Structure optimization of cycloid gear based on the finite element method

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Abstract

A numerical model of cycloidal gear is created by using three-dimensional software and finite element analysis is applied with ANSYS platform. The first six natural frequencies and mode shapes are obtained, as a result. Influences from structure, material and thickness of the gear are investigated. Analysis shows that, modal shapes of cycloid gear are mainly circumferential modes, umbrella-type modes, torsional vibration mode and radial modes. The first six natural frequencies of 5 kinds of cycloid gear with variable cross-section were smaller than those of ordinary cycloid gears, and cycloid gears with variable cross-section can avoid resonance frequencies easily. Dynamics of five new cycloidal gears with variable cross-sections are consistent with ordinary cycloid gears. Modal frequencies of ordinary cycloid gears increases in accordance with materials, such as bearing steel, alloy steel and plastics; also, natural frequency increases with the increase of the thickness of the gear. Conclusions of this paper provide a basis for dynamic designing of cycloid gears.

Keywords: cycloid gear, finite element modal analysis, free modal, constraint modal

1 Introduction

In recent decades, many scholars have done a lot of researches in terms of cycloid drive. Yan Hong Sen and Ta Shi Lai [1] established a basic planetary gear system based on a cylindrical tooth profile. Laita-Shi [2-4] proposed a mathematical model and design procedures for planetary transmission systems with cycloid gears and derived the equation of meshing cycloid drive from conjugate surface theory. Leilei et al [5] built a finite element model of the cycloid gear and needle teeth based on ANSYS software and conducted three-dimensional contact analysis. Thube S. V and TR Bobak [6] established a finite element model of the rotating parts of the cycloid reducer and conducted a dynamic analysis. Zhang Xiu Yan and Xiao Jun Dai [7-8] designed a new type of tetracyclic cycloid gear, conducted meshing stiffness analysis and finite element modal analysis based on the finite element software. Biernacki and Krzysztof [9] designed a plastic cycloidal gear and conducted its finite element analysis. Nam, WK, JW Shin, and SH Oh [10] designed a thin ball reducer for a robot. Xiaojun Jun and Wei-Dong [11] established solid model of RV cycloid reducer using Pro/E software and built dynamic analysis models, analysed the inherent characteristics of cycloid using ANSYS software. Liji Shun et al [12] established solid model of cycloid gear with two teeth difference using Pro/E software and analysed the inherent characteristics of cycloid including non-binding modes and modal with actual boundary constraints using ANSYS software. Qin Guang Yue et al [13] conducted modal analysis for Shearer cycloidal gear based on ANSYS software. Woody and Zhang you Chen [14] established cycloid gear model for multiple teeth difference using ANSYS analysis software, derived root stress calculation formula of cycloid gear with different meshing phase angles and calculated and analysed root stress of cycloid gear with different meshing phase angles.

Cycloid planetary drive is a drive with broad application prospects. Cycloid planetary transmission has significant advantages including transmission ratio range, hardened (bearing steel) multiple-gear meshing, small, smooth motion, long life, low noise, high load capacity and high transmission efficiency. Cycloid planetary transmission has been widely used in many industries and occupies a very large portion in reducer industry. Tooth profile of cycloid gear is composed of hypocycloidal or epicycloid and is a key component of cycloidal gear reducer. As a part of the main transmission, cycloid gear bears the coupling of multiple parts in the meshing process. If its own natural frequency coincides with the drive frequency, it may cause forced vibration of whole reducer. So modal frequencies for each cycloid is calculated in this paper, it can provide a theoretical basis to avoid resonance and the emergence of harmful vibration mode and study dynamics in depth.

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2 Mathematical model of tooth profile for an ordinary cycloid gear

One method of forming cycloidal tooth profile is of formation with two external circles, another method is of formation with two circles inner meshing. Formation method with two external circles meshing is described here. The two circles are named a roll circle and a base circle, respectively (insert a figure, Figure 1). When the roll circle and the base circle are tangent externally, and the roll circle makes pure rolling against the base circle, the trajectory of any fixed point on the roll circle is a epicycloid. Equations of a cycloid gear in Cartesian coordinate system are as follows.

$$X_1 = (r_1 + r_2)\sin\phi - r_2 \sin\left(\frac{r_1}{r_2}\phi\right),$$
$$Y_1 = (r_1 + r_2)\cos\phi - r_2 \cos\left(\frac{r_1}{r_2}\phi\right), \quad (1)$$

where r_1 represents the radius of the base circle; r_2 represents roll circle radius; ϕ represents the angle of the roll circle rotates about the centre of the base circle in forming a cycloid.

3 Finite element modal analysis for ordinary cycloid gear

3.1 MODAL ANALYSIS THEORY

A modal is the representation of natural vibration characteristics for a mechanical structure, each modal has specific natural frequencies and mode shapes. Modal analysis in this paper is used to obtain the natural frequencies of cycloid gears. Free vibration equation is as follows:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\}, \quad (2)$$

where $[K]$ is stiffness matrix, $[M]$ is mass matrix, $\{\ddot{u}\}$ and $\{u\}$ are acceleration vector and displacement vector respectively. Algebraic equation of natural frequency is obtained by solving free vibration equation.

$$w^{2n} + a_1 w^{2(n-1)} + ... + a_n w^{2n} + a_n = 0. \quad (3)$$

Order natural frequencies and mode shapes of the structure can be obtained by solving Equation (3).

3.2 ESTABLISH FINITE ELEMENT MODEL OF ORDINARY CYCLOID GEAR

Cycloid gear geometric model is established with 3D modelling software, as shown in Figure 1. The 3D model is imported to ANSYS and finite element modal analysis is conducted. Parameters and procedures are as follows.

1) Modulus of elasticity is 2.07 × 10^5 Mpa, Poisson's ratio is 0.25 and the density is 7.8×10^3 kg/m^3.

2) Model meshing is conducted. Mesh quality and density have a very big impact on the results of finite element analysis and have direct impact on the quality of the final grid results. Because cycloidal tooth profile is complex, the sweep mesh partition method is adopted in this paper. The model grid is mainly Hexahedral elements. Final model is divided into 107,955 units and has a total of 162,553 nodes. Finite element model of ordinary cycloid gear is shown in Figure 2.

3) Analysis type is specified as modal analysis.

4) Set number of extended modal. Structure vibrating can be expressed as linear combination of vibration modes corresponding to natural frequencies. Low order modes have great impact on the vibrational structure, low natural frequencies and mode shapes have practical significance. So generally the first 5 to 10 modal analysis are performed; in this paper, the first six natural frequencies and mode shapes are calculated.

FIGURE 1 Geometry of ordinary cycloid gear

FIGURE 2 Finite element model of the ordinary cycloid gear

3.3 RESULTS ANALYSIS

According to modal analysis theory, modal characteristics under arbitrary boundary constraint conditions can be calculated by mathematical modelling methods from modal parameters calculated under free boundary conditions. On the contrary, the results obtained under the specified boundary conditions cannot be converted to the dynamic characteristics of other boundary constraints. It is necessary to analyse the differences for ordinary cycloid gear between modal analysis under free boundary conditions and modal analysis under actual boundary constraints. Figure 3 shows the comparison of natural frequency for cycloid gear between free modal and constrained modal. As can be seen, the first-order and second-order natural frequency of ordinary cycloid gear under constraint modal is bigger than that of free modal. But the third-order to sixth-order natural frequency of ordinary cycloid gear under constraint modal are smaller than that of free modal.

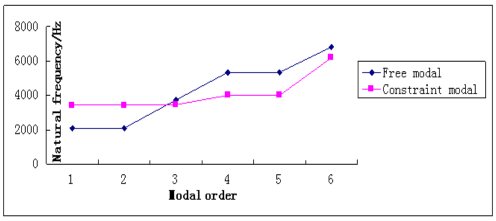
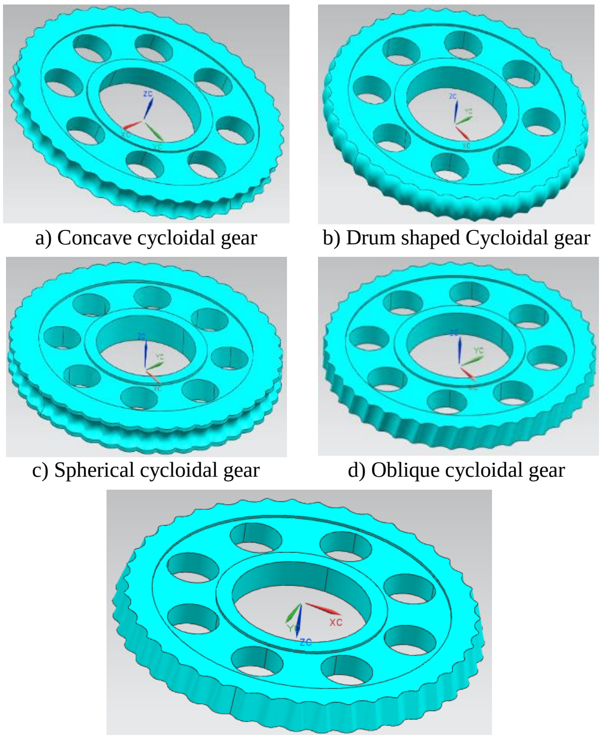


FIGURE 3 The comparison of natura3l frequency for ordinary cycloid gear between free modal and constraint modal

4 Structure impact

4.1 DESIGN FIVE NEW VARIABLE CROSS SECTION CYCLOIDAL GEAR

Cycloidal gear with variable cross-section is obtained when tooth profile changes along the axis of the gear, linearly or non-linearly. Each cross-section of variable cross-section cycloid gear is a tooth profile of ordinary cycloid gear. Five kinds of variable cross-section cycloid gear are designed and 3D structure is shown in Figure 4. These shapes include concave, drum-shaped, spherical, oblique and tapered. The outer periphery tooth surface of concave cycloidal gear (Figure 4a) is the concave surface, corresponding needle teeth are convex central drum. Central convex drum-shaped surface of needle teeth mesh with the outer periphery concave surface of cycloid gear. Drum-shaped cycloid gear (Figure 4b) is contrary to the cycloidal gear with concave structure. Its outer circumferential tooth surface of cycloid gear is convex central drum surface, corresponding needle teeth are concave surface. Needle teeth of spherical cycloid gear (Figure 4c) is spherical, corresponding outer periphery of cycloid gear is arc-shaped surface meshing with the ball needle teeth. Needle teeth of oblique cycloidal gear (Figure 4d) are inclined cylinder, their outer periphery teeth of cycloid are inclined surface. The tooth profile of Conical cycloidal gear (Figure 4e) is conical surface. Contact areas between these five new cycloidal gears and their corresponding needle teeth are large. Contact stress is uniform. They have high transmission efficiency and long service life.



a) Concave cycloidal gear b) Drum shaped Cycloidal gear

c) Spherical cycloidal gear d) Oblique cycloidal gear

e) Conical cycloidal gear

FIGURE 4 5 kinds of variable cross-section cycloid gear

4.2 MODAL ANALYSIS

If structural design of cycloidal gear is unreasonable, then the cycloid reducer at work will generate noise and vibration. It is necessary to conduct finite element modal analysis for the five kinds of new cycloid gears. Figure5 shows the comparison of natural frequency among the five kinds of variable cross-section cycloid gears and ordinary cycloid gear. As is shown, the first order natural frequency of ordinary cycloid gear is 2094Hz and the first order natural frequencies of the five new variable cross-section cycloid gears are 2068Hz, 2019Hz, 1884Hz, 2039Hz and 1963Hz, respectively. The first six natural frequencies of the 5 variable cross section cycloid gears are less than that of ordinary cycloid gear; so, it is easier to avoid the resonance for the 5 variable cross section cycloid gears. Changing trends of the first six natural frequencies of the 5 variable cross section cycloid gears are consistent with that of ordinary cycloid gear. Thus dynamics of the five new variable cross-section cycloid gears are consistent with that of ordinary cycloid gear .The front sixth modal shapes of three kinds of cycloid gear are shown in Figure 6,7 and 8 ,the shape of which is concave, drum-shaped and spherical respectively. The front sixth modal vibration shapes of these three kinds of cycloidal gear are very similar. The main deformation parts locate in the middle and edge of cycloidal gear. Deformation of spherical cycloid gear is bigger than that of concave and drum shaped cycloid gear.

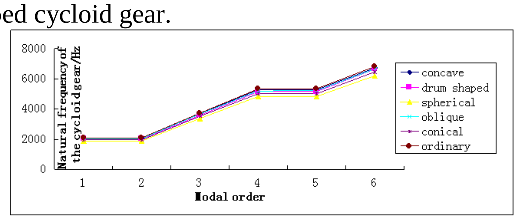
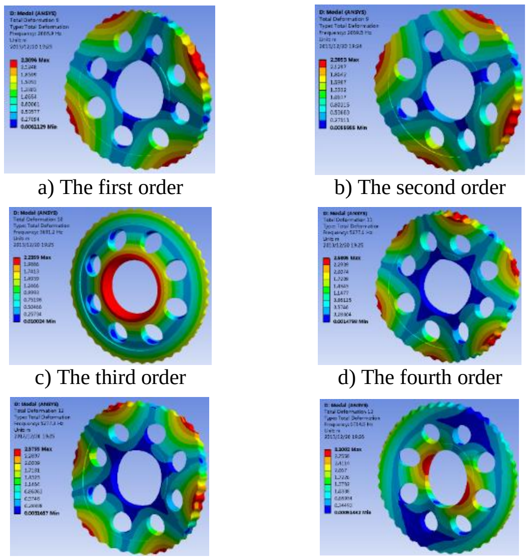


FIGURE 5 The comparison of natural frequency between variable cross-section cycloid gear and ordinary cycloid gear

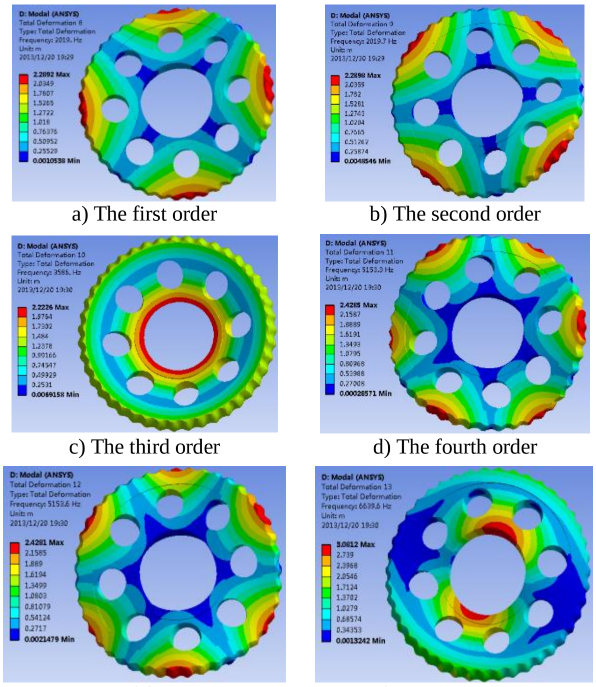


a) The first order b) The second order

c) The third order d) The fourth order

e) The fifth order f) The sixth order

FIGURE 6 The modal vibration pattern map of concave cycloidal gear from the first to the sixth order

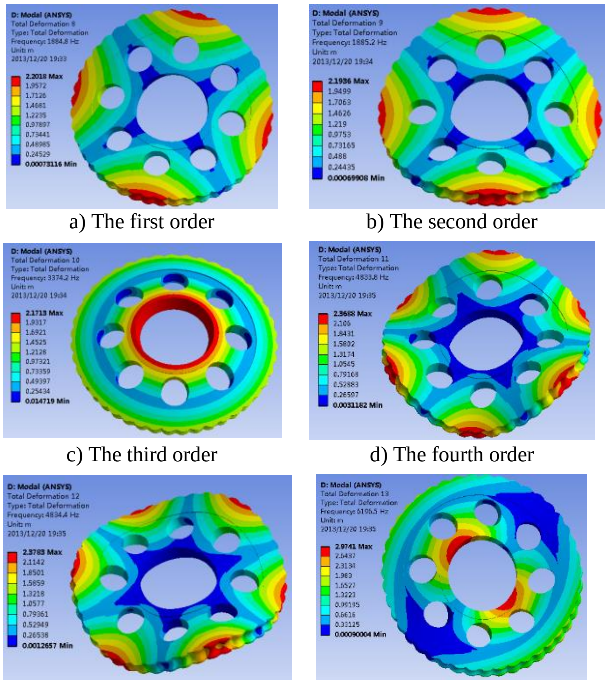


a) The first order b) The second order

c) The third order d) The fourth order

e) The fifth order f) The sixth order

FIGURE 7 The modal vibration pattern map of drum shaped cycloidal gear from the first to the sixth order



a) The first order b) The second order

c) The third order d) The fourth order

e) The fifth order f) The sixth order

FIGURE 8 The modal vibration pattern map of spherical cycloidal gear from the first to the sixth order

5 Impact of material

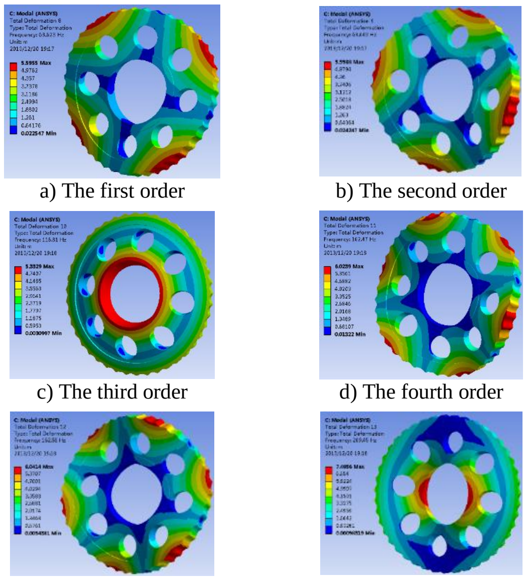
Due to the development of high-performance engineering plastics, application of plastics in industry is accelerated. Materials used to produce gears are no longer limited to metals, plastics are used as well. Finite element model analysis of ordinary cycloid gear, with different materials, including bearing steel, alloy steel and plastics, are performed. Material parameters of cycloid gear are shown in Table 1.

TABLE 1 Material parameters of cycloid gear

Material	**Modulus of elasticity (MPa)**	**Poisson's ratio**	**Density (kg/m^3)**
Bearing Steel 20CrMnTi	2.07×10^5	0.25	7.8×10^3
alloy steel SNCM220	2.2×10^5	0.3	7.8×10^3
Engineering Plastics MC901	36	0.4	1.41×10^3

TABLE 2 The first six natural frequencies of ordinary cycloid gear

	Bearing Steel 20CrMnTi	**alloy steel SNCM220**	**Engineering Plastics MC901**
Modal order	*Natural frequency (Hz)*		
1	2094	2143	63
2	2094	2144	63
3	3734	3860	116
4	5335	5463	162
5	5337	5467	162
6	6810	6996	209



a) The first order b) The second order

c) The third order d) The fourth order

e) The fifth order f) The sixth order

FIGURE 9 The first-sixth modal vibration pattern map of ordinary cycloid gear with plastic MC901

When free modal analysis is conducted the front six natural frequencies of ordinary cycloid gear are close to zero. It is the rigid modal. Research on rigid modal is of no real meaning. So each modal frequencies of the cycloid gear after the seventh modal are extracted. The front six modal nonzero modal that is after the seventh modal of cycloidal gear is researched and analysed in this article. The front six natural frequencies of cycloid gear with 3 kinds of different materials are shown in Table 2. As can be seen from Table 2, that the minimum natural frequency of the cycloidal gear with these three materials are 2094Hz, 2143Hz and 63Hz respectively. The natural frequency of cycloid gear with plastic MC901 is significantly less than that of cycloid gear with bearing steel or alloy steel. The natural frequency of ordinary cycloid gear is gradually increasing with the modal order increases. The nonzero order front six vibration modal of the ordinary cycloid gear with plastic material MC901 is shown in Figure 9. As can be seen from the Figure 6 that the first-order and the second-order vibration mode of cycloidal gear are the circumference modes, the central part of cycloidal gear has almost not distortion. The third order vibration mode of cycloidal gear is an umbrella-type vibration and the deformation of the central part is the largest. The forth order and the fifth-order vibration mode of cycloid gear is torsional vibration mode, the central part of the cycloid gear has no distortion. The sixth vibration mode of cycloid gear is radial mode; the radial deformation of the central portion is the largest, peripheral part has not distortion.

6 Impact of the thickness

The thickness change of cycloid gear affects the dynamic characteristics of the cycloidal gear. To reveal this, modal analysis of ordinary cycloid gear with thicknesses of 11mm, 13mm, 15mm, 17mm and 19mm is conducted by using finite element software and the front sixth natural frequencies of cycloid gear with six kinds of different thicknesses are obtained. Details are shown in Table 3. From the data, the natural frequencies of cycloidal gear with different thickness have an increasing trend with the increase of modal order. Simultaneously the natural frequency of the same modal order is gradually increasing with the increase of cycloid gear thickness.

TABLE 3 The front six natural frequencies of cycloid gear with different thickness

Cycloidal gear thickness (mm)	The first order (Hz)	The second order (Hz)	The third order (Hz)	The fourth order (Hz)	The fifth order (Hz)	The sixth order (Hz)
11	2860	2878	2880	3492	3493	5699
13	3750	3750	3759	4412	4413	6939
15	4546	4548	4592	5251	5253	8081
17	5279	5281	5363	6025	6027	9137
19	5948	5949	6070	6735	6737	9638

7 Conclusions

1) 5 kinds of variable cross-section cycloid gears, including concave, drum-shaped, spherical, oblique and conical cycloid gears, are studied, and finite element modal analysis are conducted. The front six natural frequencies of the 5 variable cross section cycloid gears are smaller than that of ordinary cycloid gear, so it is easier to avoid resonance for the 5 variable cross section cycloid gears. Changing trends of the front six order natural frequencies of the 5 variable cross section cycloid gears are consistent with that of ordinary cycloid gear. Thus, dynamics of the five new variable cross-section cycloid gears are consistent with that of ordinary cycloid gear.

2) The natural frequency of ordinary plastic cycloid gear is significantly lower compared to those of the bearing steel and alloy steel cycloid gears. Order modal frequencies of cycloidal gears with these three materials show an increasing trend with the increasing of modal order. Modal shape of cycloidal gear include circumferential mode, umbrella-type mode, Torsional mode and radial mode. The deformation of cycloid gears in the peripheral portion and the central portion is the greatest.

3) The natural frequencies of cycloidal gear with different thickness have an increasing trend with the increase of modal order. Simultaneously the natural frequency of the same modal order is gradually increasing with the increase of cycloid gear thickness.

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