The nonlinear vibration analysis of the fluid conveying pipe based on finite element method

Gongfa Li*, Jia Liu, Guozhang Jiang, Jianyi Kong, Liangxi Xie, Wentao Xiao, Yikun Zhang, Fuwei Cheng

College of Machinery and Automation, Wuhan University of Science and Technology, Wuhan, 430081, China
Received 1 March 2014, www.tsi.lv

Abstract

A coupling between the fluid and the structure existed almost in all industrial areas the vibration of fluid solid coupling for fluid conveying pipe was called the "dynamics of typical"[1]. Because of the physical model and mathematical description for the fluid conveying pipe was simple, especially it was easy to design and manufacturing, according to the characteristics of fluid conveying pipe, transformed the transverse vibration of the fluid conveying pipe to the beam element model of two nodes. Using Lagrangian interpolation function, the first order Hermite interpolation function and the Ritz method to obtain the element standard equation, and then integrated a global matrix equation. Used the mode decomposition method, obtained the vibration modal of the fluid conveying pipe with Matlab programming. The vibration modal of the fluid conveying pipe in four kinds of boundary conditions was analysed. The characteristics of pipes conveying fluid was obtained which the pipeline system parameters under different boundary constraints. To provide the theoretical support for the research of vibration attenuation of fluid conveying pipes.

Keywords: Fluid solid coupling, Nonlinear vibration, Modal analysis, Interpolation, The finite element algorithm

1 Introduction

A system of conveying fluid pipe was widely used in the city water supply and drainage, water power, chemical machinery, aerospace, marine engineering and the nuclear industry and other fields, it was play an important role for improving the living standards of the nation and the national economic strength. However, according to statistics, in the industrial production, the damage of the water hammer in pipeline interface and pipeline rupture accounted for over 75% in the total system failure rates, seriously affected the normal production and operation, resulting in huge economic losses. Coupling between the fluid and the structure is almost exist in all industrial areas, the fluid and solid coupling vibration of pipeline flow is called the typical "dynamic" [2], because of its simple physical model and mathematical description, especially the pipeline is easy to design and manufacture, which provides convenience to the coordinated development of the theoretical and experimental research. But the pipeline, as the application extremely widely, of the coupling vibration of pressure flow is the most representative in this field, which has a broad background in engineering applications, and a very high theoretical research value and practical significance, but also has many challenges [3].

2 The establish of mathematical model for the output response of the fluid conveying pipe

Taking into account the pipe ratio for length to diameter is relatively large, the deformation of radial is the same, just only exists a certain angle difference, which can be regarded the pipe as plane beam element to consider, using two node element, as shown in Figure 1, the node number of 1 and J. The conveying fluid pipe is only affected by the lateral force, no axial force, so analysis with the two node element, the nodal displacement model can be defined as [4]:

\[ y(t) = [y_i, \theta_i, y_j, \theta_j]^T. \]  

(1)

FIGURE 1 The deformation of two node element

* Corresponding author- E-mail: ligongfa@aliyun.com
In the node parameter of unit, in addition to the node value of field function, also contains node value for a derivative $\partial y/\partial x$ of the field function. In order to maintain the continuity of field function derivative between the public node element, and in the end nodes to keep the derivative order is first for the field function, so the first-order Hermite interpolation polynomial is used [5]:

$$N(\xi)=[N_1(\xi), N_2(\xi), N_3(\xi), N_4(\xi)],$$

(2)

where:

$$N_1(\xi) = 1 - 3\xi^2 + 2\xi^3,$$
$$N_2(\xi) = \xi - 2\xi^2 + \xi^3,$$
$$N_3(\xi) = 3\xi^2 - 2\xi^3,$$
$$N_4(\xi) = \xi^3 - \xi^2.$$

When the local dimensionless coordinate is taken, the $\xi$ is get $0 \leq \xi \leq 1$.

Using the Ritz method interpolation function to establish standard unit equation of the approximate solution of the lateral vibration we find the interpolation function:

$$[M^e][\ddot{y}] + [C^e][\dot{y}] + [K^e][y] = [Q^e],$$

(3)

where

$$[M^e] = [M^e_1] + [M^e_2],$$
$$[C^e] = [C^e_1] + [C^e_2],$$
$$[K^e] = [K^e_1] + [K^e_2] + [K^e_3],$$
$$[Q^e] = [Q^e_1] + [Q^e_2] + [Q^e_3],$$

$$[M^e_1] = \int N_i (m_p + m_f) N_j dx,$$
$$[C^e_1] = \int N_i (2m_f v_f) \frac{\partial N_j}{\partial x} dx,$$
$$[C^e_2] = \int N_i (A_f \frac{\partial P}{\partial x}) N_j dx,$$
$$[K^e_1] = \int \frac{\partial^2 N_i}{\partial x^2} (EI) \frac{\partial^2 N_j}{\partial x^2} dx,$$
$$[K^e_2] = \int \frac{\partial N_i}{\partial x} (m_f v_f^2 + (1 - 2\gamma) A_f P) \frac{\partial N_j}{\partial x} dx,$$
$$[Q^e_1] = \int (-m_f \frac{\partial v_f}{\partial t}) N_j dx,$$
$$[Q^e_2] = \int (\frac{v_f A_f}{c^2} \frac{\partial P}{\partial t}) N_j dx,$$
$$[Q^e_3] = \int (m_p g + m_f g) N_j dx.$$

2.1 THE ENTIRETY MATRIX

There are some matrix must be appropriately expanded rewrite when the unit matrix integrated to the entirety matrix so that the matrix of all elements with uniform format, then according to the superposition to assembly [6].

The usually study boundary constraint conditions include fix to hinge and fix to fix constraints, the mathematical expression of its boundary is given below, respectively: (I), (II), (III), (IV):

$$\begin{align*}
(I) \quad & y(0,t) = 0, y(L,t) = 0, \\
(II) \quad & y(0,t) = 0, \frac{\partial y}{\partial x} |_{x=0} = 0, \\
(III) \quad & y(0,t) = 0, \frac{\partial y}{\partial x} |_{x=L} = 0, \\
(IV) \quad & y(0,t) = 0, \frac{\partial y}{\partial x} |_{x=L} = 0.
\end{align*}$$

The four kinds of boundary conditions of above given all belong to the first class constraint conditions, for this kind of constraint conditions can usually use "row row column method" and "multiplied with bigger number method". The "multiplied with bigger number method" is make the main diagonal element about the specified node displacement in the overall stiffness matrix with multiply by the large number $\lambda$, at the same time, give the specified value of node displacement to the corresponding element of load matrix [7], then multiply by the same number as well as the main diagonal elements. Using the “multiplied with bigger number method ‘to deal with the boundary constraint condition by’, finally forms the whole matrix:

$$[M][\ddot{y}] + [C][\dot{y}] + [K][y] = [Q].$$

(4)

2.2 THE MODAL SOLUTION OF FLUID CONVEYING PIPES BASED ON MODE-SUPERPOSITION METHOD

Solving the modal of the system when the movement of the global matrix is obtained, we select the mode-superposition method for the model. Solving the inherent frequency and vibration type of fluid conveying pipes it is usually divided into two cases about damped and non-damped. The non-damping case is solved in the real domain, and the damping case is solved in the complex domain.

1) The free vibration equation without consider the damping:
\( M\ddot{y}(t) + Ky(t) = 0, \) (5)

using a solution:
\[ y = \phi \sin(\omega(t - t_0)), \]
where \( \phi \) is the \( n \) order vector, \( \omega \) is the vibration frequency vector, \( t \) is the time variable, \( t_0 \) is the time constant that determined by the initial conditions.

Using (6) and (5) we can get the generalized eigenvalue, i.e.:
\[ K\phi - \omega^2 M\phi = 0. \] (7)

According to the general solution of eigenvalues and eigenvectors to the identified \( \phi \) and \( \omega \), the results can be obtained as \( n \) characteristic solutions \( (\alpha_1^2, \varphi_1), (\alpha_2^2, \varphi_2), \ldots, (\alpha_n^2, \varphi_n) \), which the characteristic values \( \alpha_1, \alpha_2, \ldots, \alpha_n \) for the natural frequency of the conveying fluid pipes system take place. There is also the ordering \( 0 \leq \alpha_1 < \alpha_2 < \cdots < \alpha_n \) and the eigenvectors \( \varphi_1, \varphi_2, \ldots, \varphi_n \) for \( n \) inherent vibration feature vector, which corresponds to the inherent natural frequency.

2) The free vibration equation with a damping:
\[ M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = 0. \] (8)

A solution of the natural frequency and vibration type equations in the damped, early obtained the natural frequency and vibration model, which corresponds to the free damped vibration equation, gives:
\[ \Phi = [\varphi_1, \varphi_2, \ldots, \varphi_n], \]
\[ \Omega = \text{diag}[\omega_1, \omega_2, \ldots, \omega_n], \]
\[ \Xi = \text{diag}[^n\xi_1, ^n\xi_2, \ldots, ^n\xi_n]. \] (9), (10), (11)

Using the vibration equation of the generalized coordinates and the equation (8) we obtain:
\[ [\Phi]^T [M][\Phi]^T [\dot{y}(t)] + [\Phi]^T [C][\Phi]^T [\dot{y}(t)] + [\Phi]^T [K][\Phi]^T [y(t)] = 0. \] (12)

The generalized mass matrix, the generalized damping matrix and generalized stiffness matrix, respectively are:
\[ [M]_c = [\Phi]^T [M][\Phi], \]
\[ [C]_c = [\Phi]^T [C][\Phi], \]
\[ [K]_c = [\Phi]^T [K][\Phi]. \]

After the generalized coordinate transformation \( \{q\} = [\Phi]^{-1}\{y\} \) we obtain:
\[ \ddot{q} + 2\Omega\dot{q} + \Omega^2 q = 0. \] (13)

The equation (13) is a group of uncoupled equations in component form:
\[ \ddot{q}_i + 2\xi_i \omega_i \dot{q}_i + \omega_i^2 q_i = 0, \quad (i=1,2,\ldots,n) \] (14)

It is easy to obtain the complex eigenvalue and complex eigenvectors using the method of the generalized eigenvalues and then to obtain the natural vibration modes and natural frequencies in the damping case.

3) The effects of the system parameters on the conveying fluid pipe

The developed equation of transverse motion for conveying fluid pipes with the Matlab simulation software and all of the parameters in the simulation process are given in Table 1. During the simulation process, we make two end points of pipe as the supporting points assuming a rigid constrain with a room temperature water of the fluid and rolling copper as pipe material.

3.1 FOUR KINDS OF BOUNDARY CONDITIONS OF THE FIRST FOUR ORDER VIBRATION MODE

The first four order mode of vibration of the constraint of fix to hinge as shown in Figure 2. From Figure 2, the relative position of all nodes in the vibration pipes that reflect the inherent form vibration of the conveying fluid pipe in the constraint of fix to hinge. The first four-order vibration mode that corresponding to the first four order natural frequency of the conveying fluid pipe, the natural frequency of the first order vibration mode is the minimum and two order mode, three order mode, four order mode is increasing [8]. The constraints of fix to suspension extension and fix to hinge does not belong to the same class, so the vibration mode is also different, as shown in Figure 3. For the same boundary, damped and no damped vibration mode is basically the same.

3.2 FOUR KINDS OF BOUNDARY OF VIBRATION MODE OF DEFORMATION

For different boundary constraints, a vibration mode of deflection for conveying fluid pipe is not identical [9]. The constraint of fix to hinge is shown in Figure 4, the constraint of hinge to hinge is shown in Figure 5, the constraint of fix to fix is shown in Figure 6, the constraint of fix to suspension extension is shown in Figure 7.
From the simulation results, the boundary constraints have great influence on the vibration characteristics of the conveying fluid pipe. Compare with vibration characteristics under the various boundary conditions, it
can be concluded that the constraint of fix to hinge, hinge to hinge, fix to fix, fix to suspension extension:

a) The first-order natural frequency arrange as follows: the constraint of fix to suspension extension, hinge to hinge, fix to hinge, fix to fix;

b) Node vibration period arrange as follows: the constraint of fix to fix, fix to hinge, hinge to hinge, fix to suspension extension;

c) Node vibration amplitude arrange as follows: the constraint of fix to fix, fix to hinge, hinge to hinge, fix to suspension extension.

TABLE 1

<table>
<thead>
<tr>
<th>Name</th>
<th>Boundary conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency (Hz)</td>
<td>Fix to hinge</td>
</tr>
<tr>
<td>First order</td>
<td>23.3534</td>
</tr>
<tr>
<td>Second order</td>
<td>78.5668</td>
</tr>
<tr>
<td>Third order</td>
<td>165.251</td>
</tr>
<tr>
<td>Fourth order</td>
<td>284.1623</td>
</tr>
</tbody>
</table>

4 Conclusions

This paper transformed the fluid conveying pipe to the beam element model for two nodes, and with the Matlab to simulate the transverse motion equation of the conveying fluid pipe, summarized the simulation results and analyzed. In four kinds of boundary conditions, analysis the affect factors of pipeline system parameters for the vibration modal of the fluid conveying pipe, verified the correctness of the established vibration model of the fluid conveying pipeline. The characteristics of pipes conveying fluid is obtained which the pipeline system parameters under different boundary constraints. To provide the theoretical support for the research of vibration attenuation of fluid conveying pipes.

Acknowledgments

This research reported in the paper is supported by National Natural Science Foundation of China (71271160).

References


Authors

Gongfa Li, born on October 7, 1979, Honghu, China

Current position, grades: Associate professor. College of Machinery and Automation, Wuhan University of Science and Technology

Scientific interest: computer aided engineering, mechanical CAD/CAE, Modelling and optimal control of complex industrial process.

Publications number or main:58

Experience: Dr. Gongfa Li received the Ph.D. degree in mechanical design and theory from Wuhan University of Science and Technology in China. Currently, he is an associate professor at Wuhan University of Science and Technology, China. His major research interests include modelling and optimal control of complex industrial process. He is invited as a reviewer by the editors of some international journals, such as Environmental Engineering and Management Journal, International Journal of Engineering and Technology, International Journal of Physical Sciences, International Journal of Water Resources and Environmental Engineering, etc. He has published nearly twenty papers in related journals.

Jia Liu, born in 1990, Shanxi, China

Current position, grades: Currently occupied in his M.S. degree in mechanical design and theory at Wuhan University of Science and Technology

Scientific interest: mechanical CAD/CAE, signal analysis and processing

Experience: Jia Liu was born in Shanxi province, P. R. China, in 1990. He received B.S. degree in mechanical engineering and automation from Wuchang Institute of Technology, Wuhan, China, in 2012. He is currently occupied in his M.S. degree in mechanical design and theory at Wuhan University of Science and Technology. His current research interests include mechanical CAD/CAE, signal analysis and processing.
### Guozhang Jiang, born on December 15, 1965, Tianmen, China

**Current position, grades:** Professor of Industrial Engineering, and the Assistant Dean of the college of machinery and automation, Wuhan University of Science and Technology.  
**University studies:** He received the B.S. degree in Chang'an University, China, in 1986, and M.S. degree in Wuhan University of Technology, China, in 1992. He received the Ph.D. degree in mechanical design and theory from Wuhan University of Science and Technology, China, in 2007.  
**Scientific interest:** computer aided engineering, mechanical CAD/CAE and industrial engineering and management system.  
**Publications:** 100  
**Experience:** Guozhang Jiang was born in Hubei province, P. R. China, in 1965. He is a Professor of Industrial Engineering, and the Assistant Dean of the college of machinery and automation, Wuhan University of Science and Technology. Currently, his research interests are computer aided engineering, mechanical CAD/CAE and industrial engineering and management system.

### Jianyi Kong, born on February 19, 1961, Jiangxi, China

**Current position, grades:** The president of Wuhan University of Science and Technology, China.  
**University studies:** Jianyi Kong received the Ph.D. degree in mechanical design from Universität der Bundeswehr Hamburg, Germany, in 1995.  
**Scientific interest:** intelligent machine and controlled mechanism, dynamic design and fault diagnosis in electromechanical systems, mechanical CAD/CAE, intelligent design and control, etc.  
**Publications:** 200  
**Experience:** He was awarded as a professor of Wuhan University of Science and Technology in 1998. Currently, he is the president of Wuhan University of Science and Technology, China. He services on the editorial boards of the Chinese journal of equipment manufacturing technology. He is a director of the Chinese society for metals, etc. His research interests focus on intelligent machine and controlled mechanism, dynamic design and fault diagnosis in electromechanical systems, mechanical CAD/CAE, intelligent design and control, etc.

### Liangxi Xie, born on October 27, 1971, Honghu, China

**Current position, grades:** Associate professor, College of Machinery and Automation, Wuhan University of Science and Technology.  
**Scientific interest:** rotary vane steering gear (RVSG) and vane seals.  
**Publications:** 35  
**Experience:** He is a major in mechanical design and theory and focus on the research of rotary vane steering gear (RVSG) and vane seals. He has published more than ten papers in related journals.

### Xiao Wentao, born in 1989, Hubei, China

**Current position, grades:** Currently occupied in his M.S. degree in mechanical design and theory at Wuhan University of Science and Technology.  
**Scientific interest:** mechanical CAD/CAE, signal analysis and processing.  
**Experience:** Wentao Xiao was born in Hubei province, P. R. China, in 1989. He received B.S. degree in mechanical engineering and automation from City College of Wuhan University of Science and Technology, Wuhan, China, in 2013. He is currently occupied in his M.S. degree in mechanical design and theory at Wuhan University of Science and Technology. His current research interests include mechanical CAD/CAE, signal analysis and processing.

### Yikun Zhang, born in 1990, Hubei China

**Current position, grades:** Currently occupied in his M.S. degree in mechanical design and theory at Wuhan University of Science and Technology.  
**Scientific interest:** mechanical CAD/CAE, signal analysis and processing.  
**Experience:** Yikun Zhang was born in Hubei province, P. R. China, in 1990. He received B.S. degree in mechanical engineering and automation from Hu Bei University of Arts and Science, Xiangyang, China, in 2013. He is currently occupied in his M.S. degree in mechanical design and theory at Wuhan University of Science and Technology. His current research interests include mechanical CAD/CAE, signal analysis and processing.

### Fuwei Cheng, born in 1988, Hubei, China

**Current position, grades:** Currently occupied in his M.S. degree in mechanical design and theory at Wuhan University of Science and Technology.  
**Scientific interest:** mechanical CAD/CAE, signal analysis and processing.  
**Experience:** Fuwei Cheng was born in Hubei province, P.R. China, in 1988. He received B.S. degree in mechanical engineering and automation from Donghu college of Wuhan University, Wuhan, China, in 2012. He is currently occupied in his M.S. degree in mechanical design and theory at Wuhan University of Science and Technology. His current research interests include mechanical CAD/CAE, signal analysis and processing.