Corporate growth, liquidity assets value and financing decision

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Abstract

Based on the external financing analysis framework under asymmetric information, this paper analyzes the influence of corporate growth on liquid assets value and financing decision. Both of the theory and numerical calculations show that liquid assets value would increase with the increasing in corporate growth. For the reinvestment decision in the case of higher reinvestment demand, if the price of liquidity assets is higher than value, it is optimal not to reinvest; if the price is lower than value, it is optimal to reinvest; if the price is equal to value, there is no difference found in reinvesting or not.

Keywords: liquidity shocks, liquid assets value, corporate growth, financing decision

1 Introduction

Shortage of liquidity has been considered to be the main reason of financial crisis sweeping global. As an important part of the companies’ operation process, holding of liquidity asset may influence its operating performance, and directly impact on its management, investment and financing activities. Why do companies hold large pools of liquidity? What will holding liquidity do for the companies’ value? How to determine the value of liquidity assets? What factors will affect the value of liquid assets? Such problems have been hot issues of the academic research.

As far as factors affecting the value of liquid assets is concerned. Harford (1999) proves that mergers and acquisitions will damage the companies’ cash value by studying the relationship between policy of mergers and acquisitions and value of holding liquid assets. Faulkenber and Wang (2006) et al. consider that the marginal value of holding liquid assets increases with the increasing in financing constraints. Denis and Sibilkov (2007) deem that holding liquid assets can help the enterprise get projects with higher yields, and so the enterprise with higher financing constraints may own higher value of holding liquid assets. Pinkowitz and Williamson (2007), Kalcheva and Lins (2007) make a comparative study for multinational corporations and find that, compared to countries with poor shareholders’ protection, investors contain higher liquidity assets valuation in those with better protection. Tong (2009) believes the enterprise’s diversification can increase agency cost and reduce the value of liquid assets. Drobezt et al. (2010) think the entrepreneur’s moral hazard could reduce the value of liquid assets on account of asymmetric information.

In point of research on the growth and value of liquidity assets, Myers and Majluf (1984) believe that, compared with the enterprise with lower growth, the enterprise with higher growth may own higher cost of under-investment and more serious problem of asymmetric information, so the cash value is relatively higher held by the enterprise with higher growth. Mikkelson and Partch (2003) find that companies holding more liquidity assets usually own more investment, more research and development expenses and bigger expansion of asset scale. Saddour (2006) deems the companies' market value is positively correlated with holding level of liquid assets, and compared with the enterprise with lower growth; this kind of positive correlation is more significant for the enterprise with higher growth. Through the empirical analysis, Pinkowitz and Williamson (2007) think that for the company in high-speed growth stage, its market value of holding liquidity assets would be higher.

2 Assumptions

The basic assumptions will be given in the following: three periods: \( t = 0 \) represents ex enter period; \( t = 1 \) represents intermediate period; \( t = 2 \) represents ex post period.

Participants: an entrepreneur and investors.

At date \( t = 0 \), the entrepreneur has "assets" \( A \) and a project requiring variable investment \( I \in [0, \infty) \). To imp-
lemen the project the entrepreneur must borrow \( I - A \) from investors.

If the project can get financing, the company meets a reinvestment opportunity requiring an amount \( \rho f \) at date \( t = 1 \), where \( \rho \) means the reinvestment amount required by one unit of initial investment for reinvesting. \( \rho \) is unknown at date \( t = 1 \) and subjects to the following two-point distribution:

| per unit of reinvestment \( \rho \) | \( 1 - \lambda \) | \( \lambda \) |

Average reinvestment demand for per unit of initial investment is:

\[
\bar{\rho} = (1 - \lambda)\rho_L + \lambda\rho_H.
\]

Whether the enterprise reinvests or not, the company can continue to operate.

At date \( t = 2 \), the project either succeeds, that is, yields verifiable income \( RI \), or fails, that is, yields no income, where \( R \) is the yield of initial investment in the case of success.

The probability of success would be influenced by the effort degree of the entrepreneur and reinvestment opportunities.

At date \( t = 1 \), if the company does not reinvest, the probability of success would be \( p \); on the contrary, if the company reinvests, the probability of success would be \( p + \tau \). Where \( \tau > 0 \) indicates the size of growth opportunities. The bigger \( \tau \), the greater growth opportunities.

\( p \) is affected by the effort degree of the entrepreneur, but it is unobservable. Behaving yields probability \( p = p_H \) of success and no private benefit to the entrepreneur, and misbehaving results in probability \( p = p_L < p_H \) of success and private benefit \( BL > 0 \). Let \( \Delta p = p_H - p_L \).

Per unit investment has positive NPV if the entrepreneur behaves at date \( t = 2 \), but negative NPV even if one includes the entrepreneur’s private benefit, if he does not. In other words, the initial contract need to motivate the entrepreneur to behave at date \( t = 2 \).

Reinvestment is also optimal for the society, even if the company requires reinvesting \( \rho_H I \) for growth opportunities. That is:

\[
(p_H + \tau)RI - \rho_H I - I > p_H RI - I \Rightarrow \tau R > p_H. \quad (1)
\]

At date \( t = 1 \), the entrepreneur can raise internal liquidity \( \tau(R - B/\Delta p)I \) when facing reinvestment demand. Let:

\[
\rho_L I < \tau(R - B/\Delta p)I < p_H I. \quad (2)
\]

This means that, when faced with reinvestment needs \( \rho_L I \), the enterprise could meet his reinvestment needs through internal liquidity; otherwise, when faced with reinvestment needs \( \rho_H I \), he could not meet his reinvestment needs through internal liquidity.

There exist in the economy liquid assets. That is, 1 unit invested at \( t = 0 \) delivers a return of 1 unit at \( t = 1 \).

The price of liquid assets is \( q \) at \( t = 0 \), where \( q \geq 1 \).

In order to ensure the threshold of liquid assets price is not less than 1, let:

\[
\rho_H + \tau[(1 - \lambda)(\rho_H - \rho_L) - 1] \leq 0, \quad (3)
\]

where \( \tau' = \tau / p_H \). In fact, \( \tau' \) reflects the increasing proportion of the success probability due to growth opportunities, in the case of behaving; and it could be used to measure the marginal productivity of growth opportunities.

In order to ensure the investment scale of equilibrium is positive, let:

\[
1 + (1 - \lambda)\rho_L - \rho_0 - (1 - \lambda)\tau'\rho_L > 0, \quad (4)
\]

where \( \rho_0 = p_H(R - B/\Delta p)I \) denotes the expected pledge able income per unit of investment at \( t = 2 \) without reinvestment.

Both the entrepreneur and investors are risk neutral.

Both the entrepreneur and investors have not time preference; the riskless rate is taken to be 0.

Investors behave competitively in the sense that the loan, if any, makes zero profit.

The entrepreneur has bargaining power, and he puts forward a financing contract for investors “either accept or reject”, he is also protected by limited liability.

The timing could be summarized in Figure 1.

\[
(p_H + \tau)RI - \rho_H I - I > p_H RI - I \Rightarrow \tau R > p_H. \quad (1)
\]
3 Optimal models

Suppose that the financing contract signed at date $t = 0$ between the entrepreneur and investors takes the following state-contingent form:

$$\{I, L; (1, x); (R_0, 0), (RI - R_0, 0)\},$$

where the contract specifies an initial investment level $I$.

The entrepreneur can purchase $L$ units of liquidity assets with the price $q$ at date $t = 0$.

At date $t = 1$, if reinvestment level is $\rho_1 I$, the entrepreneur always reinvests; contrarily, if reinvestment level is $\rho_H I$, he could only reinvests with probability $x$.

At date $t = 2$, if the project succeed, the entrepreneur and investors get $R_0$ and $RI - R_0$, respectively, if the project fail, both of them get 0.

The enterprise's optimal model under this kind of contract form is like that:

$$\max \left\{ (1 - \lambda)(p_H + \tau) + \lambda [x(p_H + \tau) + (1 - x)p_H] \right\} R_0 - A$$

s.t.

1. $a_2$: $p_H R_0 \geq p_H R_0 + BI$
2. $a_3$: $L \times 1 + x(1 - \rho_H I) \geq x \rho_H I$
3. $a_4$: $\lambda \Delta \{p_H + \tau\}(RI - R_0) - \rho_H I + \lambda (1 - x)p_H (RI - R_0)$

where

$$x \Delta \{p_H + \tau\}(RI - R_0) - \rho_H I + \lambda (1 - x)p_H (RI - R_0) + (1 - \lambda)(p_H + \tau)(RI - R_0) - \rho_H I \geq I - A + (q - 1)L$$

where the objective function is the entrepreneur's net utility. When reinvestment level is $\rho_H I$, the entrepreneur always reinvests, and the probability of success is $p_H$. When reinvestment level is $\rho_H I$, the entrepreneur could only reinvest with probability $x$ (the success probability is $p_H$), and could not reinvest with probability $1 - x$ (the success probability is $p_H$).

The constraint $(a_2)$ is the entrepreneur's incentive-compatibility constraint with reinvestment. This constraint ensures the entrepreneur behaving at date $t = 2$ if he reinvests at date $t = 1$. Where the left side of the inequality is the expected profit if he behaves, the right side is the expected profit if he misbehaves, and $(a_2)$ could be simplified as Equation (5). Therefore, the entrepreneur's incentive-compatibility constraint will not be affected by reinvestment. In fact, the moral hazard happens after reinvestment, so the incentive-compatibility constraint will be the same whether the project gets growth opportunity or not.

The constraint $(a_3)$ ensures that the entrepreneur can reinvest if reinvestment level is $\rho_1 I$; $(a_4)$ ensures that the entrepreneur can reinvest if reinvestment level is $\rho_H I$. In fact, the enterprise can meet his reinvestment needs by internal liquidity if reinvestment level is $\rho_H I$. Contrarily, if reinvestment level is $\rho_H I$, the expected probability for investors getting $RI - R_0$ is $x(p_H + \tau) + (1 - x)p_H$. and the investors’ expected reinvestment is $x \rho_H + (1 - x) \cdot 0$, $I - A$ is their initial invest at date $t = 0$, $(q - 1)L$ is the liquidity premium generated by $L$ units of liquidity assets. In fact, the investors' individual-rationality constraint can be simplified as:

$$R_0 \geq BI / 4 r$$

where:

$$H = p_H R + [(1 - \lambda) + \lambda x] \rho_H - (q - 1) \lambda (\rho_H - 4 r) - 1 - (1 - \lambda) \rho_H - \lambda x \rho_H.$$

4 Optimal contracts

The optimization problem can be solved in three steps.

**Step 1:** Solve the optimal $R^\circ_0$ and $I^\circ (x)$ for a given $x$.

In fact, all the solutions satisfied the constraint Equations (5) and (6) are called the "feasible contract set" $S$. The optimization problem shows that the entrepreneur needs to find one financing contract to maximize his own profit from the feasible set $S$. Figure 2 illustrates the "feasible contract set" of the optimization problem.
We can get that the "feasible contract set "from a geometric point of view, the "feasible contract set" is constituted by the shaded area OEF surrounded by the incentive-compatibility constraint, individual-rationality constraint and non-negative constraint. The objective function shows that it is the bigger the better for entrepreneurs.

Therefore, the point $F$ constitutes the optimal contract, and then:

$$R_b = \frac{BI}{\partial p}$$

and

$$I'(x) = A \cdot \left[1 - \rho_0 - (1 + \lambda)x\right]\rho_0 + (q-1)x(1+\lambda)x$$

where

$$\rho_0 = p_H (R - B/(\partial p)).$$

In Equation (7), $(1-\lambda) + \lambda x$ represents the probability of reinvesting or getting growth opportunity at date $t = 1$. $\rho_0 + (1 - \lambda)x(1+\lambda)x\rho_0$ expresses the expected pledge able income generated by per unit of initial investment; $(1-\lambda)\rho_H + \lambda x\rho_H$ shows the expected reinvestment cost of per unit of initial investment; $x(xH - \tau'\rho_0)$ is the holding amount of liquid assets for one unit initial investment; $q - 1$ is the liquidity premium.

**Step 2:** Consider the optimal $x^*$ by taking $R_b$ and $I'$ into the optimization problem.

In conclusion, the optimization problem can be expressed as:

$$\max_{x} \quad \rho_1 - c(x, q) \cdot A,$$

where:

$$c(x, q) = \frac{(q-1)x(\rho_H - \tau'\rho_0) + (1 - \lambda)\rho_L + \lambda x\rho_H + 1}{1 + (1 - \lambda + \lambda)x\tau'},$$

as:

$$\max_{x} \quad \rho_1 - c(x, q) \cdot A \Leftrightarrow \min_{x} c(x, q),$$

this means the entrepreneur could get maximum utility when $c(x, q)$ takes the minimum.

**Suggestion 1:** When the entrepreneur needs to reinvest $\rho_1 I$ because of getting the growth opportunity, his optimal strategy is closely related to the liquidity assets $q$ it displays as that:

if $q \in [1, \bar{q})$, it is optimal to reinvest with probability $x^* = 1$;

if $q \in (\bar{q}, +\infty)$, it is optimal to reinvest with probability $x^* = 0$;

if $q = \bar{q}$, it is optimal to reinvest with probability $x^* = 0,1$.

where $\bar{q} = 1 + \lambda \tau'[1 - (1 - \lambda)(\rho_H - \rho_L)] - \rho_H \tau'$ represents the value of liquid assets.

**Proof:** Due to $\frac{\partial c(x, q)}{\partial x} = E$, where:

$$E = (q-1)(\rho_H - \tau'\rho_0)\tau' + \lambda \rho_H + \lambda x\tau'[1 - (1 - \lambda)(\rho_H - \rho_L) - 1],$$

$$F = [1 + (1 - \lambda + \lambda)x]\tau'^2,$$

let:

$$\bar{q} = 1 + \lambda \tau'[1 - (1 - \lambda)(\rho_H - \rho_L) - \rho_H] \tau',$$

which leads to:

if $q > \bar{q}$, $\frac{\partial c(x, q)}{\partial x} > 0 \Rightarrow x^* = 0$;

if $q < \bar{q}$, $\frac{\partial c(x, q)}{\partial x} < 0 \Rightarrow x^* = 1$;

if $q = \bar{q}$, $\frac{\partial c(x, q)}{\partial x} = 0 \Rightarrow x^* = [0,1].$
neur that means the cost of holding liquid assets is bigger than expected earnings. Therefore, it is irrational for the entrepreneur to hold liquid assets. Secondly, if the price of liquid assets is relatively lower (\( q < \bar{q} \)), it is optimal to hold \( \rho_u I - \tau \rho_p \) units of liquid assets, in order to ensure the reinvestment in any case by getting plenty of liquidity. Thirdly, if the price of liquid assets is equal to the entrepreneur’s expected return \( \bar{q} \) generated by one unit of liquid assets, there is no difference whether to store liquid assets or not for the entrepreneur.

Step 3: Find the optimal investment level \( I^* \).

Suggestion 1 and Equation (7) indicate that the optimal investment level is closely related to the liquid assets price \( q \), specifically as follows:

if \( q < \bar{q} \), the optimal investment level would be

\[
I^* = \frac{1}{1 - \rho_0 - \tau \rho_p + (q - 1)(\rho_u - \tau \rho_p) + \bar{q}^A};
\]

if \( q > \bar{q} \), the optimal investment level would be

\[
I^* = \frac{1}{1 - \rho_0 - (1 - \lambda) \tau \rho_p + (1 - \lambda) \rho_L} A;
\]

if \( q = \bar{q} \), the optimal investment level would be

\[
I^*(x) = A \cdot \{1 - \rho_0 - [(1 - \lambda) + \lambda x] \tau \rho_p + \}
\]

\[
(\bar{q} - 1) x (\rho_u - \tau \rho_p) + (1 - \lambda) \rho_L + \lambda x \rho_H \}^{-1}.
\]

5 Comparative static analyses

The relationship between corporate growth and liquidity assets value will be given next.

Suggestion 2: The better the corporate growth is, the higher the enterprise can get from one unit of liquid assets, that means, liquid assets value \( \bar{q} \) and corporate growth \( \tau \) are positively correlated.

Proof: Proposition 1 indicates that the value of one unit of liquid assets for the enterprise is:

\[
\bar{q} = 1 + \lambda \frac{\tau^2 [(1 - \lambda)(\rho_u - \rho_L) - \rho_H]}{(\rho_u - \tau \rho_p) [1 + (1 - \lambda) \tau^2]}.
\]

Solving the partial derivative of \( \bar{q} \) with respect to \( \tau^2 \):

\[
\frac{d \bar{q}}{d \tau^2} = -\lambda \frac{G}{J},
\]

where:

\[
J = \{[(\rho_u - \tau \rho_p) [1 + (1 - \lambda) \tau^2]]^2,
\]

\[
G = [(1 - \lambda)(\rho_u - \rho_L) - 1][\rho_u - \tau \rho_p][1 + (1 - \lambda) \tau^2] +
\]

\[
(\rho_u + \tau^2 [(1 - \lambda)(\rho_u - \rho_L) - 1]) \rho_0 [1 + (1 - \lambda) \tau^2] -
\]

\[
(\rho_u + \tau^2 [(1 - \lambda)(\rho_u - \rho_L) - 1]) (\rho_u - \tau \rho_p) (1 - \lambda).
\]

6 Numerical simulations

Now, some numerical calculations would be made for the theoretical results. Table 1 indicates the influence of corporate growth on liquid assets value \( \bar{q} \), and then the optimal financing contract could be given in the condition of a certain liquid assets price. Where basic parameters are \( q = 1.5 \), \( A = 1 \), \( \rho_u = 0.4 \), \( \rho_L = 0.2 \), \( B = 0.4 \), \( R = 3 \), \( \rho_H = 0.6 \), \( \rho_L = 0.3 \), \( \lambda = 0.5 \).

Table 1 shows that, both of the entrepreneur’s expected net utility \( U_b \) and liquid assets value \( \bar{q} \) increase with the increasing in corporate growth \( \tau \). If the price of liquidity assets is higher than value, the probability of reinvestment \( x = 0 \) in the case of higher reinvestment demand. The number of holding liquid assets \( L = 0 \); if the price is lower than value, the enterprise will store enough liquid assets to meet his reinvestment need, and the number of holding liquid assets would decrease with increasing in corporate growth.

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7 Conclusions

Based on the external financing analysis framework under asymmetric information, this paper analyzes the influence of corporate growth on liquid assets value, and then discusses the enterprise's optimal financing decision in the condition of a given liquid assets price. The study shows that liquid assets value would increase with the increasing in corporate growth, in other words, the enterprise with higher growth is willing to pay higher price to store liquid assets. If the price of liquidity assets is higher than value, the enterprise will store enough liquid assets to meet his reinvestment need in any case for getting the growth opportunity; if the price is lower than value, the enterprise will store enough liquid assets in the case of higher reinvestment demand; if the price is equal to value, there is no difference found in reinvesting or not.

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