

# An Improved K2DPCA Dimensional Reduction method for Hyper spectral Remote Sensing Image

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## Abstract

An improved kernel two-dimensional principle analysis (K2DPCA) dimensional reduction method for hyperspectral remote sensing image was proposed in this paper. It decorrelated the columns of remote sensing image by the standard K2DPCA, then used columns 2DPCA to further decorrelate the row direction. It could achieve the dimensional reduction at both widthways and lengthways for remote sensing image. The original images could be reconstructed by the principle components of extracted from each bands of remote sensing image. Experiments were verified with AVIRIS hyperspectral remote sensing image Cuprite, and the result showed that this new method could not only ensure the reconstructed image quality, but also effectively improve the image compression rate.

*Keywords:* two-dimensional principle analysis, kernel two-dimensional principle analysis, image dimensional reduction, image reconstruction, hyperspectral remote sensing image

## 1 Introduction

Hyperspectral remote sensing image(HRSI) not only provides the spatial information, but also can provide the spectral information that involves several ten bands. This characteristic is beneficial to detect and recognize ground target, but leads to the curse of the dimensionality that poses serious problems to storage and process the images. Dimension reduction can be seen as a transformation from a high order dimension to a low order dimension to conquer the curse of the dimensionality. Principal Component Analysis (PCA) is perhaps the most popular dimension reduction technique for remote sensing image. But PCA can't process the higher order statistic information of hyperspectral remote sensing image. To solve this problem, kernel method was introduced to PCA by Scholkopf et al., who put forward the Kernel Principal Component Analysis (KPCA). Dimension reduction methods based on PCA and KPCA, only can transform image matrices of every band to one dimensional image vector, and can't use the structural information from image matrix of every band.

In view of the above problems, many scholars expanded two-dimensional methods for PCA to use structural information from image matrices effectively, and applied to pattern recognition successfully. There is Two-dimensional PCA (2DPCA), Bilateral Two-dimensional PCA (Bi-2DPCA), kernel Two-dimensional PCA (K2DPCA) and so on. Yang et al. firstly put forward 2DPCA to face recognition and tested it in Yale, AR and ORL face database. Contrast to 2DPCA and Bi-2DPCA can extract row and column vectors from images, and have higher recog-

niton rate. Especially, kernel Two-dimensional PCA can extract nonlinear principal components based directly on input image matrices and nonlinear features efficiently instead of carrying out the nonlinear mapping explicitly, and get better recognition effect. However, these extension methods of PCA rarely were used in dimension reduction for hyperspectral remote sensing image. This paper tries to find an improved K2DPCA algorithm which can do dimension reduction for hyperspectral remote sensing image.

## 2 Dimensional reduction based on 2DPCA and K2DPCA

2DPCA and K2DPCA, which are different from dimension reduction based on PCA and KPCA, can obtain principal component of every band of hyperspectral imagery, and reduce the size of the original image in row direction or column direction, and realize the dimension reduction of hyperspectral imagery in spatial dimension.

### 2.1 TWO-DIMENSIONAL PRINCIPAL COMPONENT ANALYSIS

2DPCA is a feature extraction method based on image matrices. Depending on different matrix direction, 2DPCA can be divided into row direction 2DPCA and column direction 2DPCA. Row direction 2DPCA is the classical one which is introduced here.

Suppose matrix  $A$  is a cube composed of  $p$  bands hyperspectral remote sensing image, size of which is  $m$  pixels. Let  $X \in R^{n \times d}$  ( $n \geq d$ ) which is composed of mutually orthogonal column vectors. One of band matrices can be

projected into X to be a new m by d matrix  $Y_k$  ( $k=1, 2, \dots, p$ ) by Eq.(1).

$$Y_k = A_k X \tag{1}$$

In Eq.(1), X is a projection axis,  $Y_k$  ( $k=1, 2, \dots, p$ ) is a projection eigenvector of  $A_k$  ( $k=1, 2, \dots, p$ ). In 2DPCA, an optimal projection matrix X can be determined by total scatter situation of projection samples, and scatter matrix can be characterized by trace of covariance matrix of projection eigenvector. So, the optimal projection matrix X can be found by maximizing the value of Eq.(2).

$$J(X) = trace(S_t) = X^T E \{ [A_k - E(A_k)]^T [A_k - E(A_k)] \} X \tag{2}$$

In Eq.(2),  $J(X)$  is scatter matrix,  $S_t$  is covariance matrix of projection eigenvector,  $trace(S_t)$  is the trace of  $S_t$ , and  $E(\cdot)$  is expectation. Define the covariance matrix  $S_t = E \{ [A_k - E(A_k)]^T [A_k - E(A_k)] \}$  is an n by n nonnegative definite matrix.

Now, based on m by n matrix  $A_k$  ( $k=1, 2, \dots, p$ ),  $\bar{A}$  is defined to be the mean of hyperspectral remote sensing image,  $\bar{A} = \frac{1}{p} \sum_{k=1}^p A_k$ .

So,  $S_t$  can be evaluated by Eq.(3),

$$S_t = \frac{1}{p} \sum_{k=1}^p (A_k - \bar{A})^T (A_k - \bar{A}) \tag{3}$$

Accordingly,  $J(X)$  in Eq.(2), can be represented like this,

$$J(X) = X^T S_t X \tag{4}$$

If every band of hyperspectral remote sensing image is represented by row vector, Eq.(3) can be rewritten,

$$S_t = \frac{1}{p} \sum_{k=1}^p (A_k^{(i)} - \bar{A}^{(i)})^T (A_k^{(i)} - \bar{A}^{(i)}) \tag{5}$$

In Eq.(5), size of is n by n,  $A_k^{(i)}$  is the  $i$  th row in the  $j$  th band of remote sensing image. Ranging in descending order, the pre- $d$  th of eigenvalue of  $S_t$  will be obtained. According to eigenvalues ( $\lambda_1, \lambda_2, \lambda_3 \dots \lambda_d$ ), there are n orthogonal engenvectors ( $X_1, X_2, X_3 \dots X_d$ ) which constitutes the optimal projection axis  $X_{opt}$ .

## 2.2 KERNEL TWO-DIMENSIONAL PCA

In K2DPCA, hyperspectral remote sensing image is transformed into a higher dimensional or even infinite dimensional kernel space by nonlinear mapping. Then, the new image samples are processed by classical 2DPCA method in kernel space. Using a nonlinear map  $\Phi: A_i \rightarrow \Phi(A_i)$ , any band matrix  $A_i$  ( $i=1, 2, \dots, p$ ) can be projected to a Hilbert space  $R^{m \times f}$  ( $f > m$ ). Suppose  $A_i^{j*}$  is the  $i$  th row in the  $j$  th band of remote sensing image,  $j=1, 2 \dots m$ , in kernel space, image sample of the  $i$  th band can be represented like this,

$$\Psi(A_i) = \begin{bmatrix} \Phi [ (A_i^{1*})^T ]^T \\ \dots \\ \Phi [ (A_i^{m*})^T ]^T \end{bmatrix} \tag{6}$$

In new feature space,  $\Phi(A)^{[10]}$  is centralized, then  $\sum_{i=1}^p \Phi(A_i) = 0$ . The covariance matrix of image sample can be calculated by Eq.(7).

$$S_t^\Phi = \sum_{i=1}^p \Psi(A_i)^T \Psi(A_i) = \sum_{i=1}^p \sum_{j=1}^m \Phi [ (A_i^{j*})^T ] \Phi [ (A_i^{j*})^T ]^T \tag{7}$$

Then, the eigenvalue  $\lambda$  and eigenvector  $v$  of  $S_t^\Phi$  can be obtained by the formula:  $\lambda v = S_t^\Phi v$ . When  $\lambda \neq 0$  and  $v$  is located in the subspace expanded by  $\Phi(A_i^j)$  ( $i=1, 2, \dots, p; j=1, 2, \dots, m$ ), there is a coefficient  $\alpha_i^j$  ( $i=1, 2, \dots, p; j=1, 2, \dots, m$ ) which meets Eq.(8).

$$v = \sum_{i=1}^p \sum_{j=1}^m \alpha_i^j \Phi(A_i^j) \tag{8}$$

Eq.(7) and Eq.(8) are substituted into the formula:  $\lambda v = S_t^\Phi v$ .

$$p \lambda \alpha = K \alpha \tag{9}$$

In Eq.(9),  $\alpha$  are eigenvectors of kernel matrix  $K = \Phi(A_i^j)^T \times \Phi(A_i^j)$  ( $i=1, 2, \dots, p; j=1, 2, \dots, m$ ). Firstly, eigenvectors according to the pre- $l$  th bigger eigenvalues are obtained. Then, using Eq.(8), the eigenvector  $v$  can be obtained. So,  $\omega^\Phi = [v^1, v^2, \dots, v^l]$  is the optimal projection axis in K2DPCA. Principal components of hyperspectral remote sensing image is  $Y_i$  by K2DPCA, in Eq.(10).

$$Y_i = \Psi(A_i) \omega^\Phi = \begin{bmatrix} \Phi [ (A_i^{1*})^T ]^T \\ \dots \\ \Phi [ (A_i^{m*})^T ]^T \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^l \alpha^i \{ \Phi [ (A_i^{1*})^T ]^T \Phi(A) \} \\ \dots \\ \sum_{i=1}^l \alpha^i \{ \Phi [ (A_i^{m*})^T ]^T \Phi(A) \} \end{bmatrix} \tag{10}$$

### 3 An Improved K2DPCA dimensional reduction method

K2DPCA can extract the nonlinear information of band images, but it only can process band image in column direction. In order to improve compressibility of remote sensing image, this paper presents an improved K2DPCA dimensional reduction method. After transforming band images to kernel space, this method reduces the dimension of projected images in two direction using row 2DPCA and column 2DPCA. It can obtain principal components which include nonlinear information of band images and have a smaller size. The process of this dimensional reduction method likes Fig.1.

### 4 Experiment and result analysis

#### 4.1 THE DATA SET

For verifying the efficiency of this method, the data set used was obtained from the Airborne Visible Infrared Imaging Spectrometer (AVIRIS) hyperspectral instrument. For this paper, hyperspectral data was obtained from AVIRIS which has a ground pixel size of  $17m \times 17m$  and a spectral resolution of 224 channels, covering the range from 400 nm to 2500 nm, centered at 10 nm intervals. We focus on a collection of data taken in 1995 in Cuprite District of Nevada, U.S., which consists of  $350 \times 400$  pixels by 50 bands.

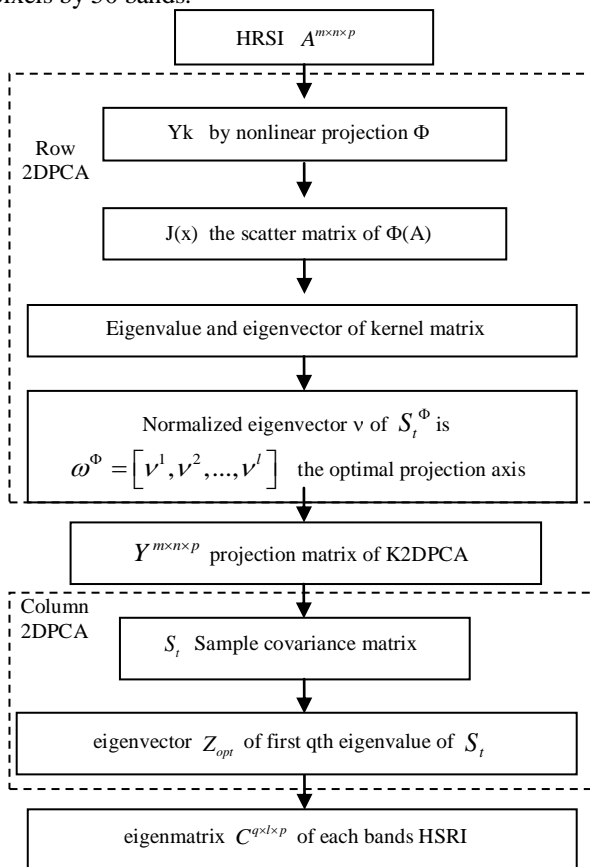


FIGURE 1. Flow of this new method

#### 4.2 RESULT ANALYSIS

TABLE 1 The result of dimension reduction using Cuprite

method	Information retention %	Size of principle component	compression rate
2DPCA	99.5	350×10×50	39.0013
	99	350×2×50	190.3520
	95	350×1×50	392.6071
	90	350×1×50	392.6071
K2DPCA	99.5	350×209×50	1.9802
	99	350×142×50	2.7546
	95	350×39×50	10.2098
	90	350×16×50	25.7658
This new method	99.5	151×209×50	4.0822
	99	92×142×50	9.2304
	95	17×39×50	127.3893
	90	5×16×50	580.2457

All methods have been verified in MATLAB and ENVI. Homogeneous polynomial was used to be kernel function in K2DPCA transformation.  $p$  is the order of polynomial and  $p=1$  here. In the real application,  $p$  can be adapted by different data set. To the Cuprite, Tab.1 is the result of dimensional reduction by 2DPCA, K2DPCA and this new method. When the information retention is 99.5%, the Cuprite in band 5, 15, 25, 35 and band 45, were reconstructed by above 3 methods, band 5 and 35 like Fig.2 From Tab.1 and Fig.2b, in 2DPCA, when information retention was 99.5%, the clarity of reconstructed image of the Cuprite was low, because the first few principle components had more low frequency information, and less high frequency information (like object edges), after feature extraction using 2DPCA. For this feature of 2DPCA, compression rate and effect of image reconstruction were affected by feature of remote sensing images. 2DPCA had good dimensional reduction effect for remote sensing images which has more gentle change, and had poor effect for remote sensing images which has more sudden change and details.

From Tab.1, K2DPCA need principle component which had higher dimension to reconstruct the original image. From Fig.2c, by K2DPCA the reconstructed image could have higher clarity and distinguish outline and texture detail of object, and K2DPCA could extract low frequency information from remote sensing images.

TABLE 2 The MSE of above 3 method

Method	Bands of Cuprite					
	5	15	25	35	45	Mean
2DPCA	400.8702	389.5298	280.4328	289.0793	255.3782	323.0581
K2DPCA	88.3789	99.4729	80.3098	95.6920	89.9832	90.7674
New method	110.3298	130.3786	109.3628	125.0987	117.1273	118.4594

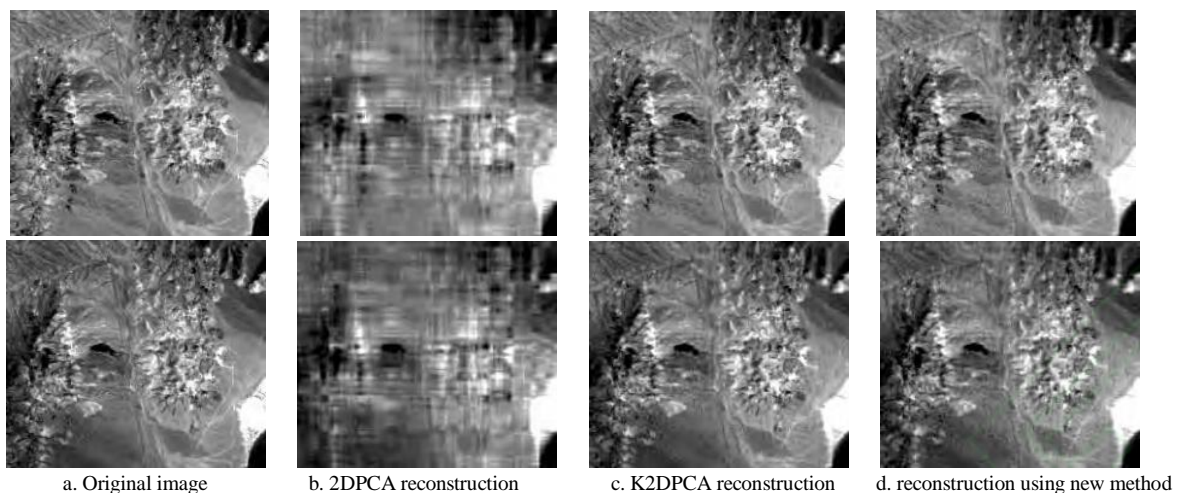


FIGURE 2 The reconstructed image of Cuprite District from AVIRIS

In this paper, the new method which was proposed can make up the short that reconstructing original image need more principle components in K2DPCA. From Tab.1, when the information retention was 95%, the size of principle component was  $350 \times 39$  by K2DPCA, but the size principle component was  $17 \times 39$  by this new method. And the compression rate was changed from 10.2098 to 127.3893.

In this paper, the mean square error (MSE) was used to evaluate the quality of reconstructed image, the value of MSE mean the difference between reconstructed image and original image. The MSE of the 5 bands and average value of above 3 method like Tab.2. As can be seen, MSE was dropped greatly, when dimensional reduction method was changed from 2DPCA to this new method, and MSE between K2DPCA and this new method had little difference.

Above all, this new method has a good capability of image dimension reduction, and has better effect of image reconstruction.

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