Financial contagion dynamics and fragility assessment of industrial complex network

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Abstract

The paper proposed a mathematical modelling of financial contagion dynamics that is tightly linked to systematic risk of industrial complex network. And the paper provides a practical method to assess fragility of industrial complex network in the context of financial contagion. To examine its function, an experimental analysis based on real data set of a Chinese textile industrial network is conducted. The experimental analysis shows that the method proposed in the paper is effective ad reliable, and is capable to assess fragility of industrial complex network in the context of financial contagion.

Keywords: financial contagion, fragility, industrial complex network, assessment method

1 Introduction

Industrial complex network, such as Industrial clusters, are complex network of lots of interconnected companies in a specific industry. These companies are bonded together with highly complicated business, technological and financial relationships. In this way, these industrial systems are essentially complex networks. And these industrial complex networks play significant roles in the process of regional development in European countries, Asian countries and other areas. In rural area of emerging countries, many township economies are essentially mixtures of several industrial clusters that are typical industrial complex networks. The performance of these industrial clusters is crucial for economic development and social stability of these developing areas.

However, these industrial complex networks are readily destructed by market fluctuation and economic crisis because their interior complex network imposed risk interdependency on each individual organization. “Robust yet fragile” is a characteristic of complex network. Idiosyncratic risk of bankruptcy of an important company in the network would be likely to trigger systematic risk by inter-company financial contagion in context of interdependent industrial network. Domino-style bankruptcy in such interdependent business cycle occurs more frequently in recent years, and receives concerns of policy-makers. The argument that inter-company financial contagion is a critical step leading to systematic crisis [1] is supported by empirical evidences and case studies in recent literatures. With financial contagion and systematic risk are widely talk about in literatures, fragility of industrial complex network in the context of financial contagion became an important topic. Recent work on physical networks, such as internet and power grid, has addressed the resilience of these networks to idiosyncratic risk. Scientific works in complex network-related field has provides ideas about how to measure fragility of physical complex network. And recent financial literatures offer in-depth insights about financial contagion [2-7]. Though similar topics are discussed by scholars from disciplines, more efforts are needed to shed some light on financial contagion and its impact on fragility of industrial complex network. In previous works, the author proposed a framework for testing stability of regional industrial cluster considering inter-company financial contagion [9], and developed a network model for inter-company financial contagion [10]. In the paper, the author tries further to propose a new method for evaluating fragility of industrial complex network in the context of financial contagion.

The paper is organized as follows. The paper discusses contagious effect of idiosyncratic risk in the complex network of an industrial network in the second section. Then the paper introduces a new method for measuring fragility of industrial complex network in third section. In fourth section, the paper would illustrate an experimental analysis in the background of a Chinese industrial cluster. Its fragility would be discussed in details in this section, along with validation of this new measuring method.

2 Modelling financial contagion dynamics

In this section, the paper derives a basic mathematical framework for modelling dynamic process of financial contagion triggered by bankruptcy of one or several companies in context of industrial complex network. The paper considers mechanisms of risk interdependence and trend reinforcement in an inter-company network in the model. The paper describes an industrial complex network as a graph (donated by $G=(N,R)$), in which a set of N companies are connected with others via financial contacts.

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The network is also associated with an adjacency matrix $M = \left[M_{ij}\right]$, where $M_{ij} = 0$ means there is not financial contract from company *i* to company *j*, and $M_{ij} = 1$ represents there is some kind of financial contract from company *i* to company *j*, i.e. company *i* invested some money or lend some money to company *j*. And the out-degree of company *i* in this network, k_i represents the number of counterparties to which company *i* is exposed. And a weight matrix $H = \left[H_{ij}\right]$ is introduced. In which, the weight H_{ij} represents the exposure of *i* to *j* relative to its portfolio of all exposures. So, the matrix H is defined as a row-stochastic matrix, and subjects to $\sum_j H_{ij} = 1$. Denote M_{ij} must equal to 1, if H_{ij} is more than zero.

According to literatures about financial contagion, the dynamic law of financial contagion process can be modelled as the two mechanisms of interdependence and trend reinforcement (or financial accelerator). The paper employs stochastic differential equation (SDE) to include mechanisms of interdependence and trend reinforcement in our framework, and to investigate their interplay in the contagious process. Denote a financial robustness indicator β_i for each node *i*, to measure its distance from actual default or to indicate its state of liability reputation. If $\beta_i = 0$, the node *i* has been on the situation of default or bankruptcy. A drift term that depends on the past realizations is introduced in the SDE for representing the mechanism of financial accelerator. The paper restricts the dependence at time t on the past only to $t' = t - T$. This is, β_i at t time is dependent on the realization of β_i at time $t' = t - T$. So, the paper models the law of motion of robustness by the following time-delayed SDE.

$$d\beta_i = L\left(\beta(t), \beta(t')\right)dt + \sigma d\varepsilon_i, \tag{1}$$

where $L\left(\beta(t), \beta(t')\right)$ is not more than 0, σ is the variance of idiosyncratic shocks that hit company *i*, and $d\varepsilon_i$ denotes the Wiener stochastic process.

Then, the expected fist passage time T_f is $T_f \sim 1/\sigma^2$, if financial accelerator is absent ($L \equiv 0$). While, the negative drift would make the expected fist passage time shorter, i.e. $T_f\left(L \prec 0\right) \prec T_f\left(L = 0\right)$. In this way, financial accelerator would increase the probability of default or bankruptcy. And the paper restricts interdependence in the context of complex network. Denote C_{ij} as the asset related to company *j* and held by company, such as a liability of company *j* to company *i*. Then the value of asset C_{ij} depends on financial robustness of company *j*, i.e. its ability to meet the obligation to company *i*. To be more specific, the paper assumes the value of asset C_{ij} is proportional to the financial robustness of β_i, i.e. $C_{ij} = C_{ij}^0 \times \beta_j$, where C_{ij}^0 is nominal value of the asset. Then the total assets of company *i* at a time *t* is determined based on the estimate of financial robustness of its counterparties at time $t - 1$.

$$C_i(t) = \sum_j C_{ij}(t-1) = C_i^0 \sum_j H_{ij}\beta_j(t-1), \tag{2}$$

where C_{ij}^0 is the nominal value of investments or obligations invested or lent by company *i*, H_{ij} is fraction of investments or obligations of company *i* that is related to company *j*. Thus, H_{ij} is a reasonable proxy of relative impact on total asset of company *i* due to a change in financial robustness of company *j*. In this way, the interdependence mechanism is modelled in the context of a network of financial contacts.

The paper assumes financial robustness is linearly dependent on its counterparties in absence of bankruptcies:

$$\beta_i(t) = \sum_j H_{ij}\left(\beta_j(t-1) + \sigma\varepsilon_i(t-1)\right). \tag{3}$$

And assume financial robustness is subject to independently and identically normal distribution. If company *i* is hit by idiosyncratic shock, its own loss is only proportionally to the term H_{ij} due to this shock hit. Other loss would be shared among its counterparties. And company *i* also would be proportionally affected by the shocks hitting its counterparties. Then, the change of financial robustness is express by following equation:

$$\begin{aligned}\beta_i(t) - \beta_i(t-1) = \sum_j H_{ij}\left(\beta_j(t-1) - \beta_i(t-1)\right) + \\ \sigma\sum_j H_{ij}\varepsilon_j(t-1)\end{aligned}. \tag{4}$$

Passing to the limit of continuous time and introducing Equation (1) into Equation (4) to capture the financial accelerator, then:

$$\begin{aligned}d\beta_i = \left[\sum_j H_{ij}\beta_j(t) - \beta_i(t) + L\left(\beta(t), \beta(t')\right)\right]dt + \\ \sigma\sum_j H_{ij}d\varepsilon_j,\end{aligned}, \tag{5}$$

where $d\varepsilon_j$ is an independent Wiener stochastic process. Since $d\varepsilon_j$ is independent, its linear combination is still a Wiener stochastic process. So, denoted as $d\tau_i = \sum_j H_{ij}d\varepsilon_j$, Equation (5) could rewrite as following:

$$d\beta_i = \left[\sum_j H_{ij}\beta_j(t) - \beta_i(t) + L\left(\beta(t), \beta(t')\right)\right]dt + \sigma d\tau_i. \tag{6}$$

A company goes bankrupt when its robustness falls below a given threshold. If the bottom threshold is hit then the robustness value is initiated to Zero. Also, for the sake of simplicity, the paper assumes the robustness cannot exceed an upper barrier at $\beta_i = 1$. Then the dynamics of financial contagion process in the context of industrial complex network can be captured by the theoretical model.

3 Assessing fragility of industrial complex network

To simulate financial contagion process as described in the above theoretical model, the paper formulated a simulation procedure as illustrated in Figure 1. The simulation procedure is realized in MATLAB circumstance, operated with the help of software MATLAB 2010(a). Please see Appendix A for details of the simulation procedure.

Recent literatures of scale-free and small-world complex network research have deepened understanding of vulnerability of real-world networks [10,11]. In these researches, fragility of a complex network is generally evaluated by damage of the complex network after removing some most important nodes. To measure fragility of an industrial complex network, the paper simulates a deliberate attack at initial stage, pick up a number of top systematically-weighted nodes as bankrupts, and removes these nodes simultaneously from the network at initial stage. Then a contagion process triggered by this deliberate attack is simulated according to above mentioned procedure. Top 1, top 2 and top 3 systematically weighted nodes are respectively removed in specific simulation circumstances. For simplicity, systemically importance is evaluated by node degree. And the fragility of industrial complex network in such a deliberate attack is measured by the ratio of nodes going bankrupt and total node number.

$$F_T = B/N \,, \tag{7}$$

where F denotes fragility of industrial cluster, T represents simulation circumstance in which top 1, top 2 or top 3 systematically-weighted important nodes are removed by risk shock. B is the number of companys in the industrial complex network that go bankrupt after the ending of contagion process. N is the total number of companys at initial stage.

4 Experimental analysis

4.1 EXPERIMENTAL DATA

To examine the effectiveness of the method for assessing fragility of industrial complex network, this section evaluates fragility of an industrial complex network in Huzhou City of P.R. China. Huzhou city is located in the centre of Yangtze River Delta Economic Area. The city is composed of 6 districts with a total area of 5817 square Kilometres and a population of 2.54 million.

In its history, the city is known for its silk products and other textile products. A network dataset of Huzhou textile industrial cluster is employed in the experimental analysis. And visualization of the industrial complex network is illustrated in Figure 1.

All 27 nodes represent 27 textile companies in the city, which is interconnected with financial bonds. In here, financial bonds include share-holding relationship, debtor-creditor relationship and mutually credit guarantee. Financial information is collected from annual financial reports of these 27 companies, and their capacity to resist risk impact and financial robustness are evaluated based on their asset data and financial indicators.

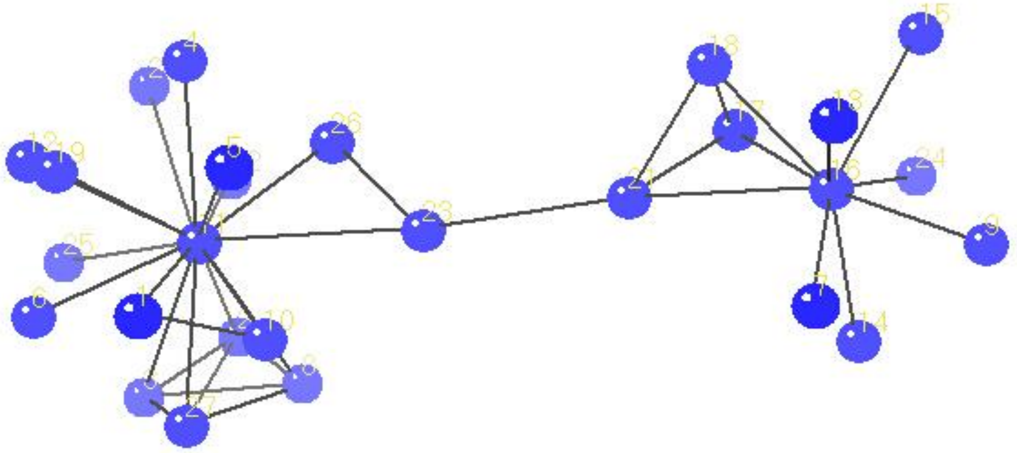

FIGURE 1 Visualization of the industrial complex network in Huzhou City

The strengths of financial bonds are calculated based on their financial releases about investments, liabilities and credit guarantees. The structure characteristics of the network of financial bonds are analyzed by UCINET 6.0, please see Table 1 for details.

TABLE 1 Network characteristics of the industrial complex network in Huzhou city

Network Characteristics	Value
Network centralization	55.08%
Network scale	27 (nodes)
Financial Bonds	73 (ties)
Average Degree	2.71
Network Density	0.1054
Clustering Coefficient	0.370

In the network, node code# 11 and node code# 9 is listed as systematically important nodes. Their share of degree is respectively 21.6% and 12.2% (see Appendix A). And there are many nodes are connected with the two nodes. If the two nodes are removed, these nodes will be disconnected.

4.2 RESULTS

In the experimental analysis, financial contagion process is simulated in a given experimental environment that is similar to real conditions in the Huzhou textile industrial cluster. Contagion effects trigged by each nodes are calculated (Table 2).

In the experimental analysis, the companies that are infected in the contagious process and finally go to bankrupt in the process are counted as infected number. These infected companies that have direct financial contact with the impacted node that is knock out by risk shock at initial stage are listed as number of directly infected. And in the experimental analysis, the industrial complex network is fragile, because 62.96% of nodes in the industrial cluster will go bankrupt in the contagion process if the node code # 11 is removed by risk shock. Furthermore, the whole industrial cluster will be crashed down if only the two nodes (code #11 and 16) are removed by risk shocks. In other aspect, the network is also robust because any other individual bankruptcy, excluding the two nodes (code #11 and 16), will not cause significant impact. Their *F* values are lower than 20%. Only deliberately risk attacks to node code #11 and 16 would cause serious financial panics leading to collapse of the whole network (Table 3).

TABLE 2 Simulation results of contagion process triggered by single bankruptcy

impacted node #	Infected number	Directly infected	Indirectly infected	*F* value	impacted node #	Infected number	Directly infected	Indirectly infected	*F* value
11	16	16	0	62.96%	6	0	0	0	3.70%
16	11	9	2	44.44%	12	0	0	0	3.70%
18	4	3	1	18.52%	9	0	0	0	3.70%
17	4	3	1	18.52%	7	0	0	0	3.70%
27	3	3	0	14.81%	5	0	0	0	3.70%
3	3	3	0	14.81%	19	0	0	0	3.70%
2	3	3	0	14.81%	20	0	0	0	3.70%
8	3	3	0	14.81%	15	0	0	0	3.70%
21	2	2	0	11.11%	22	0	0	0	3.70%
26	1	1	0	7.41%	4	0	0	0	3.70%
10	1	1	0	7.41%	24	0	0	0	3.70%
23	1	1	0	7.41%	25	0	0	0	3.70%
1	1	1	0	7.41%	13	0	0	0	3.70%
14	0	0	0	3.70%					

5 Conclusions

The paper has proposed a methodology to evaluate fragility of industrial cluster in the context of financial contagion. The evaluation is based on calculation of contagious effects triggered by bankruptcy of one or several companies and simulation of the contagion process. Huzhou textile industrial cluster is analyzed using the methodology proposed by the paper. The paper concludes that the industrial cluster is fragile, because deliberate attacks targeting only two specific companies will lead to collapse of the whole industrial cluster in the contagion process. And this experimental result supports existing evidence about risk evaluation of similar industrial clusters [6,11,12]. So, the methodology proposed in the paper is an effective way to evaluate fragility of industrial cluster in the context of financial contagion.

TABLE 3 Fragility assessments of deliberate attacks

Deliberate Attacks	Infected nodes (Bankruptcy)	F value Fragility Assessment
Top 1	16 nodes +1 nodes	62.96%
Top 2	25 nodes+2 nodes	100%
Top3	24 nodes+3nodes	100%

In the experimental analysis, all nodes are given an initial financial robustness value ($\beta_i(0)=1$ for all nodes). The financial robustness will be changed rapidly in the dynamic process of financial contagion. The simulation shows that 25 nodes out of total 27 nodes will not trigger substantial financial contagion. This is, the bankruptcy of these companies is not likely to cause financial panics in the industrial complex network, and lead to systematic risk.

In additional tests, the author found that in more severe circumstance, which means more average lower financial robustness, these nodes is still less likely to cause systematic risk. While, the two systematically important nodes (code No.11 and 16) is still more likely to cause large-scale financial contagion and lead to systematic risk. So, in the case of the experimental analysis, network structure is more important factor in determining the fragility of an industrial complex network. This argument is supported by many evidences in related literatures. In the experimental analysis, the paper ranks systematic importance of nodes according its position, especially its centrality degree. The experimental result shows this method is effective. And the logic underlying the method is also connected with above argument, and supported by literatures.

In the experimental analysis, direct contagion is major effect in the contagion process. Only 3 nodes are cause indirect contagion. However, the author thinks indirect contagion is restricted by its star-centred structure in the experimental analysis. The trend reinforcement effect or financial accelerator plays important role in the contagion dynamics simulated in the experimental analysis. Indirect effects should not be undervalued.

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Appendix A Simulation procedure based on theoretical model (Figure 2)

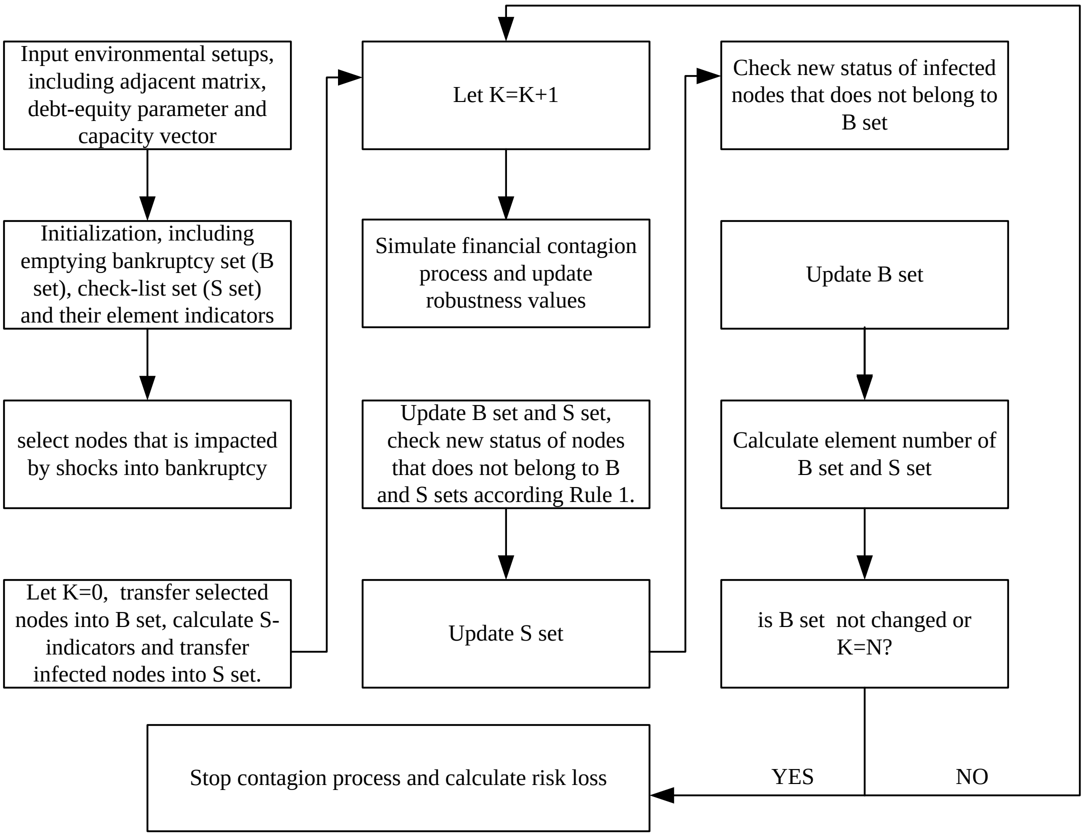

FIGURE 2 A simulation procedure based on theoretical model

Appendix B Degree distribution in the industrial complex network (Table 4)

Table 4 Degree ranking in the network of financial bonds in Huzhou city

Rank	Node Code	Degree	share	Rank	Node Code	Degree	share
1	11	16	0.216	15	12	1	0.014
2	16	9	0.122	16	9	1	0.014
3	27	4	0.054	17	7	1	0.014
4	3	4	0.054	18	5	1	0.014
5	2	4	0.054	19	19	1	0.014
6	21	4	0.054	20	20	1	0.014
7	8	4	0.054	21	15	1	0.014
8	17	3	0.041	22	22	1	0.014
9	23	3	0.041	23	4	1	0.014
10	18	3	0.041	24	24	1	0.014
11	26	2	0.027	25	25	1	0.014
12	1	2	0.027	26	13	1	0.014
13	10	2	0.027	27	14	1	0.014
14	6	1	0.014				

Appendix C Degree distribution in the industrial complex network (Table 5)

Table 5 Financial capacity and financial robustness used in the experimental analysis

Rank	Node Code	Financial capacity	Initial financial robustness	Rank	Node Code	Financial capacity	Initial financial robustness
1	16	20	1.00	15	9	1.25	1.00
2	11	11.25	1.00	16	17	1.25	1.00
3	12	5	1.00	17	23	1.25	1.00
4	18	5	1.00	18	7	1.25	1.00
5	6	5	1.00	19	15	1.25	1.00
6	1	5	1.00	20	4	1.25	1.00
7	22	5	1.00	21	24	1.25	1.00
8	5	5	1.00	22	25	1.25	1.00
9	2	3.75	1.00	23	13	1.25	1.00
10	19	3.75	1.00	24	14	1.25	1.00
11	20	3.75	1.00	25	26	1.25	1.00
12	3	2.5	1.00	26	27	1.25	1.00
13	21	2.5	1.00	27	10	1.25	1.00
14	8	1.25	1.00				

Note: Financial capacity is calculated and normalized based on financial information, including total asset and net asset of every nodes in the industrial complex network

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