The simulation and analysis of fish school behaviours with different body lengths

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Received 20 April 2014, www.cmnt.lv

Abstract

In this paper, through simulating and modelling of fish school with different body length, we study the influences of the body length difference of fish school on spatial structure and group behaviour. Based on attraction/repulsion model, we obtain three typical spatial structure of group with different model parameters: mixture structure, periphery structure and front-back structure. Moreover, we analyse the polarization index and average angular speed of group with different model parameters and get the corresponding relationship between these indices and model parameters. The results obtained in this paper coincide with the phenomenon observed in the natural world and the methods provide an effective way to study the fish school behaviour.

Keywords: computer simulation, attraction/repulsion model, aggregation behaviour, spatial structure

1 Introduction

Many biology systems are self-organizing in nature, such as fish school, bird flocks, grasshopper, ants etc. The aggregation behaviours in animals not only help protect themselves against predators, but also facilitate mate choice and access to information (such as the location of food sources or predators, migratory routes, etc.). Schooling of fish as a common aggregation phenomenon in nature is attracting a lot of biology, cognitive science, psychology, computer science, physics and other related fields of scientists to attempt to describe, explain and predict the structure and behaviour of aggregation. However, it is very difficult to study the fish schooling in the wild. Recently, computer simulation technology has become important tools in fish school behaviour research. Many models of aggregation offer researchers a way to investigate how the interplay of individual behaviour makes aggregation possible and leads to different aggregate-level behaviours [1-7].

Many fish school behavioural models stand on Breder's model [8, 9], Breder considered the individual in the group as atoms in the crystal and described the mechanical model using atomic attraction and repulsion analogue. Inspired by Breder's work, many Attraction/Repulsion (AR) models and their variants emerged. AR models consider the individuals in the group as independent agent, the agent attempt to maintain a minimum distance (within zone of repulsion) between themselves and others at all times, and they tend to be attracted towards other individuals and to align themselves with neighbours [1, 6]. For example, Couzin et al. [1] modelled the collective behaviour using three simple rules (repulsion, attraction, alignment) and four common collective behaviour patterns are produced in the model. Jennifer et al. [7] extended the AR model from discrete to continuous. Vincent [10] presented numerical simulations of an animal grouping model based on individual behaviours of attraction, alignment and repulsion, and investigated how some factors such as the number of individuals, the number of influential neighbours and the strength of the alignment behaviour impacting on internal spatial structures. Rune et al. [11] studied how to adjust the parameters of the herring spawning behaviour model, resulting in some particular spatial structure. Many studies have been verified that the simulation results of AR model are consistent with field experiments in many fish and other animal aggregations, e.g. Starling [12], mosquito fish [13], surf scoters [11], golden shiners [14].

All the above models assumed that fish schools composed of individuals with the same body length, and thus their interaction zones are the same as well. But a large number of observations suggested that there are individual differences exist in fish school [15, 16]. In addition, fisheries often capture the uniform body length fishes at the beginning of fishing season, but get the varying body length individuals in late. This indicates that the fish school composed of diverse body length individuals is a common phenomenon in nature. However, the spatial structure and behaviour of such diverse collective systems are unclear.

In this paper, we consider the body length of individuals in fish school as a continuous random variable,

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and simulate the fish school behaviours with different body lengths. The main objective are to:

1) to develop a mathematics model for fish school which is composed of different body length individuals.

2) to analyse the spatial structure and aggregation behaviour generated by the model.

3) to reveal the transition rules of spatial structure and behaviour when the model parameters are changed.

2 The Model

We assume that the N number of individuals with different body length in continuous three-dimensional space. Time is partitioned into discrete time steps with a regular spacing τ. In each time step t, the *i-th* individual with body length BL_i access the position $r_i(t)$ with direction vector $d_i(t)$. The individual's length is assumed to obey the Gaussian distribution with mean m and standard deviation σ. Interaction between individuals is restricted to the directional component, and the speed of individuals is configured to constant S.

In this research, we adopt a biological model concept by Couzin [1], which is based on the observational and empirical investigation of interaction of animal behaviour with its neighbors in the aggregation phenomenon. We divide the individual's perception range into three zones: zone of repulsion (ZOR), zone of attraction (ZOA) and zone of orientation (ZOO). These zones are spherical, except for a volume behind the individual within which neighbours are undetectable. This "blind area" is defined as a cone with interior angle $(360-\varphi)$, where φ is defined as the field of perception (see, Figure 1) [1]. Unlike fixed the perception field for all individuals in the Couzin's model, we assume that the individual perception range related to their body length. Let:

$$r_{ri} = \alpha \times BL_i, \tag{1}$$

$$r_{oi} = \beta \times BL_i, \tag{2}$$

$$r_{ai} = \gamma \times BL_i, \tag{3}$$

where r_{ri}, r_{oi}, r_{ai} represent the radius of three zones, and, α, β, γ are the perception coefficient of each zones. The interaction rules of individuals in our model extended from refs. [1, 8, 9]:

1) If n_r neighbours are present in the zone of repulsion at time t, individual i responds by moving away from neighbours within this zone:

$$d_{ri}(t+\tau) = -\sum_{j=1, j\neq i}^{n_r} \frac{r_{ij}(t)}{|r_{ij}(t)|}, \tag{4}$$

where $r_{ij} = (r_j - r_i)/|r_j - r_i|$ is the unit vector in the direction of neighbour j. This rule has the highest priority in the model, so the desired direction of next time is:

$$d_i(t+\tau) = \frac{d_{ri}(t+\tau)}{|d_{ri}(t+\tau)|}. \tag{5}$$

2) If no neighbours are within the zone of repulsion, the individual responds to other within the zone of orientation and the zone of attraction. An individual will attempt to align itself with neighbours within the zone of orientation, giving the direction:

$$d_{oi}(t+\tau) = \sum_{j=1}^{n_o} \frac{d_j(t)}{|d_j(t)|}, \tag{6}$$

and towards the positions of individuals within the zone of attraction

$$d_{ai}(t+\tau) = \sum_{j=1}^{n_a} \frac{r_{ij}(t)}{|r_{ij}(t)|}, \tag{7}$$

where n_o and n_a are the individuals' number in the zone of orientation and attraction. If neighbours are only found in the zone of orientation, then the desired direction of next time is

$$d_i(t+\tau) = \frac{d_{oi}(t+\tau)}{|d_{oi}(t+\tau)|}. \tag{8}$$

If neighbours are only found in the zone of attraction, then the desired direction of next time is

$$d_i(t+\tau) = \frac{d_{ai}(t+\tau)}{|d_{ai}(t+\tau)|}. \tag{9}$$

If neighbours are found in both zones, then the desired direction of next time is

$$d_i(t+\tau) = \frac{(d_{oi}(t+\tau) + d_{ai}(t+\tau))}{2}. \tag{10}$$

After the above process has been performed for every individual they turn towards the direction vector $d_i(t+\tau)$ by maximum turning rate θ. Provided the angle between $d_i(t)$ and $d_i(t+\tau)$ is less than the maximum turning angle $\theta\tau$, then $d_i(t+\tau) = d_i(t)$. In our model, the initial position and initial direction are generated by random and the next position of individual are computed by:

$$r_i(t+\tau) = r_i(t) + d_i(t+\tau) \times \tau \times s + \varepsilon, \tag{11}$$

where ε is the random disturbance taken from a spherically wrapped Gaussian distribution with standard deviation.

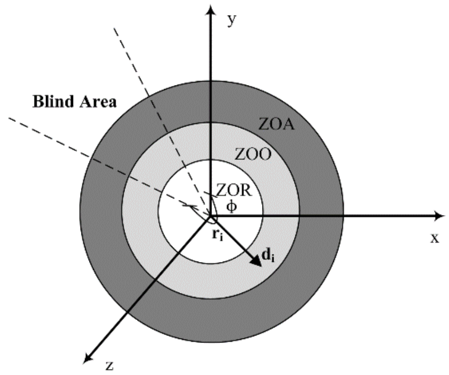


FIGURE 1 Diagram of three zones of interaction and the blind area associated with individual *i*

3 Simulation Experiments

According to previous model and the parameters showed in Table 1, the experiments were simulated through fixed the perception coefficient of ZOR α and changed the perception coefficient of ZOO β, the perception coefficient of ZOA γ and the standard deviation σ. Each experiment was repeated 5 times with 1000 time steps. The following analyses describe the influence in spatial structure and aggregation behaviour as the body length changed.

TABLE 1 Model parameters

Symbol	Parameter	Values
N	Number of individuals	256
α	Radius coefficient of ZOR (Unit[a])	1
β	Radius coefficient of ZOO (Unit[a])	2-10
γ	Radius coefficient of ZOA (Unit[a])	4β
m	The mean body length of individuals (Unit[a])	10
σ	The standard deviation of individuals (Unit[a])	0-5
φ	Perception range (Deg.)	330
τ	Time step(Sec.)	0.1
θ	Maximum turn rate (Deg./Sec.)	440
s	Speed (Unit[a]/Sec.)	2

[a] Similar to ref. [1], the "unit" relates to the non-dimensionality of certain parameters in the model with the characteristic length scale being associated with the body length of fishes, and the rest of the model parameters can be scaled appropriately.

4 Spatial Structures

As the standard deviation of body length changed, the aggregation behaviour of the system exhibits sharp transitions between three aggregation spatial structures (Figure 2), which we have labelled as follows:

Mixture structure: different sizes of fish appear randomly anywhere in aggregation. This occurs when individuals have little body length standard deviation (Figure 2a).

Periphery structure: the individuals with greater body length distribute in the group centre, but the individuals with smaller body length aggregate around the centre. This occurs when individuals have middle level body length standard deviation (Figure 2b).

Front-back structure: with further increase the standard deviation of body length, some individuals with larger body size aggregate in the front and back of fish school (Figure 2c). This can be understood as the larger individual in the group to play a "leadership" role.

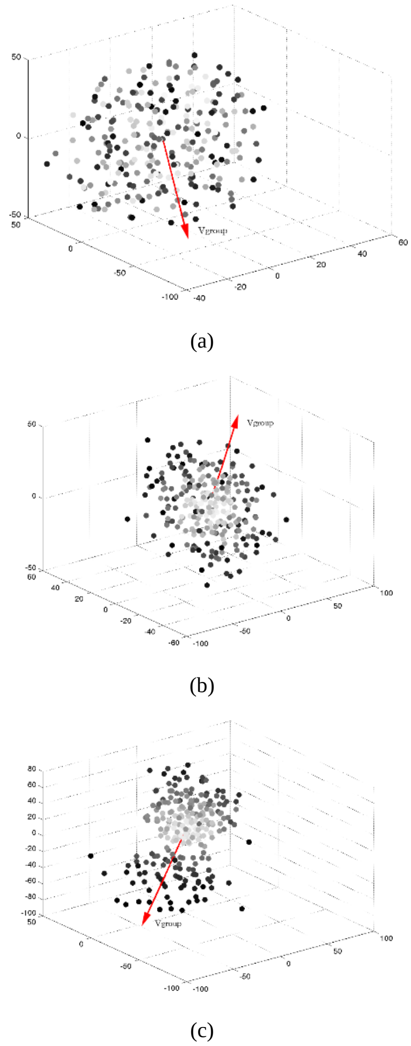
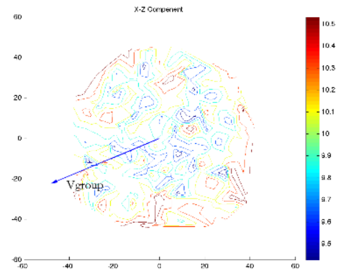
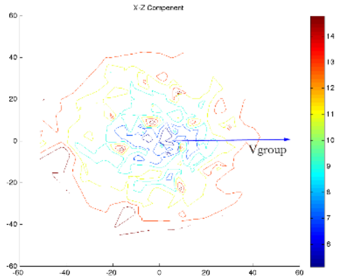


FIGURE 2 The aggregation spatial structure obtained by simulating (a) mixture structure (b) periphery structure (c) front-back structure. Small dots indicate individuals, each dot assign different level of gray color according to the fish's body length, the darker color indicate greater individual, the lighter color represents smaller individual, and the arrow indicates the direction of movement of fish school.

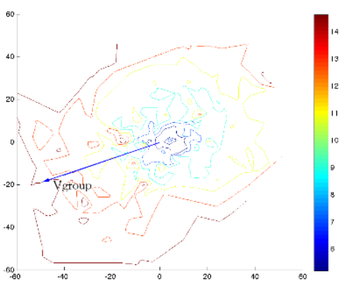
Figure 3 illustrates the contour of individual's body length after projected the spatial structure (Figure 2) to x-y plane. As shown in the Figure 3, the contour of mixture structure contains each other (Figure 3a), but the contour of periphery structure exhibits the obvious hierarchy structure (Figure 3b). In the contour of front-back structure, we can see that the contour which represents the largest body length only distribute in front of the most peripheral position of fish school (Figure 3c). In the y-z and x-z projection plane, similar results can be obtained.



(a)



(b)



(c)

FIGURE 3 The contour of individual's body length projected to x-y plane (a) the contour of mixture structure (b) the contour of periphery structure (c) the contour of front-back structure. The arrow indicates the direction of movement of fish school, and different color represents the contour of different body length.

In order to further characterize the spatial structure of fish school with different body length standard deviation, we introduce the concepts of Average Center Distance of Group and p-Quintile Center Distance Ratio. Let the body length of individuals as continuous random variable *X*, the probability density function $p(x)$, X obey the Gaussian distribution $N(m,\sigma^2)$, where *m* and σ are the parameters that have showed in the Table 1. Let $0 < p < 1$, m_p is the p-Quintile of random variable *X*, then the below equation should be satisfied:

$$P(X > m_p) = p. \tag{12}$$

We define N_p as the set of individuals which body length is greater than or equal(s) m_p and:

$$\bar{r}(t) = \frac{1}{N}\sum_{i=1}^{N} r(t), \tag{13}$$

then, the Average Center Distance of Group and p-Quintile Center Distance Ratio can be written as

$$c(t) = \frac{1}{N}\sum_{i=1}^{N} \left\| \bar{r}(t) - r_i(t) \right\|, \tag{14}$$

$$c_p(t) = \frac{1}{c(t)} \frac{1}{|N_p|} \sum_{i \in N_p} \left\| \bar{r}(t) - r_i(t) \right\|, \tag{15}$$

where $|N_p|$ is the number of elements in the N_p .The Average Center Distance of Group represents the average distance from individual to group centre. The p-Quintile Center Distance Ratio indicates the ratio of individual's with greater body length to the average centre distance.

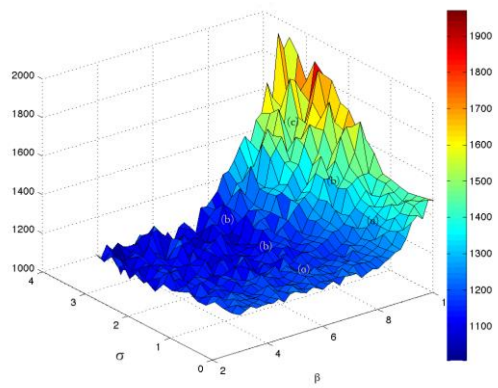


FIGURE 4 The average centre distance of group

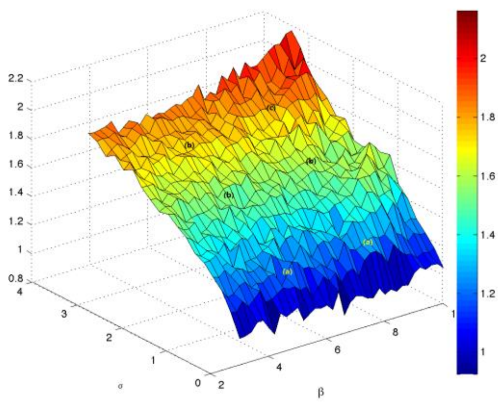


FIGURE 5 The centre distance ratio of p-Quintile

We can see from Figure 4, when the σ and β are smaller, the Average Center Distance of Group is also smaller. This indicates that the group has a higher degree of aggregation in this situation. When the σ and β increasing, the Average Center Distance of Group increases significantly. This shows that the group is more scattered. As shown in Figure 5, the value of p-Quintile Center Distance Ratio becomes larger with the σ and β increasing. The individuals that have bigger body length tend to aggregate in the peripheral position. Obviously, the variation of p-Quintile Center Distance Ratio is more influenced by σ compared with β. The markers (a-c) labelled in Figure 4 and Figure 5 correspond to the three spatial structures respectively.

5 Collective behaviour analyses

5.1 POLARIZATION INDEX

The polarization index [1] is defined as:

$$p_{group}(t) = \frac{1}{N}\left|\sum_{i=1}^{N} d_i(t)\right|. \qquad (16)$$

The polarization index ($0 \le p_{group} \le 1$) characterizes the consistent level of individuals in the group. Figure 6 shows that polarization index exhibits lower values as the β and σ in lower level, and the fish school is poor consistency. As the β and σ increasing, the polarization index is also increase, and the group motion tend to be consistent. Especially, when $8.8 \le \beta \le 10$ and σ in lower level, the polarization index equals to 1, and the motion direction are consistent completely.

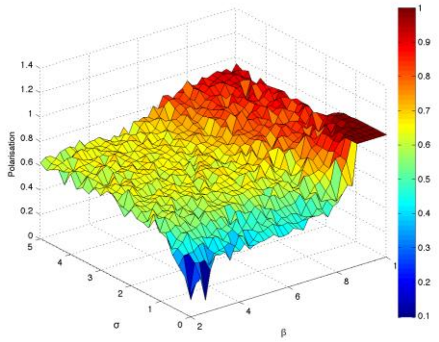


FIGURE 6 The polarization index of group

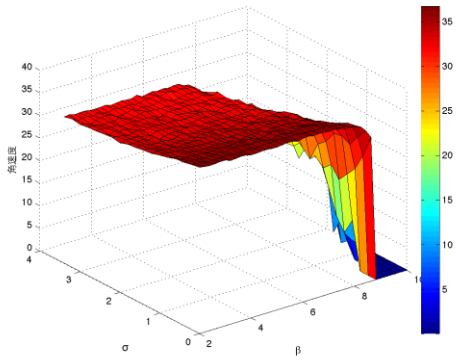


FIGURE 7 The average angular speed of group

5.2 AVERAGE ANGULAR SPEED

Let:

$$\omega_i(t) = \angle(v_i(t+\tau), v_i(t))/\tau\ , \qquad (17)$$

represents the angular speed of individual i at time t, and $\angle(v_i(t+\tau), v_i(t))$ is the angular separation of two velocity direction. The average angular speed is defined as

$$\omega_{group}(t) = \frac{1}{N}\sum_{i=1}^{N}\omega_i(t). \qquad (18)$$

As shown in the Figure 7, when $\beta < 8.8$, the average angular speed did not changed significantly as the σ changed, and the average angular speed is about 310-340 Deg./Sec.; when $8.8 \le \beta \le 10$,the average angular speed increase as the σ increasing. Especially, when the σ is in lower level, the average angular speed equals to 0, and this leads to the same conclusion with polarization index (the individuals tend to move in the same direction).

6 Conclusions

In this paper, we present a self-organizing fish school model with different body length, and use it to investigate how the body length difference (combined with other model parameters) affects the spatial structure and behavior. The results are consisting with many observational investigations in nature [17, 18]. The model can explain the spatial structure and dynamical behavior occurred in some fish school, and we also hope that our results may inspire empirical scientists to study spatial structure in schools of real fish.

This approach can be extended in future by:

1) adding other features that can lead to spatial structure or behavior changing, such as sex, hunger, disease etc.

2) inferring the model parameters from the real fish school data.

Acknowledgments

This study was supported partially by National Natural Science Foundation of China (30972267, 31201446, and 11231007), Zhejiang Provincial Natural Science Foundation of China (Y6110751, LQ12C20006).

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