, r, n, a > 0$). We are now concerned with optimization of the prediction interval for X_r under the loss function (2). Let $\mathbf{X} = (X_1, X_2, \ldots, X_k)$ and $X_r > X_k$ for $r \le n$. Then the joint probability density function of $\mathbf{X}$ and X_r is given by

$$f(x_1, x_2, \ldots, x_k, x_r \mid \sigma) = \frac{n!}{(r-k-1)!(n-r)!}[F(x_r \mid \sigma) - F(x_k \mid \sigma)]^{r-k-1}[1 - F(x_r \mid \sigma)]^{n-r}$$

$$\times \prod_{i=1}^{k} f(x_i \mid \sigma) f(x_r \mid \sigma) = \frac{n!}{(r-k-1)!(n-r)!} \frac{1}{\sigma^{k+1}} \exp\left(- \frac{\sum_{i=1}^{k} x_i + (n-k)x_k}{\sigma} \right)$$

$$\times \left[1 - \exp\left(- \frac{x_r - x_k}{\sigma} \right) \right]^{r-k-1} \left[\exp\left(- \frac{x_r - x_k}{\sigma} \right) \right]^{n-r+1}. \tag{46}$$

Let

$$V_1 = \frac{X_r - X_k}{\sigma}, \; V_2 = \frac{S_k}{\sigma} = \frac{\sum_{i=1}^{k} X_i + (n-k)X_k}{\sigma}. \tag{47}$$

Using the invariant embedding technique [13–18], we then find in a straightforward manner that the joint density of V_1, V_2 is

$$f(v_1, v_2) = f_1(v_1) f_2(v_2), \tag{48}$$

where

$$f_1(v_1) = \frac{[1 - e^{-v_1}]^{r-k-1}[e^{-v_1}]^{n-r+1}}{\mathrm{B}(r-k, n-r+1)} = \frac{1}{\mathrm{B}(r-k, n-r+1)} \sum_{j=0}^{r-k-1} \binom{r-k-1}{j} (-1)^j e^{-v_1(n-r+1+j)}, \; v_1 > 0, \tag{49}$$

and

$$f(v_2) = \frac{1}{\Gamma(k)} v_2^{k-1} e^{-v_2}, \; v_2 > 0. \tag{50}$$

It follows from (15), (49) and (50) that

$$q(w) = \frac{\int_0^\infty v_2^2 f(wv_2, v_2) dv_2}{\int_0^\infty \int_0^\infty v_2 f(v_1, v_2) dv_1 dv_2} = \frac{1}{k} \int_0^\infty v_2^2 f_1(wv_2) f_2(v_2) dv_2 = \frac{k+1}{\mathrm{B}(r-k, n-r+1)}$$

$$\times \sum_{j=0}^{r-k-1} \binom{r-k-1}{j} (-1)^j \frac{1}{[1 + w(n-r+1+j)]^{k+2}}. \tag{51}$$

It follows from (25) and (48) that

$$h(w) = \int_0^\infty v_2 f(wv_2, v_2) dv_2 = \int_0^\infty v_2 f_1(wv_2) f_2(v_2) dv_2$$

$$= \frac{k}{\mathrm{B}(r-k, n-r+1)} \sum_{j=0}^{r-k-1} \binom{r-k-1}{j} (-1)^j \frac{1}{[1 + w(n-r+1+j)]^{k+1}}. \tag{52}$$

If $c / c_1 + c / c_2 < 1$ then the optimal invariant linear-loss interval predictor of X_r based on $\mathbf{X}$ is given by

$$\mathbf{d}^* = (X_k + \eta_1 S_k, X_k + \eta_2 S_k), \tag{53}$$

where

$$\eta_1 = \arg\left(\int_0^{\eta_1} q(w)dw = \frac{c}{c_1} \right), \quad \eta_2 = \arg\left(\int_0^{\eta_1} q(w)dw = 1 - \frac{c}{c_2} \right). \tag{54}$$

The confidence coefficient associated with the optimum prediction interval $\mathbf{d}^* = (d_1,d_2)$, where $d_1 = X_k + \eta_1 S_k$, $d_2 = X_k + \eta_2 S_k$, is given by

$$\Pr\{\mathbf{d}^* : d_1 < X_r < d_2 \mid \mu,\sigma\} = H[\eta_2] - H[\eta_1] = \int_{\eta_1}^{\eta_2} h(w)dw. \tag{55}$$

5. Conclusions

In many statistical decision problems it is reasonable confine attention to rules that are invariant with respect to a certain group of transformations. If a given decision problem admits a sufficient statistic, it is well known that the class of invariant rules based on the sufficient statistic is essentially complete in the class of all invariant rules under some assumptions. This result may be used to show that if there exists a minimax invariant rule among invariant rules based on sufficient statistic, it is minimax among all invariant rules.

Remark. It should be remarked that if we deal, for instance, with within-sample prediction and wish to obtain the best invariant prediction interval $\mathbf{d} = (d_1,d_2)$ for X_r, which has the prescribed confidence coefficient (or level) γ, we have to minimize the risk function $R(\boldsymbol{\theta},\mathbf{d}) = E_{\boldsymbol{\theta}}\{r(\boldsymbol{\theta},\mathbf{d})\}$ under constraint

$$\Pr\{d_1 \le X_r \le d_2\} = \gamma. \tag{56}$$

It can be shown that this problem is reduced to the following one:

Minimize

$$R(\eta_1,\eta_2) = c_1 \int_0^{\infty} \int_0^{\eta_1 v_2} (-v_1 + \eta_1 v_2) f(v_1,v_2)dv_1dv_2$$

$$+ c_2 \int_0^{\infty} \int_{\eta_2 v_2}^{\infty} (v_1 - \eta_2 v_2) f(v_1,v_2)dv_1dv_2 + c(\eta_2 - \eta_1) \int_0^{\infty} \int_0^{\infty} v_2 f(v_1,v_2)dv_1dv_2 \,. \tag{57}$$

Subject to

$$\int_0^{\infty} \int_{\eta_1 v_2}^{\eta_2 v_2} f(v_1,v_2)dv_1dv_2 = \gamma, \tag{58}$$

i.e., the unknown parametric vector $\boldsymbol{\theta}$ is eliminated from the problem.

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OPTIMAL PROPERTIES OF PARAMETRIC SHIRYAEV-ROBERTS STATISTICAL CONTROL PROCEDURES

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Parametric change point detection schemes based on the Shiryaev-Roberts approach have been well addressed in the statistics and engineering literature that consider sequential techniques. High efficiency of such procedures can be partially explained by their known asymptotic optimal properties. Recently, Shiryaev-Roberts based procedures were proposed and examined in applications to the standard AMOC (at most one change) retrospective change point detection problems. The main aim of this article is to review and extend parametric retrospective and sequential Shiryaev-Roberts based policies, carrying out different contexts of the procedures' non-asymptotic optimal properties. We utilize the general principle of the Neyman-Pearson fundamental lemma to show that the Shiryaev-Roberts approach implies the average most powerful procedures. We also propose techniques to construct novel and efficient retrospective tests for multiple change points detection. A real data example based on biomarker measurements is provided to demonstrate implementation and effectiveness of new tests in practice.

Keywords: Shiryaev-Roberts sequential procedure; non-asymptotic optimality; retrospective change point detection; average most powerful

1. Introduction

In this article, we study parametric Shiryaev-Roberts type procedures applied to key problems of the statistical process control issues that include retrospective and sequential change point detection problems. Considerations of these problems are very important in the context of quality and reliability controls, special topics of statistical inference, as well as in experimental and mathematical sciences (e.g., Lai [13]; Gurevich and Vexler [8]).

Firstly, we outline a main principle of the proof related to the Neyman-Pearson fundamental lemma (e.g., Vexler and Gurevich [26]. To this end, let us define $\delta \in [0,1]$ and A, B to be any real numbers. Then, it is clear that

$$(A-B)(I\{A \geq B\}-\delta) \geq 0, \tag{1}$$

where $I(\cdot)$ is the indicator function. This inequality can be easily applied to evaluate optimal properties of decision rules. For example, consider the simple classification problem, where given a sample of k independent and identically distributed observations $X_1,...,X_k$, we want to test the hypothesis

$$H_0 : \mathrm{X}_1,...,X_k \sim F_0 \text{ versus } H_1 : \mathrm{X}_1,...,X_k \sim F_1. \tag{2}$$

Here F_0 and F_1 are known distributions with the density functions $f_0(x)$ and $f_1(x)$, respectively. The inequality (1) determines that the most powerful test for (2) is the likelihood ratio test that rejects H_0 if

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$\prod_{i=1}^{k} f_1(X_i) / \prod_{i=1}^{k} f_0(X_i) \geq C$, where C is a fixed threshold. This classical proposition directly follows from (1), when $A = \prod_{i=1}^{k} f_1(X_i) / \prod_{i=1}^{k} f_0(X_i)$, $B = C$, δ is considered as any decision rule based on the observed sample, and the expectation, under H_0, is derived from both the sides of (1). Although the example mentioned above is very simple, this way of the use of (1) can be applied to show different aspects of optimality related to operating characteristics of complex test-statistics. In this article, we utilize the trivial inequality (1) to provide simple proofs of non-asymptotic optimal properties of retrospective Shiryaev-Roberts procedures. We consider situations related to the retrospective change points detections, proposing accordingly adjusted forms of the retrospective Shiryaev-Roberts procedure. The problem to detect more than one change-point is also analysed in this article. The real data example is provided to demonstrate applicability of the proposed approach in practice. Considering sequential change point problems, we show that any given sequential test can be evaluated via an application of (1) type inequalities that provide optimal properties of this test, but explanations of this optimality corresponding to the classical operating characteristics of tests are complicated tasks. The presented analysis of the sequential Shiryaev-Roberts procedure and its non-asymptotic optimal property clearly demonstrate this issue.

All sections of this article are supplied with brief introductions related to the corresponding problem statements. In Section 2, we consider the retrospective AMOC change point problem and review the techniques addressed in the literature. Theoretical results, which show a non-asymptotic optimal property of the retrospective Shiryaev-Roberts procedures, are also presented. In Section 3, we propose and analyse in details adjusted forms of the retrospective Shiryaev-Roberts procedure for detecting two changes in distributions of independent observations. This section clearly demonstrates how this procedure can be adapted to be used for the multiple change point detection. A real data example introduced in Section 3 demonstrates that the proposed generalized Shiryaev-Roberts procedures can be easily applied in practice. Section 4 contributes results related to a sequential change point problem. We outline here the proof of a non-asymptotic optimality of the sequential Shiryaev-Roberts procedure. We present main conclusions in Section 5.

2. Retrospective Change Point Detection

The scientific literature has shown a significant interest in investigations related to retrospective change point detection problems (e.g., Page [18], [19]; Chernoff and Zacks [1]; James et al. [10]; Gombay and Horvath [2]; Gurevich and Vexler [7], [8]). These problems are directly associated with process capability and are important in biostatistics, engineering, education, economics and other fields (see, e.g., Sen and Srivastava [21]). The literature presents change point detection problems as key issues that belong to testing statistical hypotheses. This section focuses on the problem to detect a change in a distribution of independent data points. These data points are assumed to be observed before that we run requested procedure to analyse the homogeneity of the data points.

Thus, let us set up $X_1,...,X_n$ to be independent observations with density functions $g_1,...,g_n$, respectively. The retrospective change point problem can be formulated via the notation related to hypothesis testing, when we want to test the null hypothesis:

$$H_0 : g_i = f_0 \text{ for all } i = 1,...,n \tag{3}$$

versus the alternative hypothesis:

$$H_1 : g_1 = ... = g_{\nu-1} = f_0 \neq g_\nu = ... = g_n = f_1,\ \nu \text{ is unknown.}$$

The unknown parameter ν, $2 \leq \nu \leq n$, is called a change point. The statistical literature has investigated the problem (3) in parametric and nonparametric settings. In the parametric case of (3), it is assumed that the density functions f_0 and f_1 have known forms that can contain certain unknown parameters (e.g., Chernoff and Zacks [1]; Kander and Zacks [11]; Sen and Srivastava [21]; James et al. [10]; Gombay and Horvath [2]; Gurevich [6]). In the nonparametric case of (3), the functions f_0, f_1 are assumed to be completely unknown (e.g., Wolfe and Schechtman [28]; Gurevich [5]; Vexler and Gurevich [25]). The common distribution-free procedures for change point detection are based on signs and/or ranks and/or U-statistics (e.g., Gombay [3]; Gurevich [5]).

In this article, we attend to the parametric case of the change point problem (3). Such situations are widely addressed in both the theoretical and applied literature. Chernoff and Zacks [1] considered the problem (3) with normally distributed observations. They assumed a uniform prior distribution for the change point ν and suggested a Bayesian approach to construct the detection rule. Kander and Zacks [11] adapted the Chernoff and Zacks's method to be applied to a case based on data from one-parameter exponential families. In this framework, Sen and Srivastava [21] presented a test-statistic utilizing the maximum likelihood methodology; James et al. [10]) proposed decision rules based on likelihood ratios and recursive residuals. Gombay and Horvath [2] considered the general case, defining density functions $f_0(u) \equiv f(u;\theta_0)$, $f_1(u) \equiv f(u;\theta_1)$, $\theta_0 \neq \theta_1$, where the parameters θ_0, θ_1 are unknown. They suggested using the maximal likelihood ratio: $Z_n = \max_{2\le k\le n}(2\log\Lambda_k)$, where

$$\Lambda_k = \frac{\sup_{\theta_0\in\Theta}\prod_{i=1}^{k-1} f(x_i;\theta_0)\sup_{\theta_1\in\Theta}\prod_{j=k}^{n} f(x_j;\theta_1)}{\sup_{\theta_0\in\Theta}\prod_{i=1}^{n} f(x_i;\theta_0)}. \tag{4}$$

The Gombay and Horvath's test rule is to reject H_0 for large values of Z_n.

Following the aims of this paper, let us begin with a consideration related to a simple situation, where density functions f_0 and f_1 are known. In this case, the maximum likelihood estimation of the change point parameter ν employed in the likelihood ratio $\prod_{i=\nu}^{n} f_1(X_i)/f_0(X_i)$ leads us to the well-known CUSUM test (e.g., Gurevich and Vexler [8]). That is, we should reject H_0 if and only if $\max_{1\le k\le n}\prod_{i=k}^{n}\left(f_1(X_i)/f_0(X_i)\right) \ge C$, where $C>0$ is a threshold. Alternatively, Vexler [23] proposed and examined a test based on the Shiryaev-Roberts approach: to reject H_0 if

$$\frac{1}{n}R_n \equiv \frac{1}{n}\sum_{k=1}^{n}\prod_{i=k}^{n}\frac{f_1(X_i)}{f_0(X_i)} \ge C, \tag{5}$$

where $C>0$ is a threshold. Optimal properties of the CUSUM procedure have not been addressed in the retrospective change point literature. Vexler [23] and Vexler and Gurevich [9] showed the following non-asymptotic optimal property of the test (5) for the problem (3). Let P_k and E_k ($k=0,...,n$) respectively denote probability and expectation conditional on $\nu=k$ (the case $k=0$ corresponds to H_0). Setting in (1.1) $A = R_n/n$ and $B = C$ implies

$$\left(\frac{1}{n}R_n - C\right) I\left\{\frac{1}{n}R_n \ge C\right\} \ge \left(\frac{1}{n}R_n - C\right)\delta. \tag{6}$$

Without loss of generality, we suppose that $\delta = 0,\ 1$ is any decision rule based on $X_1,...,X_n$ such that the event $\{\delta=1\}$ leads us to reject H_0. Because of

$$\frac{1}{n}E_0(R_n\delta) = \frac{1}{n}\sum_{k=1}^{n}E_0\left(\prod_{i=k}^{n}\frac{f_1(X_i)}{f_0(X_i)}\delta\right) = \frac{1}{n}\sum_{k=1}^{n}\int\cdots\int\prod_{i=k}^{n}\frac{f_1(x_i)}{f_0(x_i)}\delta\prod_{i=1}^{n}f_0(x_i)\prod_{i=1}^{n}dx_i$$

$$= \frac{1}{n}\sum_{k=1}^{n}\int\cdots\int I\{\delta=1\}\prod_{i=1}^{k-1}f_0(x_i)\prod_{i=k}^{n}f_1(x_i)\prod_{i=1}^{n}dx_i = \frac{1}{n}\sum_{k=1}^{n}P_k(\delta=1),$$

the derivations of H_0 – expectation applied to both the left and right side of (6) provide the next proposition.

Proposition 2.1: The test (5) is the average most powerful test, i.e.

$$\frac{1}{n}\sum_{k=1}^{n} P_k\left(\frac{1}{n}R_n \geq C\right) - CP_0\left(\frac{1}{n}R_n \geq C\right)$$
$$\geq \frac{1}{n}\sum_{k=1}^{n} P_k\left(\delta \text{ rejects } \mathrm{H}_0\right) - CP_0\left(\delta \text{ rejects } \mathrm{H}_0\right),$$

for any decision rule $\delta \in [0,\ 1]$ based on the observations $X_1, X_2, ..., X_n$.

Remark 2.1. The test statistic (5) can be easily modified when the density functions f_0 and f_1 are known up to parameters. For example, one can use an approach to adapt (5) to this case via the mixture technique described below (e.g., Krieger et al. [12]). For instance, consider the problem (3) with a known density function f_0, and $f_1(u) \equiv f_1(u;\theta)$, where θ is an unknown parameter. In this case, we can represent the test (5) following the mixture methodology. That is, we choose a prior $\pi(\theta)$ and pretend that $\theta \sim \pi(\theta)$. Hence, the mixture Shiryaev-Roberts type statistic has the form of

$$\frac{1}{n}R_n^{(1)} \equiv \frac{1}{n}\sum_{k=1}^{n}\int\prod_{i=k}^{n}\frac{f_1(X_i;\theta)}{f_0(X_i)}d\pi(\theta).$$

This definition provides to show the following property of the adapted change point detection scheme:

$$\frac{1}{n}\sum_{k=1}^{n}\int P_k\left\{\frac{1}{n}R_n^{(1)} \geq C \middle| \{X_j\}_{j\geq k} \text{ are from} f_1(X_i;\theta)\right\}d\pi(\theta) - CP_0\left\{\frac{1}{n}R_n^{(1)} \geq C\right\}$$
$$\geq \frac{1}{n}\sum_{k=1}^{n}\int P_k\left\{\delta \text{ rejects } \mathrm{H}_0 \middle| \{X_j\}_{j\geq k} \text{ are from} f_1(X_i;\theta)\right\}d\pi(\theta)$$
$$-CP_0\{\delta \text{ rejects } \mathrm{H}_0\},$$

for any decision rule $\delta \in [0,1]$ based on the observations $X_1, ..., X_n$. This optimality is again obtained using the inequality (1). In this case, the meaning of optimality mentioned in Proposition 2.1 is modified to be integrated over values of the unknown parameter θ with respect to the function π.

The different approach for the case, where $f_0(u) \equiv f(u;\theta_0)$, $f_1(u) \equiv f(u;\theta_1)$, $\theta_0 \neq \theta_1$, is to adapt the CUSUM and Shiryaev-Roberts tests to be the next rules: to reject H_0 if

$$\max_{2\leq k\leq n}\Lambda_k \geq C_1, \tag{7}$$

and

$$\frac{1}{n}\sum_{k=2}^{n}\Lambda_k \geq C_2, \tag{8}$$

respectively, where $C_1, C_2 > 0$ are thresholds and the ratios Λ_k, $k = 2, ..., n$, are denoted in (4). Gurevich and Vexler [8] conducted an extensive Monte Carlo study to compare various change point procedures. The powers of the modified CUSUM test (7) and the modified Shiryaev-Roberts test (8) were compared for different families of the null and the alternative distributions. It was shown that the test (8) is more powerful (not just more powerful in average) than the test (7) in most of considered scenarios. However, when the change point location ν is relatively very close to 1, the test (7) is weakly superior to the test (8). Monte Carlo experiments presented in Gurevich and Vexler [8] also confirmed that the modified Shiryaev-Roberts test statistic (8) is usually more robust than the CUSUM test statistic (7) with respect to misclassifications regarding the data distributions.

Remark 2.2. Considering the change point problem (3) with completely unknown density functions f_0 and f_1, Gurevich and Vexler [8] proposed distribution-free forms of the CUSUM and Shiryaev-Roberts procedures approximating nonparametrically the likelihood ratio's components of the parametric CUSUM and Shiryaev-Roberts test statistics. The authors used Monte Carlo studies to show that comparisons of the nonparametric CUSUM and Shiryaev-Roberts tests give the results similar to those of that related to the parametric tests' comparisons. The nonparametric form of the Shiryaev-Roberts test is more powerful (and always more powerful in average) than that of the CUSUM test in most of scenarios dealt with different data distributions.

The Shiryaev-Roberts procedure (5) can be easily adapted to be a multiple change point detection procedure. In the next section, we propose an extended Shiryaev-Roberts procedure for the two change point detection problems, presenting in details its non-asymptotic properties. We demonstrate an application of the proposed procedure in this section to a real data example.

3. Retrospective Detection of Two Change Points

In this section, we consider the problem to develop a test for

$$H_0 : \mathrm{g_i} = f_0 \text{ for all } i = 1,...,n \tag{9}$$

versus

$$H_1 : \mathrm{g_1} = ... = g_{\nu_1 - 1} = f_1; g_{\nu_1} = ... = g_{\nu_2 - 1} = f_2; g_{\nu_2} = ... = g_n = f_3,$$

ν_1, ν_2 are unknown change points, $2 \le \nu_1 < \nu_2 \le n$. The density functions f_0, f_1, f_2 and f_3 are not necessary known. We propose to apply the following adjusted Shiryaev-Roberts statistic for the problem (9):

$$\frac{1}{n} R_n^{(2)} = \frac{1}{n} \sum_{k_1=1}^{n} \sum_{k_2=k_1}^{n} \frac{\prod_{i=1}^{k_1-1} f_1(X_i) \prod_{j=k_1}^{k_2-1} f_2(X_j) \prod_{l=k_2}^{n} f_3(X_l)}{\prod_{i=1}^{n} f_0(X_i)}.$$

Then, we reject H_0 if

$$\frac{1}{n} R_n^{(2)} > C_\alpha, \tag{10}$$

where C_α is a test threshold at the significance level of α.

3.1. Non-Asymptotic Optimal Properties of the Shiryaev-Roberts Test (10)

Let P_{k_1,k_2} denote probability conditional on $\nu_1 = k_1$ and $\nu_2 = k_2$. The next proposition presents a non-asymptotic property of the proposed test (10).

Proposition 3.1: The proposed test (10) is the average most powerful test for (9) with known density functions f_0, f_1, f_2 and f_3, i.e.

$$\frac{1}{n} \sum_{k_1=1}^{n} \sum_{k_2=k_1}^{n} \mathrm{P}_{k_1,k_2} \left\{ \frac{1}{n} R_n^{(2)} > C_\alpha \right\} \ge \frac{1}{n} \sum_{k_1=1}^{n} \sum_{k_2=k_1}^{n} \mathrm{P}_{k_1,k_2} \{ \delta \text{ rejects } H_0 \},$$

for any decision rule $\delta \in [0,\ 1]$ with fixed $\mathrm{P}_{H_0}\{\delta \text{ rejects } H_0\} = \alpha$ based on the observations $X_1,...,X_n$.

Proof. The corresponding proof scheme is similar to that of Proposition 2.1. That is, using Equation (1) with $A = R_n^{(2)} / n$ and $B = C_\alpha$, we can write

$$\left(\frac{1}{n} R_n^{(2)} - C_\alpha \right) \left(I\left\{ \frac{1}{n} R_n^{(2)} \ge C_\alpha \right\} - \delta \right) \ge 0. \tag{11}$$

Taking the H_0 – expectation on both the sides of (11), we have

$$\frac{1}{n}E_{H_0}\left(R_n^{(2)}I\left(\frac{1}{n}R_n^{(2)}>C_\alpha\right)\right)-C_\alpha E_{H_0}\left(I\left(\frac{1}{n}R_n^{(2)}>C_\alpha\right)\right)\geq\frac{1}{n}E_{H_0}\left(R_n^{(2)}\delta\right)-C_\alpha E_{H_0}\left(\delta\right). \tag{12}$$

It is enough to note that utilizing Equation (12), we can complete the proof, since

$$E_{H_0}\left(R_n^{(2)}\delta\right)=\sum_{k_1=1}^{n}\sum_{k_2=k_1}^{n}E_{H_0}\left\{\frac{\prod_{i=1}^{k_1-1}f_1(X_i)\prod_{j=k_1}^{k_2-1}f_2(X_j)\prod_{l=k_2}^{n}f_3(X_l)}{\prod_{i=1}^{n}f_0(X_i)}\delta\right\}$$

$$=\sum_{k_1=1}^{n}\sum_{k_2=k_1}^{n}\int\cdots\int\frac{\prod_{i=1}^{k_1-1}f_1(X_i)\prod_{j=k_1}^{k_2-1}f_2(X_j)\prod_{l=k_2}^{n}f_3(X_l)}{\prod_{i=1}^{n}f_0(X_i)}\delta\prod_{i=1}^{n}f_0(X_i)\prod_{i=1}^{n}dx_i$$

$$=\sum_{k_1=1}^{n}\sum_{k_2=k_1}^{n}\int\cdots\int 1)\prod_{i=1}^{k_1-1}f_1(X_i)\prod_{j=k_1}^{k_2-1}f_2(X_j)\prod_{l=k_2}^{n}f_3(X_l)\prod_{i=1}^{n}dx_i$$

$$=\sum_{k_1=1}^{n}\sum_{k_2=k_1}^{n}\mathrm{P}_{k_1,k_2}\left\{\delta=1\right\}=\sum_{k_1=1}^{n}\sum_{k_2=k_1}^{n}\mathrm{P}_{k_1,k_2}\left\{\delta\text{ rejects }H_0\right\}.$$

When forms of the density functions f_0, f_1, f_2 and f_3 depend on unknown parameters, one can apply:

a. The mixture approach, in which a class of likelihood ratio test statistics is constructed via the Bayesian methodology (see, e.g., Vexler and Gurevich [26] and Remark 2.1). Let

$$f_s\left(u\right)\equiv f\left(u;\theta_s\right),\ s=0,...,3, \tag{13}$$

where θ_0 is an unknown parameter and the vector of unknown parameters $(\theta_1,\theta_2,\theta_3)$ has a known prior $\pi(\theta_1,\theta_2,\theta_3)$. Then the mixture Shiryaev-Roberts statistic takes the form of

$$\frac{1}{n}R_n^{(3)}=\frac{1}{n}\sum_{k_1=1}^{n}\sum_{k_2=1}^{n}\int\int\int\frac{\prod_{i=1}^{k_1-1}f_1(X_i;\theta_1)\prod_{j=k_1}^{k_2-1}f_2(X_j;\theta_2)\prod_{l=k_2}^{n}f_3(X_l;\theta_3)}{\prod_{i=1}^{n}f_0(X_i;\hat{\theta}_0)}d\pi(\theta_1,\theta_2,\theta_3), \tag{14}$$

where $\hat{\theta}_0=\arg\max_{\theta}\prod_{i=1}^{n}f_0(X_i;\theta)$ is the maximum likelihood estimator, under the null hypothesis, of θ_0 based on the observations $X_1,...,X_n$. The appropriate test rejects H_0 if

$$\frac{1}{n}R_n^{(3)}>C_\alpha, \tag{15}$$

where C_α is a test threshold at the significance level of α. The following proposition presents an optimal property of the test (15).

Proposition 3.2: In a class of any detection rules $\delta\in[0,\ 1]$ based on the observations $X_1,...,X_n$ for the problem (9) with the density functions f_0, f_1, f_2, f_3 presented at (13), the test (15) is the average integrated most powerful test with respect to a prior $\pi(\theta_1,\theta_2,\theta_3)$ for a fixed estimate of the significance level $\hat{\alpha}=\int\delta(x_1,x_2,...,x_n)f_0(x_1,x_2,...,x_n;\hat{\theta}_0)\prod_{i=1}^{n}dx_i$, i.e.

$$\frac{1}{n}\sum_{k_1=1}^{n}\sum_{k_2=k_1}^{n}\int\int\int \mathrm{P}_{k_1,k_2}\left\{\frac{1}{n}R_n^{(3)} > C_{\hat{\alpha}}\right\}d\pi(\theta_1,\theta_2,\theta_3) - C_{\hat{\alpha}}\,\mathrm{P}_{H_0}\left\{\frac{1}{n}R_n^{(3)} > C_{\hat{\alpha}}\right\}$$

$$\geq \frac{1}{n}\sum_{k_1=1}^{n}\sum_{k_2=k_1}^{n}\int\int\int \mathrm{P}_{k_1,k_2}\{\delta \text{ rejects } H_0\}d\pi(\theta_1,\theta_2,\theta_3) - C_{\hat{\alpha}}\,\mathrm{P}_{H_0}\{\delta \text{ rejects } H_0\}\,.$$

Proof. The proof is similar to those of Propositions 2.1 and 3.1.

Example 3.1.1: Let $f_s(x) = f_{N(\mu_s,\sigma_s^2)}(x)$, $s = 0,1,2,3$, where σ_s^2, $s = 0,1,2,3$, are fixed known parameters, μ_0 is an unknown parameter. We assume that the priors for the parameters $\mu_j, j = 1,2,3$, under the alternative hypothesis, are normal densities, i.e. $\mu_j \sim N(\lambda_j, \varphi_j{}^2)$, $j = 1,2,3$. Then the mixture Shiryaev-Roberts statistic (14) is

$$\frac{1}{n}R_n^{(3)} = \frac{1}{n}\frac{\sum_{k_1=1}^{n}\sum_{k_2=1}^{n}\left(\frac{\sigma_1^2}{(k_1-1)}+\varphi_1{}^2\right)^{\frac{-(k_1-1)}{2}}\left(\frac{\sigma_2^2}{(k_2-k_1)}+\varphi_2{}^2\right)^{\frac{-(k_2-k_1)}{2}}\left(\frac{\sigma_3^2}{(n-k_2+1)}+\varphi_3{}^2\right)^{\frac{-(n-k_2+1)}{2}}\exp\left\{-\frac{A_{k_1,k_2}}{2}\right\}}{(\sigma_0)^{-n}\exp\left\{-\sum_{i=1}^{n}(X_i-\bar{X}_0)^2\Big/2\sigma_0^2\right\}},$$

where

$$A_{k_1,k_2} = \frac{(\bar{X}_1-\lambda_1)^2}{(\sigma_1^2/(k_1-1)+\varphi_1{}^2)} + \frac{(\bar{X}_2-\lambda_2)^2}{(\sigma_2^2/(k_2-k_1)+\varphi_2{}^2)} + \frac{(\bar{X}_3-\lambda_3)^2}{(\sigma_3^2/(n-k_2+1)+\varphi_3{}^2)}, \text{ and } \bar{X}_0 = \sum_{i=1}^{n}X_i\Big/n,$$

$$\bar{X}_1 = \sum_{i=1}^{k_1-1}X_i\Big/(k_1-1),\ \bar{X}_2 = \sum_{i=k_1}^{k_2-1}X_i\Big/(k_2-k_1),\ \bar{X}_3 = \sum_{i=k_2}^{n}X_i\Big/(n-k_2+1).$$

b. The estimation approach**,** in which a class of likelihood ratio test statistics, is constructed via the maximum likelihood estimation of the parameters (see Remark 2.1). Let $f_s(u) \equiv f(u;\theta_s)$, $s = 0,...,3$. Then the proposed modified Shiryaev-Roberts statistic has the form of

$$\frac{1}{n}R_n^{(4)} = \frac{1}{n}\sum_{k_1=1}^{n}\sum_{k_2=k_1}^{n}\frac{\sup_{\theta_1\in\Theta}\prod_{i=1}^{k_1-1}f_1(X_i;\theta_1)\sup_{\theta_2\in\Theta}\prod_{j=k_1}^{k_2-1}f_2(X_j;\theta_2)\sup_{\theta_3\in\Theta}\prod_{l=k_2}^{n}f_3(X_l;\theta_3)}{\sup_{\theta_0\in\Theta}\prod_{i=1}^{n}f_0(X_i;\theta_0)}. \tag{16}$$

The appropriate test rejects H_0 if

$$\frac{1}{n}R_n^{(4)} > C_\alpha, \tag{17}$$

where C_α is a test threshold at the significance level of α.

Example 3.1.2. Assume $f_s(x) = f_{N(\mu_s,\sigma_s^2)}(x)$, $s = 0,1,2,3$, where expectations μ_s and variances σ_s^2, $s = 0,1,2,3$, are unknown. Then, the statistic (16) has the form of

$$\frac{1}{n}R_n^{(4)} = \frac{1}{n}\sum_{k_1=1}^{n}\sum_{k_2=k_1}^{n}\frac{\hat{\sigma}_0^{\frac{n}{2}}}{\hat{\sigma}_1^{\frac{(k_1-1)}{2}}\hat{\sigma}_2^{\frac{(k_2-k_1)}{2}}\hat{\sigma}_3^{\frac{(n-k_2+1)}{2}}}, \tag{18}$$

where $\hat{\sigma}_0 = \dfrac{\sum_{i=1}^{n}(X_i - \hat{\mu}_0)^2}{n}$, $\hat{\mu}_0 = \dfrac{\sum_{i=1}^{n} X_i}{n}$; $\hat{\sigma}_1 = \dfrac{\sum_{i=1}^{k_1-1}(X_i - \hat{\mu}_1)^2}{k_1 - 1}$, $\hat{\mu}_1 = \dfrac{\sum_{i=1}^{k_1-1} X_i}{k_1 - 1}$; $\hat{\sigma}_2 = \dfrac{\sum_{j=k_1}^{k_2-1}(X_j - \hat{\mu}_2)^2}{k_2 - k_1}$,

$\hat{\mu}_2 = \dfrac{\sum_{j=k_1}^{k_2-1} X_j}{k_2 - k_1}$; $\hat{\sigma}_3 = \dfrac{\sum_{\ell}^{n}(X_\ell \quad \hat{} \quad)^2}{n - k_2 + 1}$, $\hat{\mu}_3 = \dfrac{\sum_{\ell}^{n} X_\ell}{n - k_2 + 1}$.

3.2. A Real Data Example

In this example, we demonstrate that the proposed test (17) can be easily applied in practice. We apply the proposed method to analyse data from a study that was shown in Wians et al. [27]. The same data set was utilized by Obuchowski [16] and Tian et al. [22]. These authors compared the diagnostic abilities of different rapid blood test-scores, including per cent transferrin saturation (%TS) and total iron binding capacity (TIBC), for determining blood iron concentrations. The data set was composed of 134 patients (55 females and 79 males) with anaemia who underwent the series of blood test-scores. Following previous works of Obuchowski [16] and Tian et al. [22], we focus on only the %TS and TIBC blood test-scores and limit the analysis to 55 female anaemia patients. The plots and the empirical histograms based on the %TS and TIBC data are displayed on Figure 1. Tian et al. [22] categorized the study subjects into three groups based on the results of ferritin concentration that provides a useful screening test for iron deficiency anaemia (IDA). Non-pregnant women with anaemia and a ferritin concentration less than 20 $(\mu g / L)$ were assigned to the IDA group, while those with anaemia and a ferritin concentration greater than 240 $(\mu g / L)$ were assigned to be in the anaemia of chronic disease (ACD) group. The intermediate group consists of the women with ferritin concentration between 20 and 240 $(\mu g / L)$. There were 29, 14, 12 female anaemia patients in IDA, intermediate, ACD groups, respectively. The histograms of the %TC data and those of the TIBC measurements in each group are shown on Figures 2 and 3, respectively. Our interest is to detect if the underlying distributions of the %TC data as well as the distributions of the TIBC measurements change at two different points. In this section, we will formally test for the assumption made by Tian et al. [22] that suggested to consider the %TS measurements as three groups as well as the TIBC measurements split into the three groups, i.e. there are two change points in the distribution of the %TS and also two change points in the TIBC measurements' distribution.

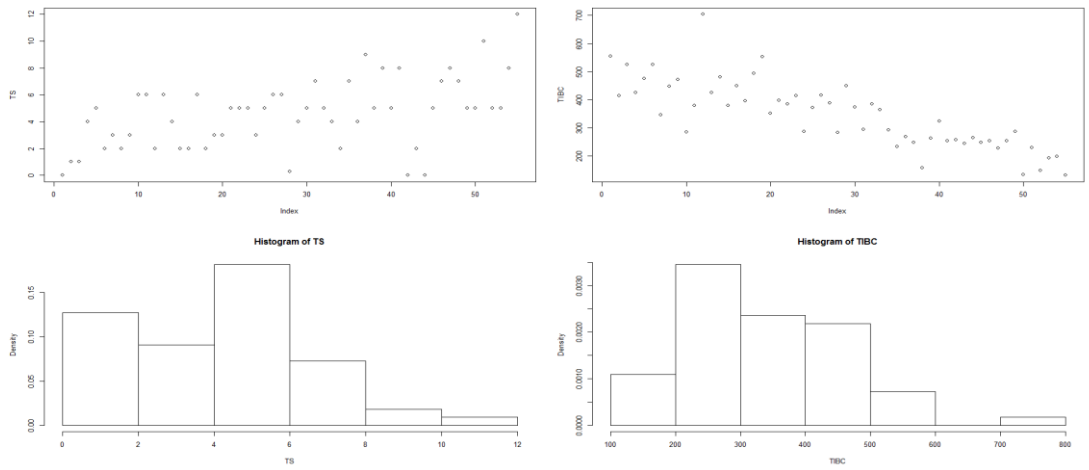

Figure 1. The left-hand side of the Figure 1 shows plots and histograms of %TS data; the right-hand side of the Figure 1 shows plots and histograms of TIBC data

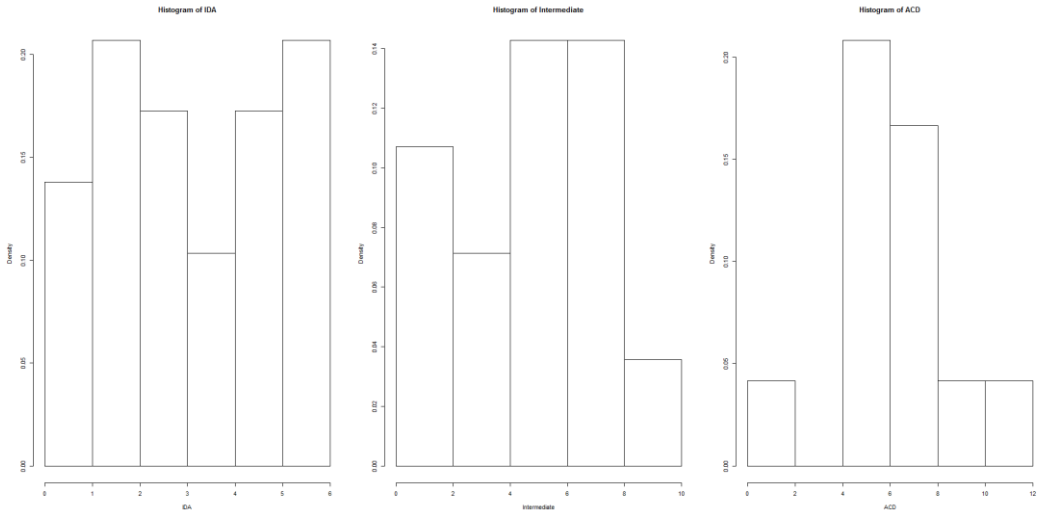

Figure 2. .Histograms of %TS data in each group

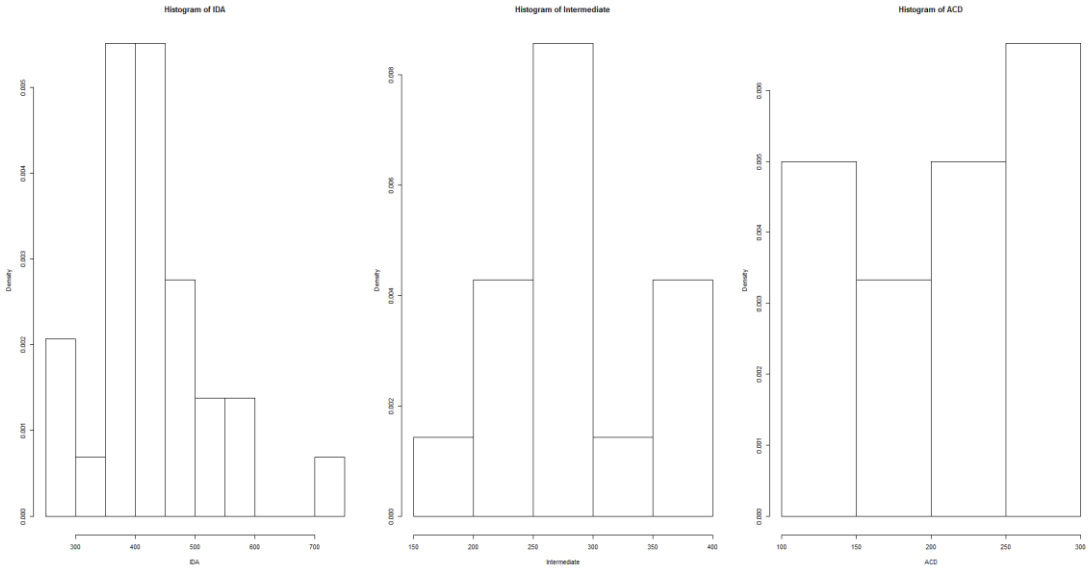

Figure 3. Histograms of TIBC data in each group

Following the publications mentioned above in this section, we assume the %TS and TIBC data distributed normally. To this end, we apply the test based on the statistic (18). The mean and standard deviation of the %TS data are 4.55 and 2.59, respectively, whereas the mean and standard deviation of the TIBC observations are 345 and 120 $(\mu g / L)$, respectively. The means and standard deviations of the %TS and TIBC data in each group are presented in Table 1.

Table 1. Means and standard deviations of the %TS data and the TIBC data in each group

Group	IDA	intermediate	ACD
Sample size *n*	29	14	12
Mean	3.5276	5.0714	5.7500
Standard deviation	1.8820	2.5859	2.0505

To approximate the *p*-value of the test (17), where the statistic $R_n^{(4)} / n$ is defined by (18), we propose the following methods.

3.2.1. The methods for the test (17)'s p-value approximation

In this section, we propose and apply three different methods for *p*-value approximation related to the test (17) with the statistic $R_n^{(4)} / n$ by (18).

1) The Monte Carlo technique. Since, given that observations follow a normal distribution, the construction of the test statistic (18), under the null hypothesis, does not depend on parameters μ_0 and σ_0^2 of the null normal distribution, we can conduct the Monte Carlo study to obtain the *p*-value of the test. To execute the Monte Carlo experiment, we first draw 50,000 replicate samples of 55 observations $X_i \sim N(0,1)$, $i = 1,...,55,$ and accordingly evaluate the generated values of the test statistics, say, $r_j = R_{n=55}^{(4)} / 55$ at one generation of $X_1,...,X_{55}$, $j = 1,...,50,000$. Let r be the observed test statistic value based on the data. Then we determine the approximate *p*-value of the test as the proportion of cases when values of r_j, $j = 1,...,50,000$, exceed the value of r. Following the procedures mentioned above, we obtain the *p*-value of 0.0244 based on the %TS data and the *p*-value close to zero based on the TIBC measurements (*p*-value < 0.0001). Both the *p*-values are less than the significance level of $\alpha = 0.05$; therefore, we recommend to reject the null hypothesis, implying that there are changes at two time points in both the %TS and TIBC observation distributions.
2) Bootstrap calibration. The procedure of the bootstrap calibration (e.g., Owen [17]) is defined as follows. Let X_i^{*b}, $i = 1,...,n$, $b = 1,...,B$, be independent random vectors sampled from the empirical distribution function F_n of the data X_i, $i = 1,...,n$. This resampling can be implemented by drawing n random integers ℓ independently from the uniform distribution *Unif* [1, n], and setting $X_i^{*b} = X_\ell$. We use $n = 55$ and $B = 10,000$. Now let $H_b = R_{n=55}^{(4)}(X_1^{*b},...,X_{55}^{*b}) / 55$. This defines the order statistics $H_{(1)} \le H_{(2)} \le ... \le H_{(B)}$. Then, the critical value of the test at the significance level of $\alpha = 0.05$ is $H_{(9,500)}$. The *p*-value of the test can be evaluated by obtaining $q : H_{(q-1)} \le r \le H_{(q)}$, where r is a value of the test statistic based on the original data set, and $(1 - q/n)$ approximates the *p*-value. The bootstrap procedure gives the corresponding *p*-values based on the %TS data and the TIBC measurements as 0.003 and 0.0001, respectively. Both the *p*-values are less than the significance level of $\alpha = 0.05$, supporting the conclusion that the underlying distributions of the %TC and TIBC measurements have significant changes at two different points.
3) The permutation method. This procedure (e.g., Good [4]) is defined as follows. We randomly sample data with 55 variables from the 55 observations without replacement 10,000 times. Then we compute the value of the test statistic in each sample, which is denoted by M_i, $i = 1,...,10,000$. Define the corresponding order statistics by $M_{(1)} \le M_{(2)} \le ... \le M_{(10,000)}$. Consequently, the *p*-value of the test can be estimated by using the value of *w* such that $w : M_{(w-1)} \le r \le M_{(w)}$, where r is a value of the test statistic based on the original data set, and $(1 - w/n)$ approximates the *p*-value. By using the permutation test, the *p*-value based on the %TS data is 0.0008 and the *p*-value based on the TIBC measurements is close to zero (*p*-value < 0.0001). Again, both the *p*-values that are less than the significance level $\alpha = 0.05$ confirm the rejection of the null hypothesis that the %TC data has no change in the distribution as well as the null hypothesis that there is no change in the TIBC data distribution.

Therefore, three methods suggest rejecting the null hypothesis. Note that, for the method 1), it is important that the observations under the null hypothesis are independent and identically normally distributed, whereas for methods 2) and 3), the observations, under the null hypothesis, are assumed to be just independent and identically distributed (*i.i.d.*). Hence, in the case where data are close to being normally distributed, the type I errors of methods 2) and 3) will be very close to results for method 1).

3.2.2. Additional study

In this subsection, we consider a situation when no change is expected in the real data distributions. Now we test the hypotheses (9) based on the %TS observations in the IDA group (n = 29). By using the Monte Carlo study, the bootstrap calibration, and the permutation method as mentioned above, we obtain that the corresponding p-values are 0.4118, 0.1136, and 0.0921, respectively. These results suggest that the distribution of the %TS data in the IDA group has no significant change in this case. Similarly, by applying the Monte Carlo study, the bootstrap calibration, and the permutation method, the p-values based on the TIBC observations in the IDA group, are 0.5376, 0.6533, and 0.6662, respectively. These p-values indicate that there is no significant change in the distribution of the TIBC data in the IDA group in this case.

4. Sequential Change Point Detection

There are extensive references in the statistics and engineering literature on the subject of quick detection, with low false alarm rate, of changes in stochastic systems on the basis of sequential observations from the system. These problems are very important in the context of quality and reliability controls (e.g., Lai [13]).

In many common situations, we assume that we survey sequentially independent observations $X_1, X_2, \ldots$. Initially, the observations follow an in-control distribution with a density function f_0. It is possible that at ν -time, an unknown point in time, an accident is in effect, causing the distribution of the observations to change to an out-of-control distribution with a density function f_1.

A common performance measure for any inspection scheme is the in-control average run length (ARL). Let T be the random variable corresponding to the time when the scheme signals that the process is out of control (distribution of the observations has changed), which henceforth will be referred to as the stopping time. Thus, T is the number of observations until the alarm signal. The in-control ARL is defined by $E_{f_0}(T)$, whereas the out-of-control ARL is defined by $E_{f_1}(T)$, where we define by $E_f(T)$ the expectation of the stopping time T under the assumption that the observations come from a distribution with a density function f. Clearly, one desires $E_{f_0}(T)$ to be large and $E_{f_1}(T)$ to be small. In the literature, a proposed index of the speed of detection is $E_\nu(T-\nu+1|T\geq\nu)$. The latter is the expectation of the delay in detection given that the change is at point ν in time, and given that the stopping time T is larger than ν.

In this section, we consider the observations $X_1, X_2, \ldots, X_{\nu-1}$ to be distributed according to a density function f_0, whereas $X_\nu, X_{\nu+1}, \ldots$from a density function f_1, with an unknown ν $(1\leq\nu\leq\infty)$. The case $\nu=\infty$ indicates the situation, when all observations are distributed according to f_0. In this case, the notations P_∞ and E_∞ denote probability and expectation, respectively, when all observations are distributed according to f_0. The sequential change point detection procedures are assumed to raise an alarm as soon as possible after the change, avoiding false alarms. It is well known that CUSUM and Shiryaev-Roberts procedures are efficient detection methods for this stated problem (e.g., Moustakides [15]; Mei [14]; Gurevich and Vexler [9]). The CUSUM policy is: we stop sampling of X s and report that a change in distribution of X has been detected at the first time $n\geq 1$ that $\max_{1\leq k\leq n}\prod_{i=k}^{n}(f_1(X_i)/f_0(X_i))\geq C$, for a given threshold C; similarly, the Shiryaev-Roberts procedure can be defined via the stopping time

$$T_C=\inf\{n\geq 1: R_n\geq C\}, \tag{19}$$

where the Shiryaev-Roberts test-statistic R_n is

$$R_n=\sum_{k=1}^{n}\prod_{i=k}^{n}\frac{f_1(X_i)}{f_0(X_i)}. \tag{20}$$

The sequential CUSUM detection procedure has a non-asymptotic optimal property (e.g., Moustakides [15]). That is, if the initial and the final distributions of the observations are known, then the CUSUM control procedure most rapidly detect a change in distribution among all procedures with a common bound specifying an acceptable rate of false alarms, i.e. in-control ARL. For the Shiryaev-Roberts procedure, an asymptotic (as $C \to \infty$) optimality has been shown (Pollak [20]). To demonstrate the optimality of the Shiryaev-Roberts detection scheme, Pollak [20]) proved an asymptotic closeness of the expected loss using a Bayes rule for the change problem, with a known prior distribution of ν, to that using the rule T_C.

However, in the context of simple application of the inequality (1), the procedure (19) declares loss functions for which that detection policy is optimal. That is, setting $A = R_{\min(T_C,n)}$ and $B = C$ in (1) leads to

$$\left(R_{\min(T_C,n)} - C\right)\left(I\left\{R_{\min(T_C,n)} \geq C\right\} - \delta\right) \geq 0,$$

for all $\delta \in [0,\ 1]$. Because of $\left\{R_{\min(T_C,n)} \geq C\right\} = \left\{T_C \leq n\right\}$, we have

$$\left(R_{\min(T_C,n)} - C\right)\left(I\left\{R_{\min(T_C,n)} \geq C\right\} - \delta\right)$$

$$= \sum_{k=1}^{n}\left(R_k - C\right)\left(1-\delta\right) I\left\{T_C = k\right\} + \left(R_n - C\right)\left(-\delta\right) I\left\{T_C > n\right\} \geq 0. \tag{21}$$

It is clear that (21) can report an optimal property of the detection rule T_C. For simplicity, noting that every summand in the left side of the inequality (21) is non-negative, we can focus only on $\left(R_n - C\right)\left(-\delta\right) I\left\{T_C > n\right\} \geq 0$.Thus, if τ is defined to be a stopping time and $\delta = I\left\{\tau \leq n\right\}$, then $E_\infty\left(C - R_n\right) I\left\{\tau \leq n,\ T_C > n\right\} \geq 0$.

This and definition (20) imply that

$$C\left(P_\infty\left\{T_C > n\right\} - P_\infty\left\{\min\left(\tau, T_C\right) > n\right\}\right) - \sum_{k=1}^{n}\left(P_k\left\{T_C > n\right\} - P_k\left\{\min\left(\tau, T_C\right) > n\right\}\right) \geq 0.$$

Therefore,

$$C\sum_{n=1}^{\infty}\left(P_\infty\left\{T_C > n\right\} - P_\infty\left\{\min\left(\tau, T_C\right) > n\right\}\right) - \sum_{n=1}^{\infty}\sum_{k=1}^{n}\left(P_k\left\{T_C > n\right\} - P_k\left\{\min\left(\tau, T_C\right) > n\right\}\right) \geq 0, \tag{22}$$

where

$$\sum_{n=1}^{\infty}\sum_{k=1}^{n}\left(P_k\left\{T_C > n\right\} - P_k\left\{\min\left(\tau, T_C\right) > n\right\}\right) \tag{23}$$

$$= \sum_{k=1}^{\infty}\sum_{n=k}^{\infty}\left(P_k\left\{T_C > n\right\} - P_k\left\{\min\left(\tau, T_C\right) > n\right\}\right)$$

$$= \sum_{k=1}^{\infty}\left(E_k\left\{T_C - k + 1\right\}^+ - E_k\left\{\min\left(\tau, T_C\right) - k + 1\right\}^+\right),$$

$\left(a^+ = aI\left\{a \geq 0\right\}\right)$. The inequality (22) with (23) gives the next proposition.

Proposition 4.1: The Shiryaev-Roberts policy (19) satisfies

$$\sum_{n=1}^{\infty} E_n\left(T_C - n + 1\right)^+ + \left(-CE_\infty T_C\right) = \min_{\tau}\left[\sum_{n=1}^{\infty} E_n\left(\min\left(\tau, T_C\right) - n + 1\right)^+ + \left(-CE_\infty \min\left(\tau, T_C\right)\right)\right].$$

Here, $E_\infty(\tau)$ presents the average run length to false alarm of a stopping rule τ. Small values of $-E_\infty(\tau)$ are privileged, whereas small values of $E_n(\tau - n + 1)^+$ are also preferable (because $E_n(\tau - n + 1)^+$ relates to fallibility of the sequential detection in the case $\nu = n$). It is clear that, if $\nu < \infty$, then $\min(\tau, T_C)$ detects that $\nu < \infty$ faster than the stopping time T_C. Consequently, Proposition 4.1 states the non-asymptotic optimal property of the Shiryaev-Roberts sequential procedure in the context of series of delays in the detection, considering the expectation $E_\nu(T - \nu + 1)^+$ as the index of the speed of the detection.

5. Conclusions

The main goal of this article has been to review the parametric Shiryaev-Roberts procedures. Firstly, we introduced the general principals related to the retrospective change point problems. We provided schemes to construct the Shiryaev-Roberts type procedures corresponding to different change point problems. Although we consider the relatively simple statement of the problem (3), with independent observations, in a similar manner to constructions of Shiryaev-Roberts procedures mentioned in this article, complex regression models (see, e.g., Vexler and Gurevich [24]) can be evaluated. The Shiryaev-Roberts based procedures are appropriate to replace the classical CUSUM policies in many practical applications, because the Shiryaev-Roberts based procedures are shown to demonstrate optimal properties. We also proposed the Shiryaev-Roberts procedure for detecting two changes in the sequence of independent observations, representing a way of developing Shiryaev-Roberts-type procedures for multiple change point detection. In this article, the real data example presents the applicability of the proposed technique for detecting two possible changes in the biomarker measurements' distributions. In this example, we pointed out three different methods for estimating the *p*-values of the Shiryaev-Roberts-type test. These methods are general and can be applied to estimate the *p*-values of other Shiryaev-Roberts-type tests.

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CORRELATION BETWEEN COEFFICIENT OF FRICTION AND SURFACE ROUGHNESS IN DRY SLIDING WEAR OF AISI 316 L (N) STAINLESS STEEL AT ELEVATED TEMPERATURES

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In this paper, coefficient of friction in dry sliding wear at different temperatures has been correlated with surface roughness (R_a) of the wear tracks. Unlubricated pin-on-disc sliding wear tests were carried out on AISI Type 316 L (N) austenitic stainless steel up to 550ºC at constant load (20 N) and sliding speed (0.8 m/s) as per the ASTM standard G99-05. Line profiling along radial directions across the wear track was carried out and the line profiles were analysed to calculate surface roughness of the wear tracks. AISI Type 316 L (N) austenitic stainless steel is a major structural material in the prototype fast breeder reactor (PFBR), Kalpakkam because of its good high temperature properties and compatibility with liquid sodium. This experimental work revealed a one to one correlation between coefficient of friction and surface roughness of wear tracks. Coefficient of friction as well as surface roughness during dry sliding wear increased with increase in temperature.

Keywords: Coefficient of friction; AISI 316 L (N) stainless steel; high temperature, roughness

1. Introduction

AISI Type 316 L (N) austenitic stainless steel is a major structural material in the prototype fast breeder reactor (PFBR), Kalpakkam because of its good high temperature properties and compatibility with liquid sodium. The frictional coefficient of unlubricated type 316 austenitic stainless steel tested in room temperature was in the range 0.4–0.6. The pattern of variation had sudden amplitude fluctuations and no systematic trend could be detected [1]. The sliding wear of 304 and 310 stainless steels against M2 tool steel rings showed that the strain-induced martensite transformation. The formation of α′-martensite in 304 steel gives a lower average value of friction coefficient with large fluctuations [2] (Yang et al., 1985). The sum of the wear rates on a pin and a mated disk, the wear rate on a pin and the mean coefficient of friction increased with the mean acoustic emission event counting rates under various lubricated conditions [3] (Hisakado and Warashina, 1998). The load and sliding speed dependencies of the coefficient of friction and temperature were obtained on specimens from austenitic and martensitic steels and severe adhesive wear (seizure) conditions were resulted [4] (Tarassov and Kolubaev, 1999). Measurements of friction coefficients made in dry sliding pin-on-disc tests with steel specimens revealed that coefficient of friction increased as debris increased [5] (Sherrington and Hayhurst, 2001). Impact/sliding methodological wear tests were performed at room temperature on stainless steel claddings (304 L). Worn samples were examined by 2D profilometry [6] (Van Herpen et al., 2001). Friction tests results of AISI 1006 low-carbon steel and AISI 52100 bearing steel using a spiral pin-on-disk apparatus concluded that metals with high hardness resulted in low friction coefficient values [7] (Pintaude et al., 2003). The study of tribological properties of AISI 304 austenitic stainless showed that the decrease of relative humidity in wear tests promoted increase in weight loss and friction coefficient[8] (Bregliozzi et al., 2003). The coefficient of friction, formation of transfer layer, and the presence of stick–slip motion depend on the surface texture of hard surfaces in 080 M40 (EN8) steel plates [9] (Pradeep, 2006). Friction behaviour study of SUS 304 austenitic stainless steel (ASS) disc against GCr15 bearing steel ball showed that there was an initial gestation period and later friction coefficient increased rapidly with the normal load [10] (Hua et al., 2008). The wear behaviour at high sliding speeds of metal injection moulded 316 L stainless steel under dry sliding conditions resulted with adhesive-induced delamination [11] (Kanchanomai et al., 2009). The microstructure evolution of the worn surface layer of SUS 304 austenitic stainless steel (ASS) disk against Al_2O_3 ceramic ball revealed the martensitic transformation [12] (Xicheng et al., 2009). Comparative friction wear tests to assess the wear resistance and wear mechanism

of Fe–14Mn–5.5Si–12Cr–5Ni–0.10C and AISI 321 stainless steel were carried out [13](Jianjian et al., 2009). The sliding friction and wear behaviour of D9 alloy showed that damage is more in specimens tested at 823K due to the softening of surfaces. Friction coefficient was lower at 473K than at 823 K [14] (Hemant Kumar et al., 2010). Six common types of metallic wear particles, i.e., cutting, spherical, rubbing, laminar, fatigue chunk and severe sliding particles, have been studied and reported on the features of their boundary morphology and surface topography [15] (Peng and Kirk, 1998). Surface roughness parameters were categorised into three groups according to its functionality namely amplitude, spacing and hybrid parameters [16] (Gadelmawla, 2002). In the knowledge of 3D microtopography, asperities were statistically processed. Asperities were replaced by paraboloid and pyramidal surfaces, in order to determine the distribution of the direction angle of asperities and other related parameters [17] (Palásti-Kovács et al., 2004). Operational characteristics of technical surfaces are greatly influenced by microtopographical features [18] (Váradi et al., 2004). Numerical parameters, *R*a, *R*q and *R*sk were used to measure the evolutions of the surface alternations from the running-in to steady state wear stage [19] (Yuan et al., 2004). The evolution of the surface morphology of wear debris in relation to change in the surface morphology of wear components in sliding wear process showed a good correlation [20] (Yuan et al., 2008). Surface microtopography plays a dual role in the course of friction and wear processes. Fractal dimension (D_f), root mean square gradient (S_{dq}), surface area ratio (S_{dr}) and surface kurtosis (S_{ku}) parameters of microtopographies helps in correlating wear processes with amplitude of the roughness [21] (Bárányi et al., 2010). In this paper, dry sliding wear experiments were carried in various temperatures with constant load and sliding velocity. The variation of Coefficient of friction with respect to different operating temperatures was recorded. The worn track surfaces were correspondingly analysed by a profilometer. The measured roughness parameters were correlated with coefficient of friction during dry sliding at various test temperatures. The experimental details are given in Section 2. Results of wear experiments and surface profiling are discussed in Section 3 followed by conclusions in Section 4.

2. Experimental Procedure

2.1. Materials

AISI 316 L (N) stainless steel has faced centred cubic (FCC) crystalline structure. Type 316 L (N) is a low carbon, nitrogen-enhanced version of Type 316 austenitic stainless steel. Lower carbon content brings in enhanced resistance to sensitization and addition of nitrogen provides some solid solution hardening, raising its minimum specified yield strength compared to Type 316 L stainless steel. The chemical composition of the tested material is given in Table 1.

Table 1. Chemical composition of AISI type 316 L (N) stainless steel

Elements	C	Cr	Ni	Mo	Mn	Si	N	S	P	Cu
Weight, %	0.024	17.8	13	2.38	1.84	0.25	0.05	0.005	0.03	0.519

2.2. Wear Testing Procedure

Pin-on-Disc (POD) sliding wear experiments as per ASTM G99-05 standard were carried out in DUCOM make TR- 20-M12EV high temperature pin on disk Tribometer. AISI 316 L (N) Hemispherical pins of 5 mm radius of curvature were mated against 130 mm diameter and 10 mm thick discs of the same material. The pin was held stationary and the disc was rotated. All the tests were carried out at a constant load of 20 N for 800 seconds each at constant sliding velocity of 0.8 m/s resulting in total sliding distance of 640 m in each test. The test temperatures were room temperature (25°C), 100°C, 150°C, 250°C, 350°C, 450°C and 550°C. The experimental parameters are summarized in Figure 1. Dead weights were applied on the pin and the frictional force between the pin and the disc was measured using a load cell positioned perpendicular to the pin loading path. Coefficient of friction was calculated as the ratio of the measured frictional force to the applied pin load. Pin and disc temperatures were measured using K-Type thermocouples.

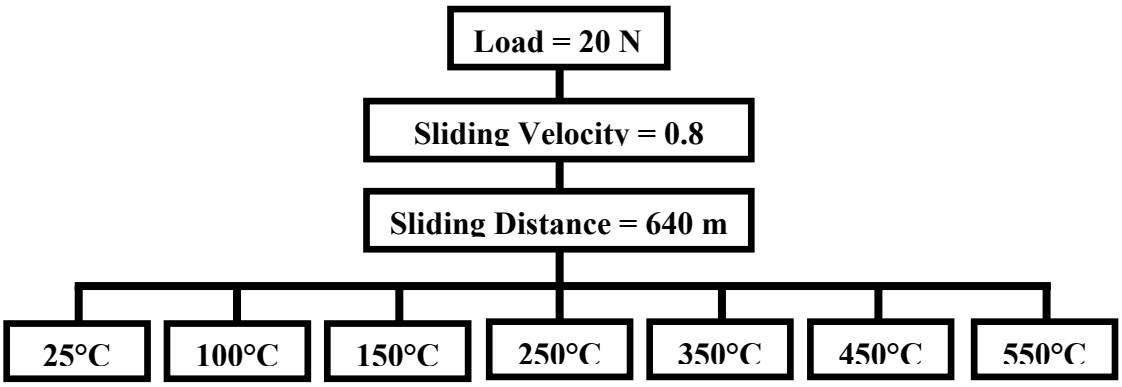


Figure 1. Experimental parameters

2.3. Surface Profilometry

Roughness is typically calculated on the high frequency, short wavelength component of a measured profile. The roughness parameter R_a represents the arithmetic mean of the profile calculated from absolute values of profile amplitudes and arithmetic mean of the measured profile. Increase in R_a represents increase in roughness of the surface. Coefficient of friction increases with increase in surface roughness and hence correlation between R_a and Coefficient of friction would be a good predictor of the performance of a mechanical component, since irregularities in the surface may form nucleation sites for cracks or corrosion. A representative radial line of 5mm length across the wear track of each tested AISI 316 L (N) disc was scanned using Talysurf CLI 1000 surface profilometer. Line profiling was done using non contact high resolution confocal point gauge. The gauge had a range of 3000 μm with 0.25 nm resolution. The gauge focussed a beam on the surface through a lens with chromatic length aberration and due to the aberration, the focus points were at different Z-positions for different wavelengths. A built-in spectrometer received the reflected light through a pin hole and provided an intensity curve depending on wavelength. The focused wavelength was the one corresponding to the maximum intensity (Chris Phillips, 2005). The surface profiles were analysed using Talymap Platinum software version 4.1.

3. Results and Discussions

3.1. Characterisation of Coefficient of Friction

Figures 2 to 8 show the Coefficient of friction recorded during dry sliding wear of AISI 316 L (N) disc and pin from room temperature to 550^{o}C. Oscillations on friction force tracings recorded during pin-on-disc experiments were found to be a result of non-uniform surface features and lubrication conditions along the circular wear track. At room temperature Coefficient of friction peaked to a higher value during initial part of the sliding and then it stabilized to a steady state with average Coefficient of friction value of 0.37. At higher temperatures the Coefficient of friction values showed oscillating pattern unlike that at room temperature. Average Coefficient of friction varied from 0.39 at 100^{o}C to 0.7 at 550^{o}C. In dry sliding wear of AISI 316 L (N) stainless steel, the primary wear mechanism is adhesive wear in which the worn surface layers adhere to the wear track. As wear increased at elevated temperatures, adhesion also increased. This resulted in increase in roughness of the wear tracks, which, in turn, led to increase in Coefficient of friction. Figure 9 presents a consolidated view of the variation of coefficient of friction during dry sliding wear with respect to the operating temperatures. With increase in temperature the contact area of disc and pin increases. The wear debris generated during wear act as abrasive material which gradually transform the two body abrasive mechanism into three body abrasive mechanism leading to more material loss at elevated temperatures.

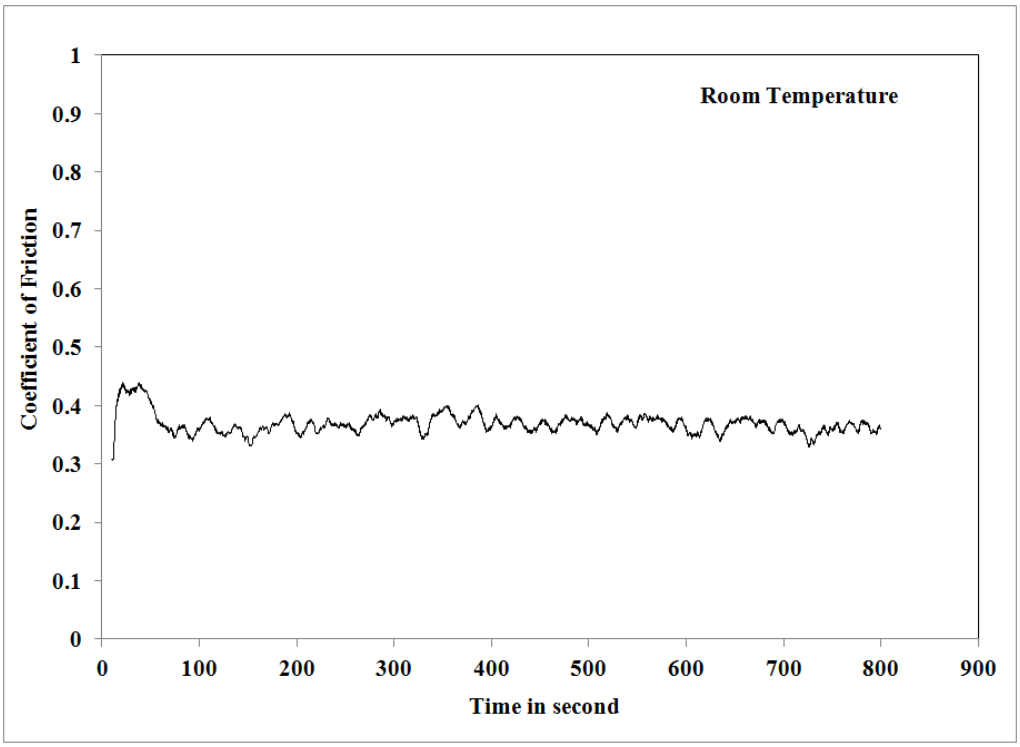


Figure 2. Variation of Coefficient of friction with respect to time at Room temperature

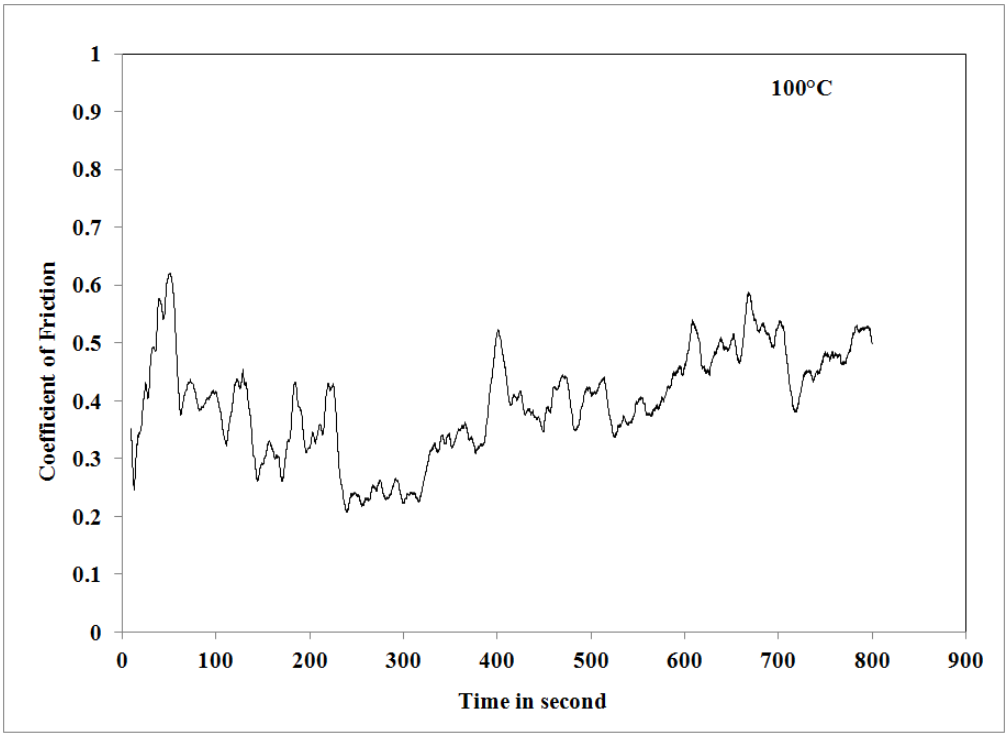


Figure 3. Variation of Coefficient of friction with respect to time at 100°C

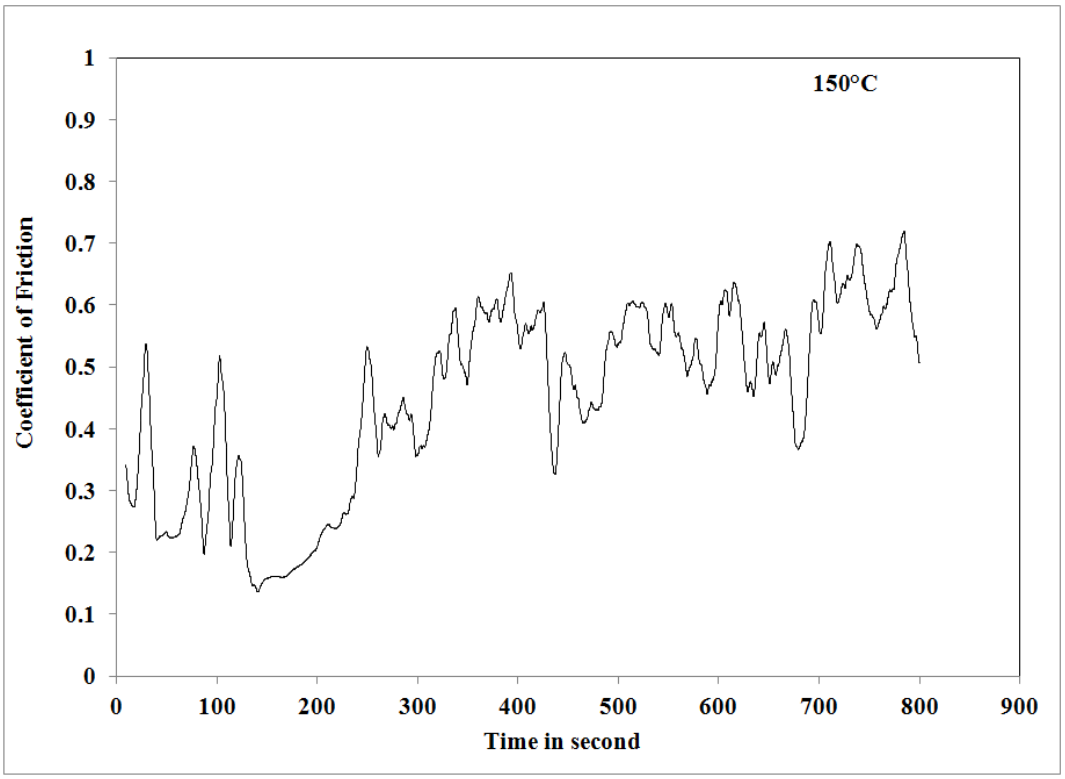


Figure 4. Variation of Coefficient of friction with respect to time at 150°C

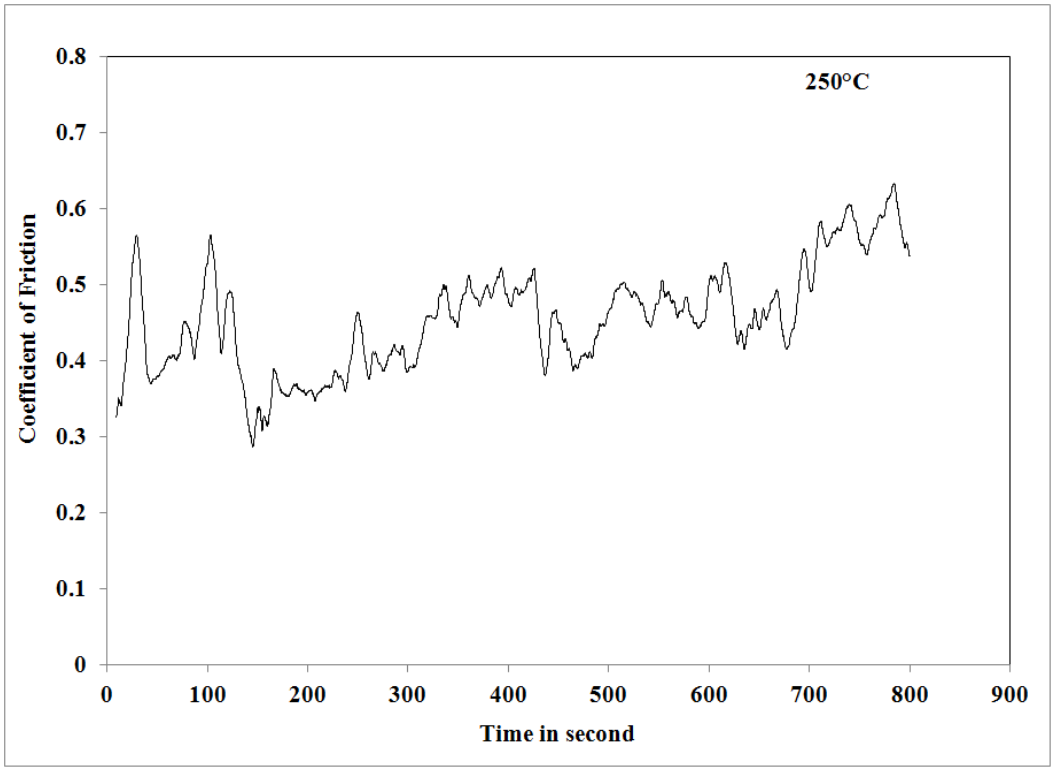


Figure 5. Variation of Coefficient of friction with respect to time at 250°C

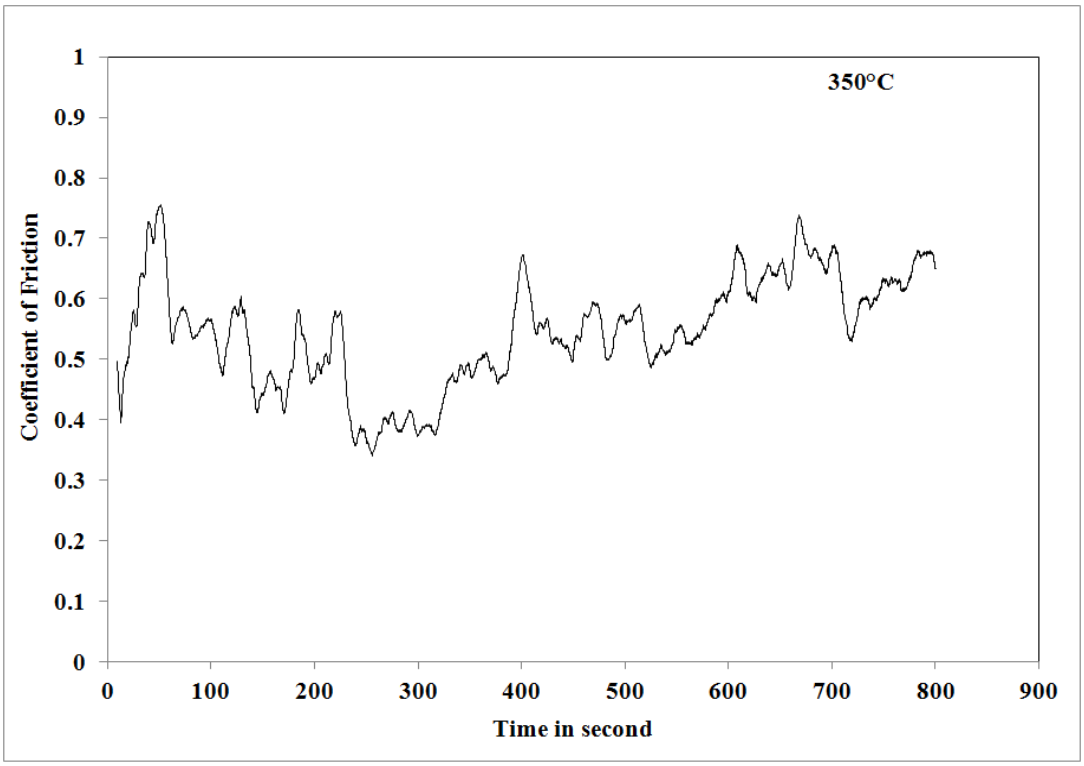


Figure 6. Variation of Coefficient of friction with respect to time at 350°C

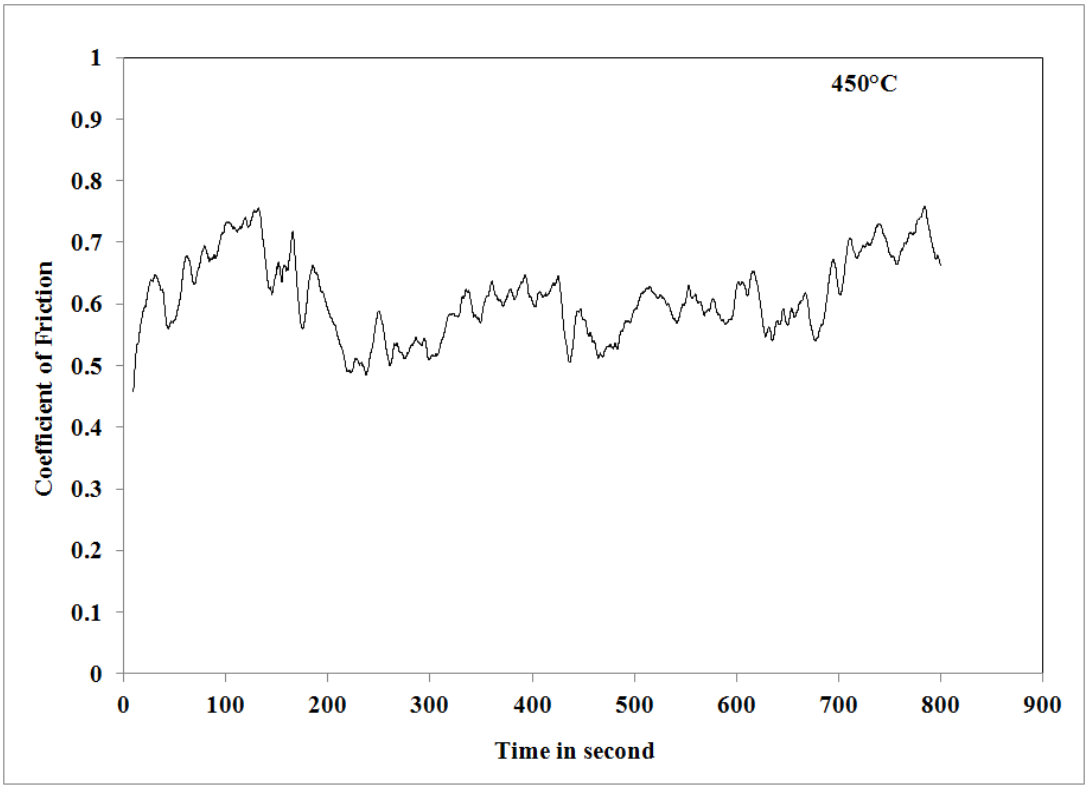


Figure 7. Variation of Coefficient of friction with respect to time at 450°C

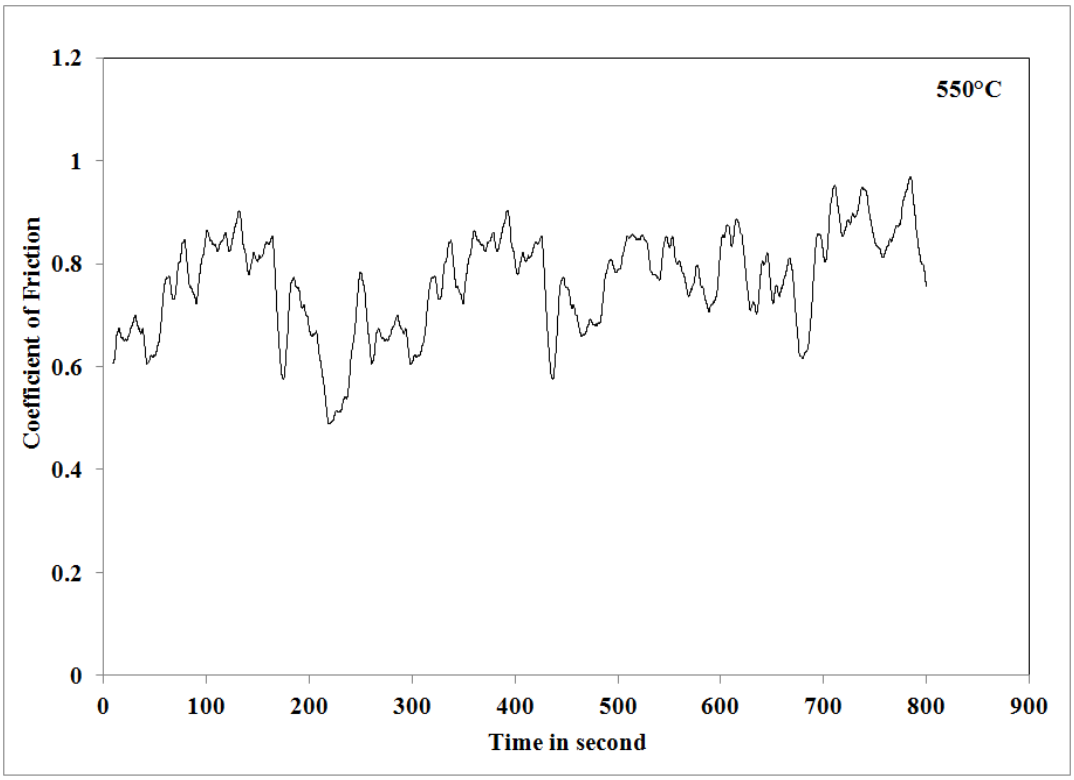


Figure 8. Variation of Coefficient of friction with respect to time at 550°C

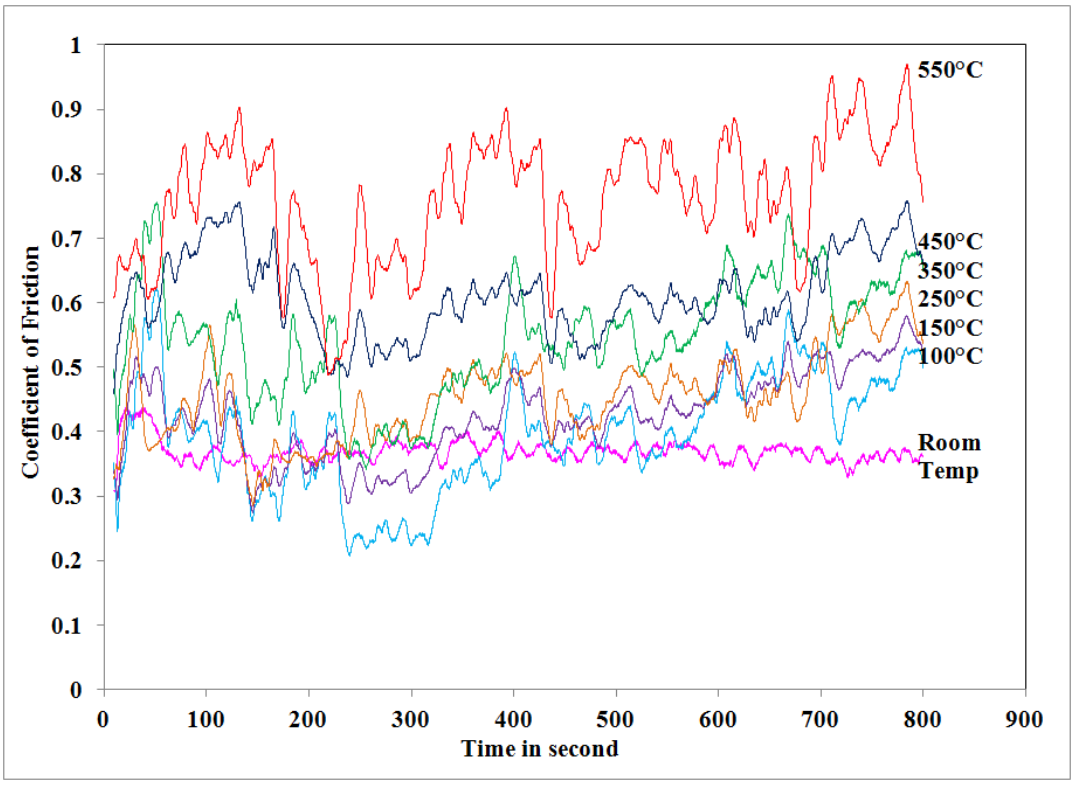
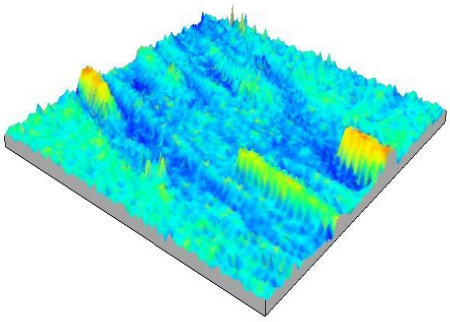


Figure 9. Variation of Coefficient of friction at different temperatures

3.2. Characterization of Measured Profiles

3.2.1. Line Profile Analysis

Figures 10 to 16 show representative line profiles of worn tracks at room temperature (25°C) and 100°C, 150°C, 250°C, 350°C, 450°C and 550°C, respectively. Constant line length of 5 mm was used for scanning the wear tracks at different temperatures. Arithmetic mean deviation of the roughness profile (R_a) was calculated for all the line profiles. The R_a values varied from 17.8 µm at room temperature to 79.3 µm at 550°C. The line profiles show higher crests and deeper troughs with increasing temperature depicting elevated wear. The crests depict adhesion and valleys characterize the amount of material removed from the wear tracks. The worn tracks also widened considerably as temperature increased. The peaks appear steeper and the pits appear deeper concluding relatively more amount of material removal. Figure 17 elaborates the variation of arithmetic mean deviation of the roughness profile (R_a) with respect to the operating temperature during dry sliding wear. The plot shows a positive slope with increasing uphill trend. The line profile measurements on the wear tracks corroborated the Coefficient of friction data calculated during the sliding wear tests on AISI 316 L (N) stainless steel at various temperatures. The profilometric feature analysis of wear tracks established the correlation between Coefficient of friction and surface roughness.



Parameters calculated on the surface data1

Sz, Sds and Ssc parameters are defined according to EUR15178N report.

Amplitude Parameters

Sa = 8.76 µm
Sa: Arithmetic Mean Deviation of the Surface.
Sq = 11.9 µm
Sq: Root-Mean-Square (RMS) Deviation of the Surface.
Sp = 63.8 µm
Sp: Maximum height of summits.
Sv = 32.6 µm
Sv: Maximum depth of valleys.
St = 96.4 µm
St: total height of the surface.
Ssk = 1.25
Ssk: Skewness of the Topography Height Distribution.
Sku = 6.51
Sku: Kurtosis of the Topography Height Distribution.
Sz = 84.7 µm
Sz: Ten Point Height of the Surface.

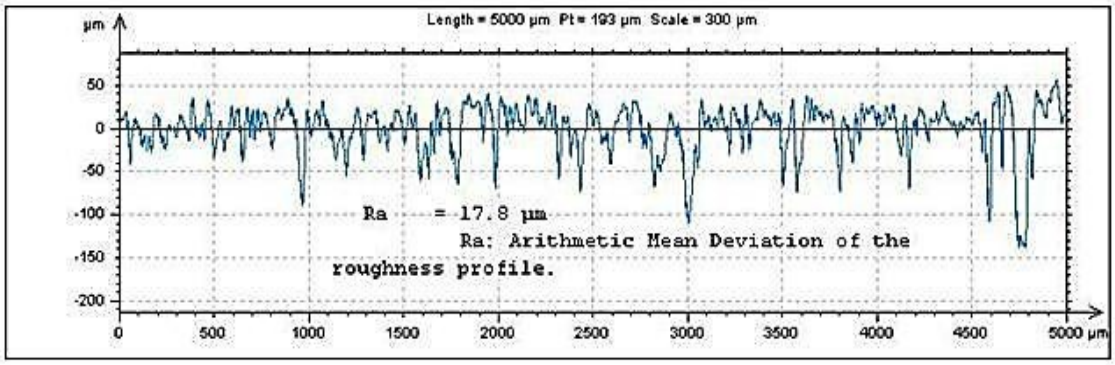


Figure 10. Surface roughness R_a for test conducted at 25°C

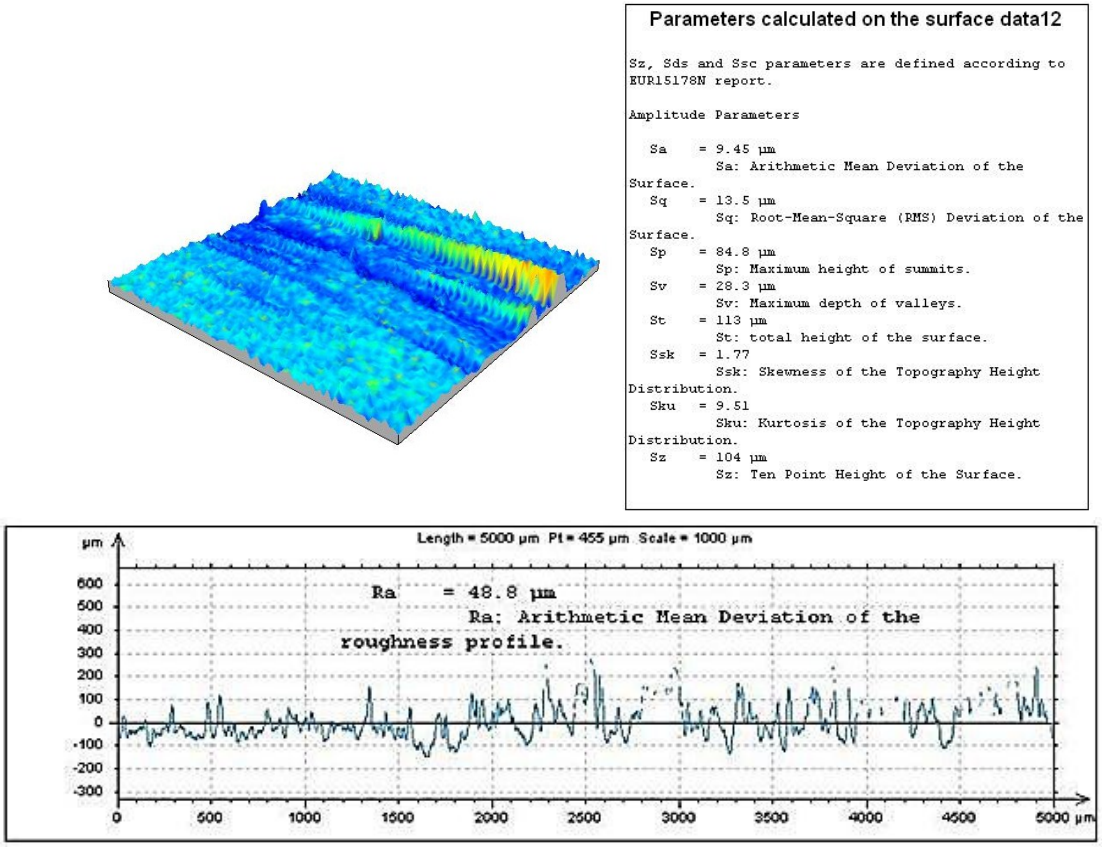


Figure 11. Surface roughness R_a for test conducted at 100°C

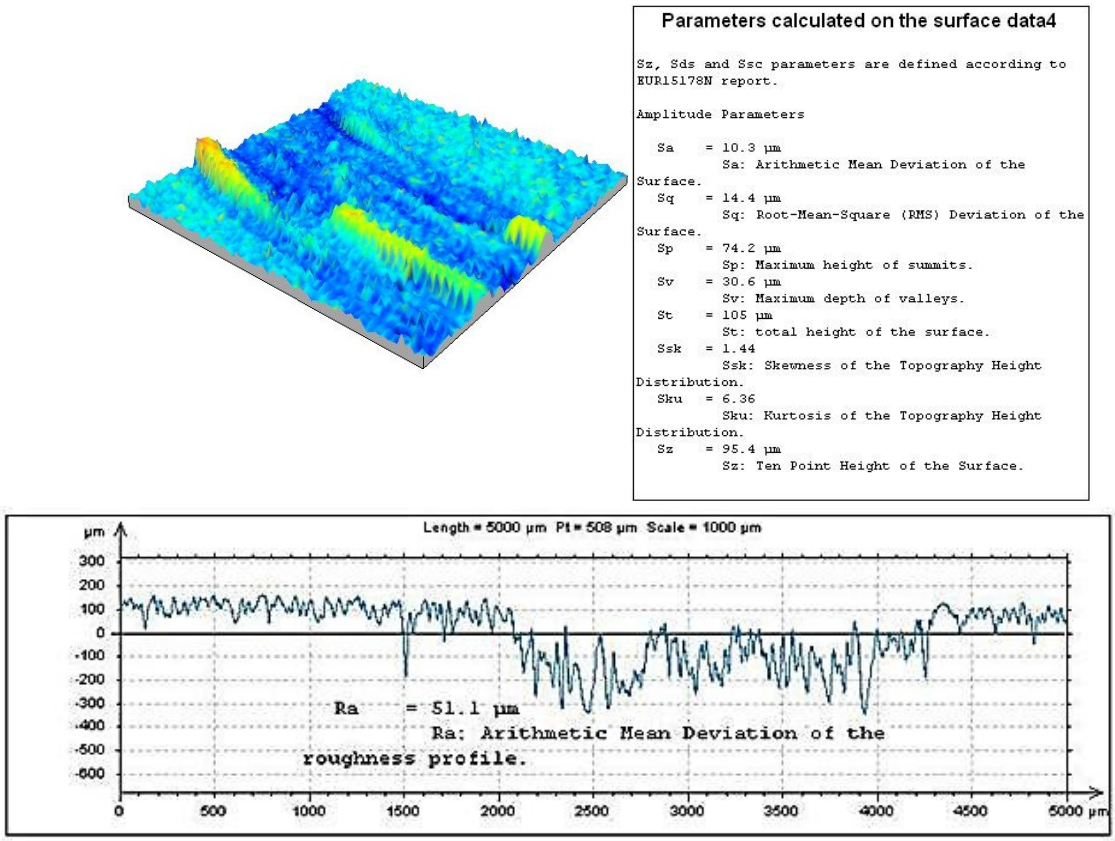
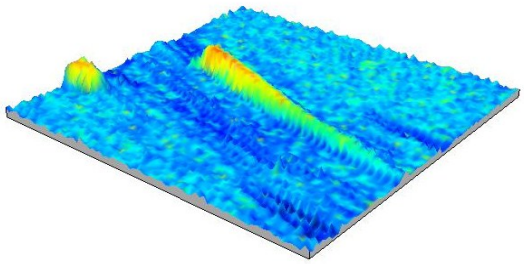


Figure 12. Surface roughness R_a for test conducted at 150°C



Parameters calculated on the surface data3

Sz, Sds and Ssc parameters are defined according to EUR15178N report.

Amplitude Parameters

Sa = 11.2 µm
Sa: Arithmetic Mean Deviation of the Surface.
Sq = 18.4 µm
Sq: Root-Mean-Square (RMS) Deviation of the Surface.
Sp = 121 µm
Sp: Maximum height of summits.
Sv = 39.6 µm
Sv: Maximum depth of valleys.
St = 160 µm
St: total height of the surface.
Ssk = 2.85
Ssk: Skewness of the Topography Height Distribution.
Sku = 14.7
Sku: Kurtosis of the Topography Height Distribution.
Sz = 142 µm
Sz: Ten Point Height of the Surface.

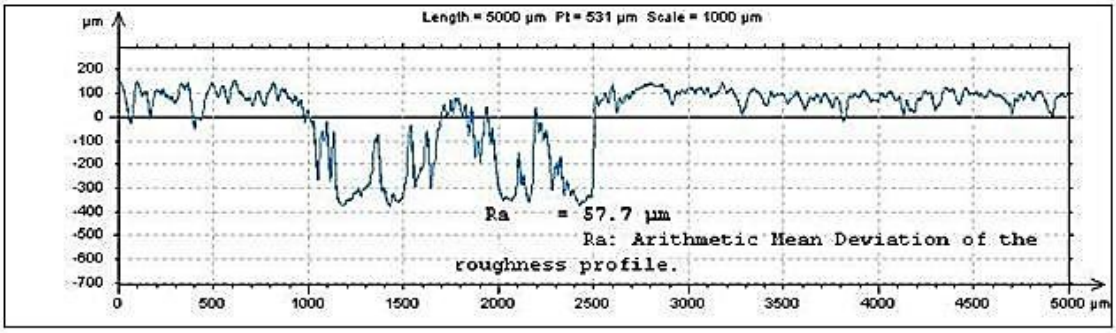
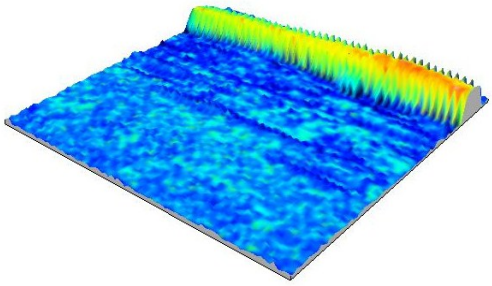


Figure 13. Surface roughness R_a for test conducted at 250°C



Parameters calculated on the surface data11

Sz, Sds and Ssc parameters are defined according to EUR15178N report.

Amplitude Parameters

Sa = 36.8 µm
Sa: Arithmetic Mean Deviation of the Surface.
Sq = 60.7 µm
Sq: Root-Mean-Square (RMS) Deviation of the Surface.
Sp = 248 µm
Sp: Maximum height of summits.
Sv = 53.3 µm
Sv: Maximum depth of valleys.
St = 301 µm
St: total height of the surface.
Ssk = 2.58
Ssk: Skewness of the Topography Height Distribution.
Sku = 8.72
Sku: Kurtosis of the Topography Height Distribution.
Sz = 274 µm
Sz: Ten Point Height of the Surface.

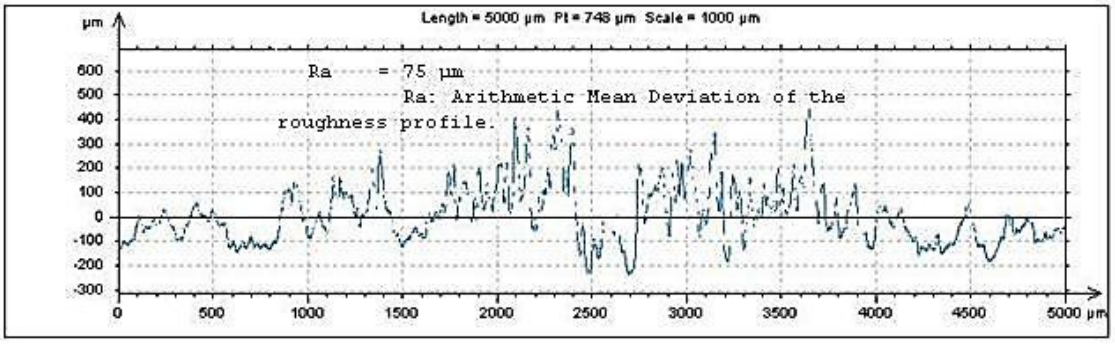
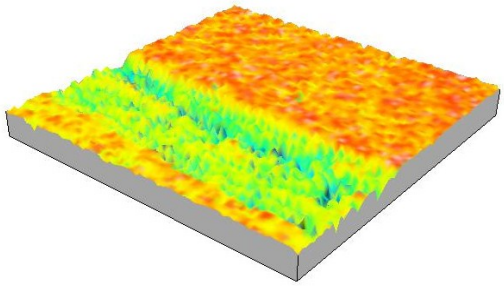


Figure 14. Surface roughness R_a for test conducted at 350°C



Parameters calculated on the surface data9

Sz, Sds and Ssc parameters are defined according to EUR15178N report.

Amplitude Parameters

Sa = 41.5 µm
Sa: Arithmetic Mean Deviation of the Surface.
Sq = 56.4 µm
Sq: Root-Mean-Square (RMS) Deviation of the Surface.
Sp = 71.4 µm
Sp: Maximum height of summits.
Sv = 299 µm
Sv: Maximum depth of valleys.
St = 370 µm
St: total height of the surface.
Ssk = -1.8
Ssk: Skewness of the Topography Height Distribution.
Sku = 6.22
Sku: Kurtosis of the Topography Height Distribution.
Sz = 293 µm
Sz: Ten Point Height of the Surface.

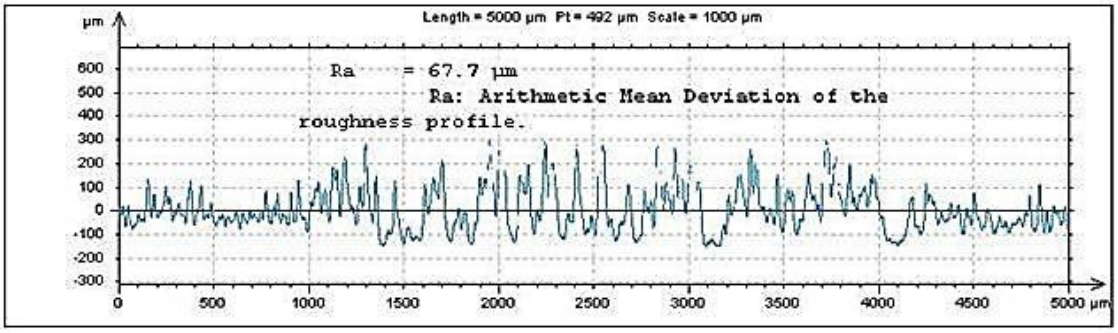
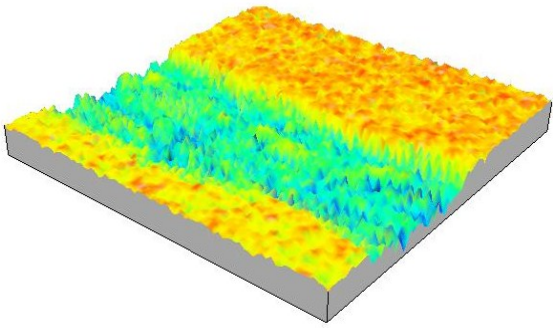


Figure 15. Surface roughness R_a for test conducted at 450°C



Parameters calculated on the surface data8

Sz, Sds and Ssc parameters are defined according to EUR15178N report.

Amplitude Parameters

Sa = 64.3 µm
Sa: Arithmetic Mean Deviation of the Surface.
Sq = 74.1 µm
Sq: Root-Mean-Square (RMS) Deviation of the Surface.
Sp = 107 µm
Sp: Maximum height of summits.
Sv = 249 µm
Sv: Maximum depth of valleys.
St = 356 µm
St: total height of the surface.
Ssk = -0.743
Ssk: Skewness of the Topography Height Distribution.
Sku = 2.38
Sku: Kurtosis of the Topography Height Distribution.
Sz = 314 µm
Sz: Ten Point Height of the Surface.

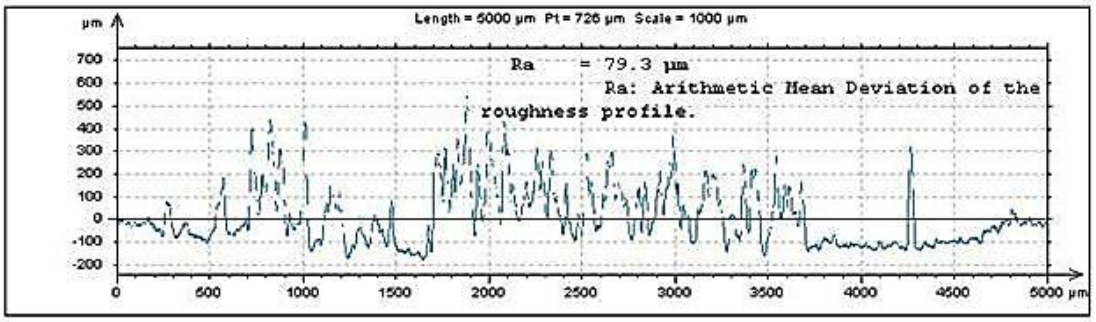


Figure 16. Surface roughness Ra for test conducted at 550°C

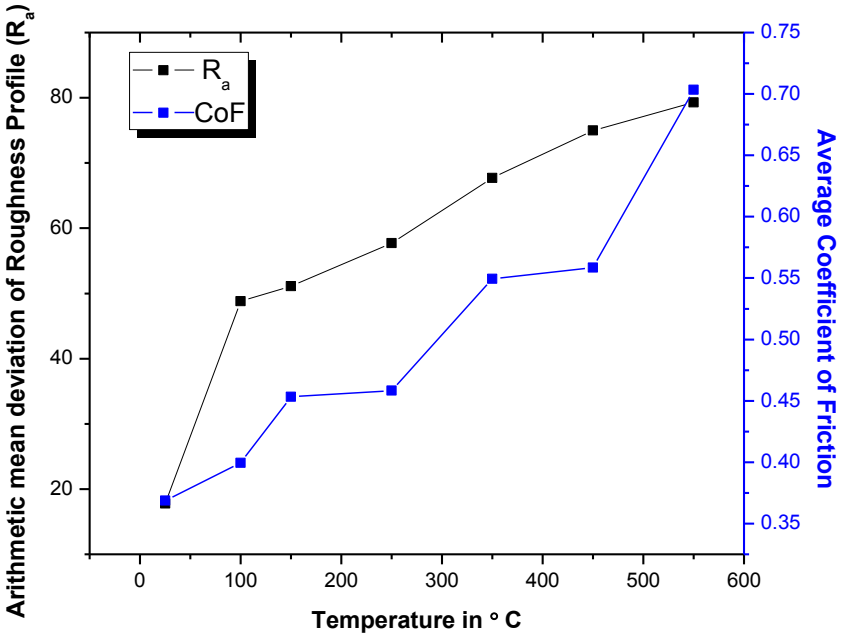


Figure 17. Variation of Surface roughness value (R_a) with respect to temperature

4. Conclusions

This study was conducted to correlate the Coefficient of friction of AISI 316 L (N) austenitic stainless steel under high temperature environment with surface roughness of the wear tracks at room temperatures (25°C) and 100°C, 150°C, 250°C, 350°C, 450°C and 550°C for a constant load of 20 N and at constant sliding velocity of 0.8 m/sec. The following conclusion can be drawn from this study:

- Coefficient of friction increased with increase in temperature.
- For room temperature the Coefficient of friction was 0.36 and at 550°C the Coefficient of friction recorded was 0.7. The Profilometric analysis of the surface revealed that the arithmetic mean deviation of the profile (R_a) increased with the increase in operating temperature during dry sliding wear.
- The measured profiles clearly indicated increase in the depth of the valleys and widening of the wear track with the increase in operating temperatures.
- There exists a strong correlation between surface roughness and Coefficient of friction in dry sliding wear of AISI 316 L (N) austenitic stainless steel with positive temperature dependence.

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NETWORKS OF NONLINEAR PROJECTORS

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Differentiation of system of the differential equations by a task becomes much easier, if system to simplify, i.e. to lead it to such look when it breaks up to blocks, in each of which – the independent system of unknown functions. In this case use special matrixes – projectors. Such projectors generate a network of surfaces in Euclidean space: surfaces of projections and projecting networks. For each set system of the equations there are networks, and not the unique. Properties of such networks are considered in work.

Keywords: systems of the differential equations, a projecting network of surfaces, Jordan's cell

1. Introduction

Both in the theory, and in many practical questions, the all–important role is played by problems of the solution of systems of the ordinary differential equations. One of such problems consists in splitting of this system on blocks, i.e. in such subsystems, each of which contains smaller number of the functions entering at the same time and under a sign of a derivative, and in the right parts. Abundantly clearly that integration of such subsystems is a task simpler, than integration of initial system. Splitting of this system on blocks appeared equivalent to reduction of a square matrix to a normal Jordan form. To Jordan's each cell there corresponds a certain block of the split system. In certain cases such splitting can be carried out by means of special degenerative matrixes – projectors.

2. Geometry of Splitting of Systems

Set of the degenerative matrixes P_α, satisfying to conditions

$$P^2{}_\alpha = P_\alpha, \quad P_\alpha P_\beta = 0, (\alpha \neq \beta), \quad \sum_{i=1}^{\alpha} P_i = E \quad (E - \text{unity matrix})$$

is called as system of linear (nonlinear) projectors.

However, thus there is open a question of the geometrical processes occurring at such splitting.

The independent system of the differential equations is considered

$$\frac{dy^i}{dt} = f^i(y^j), \; i, j = 1,...,n \,. \tag{1}$$

Variables y^i are considered as rectangular coordinates of a point in n – measured Euclidean space.

Possibility to break system (1) on blocks, each of which contains smaller number of the functions entering at the same time and under a sign of derivatives in the left parts, and in the right parts of each block, conducts to simplification of initial system, and thereby – to simplification of its integration. Let

$$y^i = F^i(z^j) \quad \rightarrow \quad z^j = \overline{F^j}(y^i) \tag{2}$$

- the no degenerative transformation bringing system (1) to the split look

$$\begin{cases} \dfrac{dz^{i_1}}{dt} = f_1^{\,i_1}(z^{j_1}), & i_1, j_1 = 1,2,3,..,s_1 \\ \dfrac{dz^{i_2}}{dt} = f_2^{\,i_2}(z^{j_2}), & i_2, j_2 = s_1 + 1,\ldots,s_1 + s_2, \\ ---------- & \\ \dfrac{dz^{i_p}}{dt} = f_p^{\,i_p}(z^{j_p}), i_p, j_p = s\ + s_2 + \ldots + 1,\ldots,s_1 + \ldots + s_p & \end{cases}, \tag{3}$$

- where $s_1 + \ldots + s_p = n.$ Each group of the equations of the system (3), containing independent system of variables call the block. No degenerative transformation (2), reformative system (1) to a look (3), generates p degenerative transformations, each of which transfers all space to some surface of dimension s_α $(\alpha = 1,\ldots,p)$ and system (1) – to the corresponding block of system (3). Really, having substituted values from (2) in (1), we will come to system of the equations $\dfrac{\partial F^i}{\partial z^j}\dfrac{dz^j}{dt} = f^i(F^j(z^k))$.

Designating through $\dfrac{\partial \overline{F^j}}{\partial y^i}$ matrix elements, to a return matrix $\left(\dfrac{\partial F^i}{\partial z^j}\right)$ (owing to a transformation (2) no degenerative, the last also no degenerative), we have

$$\frac{dz^j}{dt} = \frac{\partial \overline{F^j}}{\partial y^i} f^i . \tag{4}$$

On a condition the last system looks like (3), i.e. equalities take place

$$\begin{aligned} &\frac{\partial \overline{F^{i_1}}}{\partial y^i} f^i = f_1^{\,i_1}(\overline{F^{j_1}}) \\ &---------- \\ &\frac{\partial \overline{F^{i_p}}}{\partial y^i} f^i = f_p^{\,i_p}(\overline{F^{j_p}}). \end{aligned} \tag{5}$$

Let's break nondegenerate transformation (2) into p of blocks

$$\begin{aligned} &z^{j_1} = \overline{F^{j_1}}(y^i), \\ &------, \\ &z^{j_p} = \overline{F^{j_p}}(y^i). \end{aligned} \tag{6}$$

With each line of transformation (6) we will connect degenerative transformation which we will determine by equalities

$$\begin{aligned} &z^{j_1} = \overline{F_1^{j_1}}(y^i), && z^{j_p} = \overline{F^{j_p}}(y^i), \\ &z^{j_2} = \varphi_1^{\,j_2}(z^{j_1}), && z^{j_1} = \varphi^{j_1}(z^{j_p}), \\ &----------, & & \\ &z^{j_p} = \varphi_1^{\,j_p}(z^{j_1}), && z^{j_{p-1}} = \varphi_p^{\,j_{p-1}}(z^{j_p}), \end{aligned} \tag{7}$$

where $\varphi_1^{j_2},...,\varphi_1^{j_p},...,\varphi_p^{j_1},...,\varphi_p^{j_{h-1}}$ – some functions of the corresponding number of variables. Each transformation defined by the corresponding column of system (7), we will designate the transformation, associated to the corresponding block of system (3). The associated transformations (7) we will designate p_α $(\alpha = 1,...,p)$. Each of transformations P_α transfers all space E_n to the corresponding surface which dimension is equal respectively $s_1,...,s_p$. Let's designate these surfaces symbols σ_α. The equations of surfaces σ_α have respectively a look

$$\begin{aligned} &z^{j_2} = \varphi_1^{j_2}(z^{j_1}), \qquad z^{j_1} = \varphi_p^{j_1}(z^{j_2}), \\ &----------, \\ &z^{j_p} = \varphi_1^{j_p}(z^{j_1}), \qquad z^{j_{p-1}} = \varphi_p^{j_{p-1}}(z^{j_p}). \end{aligned} \tag{8}$$

Surfaces σ_α we will call surfaces of projections (at $s_\alpha = 1$ we have the line of projections, at $s_\alpha = n-1$ – a hyper surface of projections. Generally we speak $-s_\alpha$ – surfaces of projections). Let's assume that all surfaces pass through the beginning of coordinates. Let $M(y^i)$ – any point of space E_n. Let's take any of surfaces of projections, for example, σ_1. Let's include σ_α surfaces in a network of the Σ_α surfaces having the same dimensions, as σ_α surfaces. Let's call such network a projecting network. Let's designate it a symbol S. Through a point $M(y^i)$ of space pass p surfaces Σ_α (through the beginning of coordinates pass the σ_α surfaces which are also belonging to a network). Let's take the $\Sigma_2,...,\Sigma_p$ surfaces passing through a point M. They lie on some $H_{n-s_{12}}$ surface. Surface H_{n-s_1} we will call a projecting surface. It crosses a σ_1 surface in some point N_1. In a N_1 point all points of E_n space belonging to the projecting H_{n-s_1} surface are projected. A point N_1 we will call a projection H_{n-s_1} of a H_{n-s_1} surface to a surface σ_1 of projections. Projections $N_2,...,N_p$ on other surfaces of projections can be similarly defined.

How the projecting network is connected with the set system of the differential equations (1) split by system (3) and system of σ_α surfaces?

Theorem. At the set system of the equations (1) and beforehand set its split look (3) (functions f^i and $f_\alpha^{i_\alpha}$ – are any) exists (and thus not unique) the projecting network S containing σ_α surfaces as that it's forming, which pass through the beginning of coordinates.

Really, let the transformation bringing system (1) to the split look (3) is presented by equalities

$$y^i = y^i(z^j) \tag{9}$$

or, the equalities resolved z^j relatively

$$z^j = z^j(y^i). \tag{10}$$

In that case the first line of system (5) takes a form

$$\frac{\partial z^{i_1}}{\partial y^i}\frac{dy^i}{dt} = f_1^{i_1}(z^{j^1})$$

or

$$\frac{\partial z^{i_1}}{\partial y^i} f^i(y^j) = f_1^{i_1}(z^{j_1}). \tag{11}$$

It is system of the equations in partial derivatives of the first order concerning s_1 functions z^{i_1}. Let's carry out through surfaces $\sigma_2,...,\sigma_p$ any hyper surface H_1. Let its equation $y^n = H_1(y^1,...,y^{n-1})$. We will assume that $f^n(0) \neq 0$. The equations (11) we will write down in a look

$$\frac{\partial z^{i_1}}{\partial y^n} = \frac{f_1^{i_1}}{f^n} - \frac{f^1}{f^n}\frac{\partial z^{i_1}}{\partial y^1} - ... - \frac{f^{n-1}}{f^n}\frac{\partial z^{i_1}}{\partial y^{n-1}}. \qquad (12)$$

It is Cauchy type system. Let's integrate it so that equalities took place

$$z^{i_1} = z^{i_1}(y^1,...,y^{n-1}, H_1(y^1,...,y^{n-1})) = y^{i_1}.$$

According to the theorem Cauchy–Kovalevsky such solution $z^{i_1} = z^{i_1}(y^j)$ is unique.

As the hyper surface H_1 contains $\sigma_2,...,\sigma_p$ surfaces that

$$\varphi_2{}^n(y^{i_2}) = H_1(y^{i_2}, \varphi_2{}^{i_1}(y^{i_2}), \varphi_2{}^{i_3}(y^{i_2}),..., \varphi_2{}^{i'_p}(y^{i_2})), \quad i'_p = s_1 + ... + s_{p-1,} + 1,..n-1.$$

In particular, equalities (12) will be executed and at the following private values

$$z^{i_1} = z^{i_1}(y^{j_1}, \varphi_1{}^{j_2}(y^{j_1}),..., \varphi_1{}^{j_{p-1}}(y^{j_1}), \varphi_1{}^{j_p}(y^{j_1}) = z^{i_1}(y^{j_1}, \varphi 1^{s_1+1}(y^{j_1}), \varphi_1{}^n(y^{j_1})) = y^{i_1}.$$

Solutions of the equations entering into other lines of system (5) can be similarly found. Thus it is necessary to take other hyper surfaces $H_2,...,H_p$.

Collecting all found solutions, we will receive transformation (10) (or that the same, (9)), bringing system (1) to the split look (3).

The special place occupies a case $n = 2$. In this case we have splitting on two equations. σ_1, σ_2 are curves on the planes passing through the beginning of coordinates. Hyper surfaces H_1, H_2 coincide according to curves σ_2 and σ_1. The system's (5) solution – is unique.

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CUMULATIVE INDEX

COMPUTER MODELLING and NEW TECHNOLOGIES, volume 17, no. 1, 2013

(Abstracts)

S. V. Anfilets, V. N. Shuts. Optimizing the Management of Traffic Light Object Based on Natural Algorithms, *Computer Modelling and New Technologies,* vol. 17, no. 1, 2013, pp. 7–13.

The article proposes an adaptive control method for traffic lights, which operates at the strategic level of management. The algorithm uses data on changes in the intensity during the day, which provides forecasting module (for the experiments used neural network prediction). An adaptive algorithm is based on finding the minimum of delay at the crossroad, based on genetic algorithm and the method of "swarm of bees".

Keywords: traffic light, adaptive control, genetic algorithm, artificial bee colony algorithm

J. Sugier. Computational Methods for Adaptation of Markov Models to Requested Maintenance Policies, *Computer Modelling and New Technologies,* vol. 17, no. 1, 2013, pp. 14–24

State-transition models are often used in reliability analysis and one specific approach of this kind is the subject of this paper. By incorporating elements of both the deterioration process and the maintenance activities (inspections and repairs) in a semi-Markov model, a common computational platform has been created which serves as a foundation for various dependability studies that can investigate different maintenance scenarios. Having available some basic model it is possible to adjust its parameters so that it represents some hypothetical new maintenance policy and then to examine an impact which changing to the new policy has on various reliability characteristics of the system. Particularly, this paper discusses an extension of the generic adjustment algorithm to specific situations of the so-called model saturation when, as a result of tweaking the model towards higher repair frequencies, sum of repairs probabilities in the states reach the maximum value and there is no room for further increase. The general idea is to modify the model in such cases by forcing some non-zero value of a repair probability in those states where it is zero initially but in a manner that will not destroy the overall model behaviour.

After theoretical presentation of the modified method its effectiveness is illustrated on practical examples. It is shown that the proposed extension allows to successfully evaluating a class of cases that has not been properly handled by the generic method and thus broadens the range of dependability studies that can be effectively evaluated.

Keywords: state-transition deterioration model, semi-Markov process, model adaptation, maintenance analysis, model adjustment

K. N. Nechval, N. A. Nechval, M. Purgailis, U. Rozevskis, V. F. Strelchonok, M. Moldovan. Predictive Inferences for Future Order Statistics under Parametric Uncertainty, *Computer Modelling and New Technologies,* vol. 17, no. 1, 2013, pp. 25–36.

Prediction intervals for order statistics are widely used for reliability problems and other related problems. The determination of these intervals has been extensively investigated. But the optimality property of these intervals has not been fully explored. In this paper, in order to discuss this problem, a risk function is introduced to compare prediction intervals. In particular, new-sample prediction based on a previous sample (i.e., when for predicting the future observation in a new sample there are available the data only from a previous sample), and within-sample prediction based on the early observed data from a current experiment (i.e., when for predicting the future observation in a sample there are available the early observed data only from that sample). We restrict attention to families of distributions invariant under location and/or scale changes. The technique used here for optimization of prediction intervals based on censored data emphasizes pivotal quantities relevant for obtaining ancillary statistics. It allows one to solve the optimization problems in a simple way. An illustrative example is given.

Keywords: order statistic, prediction interval, risk function, optimisation

W-M. Tsai, G. Gurevich, A. Vexler. Optimal Properties of Parametric Shiryaev-Roberts Statistical Control Procedures, *Computer Modelling and New Technologies,* vol. 17, no. 1, 2013, pp. 37–50.

Parametric change point detection schemes based on the Shiryaev-Roberts approach have been well addressed in the statistics and engineering literature that consider sequential techniques. High efficiency of such procedures can be partially explained by their known asymptotic optimal properties. Recently, Shiryaev-Roberts based procedures were proposed and examined in applications to the standard AMOC (at most one change) retrospective change point detection problems. The main aim of this article is to review and extend parametric retrospective and sequential Shiryaev-Roberts based policies, carrying out different contexts of the procedures' non-asymptotic optimal properties. We utilize the general principle of the Neyman-Pearson fundamental lemma to show that the Shiryaev-Roberts approach implies the average most powerful procedures. We also propose techniques to construct novel and efficient retrospective tests for multiple change points detection. A real data example based on biomarker measurements is provided to demonstrate implementation and effectiveness of new tests in practice.

Keywords: Shiryaev-Roberts sequential procedure; non-asymptotic optimality; retrospective change point detection; average most powerful

N. L. Parthasarathi, U. Borah, Sh. K. Albert. Correlation Between Coefficient of Friction and Surface Roughness in Dry Sliding Wear of AISI 316 L (N) Stainless Steel at Elevated Temperatures, *Computer Modelling and New Technologies,* vol. 17, no. 1, 2013, pp. 51–63.

In this paper, coefficient of friction in dry sliding wear at different temperatures has been correlated with surface roughness (R_a) of the wear tracks. Unlubricated pin-on-disc sliding wear tests were carried out on AISI Type 316 L (N) austenitic stainless steel up to 550^{o}C at constant load (20 N) and sliding speed (0.8 m/s) as per the ASTM standard G99-05. Line profiling along radial directions across the wear track was carried out and the line profiles were analysed to calculate surface roughness of the wear tracks. AISI Type 316 L (N) austenitic stainless steel is a major structural material in the prototype fast breeder reactor (PFBR), Kalpakkam because of its good high temperature properties and compatibility with liquid sodium. This experimental work revealed a one to one correlation between coefficient of friction and surface roughness of wear tracks. Coefficient of friction as well as surface roughness during dry sliding wear increased with increase in temperature.

Keywords: Coefficient of friction; AISI 316 L (N) stainless steel; high temperature, roughness

A. Kovantsov. Networks of Nonlinear Projectors, *Computer Modelling and New Technologies,* vol. 17, no. 1, 2013, pp. 64–67.

Differentiation of system of the differential equations by a task becomes much easier, if system to simplify, i.e. to lead it to such look when it breaks up to blocks, in each of which – the independent system of unknown functions. In this case use special matrixes – projectors. Such projectors generate a network of surfaces in Euclidean space: surfaces of projections and projecting networks. For each set system of the equations there are networks, and not the unique. Properties of such networks are considered in work.

Keywords: systems of the differential equations, a projecting network of surfaces, Jordan's cell

COMPUTER MODELLING and NEW TECHNOLOGIES, 17. sējums, Nr. 1, 2013

(Anotācijas)

S. Anfilets, V. Šuts. Luksofora vadības optimizēšana, pamatojoties uz īstu algoritmu, *Computer Modelling and New Technologies,* 17. sēj., Nr. 1, 2013, 7.–13. lpp.

Raksts ierosina adaptīvo kontroles metodi luksoforiem, kas darbojas ar vadības stratēģisko līmeni. Algoritms izmanto datus par izmaiņām intensitātē dienas laikā, kas nodrošina prognozēšanas moduli (eksperimentiem, kas izmanto neironu tīkla prognozes). Adaptīvais algoritms balstās, lai atrastu kavējuma minimumu krustojumos, pamatojoties uz ģenētisko algoritmu un metodi „bišu spiets".

Atslēgvārdi: satiksmes gaismas, adaptīvās vadības, ģenētiskais algoritms, mākslīgo bišu kolonijas algoritms

J. Sugiers. Skaitļošanas metodes Markova modeļu adaptācijai pieprasītai uzturēšanas politikai, *Computer Modelling and New Technologies,* 17. sēj., Nr. 1, 2013, 14.–24. lpp.

Stāvokļa pārejas modeļus bieži izmanto drošuma analīzē, un viena īpaša šāda veida pieeja ir šī raksta tēma. Iekļaujot abus elementus gan pasliktināšanās procesu, gan uzturēšanas darbības (inspekcijas un remonti), daļējā-Markova modelī, ir radīta kopīga skaitļošanas platforma, kas kalpo par pamatu dažādiem uzticamības pētījumiem, kas var izpētīt dažādus uzturēšanas scenārijus.

Konkrēti šajā rakstā tiek aplūkots vispārēja korekcijas algoritma pagarinājums, konkrētām situācijām t.s. modeļa piesātinājumam, kad piedares rezultātā modelis uz augstākām remonta frekvencēm, remonta varbūtības summa stāvokļos sasniedz maksimālo vērtību, un nav vietas tālākam palielinājumam. Vispārējā ideja ir mainīt modeli šādos gadījumos, spēcinot kādu remonta varbūtības ne-nulles vērtību šajos stāvokļos, kur tas sākumā bija nulle, bet tādā veidā, lai neiznīcinātu kopējo modeļa uzvedību.

Pēc modificētās metodes teorētiskās prezentācijas tās efektivitāte ir ilustrēta ar praktiskiem piemēriem.

Atslēgvārdi: stāvokļa pārejas pasliktināšanās modelis, daļējs-Markova process, modeļa pielāgošana, uzturēšanas analīze, modeļa korekcijas

K. Nečvals, N. Nečvals, M. Purgailis, U. Rozevskis, V. Strelčonoks, M. Moldovans. Prognostiski slēdzieni turpmākajai pasūtījuma statistikai saskaņā ar parametrisko nenoteiktību, *Computer Modelling and New Technologies,* 17. sēj., Nr. 1, 2013, 25.–36. lpp.

Prognozēšanas intervāli pasūtījuma statistikai tiek plaši izmantoti uzticamības problēmu risināšanā, kā arī citās saistītās problēmās. Šo intervālu noteikšana ir plaši pētīta. Bet šo intervālu optimalitātes raksturs nav pilnībā izpētīts. Šajā rakstā, lai apspriestu šo problēmu, ir ieviesta riska funkcija, lai salīdzinātu prognozes intervālus; jo īpaši, jauna parauga prognozes, pamatojoties uz iepriekšējo paraugu, un iekš-parauga prognozes, pamatojoties uz agrīni novērotiem datiem pēc pašreizējā eksperimenta.

Paņēmiens, ko šeit izmanto prognozēšanas intervālu optimizācijai, pamatojoties uz necenzētiem datiem, uzsver noteicošos daudzumus, attiecīgus papildu statistikas saņemšanai. Tas ļauj atrisināt optimizācijas problēmas vienkāršā veidā; ir dots ilustratīvs piemērs.

Atslēgvārdi: pasūtījuma statistika, prognozēšanas intervāls, riska funkcija, optimizācija

V.-M. Tsai, G. Gurevičs, A. Vekslers. Parametriskās Širjajeva-Robertsa statistiskās kontroles procedūru optimālās īpašības, *Computer Modelling and New Technologies,* 17. sēj., Nr. 1, 2013, 37.–50. lpp.

Parametru maiņas punktu atklāšanas shēmas, kas balstītas uz Shirjaeva-Robertsa pieeju ir labi risinātas statistikā un inženierzinātnes literatūrā, kas izskata secīgas metodes. Šādu procedūru augsto efektivitāti var daļēji izskaidrot ar to zināmām asimptotiskām optimālām īpašībām. Nesen, uz Shirjaeva-Robertsa teoriju balstītās procedūras, tika ierosinātas un izskatītas pieteikumos uz standarta AMOC (ne vairāk kā viena izmaiņa) retrospektīvo izmaiņas punktu noteikšanas problēmas.

Galvenais šī raksta mērķis ir pārskatīt un paplašināt parametrisko retrospektīvu un secīgas, uz Shirjaeva-Robertsa teoriju balstītas, politikas, kas veic dažādus kontekstus no procedūru ne-asimptotiskām optimālām īpašībām.

Atslēgvārdi: Shirjaeva-Robertsa secīgā procedūra, ne-asimptotiskā optimalitāte, retrospektīvā maiņas punkta atklāšana, vidējais jaudīgākais

N. L. Parthasarathi, U. Borahs, Sh. K. Alberts. Korelācija starp berzes koeficientu un virsmas raupjumu AISI 316 L (N) nerūsējošā tērauda paaugstinātā temperatūrā sausās slīdes nodilumā, *Computer Modelling and New Technologies,* 17. sēj., Nr. 1, 2013, 51.–63. lpp.

Šajā rakstā berzes koeficients sausās slīdes nodilumā pie dažādām temperatūrām ir saistīts ar nodiluma sliežu virsmas raupjumu (Ra). Neieeļļoti uz diska piesprausti slīdoši nodiluma testi tika veikti uz AISI tipa 316 L (N) austenīta nerūsējošā tērauda līdz 550^{o}C pie konstantas slodzes (20 N) pie slīdoša ātruma (0,8 m/s), saskaņā ar ASTM standartu G99-05. Tika veikta līnijas profilēšana gar radiālo virzienu visā nodiluma ceļā, un līnijas profili tika analizēti, lai aprēķinātu nodiluma ceļa virsmas raupjumu. AISI tipa 316 L (N) austenīta nerūsējošais tērauds ir būtisks strukturāls materiāls reaktora uz straujiem neitroniem prototipā, *Kalpakkam*, dēļ tā labām augstas temperatūras īpašībām un ar šķidrā nātrija savietojamību. Šis eksperimentālais darbs atklāja viens pret vienu korelāciju starp berzes koeficientu un nodiluma ceļa virsmas raupjumu. Berzes koeficients, kā arī virsmas nelīdzenums sausās slīdes nodiluma laikā pieauga, palielinoties temperatūrai.

Atslēgvārdi: berzes koeficients, AISI tipa 316 L (N) austenīta nerūsējošais tērauds, augsta temperatūra, raupjums

A. Kovantsovs. Nelineāru projektoru tīkli, *Computer Modelling and New Technologies,* 17. sēj., Nr. 1, 2013, 64.–67. lpp.

Diferenciālvienādojumu sistēmas diferencēšana ar uzdevumu kļūst daudz vieglāka, ja sistēma, kura jāvienkāršo, t.i., jāieved tā tādā izskatā, kad tā salūst blokos, katrā no tiem – nezināmu funkciju neatkarīga sistēma. Šajā gadījumā jāizmanto īpašas matricas – projektori. Šādi projektori rada virsmu tīklu Eiklīda telpā: projekciju virsmas un projicētus tīklus. Katrai vienādojumu sistēmas rindai ir tīkli, un ne unikāli. Šādu tīklu īpašības tiek izskatītas dotajā rakstā.

Atslēgvārdi: diferenciālvienādojumu sistēmas, virsmu projicēšanas tīkls, Džordana šūna

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