Tensor modular sparsity preserving projections for dimensionality reduction

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Abstract

In order to reduce the computational complexity and promote the classification performance of Modular Weighted Global Sparse Representation (MWGSR), Tensor Modular Sparsity Preserving Projections (TMSPP) for dimensionality reduction is proposed. The algorithm firstly partitions an image into several equal-sized modules and constructs these modules into a third-order tensor image; then, the algorithm makes module sparse reconstructions and some modules with less reconstruction errors are selected. These selected modules are recombined into a dataset with fewer dimensions and a new sparse reconstruction weight is gotten on the new dataset, which is denoted as the sparse reconstruction weight of original samples; finally, projection matrices are gotten with steps of tensor sparsity preserving projections on the reconstructed tensor images. The algorithm promotes the computational efficiency and the robust performance of sparse preserving projections on high-dimensional datasets. Experimental results on YaleB and AR face datasets demonstrate effectiveness of proposed algorithm.

Keywords: dimensionality reduction, modular sparsity preserving projections, sparse reconstruction, the third-order tensor

1 Introduction

The destination of dimensionality reduction is to preserve certain property as far as possible in the process of projecting data from high-dimensional data space into low-dimensional data space, reducing the complexity of disposing high-dimensional data in data. Therefore dimensionality reduction is an important step in applications of data mining. In recent years sparse representation has been widely used in classification and reduces dimensionality of machine learning [1-8] thanks to its strong representation performance. Wrightet al. [12] employed the sparse representation based classification (SRC) for robust face recognition. Based on the sparse representation, Qiao et al. [3] proposed Sparsity Preserving Projections (SPP) dimensionality reduction algorithms. The main purpose of SPP is to preserve the relationship of the sparse reconstruction in high-dimensional data into low-dimensional data in the process of dimensionality reduction. However, when the number of training high-dimensional data is large, sparse reconstruction calculation of them is very large, and even harder to complete [1]. Therefore, how to improve sparse reconstruction computational efficiency of large high-dimensional data is an important issue. Some solution way is to achieve sparse reconstruction based on Principal component analysis (PCA), gabor feature and so on. However, these methods are easy to lose the realness of the original data. So Lai et al [9] proposed a Modular Weighted Global Sparse Representation (MWGSR) method. The method firstly modularize face image and achieve sparse reconstruction of every module, and then recalculated sparse reconstruction combining modular sparse reconstruction weight with the linear weighted way. Experimental results show that the improved modular sparse learning of sparse representation improves the computational efficiency and robust performance.

In order to preserve space relationship of high-dimensional data in the process of dimensionality reduction, tensor dimensionality reduction algorithms have been introduced [10-12]. These algorithms regarded a two-dimensional face image as a second-order tensor image without transforming them into vectors. Recently, Lai et al. [13] proposed a novel modular discriminant analysis algorithm. The algorithm first modularizes uniformly face image and combined these image blocks into a three-order tensor image, and then applies Multilinear Discriminant Analysis (MDA) in built third-order tensor images.

Inspired by above analyses, a dimensionality reduction algorithm called Tensor Modular Sparsity Preserving Projections (TMSPP) is proposed in this paper. The algorithm first modularizes uniformly the two-dimensional image into several module and uses these modules to construct the corresponding three-order face tensor data; then calculate sparse reconstruction weight of each module and obtains the corresponding reconstructed image based on sparse representation; selects some module with little sparse reconstruction error and

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combine them into new training feature vectors; Finally, calculates sparse reconstruction weights of new training feature vectors and the projection matrix on third-order tensor data. Experimental results on real AR and YaleB face datasets show that the proposed algorithm not only improves the performance of dimensionality reduction but also promotes the efficiency of sparse learning.

The characteristics of the proposed algorithm are listed as follows:

1)Because an image is divided into a number of modular, the number of feature dimensions is greatly reduced, which improves the computing efficiency of sparse reconstruction for image modules. Therefore, the algorithm can adapt to the large-scale high-dimensional datasets.

2) Part of modules are selected to reconstruct into training feature vectors for sparse reconstruction on image with occlusion and disguise, which is more efficient in avoiding the external disturbance, so the algorithm has better robustness performance.

3) Apart from preserving sparsity reconstruction of samples, the algorithm not only preserve pixels relation in very module but also preserve modular spatial relation.

The paper is organized as follows: In Section 2 we will introduce SPP. A theoretical analysis of TMSPP is given in Section 3. The experimental results and analysis will be presented in Section 4 and conclusions are given in Section 5.

2 Sparsity preserving projections (SPP)

Sparsity reconstruction weight of Sparse representation reveals category relationships of signal. Given training sample $X=\{x_1,x_2,x_3,...,x_n\}\in \mathrm{R}^{d\times n}$, the destination of sparse representation is to represent $x_i\in X$ with as few other samples of x to as possible. For facilitate calculation, l_0–norm is replaced by l_1–norm in sparse learning as follows:

$$\begin{aligned}&\min_{s_i}\|s_i\|_1\\&s.t.x_i=Xs_i\,,\\&1=1^{\mathrm{T}}s_i\end{aligned}\qquad(1)$$

where $\|s_i\|_1$ denotes the l_1 norm of s_i, $s_i=[s_{i1},\ldots,s_{ii-1},0,s_{ii+1},\ldots,s_{in}]^{\mathrm{T}}\in R^n$ denotes sparsity reconstruction weight of x_i and $1\in R^n$ denotes a vector full of 1's. s_{ij} denotes reconstruction coefficients of sample x_j reconstructing x_i, x_i, namely:

$$x_i=s_{i1}x_1+...+s_{ii-1}x_{i-1}+s_{ii+1}x_{i+1}+s_{in}x_n\qquad(2)$$

For each $x_i\in X$, the sparsity reconstruction matrix $S=[S_1,S_2,...S_n]^{\mathrm{T}}$ of the training samples can be obtained by calculating the corresponding s_i of $x_i\in X$.

Sparse preserving projection aims at preserving sparsity reconstruction relations of input data in the process of dimensionality reduction. Given the sparse preserving projection matrix T, $T^{\mathrm{T}}Xs_i$ denote projected points of sparsity reconstruction in high-dimensional data space, the objective function of SPP is described as follows [3]:

$$\max_T\frac{T^{\mathrm{T}}X(S+S^{\mathrm{T}}-S^{\mathrm{T}}S)X^{\mathrm{T}}T}{T^{\mathrm{T}}XX^{\mathrm{T}}T}\,.\qquad(3)$$

3 Tensor modular sparsity preserving projections (TMSPP)

3.1 BASIC IDEA

In practical applications of image data mining, dimensions of image data are usually high and the size of them is large, which affect greatly the efficiency of sparse reconstruction and even fail to achieve sparse reconstruction. Furthermore, modular sparse learning is helpful for improvements on the performance because that deformity and disguise happen in part of images. When images are divided into some modules, there are two important spatial relations, namely, spatial relations of pixels in every module and partial relations of modules. Therefore, an efficient way is to construct third-tensor data with images modules and achieve third-tensor sparsity preserving projections. Figure 1 shows eight modules of a face image and a constructed third-order tensor image.

FIGURE 1 A face modules and a constructed third-order tensor image

3.2 OBJECTIVE FUNCTION

Given training samples $X=\{x_1,x_2,x_3,...,x_n\}\in \mathrm{R}^{d\times n}$, each image will be evenly divided into m module. Modules set $X=\left[X_1^{\mathrm{T}},X_2^{\mathrm{T}},X_3^{\mathrm{T}},...,X_m^{\mathrm{T}}\right]$ with $X_k\in R^{(d/m)\times n}\,(1\le k\le m)$ are gotten.

1) Firstly, sparse learning is achieved separately in each module set $X_k\,(1\le k\le m)$ and sparsity reconstruction weight S_k of X_k is obtained. So sparsity reconstruction set of each module set, namely $Y_k=X_kS_k\,(1\le k\le m)$ is obtained. Third-order images data is obtained by combining these module sets in Figure 1.

2) Then, sparsity reconstruction error $\varepsilon_k=\sum_{i=1}^{n}\left\|Y_k^i-X_k^i\right\|_2^2$, $(1\le k\le m)$ of each sample module $X_k\,(1\le k\le m)$ set is calculated and module sets in the first z minimal reconstruction error are chosen, which is denoted by

$\tilde{X} = \left[\tilde{X}_1^{\mathrm{T}}, \tilde{X}_2^{\mathrm{T}}, \tilde{X}_3^{\mathrm{T}}, ..., \tilde{X}_m^{\mathrm{T}}\right]$ with $X_k \in R^{(z \times d/m) \times n}$, $(1 \le k \le m)$ are obtained. The next step is to obtain sparsity reconstruction weights $\tilde{S}$ for $\tilde{X}$ as follows:

$$\begin{aligned} &\min_{s_i} \|s_i\|_1 \\ &s.t. x_i = X s_i \\ &1 = 1^{\mathrm{T}} s_i \end{aligned} \tag{4}$$

3) Finally, the projection matrix of third-order sparsity preserving projections on third-order images is calculated and $Y = X \times U_1 \times U_2 \times U_3$ is obtained. Combined with Equation (3), the objective optimization function is obtained:

$$\max_{U_1,U_2,U_3} \frac{Y(\tilde{S}+\tilde{S}^{\mathrm{T}}-\tilde{S}^{T}\tilde{S})Y^{\mathrm{T}}}{YY^{\mathrm{T}}} = \max_{U_1,U_2,U_3} \frac{(X \times U_1 \times U_2 \times U_3)(\tilde{S}+\tilde{S}^{\mathrm{T}}-\tilde{S}^{T}\tilde{S})(X \times U_1 \times U_2 \times U_3)^{\mathrm{T}}}{(X \times U_1 \times U_2 \times U_3)(X \times U_1 \times U_2 \times U_3)^{\mathrm{T}}}. \tag{5}$$

3.3 ALGORITHM STEPS

Input: training sample $X = \{x_1, x_2, x_3, ..., x_n\}$, errors ε.
Output: Projection matrix U_1, U_2 and U_3.
Steps:
1) Initialize respectively the projection matrix U_1, U_2 and U_3 as one diagonal unit matrix.
2) Build sparse reconstruction matrix $\tilde{S}$ of the modular remodeling image using Equation (4).
3) Loop $t = 1…7$
  3.1) Loop $f = 1…3$
    3.1.1) $Y_i^f = X_i \times ... \times U_{f-1} \times U_{f+1i} ... \times U_3$
    3.1.2) transform Equation (5) into the solution of the generalized matrix:
    $Y^f(\tilde{S}+\tilde{S}^{\mathrm{T}}-\tilde{S}^{\mathrm{T}}\tilde{S})Y^{f\mathrm{T}}v_i - \lambda_i Y^f Y^{f\mathrm{T}} v_i, 1 \le i \le l^f$,
    $U_f^t = \left[v_1, v_2, ..., v_{l^f}\right]$ is obtained.
    3.1.3) If $\left\|U_f^t - U_f^{t-1}\right\|_2 < \varepsilon (f = 1,2,3)$, jump out of loop t.
4) Get the projection matrix U_1, U_2 and U_3.

3.4 COMPUTATIONAL COMPLEXITY ANALYSES

Sparsity reconstruction is the main process of sparse learning, analysis on the time complexity of sparse reconstruction of our proposed algorithm. Given training sample $X = \{x_1, x_2, x_3, ..., x_n\} \in \mathrm{R}^{d \times n}$, sparsity reconstruction of sparse learning is the problem of minimization solving based on l_1 norm, which is $O\left(2d^2 n - 2d^3/3\right)$ [14]. The time complexity of TMSPP is divided into two parts:

1) TMSPP divides image X into m images modules $X = \left[X_1^{\mathrm{T}}, X_2^{\mathrm{T}}, X_3^{\mathrm{T}}, ..., X_m^{\mathrm{T}}\right]$. The time complexity of sparsity reconstruction for each module $X_k \in R^{(d/m) \times n}$, $(1 \le k \le m)$ is $O\left(\frac{2d^2 n}{m^2} - \frac{2d^3}{3m^3}\right)$, the time complexity of sparse reconstruction for all m modules is $O\left(\frac{2d^2 n}{m} - \frac{2d^3}{3m^2}\right)$.

2) The time complexity of sparsity reconstruction on choosing first z modules with minimum errors for image sparse reconstruction is $O\left(\frac{2zd^2 n}{m^2} - \frac{2zd^3}{3m^3}\right)$.

In short, the time complexity of TMSPP is $O\left(\frac{2(m+z)d^2 n}{m^2} - \frac{2(m+z)d^3}{3m^3}\right)$.

4 Experiments and analyses

4.1 FACE DATASETS

1) AR consists of over 4000 face images of 126 individuals. For each individual, 26 pictures were taken in two sessions (separated by two weeks) and each section contains 13 images. These images include front view of faces with different expressions, illuminations and occlusions. A group of face images on AR are shown in Figure 2.

2) YaleB contains 2414 front-view face images of 38 individuals. For each individual, about 64 pictures were taken under various laboratory-controlled lighting conditions. A group of face images on YaleB are shown in Figure 3.

FIGURE 2 A group of face images on AR

FIGURE 3 A group of face images on YaleB

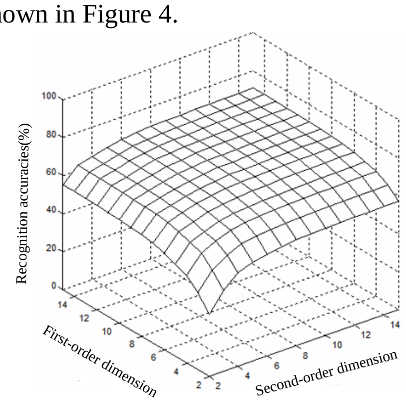
4.2 EXPERIMENTAL SETTINGS

In order to evaluate effectively the classification performance of the algorithm, MWGSR [9] and MDA [13] are selected to compare with our proposed algorithm. A certain number of images are chosen randomly as training samples from each group face images and the rest facial images as test samples. Furthermore, all the face image size is adjusted to 30×30 for computational convenience. The nearest neighbour classification algorithm is used. *m* is set to 8 and *z* is set to 3. All experiments are repeated 20 times and the average recognition accuracy is gotten as experimental results.

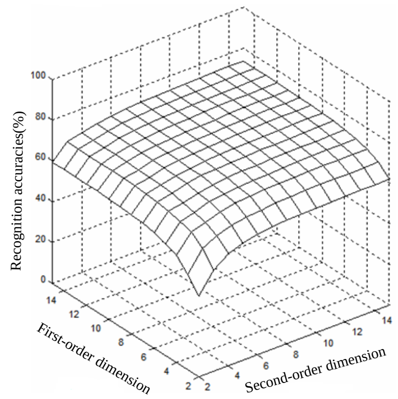
4.3 EXPERIMENTAL RESULTS AND ANALYSES

The number of modules is set to 4 and first two modules with minimum sparse reconstruction error are chosen for reconstruct feature vectors.

1) Firstly, the number of training samples in a group of samples is set to 4. With increment in feature dimension in first-order and second-order under the number of the same third-order dimension, recognition accuracies on AR are shown in Figure 4.



a) The number of the third-order dimension is 3



b) The number of the third-order dimension is 4

FIGURE 4 Recognition accuracies vs. dimensions of first-order and second-order on AR under different dimension of the third-order

From Figure 4 we can see that the recognition accuracies increase greatly with increment in dimensions of first-order and second-order and flat when the dimension exceeds certain less value. This illustrates that TMSPP can get the most classification performance in low dimension.

2) Secondly, most recognition accuracies are shown in Tables 1 2 for further verification in the performance of TMSPP under the different number of training samples.

TABLE 1 Experimental result on AR

algorithm	The training number of a group samples		
	4	**6**	**10**
MWGSR	70.35	78.50	84.24
MDA	**82.35**	**85.65**	87.56
TMSPP	75.35	84.05	**92.40**

TABLE 3 Experimental result on YaleB

algorithm	The training number of a group samples		
	10	**15**	**20**
MWGSR	80.50	85.50	90.50
MDA	**86.53**	90.65	92.45
TMSPP	81.55	**92.40**	**95.65**

Here bold data denote best and highest recognition accuracies under the same training sample.

The following conclusions can be drawn from Tables 1 and 2:

1) Although MWGSR inherited the feature of sparse learning, In contrast to MDA, the recognition accuracy of MDA is higher than MWGSR, which illustrates that third-tensor dimensionality reduction method has better classification performance.

2) Although MWGSR and TMSPP have taken advantage of some modules to guide sparse reconstruction. As a third-tensor dimension reduction algorithm, TMSPP not only preserve spatial relations of pixels in modules but also preserves spatial relations of modules, which is the reason that TMSPP has more obvious classification performance than MWGSR.

3) TMSPP and MDA share common characteristics of third-order tensor dimensionality reduction. When the size of samples is small, the classification performance of TMSPP is worse than that of MDA. When the size of samples exceeds the certain number, the classification performance of TMSPP is better than that of MDA. This shows that TMSPP inherits sparse learning robust performance, and is more suitable to face image with external disturb.

5 Conclusions

Despite the sparse learning has good performance of representation, but sparse reconstruction is not suitable for large-scale high-dimensional image datasets thanks to the computation complexity. Tensor Modular Sparse Preserving Projection (TMSPP) is proposed for dimensionality reduction. Apart from solving the problem of the computational efficiency, TMSPP improves the robustness performance through modularizing image. As third-order tensor dimensionality reduction, TMSPP not only preserves pixels spatial relation in each module but also preserves module spatial relation of modules. Experimental results on AR and YaleB show that TMSPP demonstrates the efficiency of our proposed. The next work is to study how to select the optimal number of module on different face datasets. In addition, the related semi-supervised dimensionality reduction is the future research work.

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