An improved eigenstructure method for estimating DOA in the presence of parameters uncertainties

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Abstract

This paper presents an improved eigenstructure-based method for estimating the direction of arrival (DOA) of received signal in uniform circular-array, in the presence of sensor gain and phase uncertainties. A simple sensor gain and phase uncertainties calibration method, which does not require any prior knowledge of the DOAs, but also being capable of eliminating the DOA estimation ambiguity, is proposed. The performance of the proposed method is demonstrated by some representative computer simulation.

Keywords: eigenstructure, gain and phase uncertainty calibration, DOA estimation

1 Introduction

Direction of arrival (DOA) of multiple narrowband signals estimation has widely been discussed this decade. Existing DOA estimation algorithms, such as MUSIC [1], ESPRIT [2], Capon's beam former [3], subspace-based method [4] and parametric maximum-likelihood [5] are known to be highly sensitive to the errors in the array manifold. In these algorithms prior knowledge of the signals received by the sensor array from signal sources, is required. However, in practice there always exist various degrees of perturbation in sensor array. Therefore, it is required to calibrate sensors before the DOA estimation.

Recently, the estimation of the DOA of signal emitted by narrowband sources has been widely investigated using signal processing methods. In the method of Weiss and Friedlander [6] they proposed an algorithm to calibrate the sensor array, however this method suffers from suboptimal convergence problem. Paulraj and Kailath [7] proposed method of DOA estimation in the presence of sensor gain and phase uncertainty, based on a least-squares (LS) approach using a linear equispaced (LES) array. The method does not need calibrating sources, but it suffers from high computational requirement. The method in [8] used self-calibration algorithms based on least squares approach to compensate the problem of DOA estimation using LEA in the presence of phase errors. Moreover, the method does not require any prior knowledge about signal source direction. The authors in [9] developed an eigenstructure method for DOA estimation in the presence of sensor gain and phase perturbation, which compensates for the suboptimal convergence problem, which occurs in [6]. The method in [10] considered the problem of phase autocalibration for uniform rectangular array (URA).It solves the problem of ambiguity, which arises when the phase and the DOA parameters are identified together. In [11] an iterative Maximum likelihood (ML) procedure was developed to estimate DOA using sparse sensor arrays composing of multiple widely separated sub arrays, which improve the performance achieved by Friedlander and Weiss in [12]. In [13] a MUSIC like algorithm was investigated for gain and phase estimation, assuming the source angle is known. In [14] the problem of gain and phase estimation, using the true covariance matrix was presented. The maximum likelihood calibration algorithm [15] was presented to compensate for the effect of mutual coupling, sensor gain, phase errors, and sensor position errors by estimating their calibration matrix using a set of calibrated sources at predetermined locations. Friedlander and Weiss [16] proposed an eigenstructure-based method to compensate for the mutual coupling and perturbation of gain and phase. Moreover, their method not required calibration sources. The method in [17] considered a problem of gain and phase estimation of (LEAs) based in different diagonal lines of covariance matrix.

The method in [9] studied DOA estimation problem in the presence of gain and phase uncertainties. It estimates the DOAs based on the eigendecomposition of a covariance matrix constructed from the dot product of the array output vector and its conjugate. However it has some drawbacks, such as; it is not applicable in Uniform linear arrays (ULA), and it gives ambiguous DOA estimation in circular array antennas. The proposed method solves the problem of DOA ambiguity in circular array antenna, but still not applicable in linear array antennas.

The rest of this paper is organized as follows: section II and section III describe the problem formulation and the calibration methods respectively. While section IV, discuss and present the results of computer simulations performed. Section V, gives the conclusion of the paper.

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2 Problem Formulations

Consider K narrowband far-field signals, $s_k(t)$ for k=1,2,…,K, with centre wavelength, λ, impinging on a planar array of M omnidirectional sensors labelled 1,2,…,M, from directions θ_k for k=1,2,…,K, where sensor 1 is taken as the reference point and the coordinate of sensor *m-th* is denoted by (x_m, y_m). The array output can be described as:

$$r_0(t) = \sum_{k=1}^{K} a(\theta_k) s_k(t) + n(t) = fGAs + n(t), \tag{1}$$

where

$$A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_K)], \tag{2}$$

$$a(\theta_k) = \left[1, e^{-j2\pi d_{m,k}}, \ldots, e^{-j2\pi d_{M,K}}\right]^T, \tag{3}$$

$$l_{m,k} = x_m \cos\theta_k + y_m \sin\theta_k, \tag{4}$$

$$s = [s_1(t), s_2(t), \ldots, s_K(t)]^T, \tag{5}$$

$$f = diag(e^{j\varphi_1}, e^{j\varphi_2}, \ldots, e^{j\varphi_M}), \tag{6}$$

$$G = diag(\alpha_1, \alpha_2, \ldots, \alpha_M), \tag{7}$$

α_m and ϕ_m denotes the gain and phase uncertainties of Sensor m, respectively. $n(t)$ is the vector of additive Gaussian white noise and $diag$ means diagonal matrix. In this paper, the superscripts "*", T and H denote the conjugate, transpose, and conjugate transpose, respectively.

Assuming that the additive Gaussian white noise has a zero mean, σ_n^2 variance, and $R_S = E[ss^H]$, then the covariance matrix of array output will be:

$$R = fGAR_s A^H G^H f^H + \sigma_n^2 I. \tag{8}$$

Thus, the problem addressed here is as follows: the DOA and the corresponding array gain-phase uncertainties are estimated from a given array output vector.

3 Calibration Methods

Here we propose a simple method of estimating DOA with gain and phase uncertainties. The idea is similar to that in [9], but the proposed method has better performance, and no DOA ambiguity. For comparison purpose, we review the method in [9] first.

3.1 THE DOT PRODUCT METHOD

The dot product method in [9] is described as follows.

3.1.1 The gain uncertain estimation

The eigenvalue decomposing matrix is given as follows:

$$R = \sum_{m=1}^{M} \beta_m u_m u_m^H, \tag{9}$$

where β_m represents the eigenvalues in descending order, and u_m presents corresponding eigenvectors. Then the gain uncertain can be estimated as:

$$\hat{\alpha}_m = sqrt\left(\frac{R(m,m) - \hat{\sigma}_n^2}{R(1,1) - \hat{\sigma}_n^2}\right), \tag{10}$$

where

$$\hat{\sigma}_n^2 = \frac{1}{M-K} \sum_{m=K+1}^{M} \beta_m. \tag{11}$$

3.1.2 The DOA estimation

To get an unambiguous DOA estimation, we need to make the radius of circular array antenna less than or equal to $\lambda/4$, and the two directions $\hat{\theta}_k$ and $\hat{\theta}_n$ must not closed to each other.

Let $\mathrm{Re}[\bullet]$ and $\mathrm{Im}[\bullet]$ be the real and imaginary parts of a complex number, respectively.

For complex signals, the two dimensional spatial spectrum is defined as

$$p_c(\theta, \theta') = (V_c^H \,\mathrm{Re}[a(\theta) \odot a^*(\theta')]^2 + V_c^H I_m [a(\theta) \odot a^*(\theta')]^2)^{-1}, \tag{12}$$

where V_c the noise eigenvector subspace of the dot product of the received signal with its conjugate, and $\|\bullet\|$ represents the 2-norm of a vector. Thus, the DOA pairs are given by:

$$(\hat{\theta}_k, \hat{\theta}_n) = \arg\max_{\theta' > \theta + \Delta\theta} p_c(\theta, \theta') \quad for\ k, n = 1, 2, 3, \ldots, K; k \neq n. \tag{13}$$

The subscript $\Delta\theta$ has little effect on the performance of the DOA estimation method, and it is convenient to set $\Delta\theta = 1^\circ$, while the subscript c implies complex.

For real signals, the two dimensional spatial spectrum is defined as follows:

$$p_r(\theta, \theta') = (\| V_r^H R_e [a(\theta) \cdot a^*(\theta')] \|^2)^{-1}, \tag{14}$$

$$(\hat{\theta}_k, \hat{\theta}_n) = \arg\max_{\theta' > \theta + \Delta\theta} p_r(\theta, \theta') \quad fork, n = 1, 2, 3, \ldots, K; k \neq n, \tag{15}$$

where the subscript r implies real.

3.1.3 The phase estimation

Using the notation $q=[\phi_1,\phi_2,...,\phi_M]$ and $F_\kappa = diag\{a(\theta_\kappa)\}$, the phase can be estimated as:

$$\hat{q} = \angle[z], \tag{16}$$

where $\angle[\cdot]$ represents the phase of complex number, and

$$z = \frac{Q^{-1}w}{(w^T Q^{-1} w)}, \tag{17}$$

$$Q = \sum_{k=1}^{K} F_k^H(\theta_k) V_o V_o^H F_k(\theta_k), \tag{18}$$

$$V_O = [u_{k+1},...,u_M], \tag{19}$$

$$w = [1,0,0,\ldots,0]^T. \tag{20}$$

The proposed method in [9], when a uniform circular array with a radius more than $\lambda/4$ is used to estimate the 2-D spatial spectrum of the DOA, that is $(\hat{\theta}_1,\hat{\theta}_2)$, where $\hat{\theta}_1$ and $\hat{\theta}_2$ are not closed to each other, two pairs (one is a false peak while the other is the actual peak) of results were obtained, which means there exist DOA ambiguity. The authors in [9] stated that, to solve this problem you need to make the radius $\leq \lambda/4$ according to the theorem in [9]. Using the proposed method the radius of the circular array antenna can be equal to $\lambda/2$, without DOA ambiguity.

For an antenna consisting of four sensors with sensor coordinates: $(x_1,y_1)=(-l,0)$, $(x_2,y_2)=(0,-l)$, $(x_3,y_3)=(l,0)$ and $(x_4,y_4)=(0,l)$, where $l=\lambda/2$ and the DOAs of signal are defined in [-90°;90°]. If the antenna is rotated by half the angle formed at the centre by two successive sensors (i.e, the coordinate become:

$(x_1,y_1)=(-l/\sqrt{2},-l\sqrt{2})$, $(x_2,y_2)=(l/\sqrt{2},-l/\sqrt{2})$, $(x_3,y_3)=(l/\sqrt{2},l/\sqrt{2})$, $(x_4,y_4)=(-l/\sqrt{2},l/\sqrt{2})$.

we found that, there will be no DOAs ambiguity.

To proof this consider four directions such that $(\theta_1,\theta_2)\neq(\theta_3,\theta_4)$, and $(\theta_1,\theta_2)\neq(\theta_4,\theta_3)$ from Equation (12), for complex-valued signals, the assumption that (θ_1,θ_2) and (θ_3,θ_4) are ambiguous (i.e. $(p_c(\theta_1,\theta_2)=p_c(\theta_3,\theta_4))$ leads to following four different cases:

$$a(\theta_1)\odot a^*(\theta_2) = a(\theta_3)\odot a^*(\theta_4) \tag{21}$$

$$a(\theta_1)\odot a^*(\theta_2) = j(a(\theta_3)\odot a^*(\theta_4)) \tag{22}$$

$$a(\theta_1)\odot a^*(\theta_2) = -a(\theta_3)\odot a^*(\theta_4) \tag{23}$$

$$a(\theta_1)\odot a^*(\theta_2) = -j(a(\theta_3)\odot a^*(\theta_4)) \tag{24}$$

For sensors 2, and 4

$$\begin{aligned}&\exp\left(\frac{-j2\pi l_{2,1}}{\lambda}\right)\odot\exp\left(\frac{j2\pi l_{2,2}}{\lambda}\right)= \\ &\exp\left(\frac{-j2\pi l_{2,3}}{\lambda}\right)\odot\exp\left(\frac{j2\pi l_{2,4}}{\lambda}\right),\end{aligned} \tag{25}$$

$$\begin{aligned}&\exp\left(\frac{-j2\pi l_{4,1}}{\lambda}\right)\odot\exp\left(\frac{j2\pi l_{4,2}}{\lambda}\right)= \\ &\exp\left(\frac{-j2\pi l_{4,3}}{\lambda}\right)\odot\exp\left(\frac{j2\pi l_{4,4}}{\lambda}\right),\end{aligned} \tag{26}$$

For θ_k we have $l_{2,k}=x_2\cos\theta_k - y_m\sin\theta_k$ and $l_{4,k}=-x_4\cos\theta_k + y_4\sin\theta_k$. It follows from Equations (25) and (26) that:

$$\begin{aligned}&x_2(\cos\theta_1-\cos\theta_2)-y_2(\sin\theta_1-\sin\theta_2)= \\ &x_2(\cos\theta_3-\cos\theta_4)-y_2(\sin\theta_3-\sin\theta_4)+\lambda q_1,\end{aligned} \tag{27}$$

$$\begin{aligned}&-x_4(\cos\theta_1-\cos\theta_2)+y_4(\sin\theta_1-\sin\theta_2)= \\ &-x_4(\cos\theta_3-\cos\theta_4)+y_4(\sin\theta_3-\sin\theta_4)+\lambda q_2,\end{aligned} \tag{28}$$

where q_1 and q_2 are integers.

Since

$$x_2 = x_3 = y_3 = y_4 = l/\sqrt{2},\ x_1 = x_4 = y_1 = y_2 = -l/\sqrt{2}$$

then the above two equations become:

$$\begin{aligned}&(\cos\theta_1-\cos\theta_2)+(\sin\theta_1-\sin\theta_2) \\ &=(\cos\theta_3-\cos\theta_4)+(\sin\theta_3-\sin\theta_4)+\frac{\lambda q_1}{x_2},\end{aligned} \tag{29}$$

$$\begin{aligned}&(\cos\theta_1-\cos\theta_2)+(\sin\theta_1-\sin\theta_2)= \\ &(\cos\theta_3-\cos\theta_4)+(\sin\theta_3-\sin\theta_4)+\frac{\lambda q_2}{y_4}.\end{aligned} \tag{30}$$

Since

$x_k=\lambda/2$ $y_k=\lambda/2$ $|\theta_k|\leq 90°$, $(\theta_1,\theta_2)\neq(\theta_3,\theta_4)$, and $(\theta_1,\theta_2)\neq(\theta_4,\theta_3)$ n_1 and n_2 are zero. From Equations (29) and (30), using sum-to-product identities, we can get:

$$\frac{\sin\frac{\theta_1+\theta_2}{2}}{\cos\frac{\theta_1+\theta_2}{2}} = \frac{\sin\frac{\theta_3+\theta_4}{2}}{\cos\frac{\theta_3+\theta_4}{2}}, \tag{31}$$

$$\left(\sin\frac{\theta_1-\theta_2}{2}\right)^2 = \left(\sin\frac{\theta_3-\theta_4}{2}\right)^2. \tag{32}$$

From Equations (31) and (32), we have

$$\theta_1 = \theta_3 + 2(\mathrm{n}_1+\mathrm{n}_2)\pi, \tag{33}$$

$$\theta_2 = \theta_4 + 2(n_1 - n_2)\pi, \tag{34}$$

where n_1 and n_2 are integers. Since $|\theta_k| \le 90^\circ$, $(\theta_1,\theta_2) \ne (\theta_3,\theta_4)$, $(\theta_1,\theta_2) \ne (\theta_4,\theta_3)$, Equations (33) and (34) cannot be simultaneously true. It follows that Equation (21) does not hold.

For sensors 2-4, Equations (22) leads to:

$$\begin{aligned} &2\pi\left(\frac{x_2(\cos\theta_1\text{-}\cos\theta_2)-y_2(\sin\theta_1\text{-}\sin\theta_2)}{\lambda}\right)= \\ &2\pi\left(\frac{x_2(\cos\theta_3\text{-}\cos\theta_4)\text{-}y_2(\sin\theta_3\text{-}\sin\theta_4)}{\lambda}\right)-\frac{4\pi n_1+\pi}{2}, \end{aligned} \tag{35}$$

$$\begin{aligned} &2\pi\left(\frac{x_3(\cos\theta_1\text{-}\cos\theta_2)+y_3(\sin\theta_1\text{-}\sin\theta_2)}{\lambda}\right)= \\ &2\pi\left(\frac{x_3(\cos\theta_3\text{-}\cos\theta_4)\text{-}y_3(\sin\theta_3\text{-}\sin\theta_4)}{\lambda}\right)+\frac{4\pi n_2+\pi}{2}, \end{aligned} \tag{36}$$

$$\begin{aligned} &2\pi\left(\frac{\text{-}x_4(\cos\theta_1\text{-}\cos\theta_2)+y_4(\sin\theta_1\text{-}\sin\theta_2)}{\lambda}\right)= \\ &2\pi\left(\frac{\text{-}x_4(\cos\theta_3\text{-}\cos\theta_4)+y_4(\sin\theta_3\text{-}\sin\theta_4)}{\lambda}\right)+\frac{4\pi n_3+\pi}{2}, \end{aligned} \tag{37}$$

where q_1, q_2 and q_3 are integers. Since $x_2 = x_3 = y_3 = y_4 = l/\sqrt{2}$ and $x_1 = x_4 = y_1 = y_2 = -l/\sqrt{2}$, it's clear that Equations (35) and (37) conflicts with Equation (36). It follows that Equation (22) does not hold. Similarly, neither of Equations (23) nor (24) holds.

Therefore, in the case of complex-valued signals $p_c(\theta_1,\theta_2) \ne p_c(\theta_3,\theta_4)$ i.e. (θ_1,θ_2) and (θ_3,θ_4) are unambiguous. For real-valued signals, the assumption that (θ_1,θ_2) and (θ_3,θ_4) are ambiguous, i.e. $p_c(\theta_1,\theta_2) = p_c(\theta_3,\theta_4)$ leads to the following two different cases:

$$\mathrm{Re}[a(\theta_1)\odot a^*(\theta_2)] = \mathrm{Re}[a(\theta_3)\odot a^*(\theta_4)], \tag{38}$$

$$\mathrm{Re}[a(\theta_1)\odot a^*(\theta_2)] = -\mathrm{Re}[a(\theta_3)\odot a^*(\theta_4)]. \tag{39}$$

For sensors 2 and 4, Equation (38) leads to:

$$\begin{aligned} &(\cos\theta_1\text{-}\cos\theta_2)+(\sin\theta_1-\sin\theta_2)= \\ &\pm\left[(\cos\theta_3\text{-}\cos\theta_4)+(\sin\theta_3\text{-}\sin\theta_4)\right]=\frac{\lambda q_1}{x_2}, \end{aligned} \tag{40}$$

$$\begin{aligned} &(\cos\theta_1-\cos\theta_2)+(\sin\theta_1-\sin\theta_2)= \\ &\pm\left[(\cos\theta_3-\cos\theta_4)+(\sin\theta_3-\sin\theta_4)\right]=\frac{\lambda q_2}{y_4}, \end{aligned} \tag{41}$$

where q_1 and q_2 are integers, we obtain the same result as Equations (33) and (34). Thus, it follows that Equation (38) does not hold.

For sensors 2 and 4, Equation (39) leads to:

$$\begin{aligned} &2\pi\left(\frac{x_2(\cos\theta_1\text{-}\cos\theta_2)-y_2(\sin\theta_1\text{-}\sin\theta_2)}{\lambda}\right)= \\ &2\pi\left(\frac{x_2(\cos\theta_3\text{-}\cos\theta_4)\text{-}y_2(\sin\theta_3\text{-}\sin\theta_4)}{\lambda}\right)+2\pi q_1+\pi, \end{aligned} \tag{42}$$

$$\begin{aligned} &2\pi\left(\frac{x_3(\cos\theta_1\text{-}\cos\theta_2)+y_3(\sin\theta_1\text{-}\sin\theta_2)}{\lambda}\right)= \\ &2\pi\left(\frac{x_3(\cos\theta_3\text{-}\cos\theta_4)+y_3(\sin\theta_3\text{-}\sin\theta_4)}{\lambda}\right)+2\pi q_2+\pi, \end{aligned} \tag{43}$$

$$\begin{aligned} &2\pi\left(\frac{\text{-}x_4(\cos\theta_1\text{-}\cos\theta_2)+y_4(\sin\theta_1\text{-}\sin\theta_2)}{\lambda}\right)= \\ &2\pi\left(\frac{\text{-}x_4(\cos\theta_3\text{-}\cos\theta_4)\text{-}y_4(\sin\theta_3\text{-}\sin\theta_4)}{\lambda}\right)+2\pi q_3+\pi, \end{aligned} \tag{44}$$

where q_1, q_2 and q_3 are integers.

Since $x_2 = x_3 = y_3 = y_4 = l/\sqrt{2}$ and $x_1 = x_4 = y_1 = y_2 = -l/\sqrt{2}$, it's clear that Equations (42) and (44) conflict with (43). It follows that (39) does not hold. Therefore, in the case of real-valued signals $p_c(\theta_1,\theta_2) \ne p_c(\theta_3,\theta_4)$, i.e. (θ_1,θ_2) and (θ_3,θ_4) are unambiguous.

The proposed method used the rotation technique to avoid DOA ambiguity. The steps of the proposed method can summarized as follows:

1) Rotate the uniform circular array antenna with angle $\delta/2$, so that $a(\theta_k)_{rotate} = a(\theta_k - \delta/2)$, where δ is the angle formed at the centre by two successive sensors.

2) Estimate the gain uncertain using Equation (10).

3) The DOAs of signals are estimated from Equation (13) in case of complex-valued signals or by Equation (15) in case of real-valued signals.

4) Estimate the phase uncertain using Equation (16), based on the estimated DOA.

4 Computer simulations

Consider a uniform circular array (UCA) composed of seven sensors, with the first sensor located at the origin. Two far-field narrowband signals are impinging on the array from directions $\theta_1 = 30°$ and $\theta_2 = -20°$, respectively. The range of the DOAs of the signals are defined in $[-90°, 90°]$ The unknown gain $\{a_m\}_{m=1}^{M}$ and unknown phase $\{\phi_m\}_{m=1}^{M}$ of the sensors are generated by $\alpha_m = 1 + \sqrt{12}\sigma_a C_m, \varphi_m = \sqrt{12}\sigma_\varphi b_m$, where C_m and b_m are independent and identically distributed random variables distributed uniformly over [-0.5,0.5], while σ_a and σ_ϕ are the standard deviations of α_m and ϕ_m, and are equal

0.1 and 40° respectively. Assuming the powers of the signals at different directions are equal. Let $\Delta\theta = 1^{\circ}$ for complex signals and 5° for real signals. The SNR and the sample number are 30 dB and 500, respectively. Figure 1 illustrates uniform circular array of seven sensors with radius $\lambda/2$, in which the first sensor is located at the origin. Its two-dimensional spatial spectrum $p_c(\theta_1, \theta_2)$ in the case of complex-valued signals is shown in Figure 2.

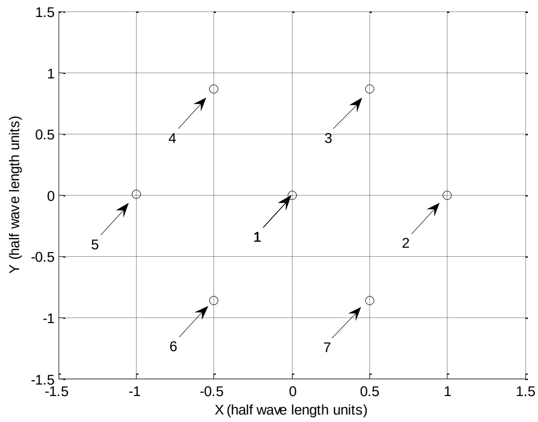


FIGURE 1 Uniform circular array of seven sensors with radius $\lambda/2$, the first sensor in the origin

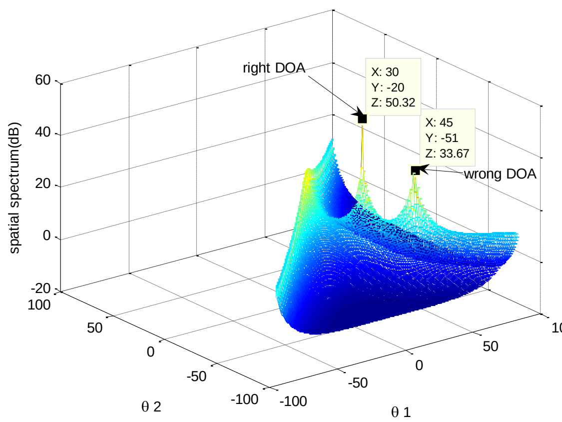


FIGURE 2 Two-dimensional spatial spectrum $p_c(\theta_1, \theta_2)$ in the case of complex-valued signals for Figure 1, $\theta_1 = 30°$ and $\theta_2 = -20°$

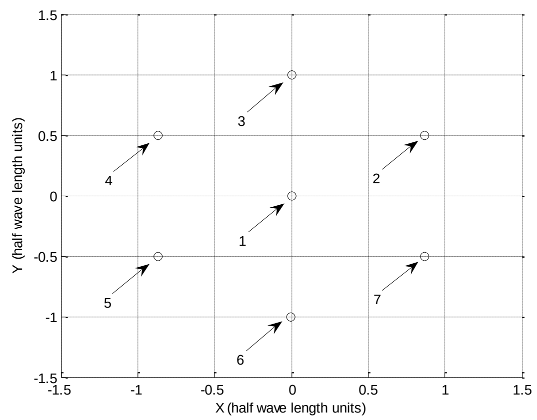


FIGURE 3 shows the uniform circular array antenna in Figure 1 after the rotation, $\delta/2 = 30°$

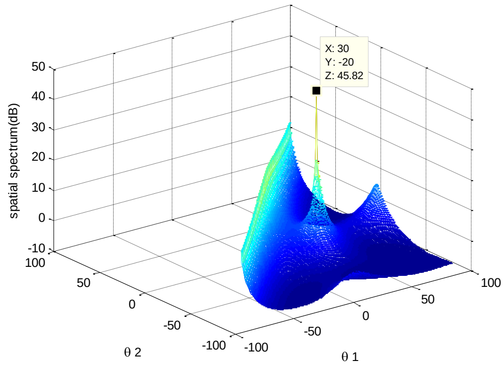


FIGURE 4 Two-dimensional spatial spectrum $p_c(\theta_1, \theta_2)$ in the case of complex-valued signals for Figure 3, $\theta_1 = 30°$ and $\theta_2 = -20°$

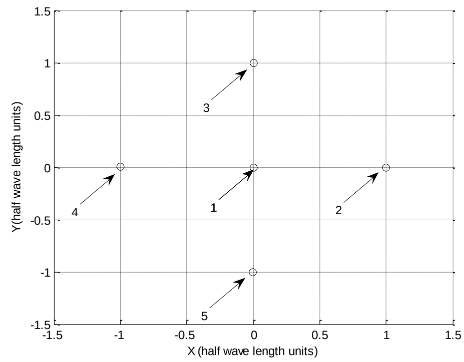


FIGURE5 Uniform circular array of five sensors with radius $\lambda/2$, first sensor in the origin

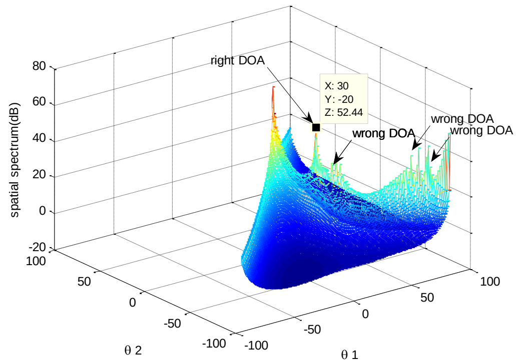


FIGURE 6 Two-dimensional spatial spectrum $p_c(\theta_1, \theta_2)$ in case of complex-valued signal for Figure 5, $\theta_1 = 30°$ and $\theta_2 = -20°$

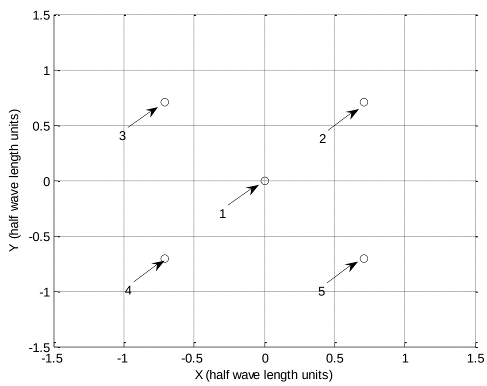


FIGURE 7 Shows the antenna in Figure (5) when rotated, $\delta/2 = 45°$

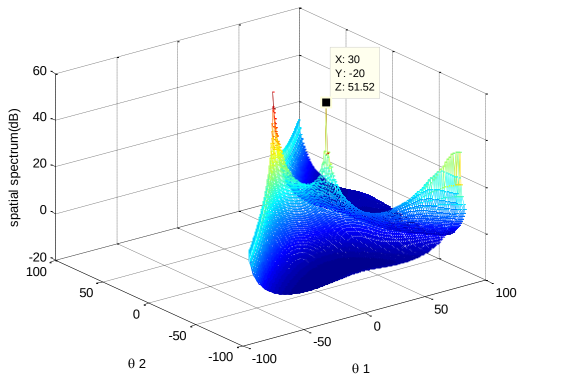


FIGURE8 Two-dimensional spatial spectrum p_c (θ_1, θ_2) in case of complex-valued signals for Figure 7, θ_1 = 30° and θ_2 = –20°

From Figure 2, it is clear that there exists a false peak at point (45°, –51°) beside the actual peak at point (30°, –20°). This is because all the inter-sensor spacing is greater than λ/4 resulting in DOA estimation ambiguity.

We can avoid this false peak by rotating the antenna by δ/2 = 30°. Figure 3 shows the uniform circular array antenna in Figure 1 after the rotation. Figure 4 shows the corresponding spatial spectrum in the case of complex-valued signals. In Figure 4, it can be observed that there is only one peak in (30°, –20°) and the false peak has been eliminated. This is due to the fact that rotating the antenna by an angle equal to half of the angle formed at the centre by two successive sensors leads to unambiguous DOA estimates, using the new DOA estimation method.

Figure 5 shows the case of a five sensors circular array antenna with radius λ/2 with first sensor at origin. Figure 6 shows its spatial spectrum in the case of complex-valued signals. From Figure 6 we can observe that, there are many peaks. However, by rotating the antenna by an angle of 45° we obtain only one peak as demonstrated next.

Figure 7 shows the antenna in Figure 5 when rotated, such that δ/2 = 45°. Figure 8 shows the corresponding spatial spectrum in the case of complex-valued signals. Figure 8 shows that there is only one peak in (30°, –20°) and the false peaks are omitted.

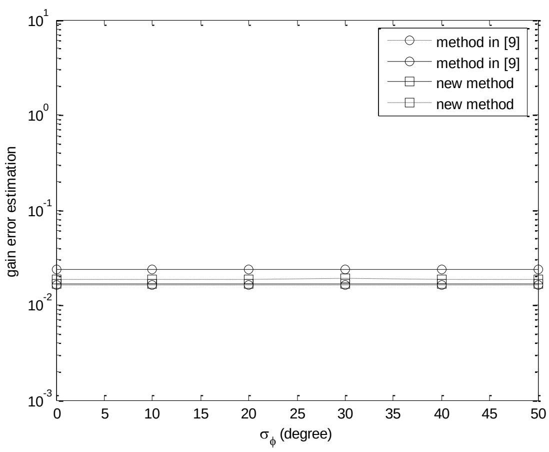


FIGURE 9a ARMSE of gain error estimates versus σ_ϕ (the dashed and solid plots represent the cases of complex-valued and real-valued signals, respectively)

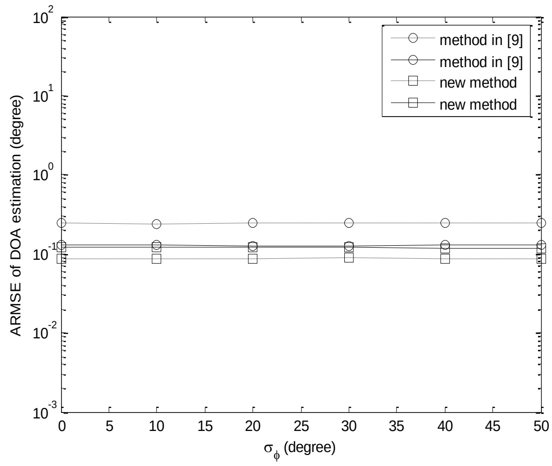
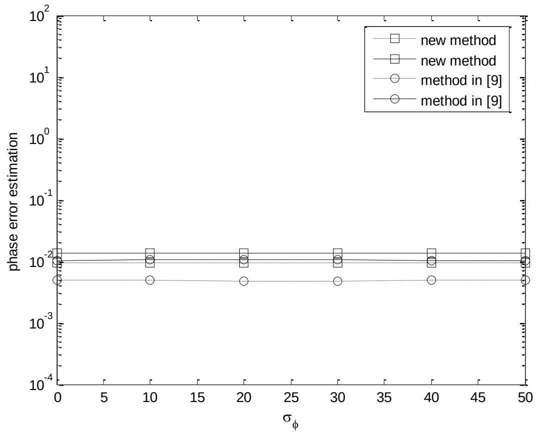


FIGURE 9b ARMSE of DOA error estimates versus σ_ϕ (the dashed and solid plots represent the cases of complex-valued and real-valued signals, respectively)

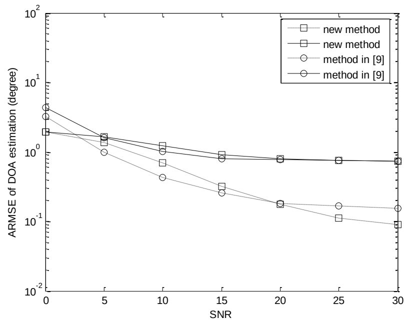


FGIRUE 9c ARMSE of phase error estimates versus σ_ϕ (the dashed and solid plots represent the cases of complex-valued and real-valued signals, respectively)

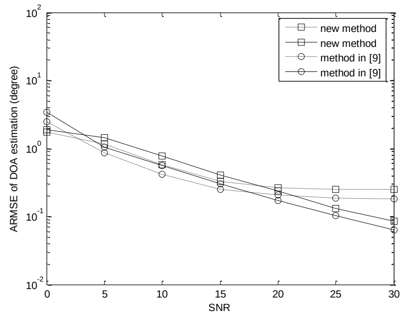
In order to examine the effect of phase error, SNR, and the number of snapshot, two signals were impinged from direction, –20° and 20° respectively. In each case the number of samples was 200 while all other simulation parameters were the same as in the previous experiment.

The effect of phase error was studied based on 500 experiments. The average root mean square error (ARMSE) [18] curves of the gain error, the DOA error, and the phase error estimates versus the standard deviation of the phase error σ_ϕ are shown in Figures 9(a-c) respectively. The Figures show that the performance of both the new method and that of the method in [9] are independent of phase errors and are approximately equal to each other.

Similarly, the effect of SNR based on σ_ϕ experiments. Figures 10(a) and (b) show the ARMSE of the DOA estimate versus the SNR, when σ_ϕ equals 5° and 50° respectively. Figure 10a shows that the performance of the high phase error work as same as small error.

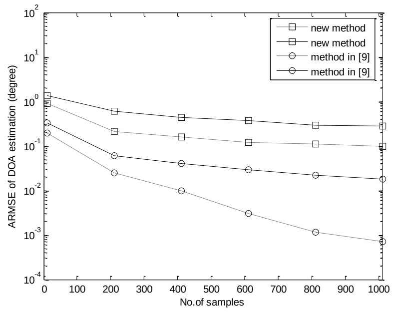


a) ARMSE of DOA error estimates versus $\sigma_\phi = 5°$

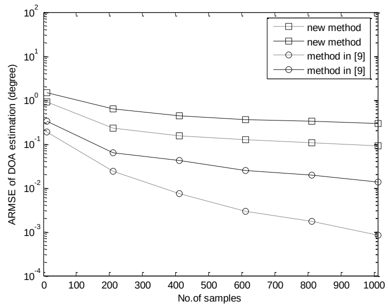


b) ARMSE of DOA error estimates versus $\sigma_\phi = 50°$

FIGURE 10 ARMSE of DOA error estimates versus SNR, σ_ϕ (the dashed and solid plots represent the cases of complex-valued and real-valued signals, respectively)



a) ARMSE of DOA error estimates versus $\sigma_\phi = 5°$



b) ARMSE of DOA error estimates versus $\sigma_\phi = 50°$

FIGURE 11 ARMSE of DOA error estimates versus number of samples, σ_ϕ (The dashed and solid plots represent the cases of complex-valued and real-valued signals, respectively)

Also, the effect of sample number based on 500 experiments. Figures 11a and 11b show the ARMSE of DOA estimates versus the number of samples when σ_ϕ equals 5° and 50° respectively. Figure 11a shows that the new method has worse performance than method in [9]. Also the new method and the method in [9] perform better as the number of samples increases. Figure 11b shows that the performance of both methods remain unchanged as the phase error increases.

5 Conclusion

In this paper, we considered the problem of DOA estimation in the presence of gain and phase errors. By improving the dot product method, we propose a method for simultaneously estimating the DOA and gain-phase errors. Therefore, the proposed method overcomes the disadvantage of the Dot product method, which fails in UCA with radius more than $\lambda/4$. The problem of DOA ambiguity can be solved by rotating the antenna with an angle equal to half the angle formed at the centre by two successive sensors. In addition, the proposed method is independent of phase errors, and it performance is almost same as that of the Dot product method. The disadvantage of the new method is that it is only applicable in UCA, and not applicable in ULA (since there is no angle between two successive sensors).

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