Analytical solution on the sensitivity of matric suction profile in soil layer which is under the condition of one-dimensional steady flow

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Abstract

To simulate the suction profile within unsaturated soil layer, which is under the condition of one-dimensional infiltration, the point that water potential energy is mainly composed of gravitational potential energy and suction potential energy was used. And this paper also takes the standpoint that the relationship between these two variables is reciprocal. Combined with the mass conservation law, *Darcy*'s law and *Gardner* empirical equation, an analytical solution was got. This solution could be used to describe the suction profile within unsaturated soil layer when the seepage field of this soil layer reaches steady stage and the rainfall infiltrates to the soil layer along with the vertical direction. Several conditions with different rainfall intensities q, different inverse values of air-entry pressure α and different ratio values of q/k_s were examined by using this analytical solution, respectively. Compared with other factors, the rainfall intensity will take more influence on the suction profile, namely, the suction profile within unsaturated soil layer is more sensitive to the change of rainfall. The results obtained in this paper could be used as one useful reference for the research work about rainfall-induce landslide and the corresponding computer simulation.

Keywords: analytical solution, unsaturated soil, matric suction, sensitivity, steady seepage

1 Introduction

The seepage field profile in one slope could be used as an important reference for the stability evaluation of slope. If the "hysteresis effect" could be ignored, thus the relationship between seepage field and suction stress field could be established by the SWCC (soil-water characteristic curve) [1]. Due to the suction is a vital factor in the growth of shear strength, the field profile of it has strong relation with the development of slope stability [2-5].

As for the soil, slope suction field profile within shallow soil layer will influence by the change of some factors such as rainfall intensity, soil porosity and saturated hydraulic conductivity. This phenomenon also acquires some researcher's attention; Li presented one type of variation of suction profile by taking the point of soil pore structure [6]. Chen completed some research on the relationship between total suction and soil saturation by considering the effect of infiltrate [7]. By using simulation method, Zhu obtained the stability development for one soil slope which under different rainfall durations [8]. But most research were conducted under the change of only one factor, and comparative study on the suction field within unsaturated soil and under the influence of factor variation are rare. For above reasons this paper will discuss the suction field profile under the condition of different rainfall intensities q, inverse value of air-entry pressure α and q/k_s (rainfall intensity/saturated hydraulic conductivity). Based on obtained results, the sensitivity of suction field to different influence factors also experienced.

2 Common suction field profile within unsaturated soil layer

Following the change of influence factor or soil property, the seepage field (i.e., water content field) will also express variation, and the field re-adjust is a typical dynamic process, which will convert into stable phase at the final, namely, the steady flow stage that the field profile is regardless of the time. Correspondingly, the suction field could be divided into two different forms, transient state and steady state [9, 10].

The steady suction field keeps at a constant state, which does not product any change as time elapse. And the profile of it will be controlled by the boundary condition of soil surface and the initial condition of the whole soil layer. In order to get the general features this paper takes study on the steady suction field within only one soil layer, which is under the condition of one-dimensional steady flow.

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3 Analytical analysis on the steady suction field

3.1 ASSUMPTIONS USED IN THIS STUDY

As for the soil layer which is in the stage of one-dimensional steady flow, the water flow in it could consider as along the vertical direction only, and there are two types of boundary conditions will be used in this section: rainfall q or evaporation presents the upper boundary condition (only rainfall considered in this paper) and the water table refers to the lower boundary condition. The water content profile or suction field within one soil layer with no upper boundary condition and steady water table will be used as initial condition for analytical analysis.

3.2 ANALYTICAL SOLUTION ON SUCTION FIELD BY CONSIDERING ENERGY POTENTIAL

According to the continuity equation of water flow, the water content of any soil unit within one soil layer keeps at a constant value, namely, the mass "break-even" in soil unit, which can be expressed as follow:

$$\frac{dq}{dz}=\frac{d(\theta)}{dt}=0\,, \tag{1}$$

where z is the flow direction, θ is water content of soil and t presents flow time.

Commonly, the solution potential, velocity potential and power potential could be ignored due to they all take very small influence on slope stability. The total potential of soil pore water at any location is composed by gravity potential and suction potential, which can be expressed as:

$$E_t=E_g-E_m=m_w gz-V_w(u_a-u_w)\,, \tag{2}$$

where E_t, E_g and E_m refers to total potential, location potential and suction potential, respectively. The negative sign in this equation presents these two potential are two shift terms when they contribute to the total potential. As the rise process of capillary water, that is, the suction potential is getting smaller and smaller with water location potential increase gradually, and the water always flow to the location with low potential.

If use h to present the total water head, the above equation can be rewritten to:

$$h=z-\frac{(u_a-u_w)}{\rho_w g}\,, \tag{3}$$

where ρ_w is water density, $(u_a-u_w)=s$ refers to matric suction. Combined the *Gardner* empirical equation $k(s)=k_s e^{[-\alpha(u_a-u_w)]}$, *Darcy* unsaturated flow equation $q=-k(s)\dfrac{dh}{dz}$ and Equation (3), we can obtain a new equation as follow:

$$\begin{aligned}&-\frac{k_s e^{-\alpha(u_a-u_w)}}{\rho_w g}\cdot\frac{d^2(u_a-u_w)}{dz^2}+\\&\frac{\alpha k_s e^{-\alpha(u_a-u_w)}}{\rho_w g}\cdot\left(\frac{d(u_a-u_w)}{dz}\right)^2-,\\&\alpha k_s e^{-\alpha(u_a-u_w)}\frac{d(u_a-u_w)}{dz}=0\end{aligned} \tag{4}$$

where α is the inverse value of air-entry pressure, k_s is a constant value which presents the saturated hydraulic conductivity, the above equation can be simplified as:

$$\begin{aligned}&\frac{d^2(u_a-u_w)}{dz^2}-\alpha\left(\frac{d(u_a-u_w)}{dz}\right)^2+\\&\alpha\rho_w g\frac{d(u_a-u_w)}{dz}=0\end{aligned}\,. \tag{5}$$

Namely:

$$\frac{d^2(u_a-u_w)}{dz^2}+\alpha\frac{d(u_a-u_w)}{dz}\left(\rho_w g-\frac{d(u_a-u_w)}{dz}\right)=0\,. \tag{6}$$

Let $f=\dfrac{d(u_a-u_w)}{dz}$ by reduced order processing, then $\dfrac{d^2(u_a-u_w)}{dz^2}=\dfrac{df}{dz}=\dfrac{df}{d(u_a-u_w)}\cdot f$, after taking this new term into Equation (6), we can get a new equation:

$$f\frac{df}{d(u_a-u_w)}+\alpha f(\rho_w g-f)=0\,. \tag{7}$$

Namely:

$$\frac{df}{d(u_a-u_w)}-\alpha f+\alpha\rho_w g=0\,. \tag{8}$$

Solving Equation (8), an expression which describes the first derivative of suction within soil layer to the soil depth is as follow:

$$f=\frac{d(u_a-u_w)}{dz}=Ce^{\alpha(u_a-u_w)}+\rho_w g\,, \tag{9}$$

where C is the undetermined coefficient and can be ensured by three factors: soil pore type, air-entry pressure and pore water retain type.

Combined with $q=-k(s)\dfrac{dh}{dz}$, $k(s)=k_s e^{[-\alpha(u_a-u_w)]}$ and $h=z-\dfrac{(u_a-u_w)}{\rho_w g}$, we can obtain the following:

$$\begin{aligned}&q=-k_s e^{[-\alpha(u_a-u_w)]}\left(1-\frac{1}{\rho_w g}\frac{d(u_a-u_w)}{dz}\right)=\\&-k_s e^{[-\alpha(u_a-u_w)]}\left(1-\frac{1}{\rho_w g}(Ce^{\alpha(u_a-u_w)}+\rho_w g)\right)\end{aligned}\,. \tag{10}$$

Accordingly, when the flow amount in the soil surface, the matric suction, the soil density and the air-entry pressure are obtained, we can determine the undetermined coefficient by using the above analytical solution.

Some experimental results show us that the suction profile can be described by one smooth monotonic function curve if there is no mutation in soil properties of the whole soil layer. According to the method provide in literature [11], we can use obtained C value to determine the suction of soil at any location, furthermore, the related profile of suction field within one soil layer which is under different conditions also can be got.

4 Sensitivity analysis on the matric suction profile to influence factor

4.1 MAIN INFLUENCE FACTORS

There are many factors can take influence on the final profile of suction field within unsaturated soil layer and under steady flow state. Overall, more important factors include the boundary condition (rainfall or evaporation), soil property and the related hydraulic features. In order to experience the sensitivity of suction field to the above issues, the study with three different conditions will conduct in this section, that are:

1) Taking different rainfall intensity values but the remaining parameters at constant (α =0.3/kPa), namely, q = 0 m/s, -2.0×10^{-8} m/s, -4.0×10^{-8} m/s and -7.2×10^{-8} m/s (negative sign on behalf of rainfall infiltration);

2) The rainfall intensity and other parameters keep constant but the inverse value of air-entry pressure is different, namely, α = 1.0/kPa, 0.5/kPa, 0.3/kPa and 0.1/kPa;

3) The inverse value of air-entry pressure α = 0.3/kPa, but the ratio of rainfall intensity to saturated hydraulic conductivity is different: q/k_s = -0.1, -0.4, -1.0 and -1.3.

The other calculation parameters are: the homogeneous isotropic soil layer is composed with unsaturated soil and the thickness of it is 2.5m. The elevation of steady water table is 0m and the saturated hydraulic conductivity is k_s = 7.2×10^{-8} m/s. Whole soil layer can be divided into 50 sub-layer by 0.05m thickness.

4.2 SENSITIVITY ANALYSIS ON THE MATRIC SUCTION PROFILE

The obtained calculation results are summarized in Figures 1 to 3 and can be used to describe the suction field profile within soil layer under different conditions. From Figures 1 to 3 we can know that rainfall intensity take greatest influence on the suction field profile of soil layer, and the variation of suction value is most evident compared with other influence factors, namely, the suction field profile will be more sensitive to the rainfall intensity value.

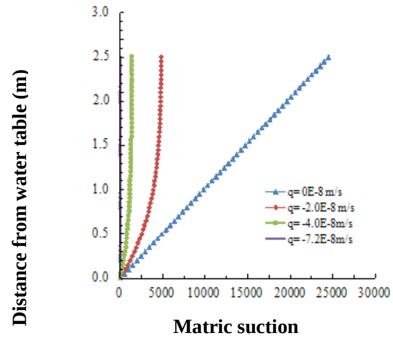

FIGURE 1 The suction profile within soil layer which is under the conditions with different rainfall intensities q

As show in Figure 1, even though at the same depth, the suction value of soil layer with no rainfall infiltration is largest, and it has linear proportional relation with the distance from water table. The suction field corresponding to this state can be considered as the initial state before rainfall starts. As the rainfall intensity increases and the seepage reaches to steady state, the suction value at the same location will be smaller and smaller, the suction field function curve will gradually move to left as well. If the rainfall intensity equal to the saturated hydraulic conductivity at the value (7.2×10^{-8} m/s), the whole soil layer will reach its saturated state when the seepage is steady, thus all the suction at any location within this soil layer are disappeared.

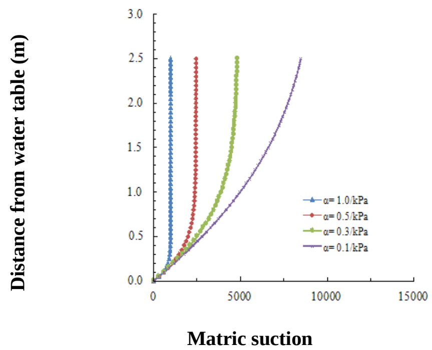

FIGURE 2 The suction profile within soil layer which is under the conditions with different inverse values of air-entry pressure

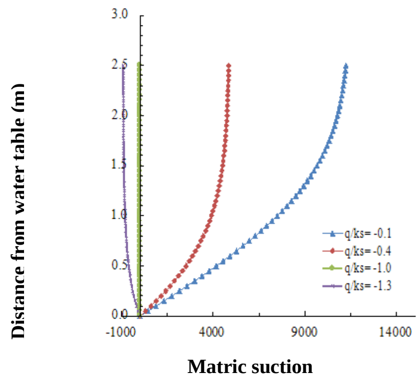

FIGURE 3 The suction profile within soil layer which is under the conditions with different ratio values of q/k_s

Compared to rainfall intensity, the change of inverse value of air-entry pressure only takes smaller influence on the suction field variation. Study on the Figure 2 we can find that even though at the same depth, the suction value related to soil with small air-entry pressure (i.e., Sandy soil) is always smaller than that of soil with large air-entry pressure (i.e., Clay soil).

Figure 3 presents the suction field profile with different ratios of rainfall intensity to saturated hydraulic conductivity. The ratio value less than 1 refers to the rainfall intensity is smaller than soil's capacity of water hold, thus all the rainfall will be accepted by the soil layer. When the seepage reaches steady state, the soil above water table also keep at saturated condition. And for the same depth, the suction value related to high ratio value (q/k_s) is smaller than other. The q/k_s=1 means that rainfall intensity is equal to soil's water hold capacity, and at this point the whole soil layer will just reach its saturated condition, correspondingly, all the suction value within it are disappeared. The soil layer cannot accept all the rainfall if the q/k_s>1, and water will occur at the surface of it. As show in Figure 3 the suction value with different depth is different for each other but all of them are less than zero.

5 Conclusions

From the perspective of potential theory, this paper considers the gravity potential and suction potential as the two major terms of pore water potential of unsaturated soil. Combined with mass conservation law, *Darcy*'s law and the *Gardner* empirical equation, an analytical solution that can be used to describe the suction field profile within unsaturated soil layer which is under one-dimensional steady flow is obtained.

With three different conditions, this paper gets the corresponding suction field profiles related to several influence factors by using the obtained. And the study on the results shows us that the suction profile within unsaturated soil layer is more sensitive to the change of rainfall than other influence factors.

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