Identification and application of investors’ risk appetite-based on the analysis of risk allocation of China multi-layer capital market system

Xiaoyuan Geng¹, Yongde Wang¹*

¹School of Accountancy, HeiLongjiang Bayi Agricultural University, Daqing, 163319, China

Received 1 October 2014, www.cmnt.lv

Abstract

In order to measure investors’ risk appetite more accurately, from the focus on the investors’ demand for the capital market, this article deduces the utility level of investors in the capital market by the inverse of the investor demand (only when the demand function satisfies integrability, then it will be deduced the utility function inversely), and thus measure and identify the investors’ risk appetite. While based on this theory approach, the paper empirically analyses the risk allocation of China multi-layer capital market, and the results show that: risk allocation of China multi-layer capital market system is non Pareto efficient, the risk allocation of each market does not meet the structure of multi-layer capital market established, but these problems can be improved by adjusting the market trading mechanisms.

Keywords: Stochastic demand; Integrability problem; Risk appetite; Risk allocation

1 Introduction

By correctly understanding the investors’ risk appetite in the multi-layer capital market, it can be provided a strong basis for accurately formulating the policy for multi-layer capital market, formulated more targeted policy, security policy role to achieve the desired effect. October 30, 2009 28 SME GEM official visit in China, which means China multi-layer capital market system, has taken shape. The multi-layer capital market is needed to establish by reforming and developing capital market in China, because the single capital market structure is caused by inefficient capital markets, speculative prominent, regulatory costs and other problems of the institutional reasons [1], only through effectively operating the multi-layer capital market system can reduce the high transaction costs due to a single hierarchy lead, narrow the scope of the transaction costs incurred [2]. The purpose of the establishment of multi-layer capital market is not only to optimize the allocation of resources, but also to optimize the allocation of risk. Optimal allocation of resources refers to it provides a direct financing platform for the different types of companies which they can maximally overcome the obstacles to credit constraints, the optimal allocation of risk refers to it provides investment environment for the investors with different characteristics of risk preferences which consistent with their risk appetite, and launch of the GEM adapts to the demand of risk lovers. That is to say, the establishment of multi-layer capital market is not only to improve the efficiency of resource allocation, but also to improve risk allocation efficiency by establishing different levels of market risk, namely the risk allocation should achieve Pareto efficiency, the various risks fall on the risk appetite of investors with such features [3]. Current research focuses on the study of efficiency of resource allocation, the study of risk allocation efficiency is relatively less, but this aspect of the research for the development of the market is how important it is.

Therefore, the focus of this paper is allocation efficiency of multi-market risk capital, combining the purpose of multi-purpose capital market system established, the paper aims to measure the allocation efficiency of risk by identifying the investors’ risk appetite in the market. Identification of investors’ risk appetite can be attributed to its utility function for the study, while investors’ utility function can be obtained by studying its demand function, which is the integrable problems, and the starting point of this study from the risk appetite is not common. That is to say, based on the analysis of investors’ market demand function, if the demand function satisfies the integrability, namely the demand function has a symmetrical, semi-negative definite substitute matrix, then we can get the investors’ utility function which consistent with their demand function, and identify the investors’ risk appetite, thereby determine whether the investors’ performance in the market consistent with the purpose of the establishment of multi-layer capital market, namely whether it is Pareto efficient.

Visibly, the establishment of investors’ demand function for the market does not like the conventionally established model, namely under the given investors’ utility function achieving the maximum utility for solving the utility function. Since the time of establishment of multi-layer capital market system is short, not mature enough, so to some extent, the current capital market is a speculative market [4]. When making investments, its purpose is not to become a corporate shareholder, with part of its title, but trying to take advantage of price movements to get the profits arising from bid-ask spread, when making investment decisions, investors don’t consider there is any
essential difference among the listed companies, but un-
steadily they see them as symbol or code, they randomly
select these symbol or code according to their own
subjective and objective constraints. Therefore, according
to investors’ performance in the market, we construct the
expected demand function of random variables. If this
function is able to satisfy the integrability, we can get the
investors’ utility function. Utility function is an important
economic analytical tool, which is real, and can be proved
by various methods [5][6]. At the same time, it is also been
widely used in the financial field[7]; used the utility
function to improve Markowitz mean – variance portfolio
model, resulting in asset allocation to meet the different
investors’ risk preferences[8]; also based on investors’
appetite for risk and return, established investors’ utility
function, and thus used the utility function in the insurance
industry[7]; certainly, also built the utility function by
direct to measure the investors’ risk attitude[9]. It can be
seen, in the use of the utility function to solve problems,
these analysis carried out under the prerequisite condition
of known investors’ risk appetite, but this article is based
on the premise of investors’ risk appetite unknown, by the
utility function to identify investors’ risk appetite; although
the article also build utility function, the utility
function is constructed based on the investors’ real need
performance in the multi-layer capital market in this paper,
rather than built directly.

Construct the investors’ risk appetite measurement
model in accordance with their utility function in this
article. In this regard, the paper does not use the standard
deviation $\sigma$ or variance $\sigma^2$ proposed by Markowitz
(1952) and $\beta$ coefficient method proposed by William
Sharpe $\beta = \frac{\text{cov}_{XY}}{\sigma_Y^2}$, in which: $\sigma_Y^2$ is the variance of the
market portfolio. Because of these two methods does not
reflect the risk difference of asset purchasing at different
prices, for investors’ risk are different facing the same
kinds of asset purchasing at different prices, the purchasing
price more higher the risk more greater. Strictly speaking, by $\sigma$ and $\beta$ measuring the risk is the
same ideologically, there is no essential difference between
them, because $\beta$ is the variant of $\sigma$.

Then, in the second part of this article, we first con-
struct stochastic demand model, and validate the integ-
ritability of this model, and then get the random utility
model which consistent with investors’ demand model; in
the third part of this article, we construct risk appetite
measurement model, in the fourth part we identify the
risk appetite of China multi-layer capital market, deter-
mine its validity, that is to say identify the current
investors’ performance on risk allocation in the multi-
layer capital market, whether it is achieved Pareto
efficient; in the five part we correspond the policy and
recommendations.

2 Construction of utility theory model – demand

Random expectation demand model raised by Becker is the
most classic, most influential. He considered individual
randomly assigned their wealth among competing goods
under linear budget constraints, constructs the probabilistic
choice models based on the assumption of uniform
distribution of two commodities, non-satiation, and found
that this model was still in line with the needs of law [10].
Since then a large number of scholars have conducted
research based on Becker’s model, Machina and Gul and
Pesendorfer proposed, even the model resulting in the non-
rational behavior can contain the satisfactory and limited
rational behavior depending on the utility function and
random selection [11][12]. Sanderson reduced the
assumptions and constraints of Becker’s model, he
expanded the Becker’s model and used the expanded
model to analyze the family output, and produced better
results. Visibly random expected demand model have been
developed more perfect. In this article, the benchmark
model is still Becker’s model, but reduces the probability
distribution assumptions, while expand the two products to
three, because this article studies the main market, small
board and GEM of capital market, according to this
method it can also be adapted to model of plurality
products certainly. Although the article reduces the
assumptions, the model still has excellent properties.

2.1 ASSUMPTIONS

Assumption 1: $X = (X_1, X_2, X_3)$ represents the market
vector, $X_i = X_1, X_2, X_3, i = 1, 2, 3$ separately represents
the three markets: main market, small board and the
GEM market, assuming that all investors who choose to
enter the $i$th market constitute a general, and the overall
homogeneity is the overall unit performance has the same
preferences, while the differences in the presence among
the overall unit makes this general heterogeneity.

Assumption 2: $p = (p_1, p_2, p_3)$ represents the price
vector relatively, and $W$ represents the individual’s total
wealth, individual randomly allocates his total wealth $W$
to three markets, $w = \sum w_i$, $w_i$ is the probability of the
total wealth randomly assigned to the $i$th market, the
wealth allocated to the $i$th market is $w_i$, $w_i = \pi_i \cdot w_i$. To
avoid degradation, we assume the income and price are
strictly positive and finite. So their budget constraint is:

$$p_i X_i + p_2 X_2 + p_3 X_3 \leq w,$$ that is to say,

$$p \cdot X = \sum_{i=1}^{3} p_i X_i \leq w \tag{1}$$

Assumption 3: individuals randomly choose their
willingness to participate in the sub-stock market, the
cumulative distribution function is:

$$F(x_1, x_2, x_3) = Pr(X_1 \leq x_1, X_2 \leq x_2, X_3 \leq x_3)$$

and the probability density function is $f(x_1, x_2, x_3)$.

To ensure its differentiability, we assume $f(x)$ is
continuous. $x \in R^3$ is a standard set of requirements.
2.2 RANDOM EXPECT DEMAND MODEL OF THE iTH MARKET

Investors’ demand for the i th market \((i=1,2,3)\), if \(p_iX_i \leq w\), then the choice of \(X_i\) is feasible. That is, if \(X_i \leq X_i^{\text{max}}, X_i^{\text{max}}\) is the maximum possible demand of \(X_i\), \(X_i^{\text{max}} = w/p_i\). In this case, the viable condition of choice \(X_i\) is need to be considered for the part of the wealth to \(X_i\). If \(X_i\) is feasible, \(p_iX_i \leq w\), then \(X_i\) is feasible, the maximum possible amount of consumption is \(X_i^{\text{max}} = w - p_iX_i\). Similarly, if \(X_1\) and \(X_2\) are feasible,

\[
F(p, w) = \Pr(0 \leq pX_i \leq w) = \int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} f(x_i, x_j, x_j)dx_jdx_i
\]

(2)

Investors’ random expect demand for the i th market to meet the:

\[
\bar{x}_i(p, w) \equiv E(X_i)0 \leq p \cdot X_i \leq w = \int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} x_i f(x_i, x_j, x_j)dx_jdx_i
\]

(3)

Model (3) is investors’ random expect demand model for the i th market. In essence, \(\bar{x}_i(p, w)\) is the truncated expectation of random variable \(X_i\), truncation occurs because all of the true value of the vector must be located within the budget set. In other words, \(\bar{x}_i(p, w)\) is the expectation of random variable from \(f(x_i) / F(p, w)\).

From (2) and (3), it can be concluded that the demand function is zero-order homogeneous for prices and wealth. This result is easily confirmed, for \(\theta > 0\), \(\bar{x}(\theta p, \theta w) = \bar{x}(p, w)\).

Define \(\bar{x}(p, w) \equiv (\bar{x}_1(p, w, \bar{x}_2(p, w, \bar{x}_3(p, w))\) as the random expect demand vector.

\[
\Sigma = \text{Cov}(X_i, X_j) = E((X_i - \bar{x}(p, w))(X_j - \bar{x}(p, w)))| X_i \leq X_i^{\text{max}}, X_j \leq X_j^{\text{max}}, X_i = X_i^{\text{max}}(x_i, x_j), X_j = X_j^{\text{max}}(x_i, x_j)]
\]

in this case if \(i \neq j\), then the elements are the covariance \(\text{Cov}(X_i, X_j)\), if \(i = j\), then the elements are the variance \(\text{Var}(X_i)\).

So \(Q(p, w) = \bar{x}_i(p, w) = \int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} x_i f(x_i, x_j, x_j)dx_jdx_i\), then \(\bar{x}_i(p, w) = \frac{Q(p, w)}{F_i(p, w)}\), and

\[
\frac{\partial Q(p, w)}{\partial p_i} = \int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} x_i f(x_i, x_j, X_i^{\text{max}}(x_i, x_j))\left(\frac{\partial X_i^{\text{max}}(x_i, x_j)}{\partial p_i}\right)dx_jdx_i
\]

(4)

\[
\frac{\partial F(p, w)}{\partial p_i} = \int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} f(x_i, x_j, X_i^{\text{max}}(x_i, x_j))\left(\frac{\partial X_i^{\text{max}}(x_i, x_j)}{\partial p_i}\right)dx_jdx_i
\]

(5)

1) Verifiction of symmetry

\[
\frac{\partial Q(p, w)}{\partial p_j} = \int_0^{X_j^{\text{max}}(x_j)} \int_0^{X_j^{\text{max}}(x_j-x_j)} f(x_i, x_j, X_j^{\text{max}}(x_i, x_j))\left(\frac{\partial X_j^{\text{max}}(x_i, x_j)}{\partial p_j}\right)dx_jdx_i,
\]

\[
\frac{\partial F(p, w)}{\partial p_j} = \int_0^{X_j^{\text{max}}(x_j)} \int_0^{X_j^{\text{max}}(x_j-x_j)} f(x_i, x_j, X_j^{\text{max}}(x_i, x_j))\left(\frac{\partial X_j^{\text{max}}(x_i, x_j)}{\partial p_j}\right)dx_jdx_i
\]

Therefore, we make \(X^{\text{max}} = (X_1^{\text{max}}, X_2^{\text{max}}(x_i), X_3^{\text{max}}(x_i, x_j))\) as the largest possible demand vector on the condition of three markets.

Define \(F(p, w) = F(X_1^{\text{max}}, X_2^{\text{max}}(x_i), X_3^{\text{max}}(x_i, x_j))\), it ensures the establishment of the budget constraint (1), given:

\[
p_iX_i \leq w-p_iX_i - p_iX_j\]

Set up, then \(X_3\) is feasible, the maximum demand is \(X_3^{\text{max}} = w - \sum_{i=1}^3 p_iX_i\).

2.3 VALIDATION OF INTEGRABILITY OF RANDOM EXPECT DEMAND MODEL

When a given set of demand function \(x(p, w)\), if it has a negative semi-definite symmetric substitution matrix, then this function can be set to satisfy the integrability, namely we can get the utility function set through the function set which consistent with.

If \(S(p, w) = \text{Slutsky substitution matrix}, \Sigma\) is the variance – covariance matrix of \(\bar{x}_i(p, w)\).

\[
S(p, w) = -\Sigma, \text{and the elements of } \Sigma \text{ as follows:}
\]

\[
Q_i(p, w) = \int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} x_i f(x_i, x_j, X_i^{\text{max}}(x_i, x_j))\left(\frac{\partial X_i^{\text{max}}(x_i, x_j)}{\partial p_i}\right)dx_jdx_i,
\]

\[
\int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} f(x_i, x_j, X_i^{\text{max}}(x_i, x_j))\left(\frac{\partial X_i^{\text{max}}(x_i, x_j)}{\partial p_i}\right)dx_jdx_i
\]

\[
\int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} x_i f(x_i, x_j, X_i^{\text{max}}(x_i, x_j))\left(\frac{\partial X_i^{\text{max}}(x_i, x_j)}{\partial p_i}\right)dx_jdx_i
\]

\[
\int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} f(x_i, x_j, X_i^{\text{max}}(x_i, x_j))\left(\frac{\partial X_i^{\text{max}}(x_i, x_j)}{\partial p_i}\right)dx_jdx_i
\]

\[
\int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} x_i f(x_i, x_j, X_i^{\text{max}}(x_i, x_j))\left(\frac{\partial X_i^{\text{max}}(x_i, x_j)}{\partial p_i}\right)dx_jdx_i
\]

\[
\int_0^{X_i^{\text{max}}(x_i)} \int_0^{X_i^{\text{max}}(x_i-x_j)} f(x_i, x_j, X_i^{\text{max}}(x_i, x_j))\left(\frac{\partial X_i^{\text{max}}(x_i, x_j)}{\partial p_i}\right)dx_jdx_i
\]
after calculation it can be obtained:  
\[
\frac{\partial \bar{X}(p, w)}{\partial p_j} = \int_0^{x_{j\max}(x_i)} \int_0^{x_{j\max}(x_i)} f(x_{i, j}, X_{j, j\max}(x_i, x_j)) \frac{\partial X_{j, j\max}(x_i, x_j)}{\partial p_j} \, dx_j \, dx_i 
\]

(6)  

Among them, \( \frac{\partial X_{j, j\max}(x_i, x_j)}{\partial p_j} = \frac{\bar{x}_j(p, w) - x_j}{p_j} \)  

(7)

Let (7) substitute into (6), we get,  
\[
\frac{\partial \bar{X}(p, w)}{\partial p_j} = \int_0^{x_{j\max}(x_i)} \int_0^{x_{j\max}(x_i)} f(x_{i, j}, X_{j, j\max}(x_i, x_j)) \frac{\partial X_{j, j\max}(x_i, x_j)}{\partial p_j} \, dx_j \, dx_i 
\]

(8)  

Similarly it can be calculated:  
\[
\frac{\partial \bar{X}(p, w)}{\partial p_i} = \int_0^{x_{j\max}(x_i)} \int_0^{x_{j\max}(x_i)} f(x_{i, j}, X_{j, j\max}(x_i, x_j)) \frac{\partial X_{j, j\max}(x_i, x_j)}{\partial p_i} \, dx_j \, dx_i 
\]

(9)  

Thus, (8)=(9), that is to say,  
\[
\frac{\partial \bar{X}(p, w)}{\partial p_j} = \frac{\partial \bar{X}(p, w)}{\partial p_i} \bigg|_{\tau} \]

This substitution matrices satisfy the symmetry.  

2) Validation of semi-negative definite  

For  
\[
\frac{\partial \bar{X}(p, w)}{\partial p_i} = \frac{\partial Q(p, w)}{\partial p_i} \bigg|_{\tau} \left( \frac{1}{F(p, w)} \right) \frac{\partial F(p, w)}{\partial p_i} \bigg|_{\tau} \left( \frac{Q(p, w)}{F^2(p, w)} \right),
\]

after arrangement we get:  
\[
\frac{\partial \bar{X}(p_i, w)}{\partial p_j} = \left( \frac{1}{F(p, w)} \right) \left( \frac{\partial Q(p_i, w)}{\partial p_j} \right) \left( \frac{Q(p_i, w)}{F^2(p, w)} \right)
\]

(10)  

Let (4) (5) substitute into (10), after arrangement, we get:  
\[
\frac{\partial \bar{X}(p_i, w)}{\partial p_j} = \int_0^{x_{j\max}(x_i)} \int_0^{x_{j\max}(x_i)} f(x_{i, j}, X_{j, j\max}(x_i, x_j)) \frac{\partial X_{j, j\max}(x_i, x_j)}{\partial p_j} \, dx_j \, dx_i 
\]

(11)  

Let (12) substitute into (11), we get:  
\[
\frac{\partial \bar{X}(p_j, w)}{\partial p_i} = \frac{\bar{x}_j(p, w) - x_j}{p_j} 
\]

(12)  

Let (12) substitute into (11), we get:  
\[
\frac{\partial \bar{X}(p_i, w)}{\partial p_j} = \left( \frac{1}{F(p, w)} \right) \left( \frac{\partial Q(p_i, w)}{\partial p_j} \right) \left( \frac{Q(p_i, w)}{F^2(p, w)} \right) 
\]

(13)  

Meanwhile,  \( Var(X_i) = E[(X_i - \bar{x}_i(p, w))^2] \)  

so  
\[
\frac{\partial \bar{X}(p, w)}{\partial p_i} = -Var(X_i) 
\]

(14)  

At this time, the price effect of expect demand is non-positive.  

And because  \( Cov(X_i, X_j) = E[(X_i - \bar{x}_i(p, w))(X_j - \bar{x}_j(p, w))] \)  

so  
\[
\frac{\partial \bar{X}(p, w)}{\partial p_i} = -Cov(X_i, X_j). 
\]

(15)  

Because of the variance – covariance matrix \( \Sigma \) is semi-positive definite, \( S(p, w) \) is semi-negative definite.  

It can be seen that this model satisfies the integrability, we can get the utility function which consistent with it, that is to say we can obtain three different investors’ utility function from the main market, small board and the GEM market, that is  
\[
U(X_i) = \int_p \bar{x}_i(t, w)dt. 
\]
3 Measurement of risk appetite

When the investors make their own choice, and at the same time they know the results they will have to bear in the future, or benefit from this choice, or bear the loss, or neither benefit nor lost, in a risk-free status. When investors choose to enter the main market, small board and GEM market, they tend to be more concerned about their possible gain or loss, if \( R^* \) represents the collection which contains all appeared unknown results after investors choose to enter the \( i \) th markets, \( R^* \) means the benefit collection from investors’ choice, \( R^- \) means loss collection from investors’ choice, it is clear that \( R^* \cap R^-=\emptyset \).

Risk is the possible loss due to the uncertainty to investors, more objective expression is that when investors choose to fall \( R^- \), investors will benefit, will not bear the loss; If when the investor’s choice falls \( R^- \), that is \( R^* \neq \emptyset \), at this time investors have to face the risk.

Investors bear the risk:

\[
E_R(U_i) = \int_{R^-} U_i dF(U_i) = \int_{R^-} U_i f(U_i) dU_i , E_R(U_i) < 0
\]

Investors obtain the benefit:

\[
E_R(U_i) = \int_{R^*} U_i dF(U_i) = \int_{R^*} U_i f(U_i) dU_i , E_R(U_i) > 0.
\]

Assuming \( r \) is risk-free return, and then we construct the measurement tools of investors’ risk appetite:

\[
\eta_i = r - E_R(U_i) - E_R(U_i) > 0.
\]

When \( \eta_i > 0 \), investors are risk lovers, and the value of \( \eta_i \) the larger indicates the degree of risk investors pursue the higher. At this point, investors are more willing to make a choice, rather than election of fair game, because the exposure may be \( -E_R(U_i) \), there is loss of revenue \( R \) (because if they don’t choose, \( R \) is the opportunity costs for investors) may outweigh the benefits obtained possibly.

When \( \eta_i < 0 \), investors are risk averse, and the value of \( \eta_i \) the smaller indicates the degree of risk adverse avoidance the higher. Because the choice may not only bring greater revenue than raised losses, but also greater than the sum of \( r \) and \( -E_R(U_i) \), they believes the risk of fair game is too high, so they should be risk averse.

When \( \eta_i = 0 \), investors consider at this time to make a choice is equal to fair game, then the investors are risk neutral, they does not behave like a risk averse too conservative, and does not behave as risk lovers too optimistic, would be more objective treatment of the problem, when \( \eta_i > 0 \), he will not select, on the contrary at the time \( \eta_i < 0 \), they will select, because at this time they believe in their favor.

Through the above theoretical analysis, we will find that: when \( \eta_i = 0 \) and when \( \eta_i < 0 \), there is the intersection between the investors’ choice. Because under certain conditions, the risk netballers’ choice is similar with the risk averse’, that is, it is difficult to identify the similar choice whether from the risk averse investors or from the risk neutral investors. Visibly, this approach is not suitable for determining the real investors’ specific type of risk appetite.

To this end, we have to be adjusted on the basis of this model, let \( \eta_i \) will be used to be replaced by \( \eta_i^{\text{max}} \), which is the measurement maximum of investors withstand, the model is:

\[
\eta_i^{\text{max}} = \max \{ r - E_R(U_i) - E_R(U_i) \}
\]

If and only if

\[
E_R(U_i) = E_i^{\text{max}}(U_i), E_R(U_i) = E_i^{\text{max}}(U_i)
\]

According to the assumptions (1) it is available that \( E_i^{\text{max}}(U_i) \) and \( E_i^{\text{max}}(U_i) \). Therefore, the model (16) can be used to identify the investors’ risk preferences in different markets.

4 Empirical analysis based on the risk allocation of China multi-layer capital market

On the basis of theoretical analysis framework, this article choose the main market, small board, the GEM market as the research object, using the method of empirical analysis to analyze the investors’ risk appetite and allocative efficiency in each market.

4.1 INDICATORS AND DATA SELECTION

According to the foregoing analysis, the model contains three corresponding variables: price, quantity, and has a wealth of purchase. To ensure the comparability, the paper selects the composite index. In the main market, we select the closing price on the Shanghai Composite Index and Shenzhen Composite Index as the price, and their volume (unit: million) as the corresponding quantity demanded; in the small board, select the small plate KLCI closing price, volume (Unit: million) as the corresponding variables; in the GEM, select the GEM KLCI closing price, volume (unit: million) as its variables; meanwhile, we sum the turnover (unit: million) corresponding to the Shanghai Composite Index and Shenzhen Composite Index, the small plate KLCI and GEM KCLI, then obtain the total wealth in demand market which investors can distribute. Since the GEM KLCI launches from August 20, 2010, the data in this paper is August 20, 2010 to November 21, 2013. Theoretically, the Shanghai Composite Index and Shenzhen Composite Index are main market index, should be unified analysis, but found that in the Shanghai Stock Exchange and Shenzhen Stock Exchange market there are significant differences in the degree of risk appetite, therefore, the paper retains the Shanghai and Shenzhen two main markets. The above data is from CSMAR database.
4.2 STATIONARY TEST OF DATA

Stationary test takes ADF test which is the most commonly to be used. To avoid the heteroscedasticity of estimation, this article deals with the data to be logarithmic treatment, and do the stationary test to these variables and the variables after the first differences, the test results are shown in table 1:

<table>
<thead>
<tr>
<th>Variables</th>
<th>ADF test statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln(Shanghai Composite Index)</td>
<td>-1.523085</td>
<td>0.8198</td>
</tr>
<tr>
<td>ln(volume of Shanghai Composite Index)</td>
<td>-5.92698</td>
<td>0.0000</td>
</tr>
<tr>
<td>ln(Shenzhen Composite Index)</td>
<td>-1.474319</td>
<td>0.8364</td>
</tr>
<tr>
<td>ln(volume of Shenzhen Composite Index)</td>
<td>-4.567777</td>
<td>0.0002</td>
</tr>
<tr>
<td>ln(Small board index)</td>
<td>-1.34279</td>
<td>0.8751</td>
</tr>
<tr>
<td>ln(volume of Small board index)</td>
<td>-4.878331</td>
<td>0.0004</td>
</tr>
<tr>
<td>ln(GEM Index)</td>
<td>-1.852633</td>
<td>0.6764</td>
</tr>
<tr>
<td>ln(volume of GEM Index)</td>
<td>-4.563277</td>
<td>0.0014</td>
</tr>
<tr>
<td>ln(Total Turnover)</td>
<td>-4.160081</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

Variables after the first differences

<table>
<thead>
<tr>
<th>ADF test statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>d ln(Shanghai Composite Index)</td>
<td>-16.90434</td>
</tr>
<tr>
<td>d ln(volume of Shanghai Composite Index)</td>
<td>-18.11173</td>
</tr>
<tr>
<td>d ln(Shenzhen Composite Index)</td>
<td>-16.06674</td>
</tr>
<tr>
<td>d ln(volume of Shenzhen Composite Index)</td>
<td>-15.20731</td>
</tr>
<tr>
<td>d ln(Small board index)</td>
<td>-15.58342</td>
</tr>
<tr>
<td>d ln(volume of Small board index)</td>
<td>-15.2738</td>
</tr>
<tr>
<td>d ln(GEM Index)</td>
<td>-15.94434</td>
</tr>
<tr>
<td>d ln(volume of GEM Index)</td>
<td>-16.01485</td>
</tr>
<tr>
<td>d ln(Total Turnover)</td>
<td>-15.44591</td>
</tr>
</tbody>
</table>

The results shows that: the variables after logarithmic treatment, at 10% significance level, the Shanghai Composite Index, Shenzhen Composite Index, the small board index and GEM index are non-stationary sequence, the rest is smooth sequence; after first differential treatment with the logarithmic variables, all variables are stationary sequence.

4.3 SELECTION AND ESTIMATION OF RANDOM EXPECT DEMAND MODEL

To ensure that the estimated demand model satisfies the integrability, based on the results of stationary test, we select the first difference variables as the variables, so the model is double logarithmic first-order differential form. Also, because of the differential model, the model does not contain a constant term. In this paper, the level of significance is $\alpha = 0.05$. The basic model is:

$$d \ln x_i = \sum_{j=1}^{3} \alpha_j d \ln p_j + \beta d \ln w + \varepsilon_i$$  \hspace{1cm} (17)

Among them, $x_i$ is the investors’ demand for $i$ th market, $p_i$ is the price for $i$ th market, $w$ is the total wealth for investors in the capital market, $\alpha, \beta$ are the parameters to be estimated.

Meanwhile, from $d \ln x_i = \sum_{j=1}^{3} \alpha_j d \ln p_j + \beta d \ln w$, we can obtain $\ln x_i = \sum_{j=1}^{3} \alpha_j \ln p_j + \beta \ln w + c$, wherein $c$ an arbitrary constant is. To facilitate the analysis, let $c = 0$, then $\ln x_i = \sum_{j=1}^{3} \alpha_j \ln p_j + \beta \ln w$, the deformation can be obtained:

$$x_i = \prod_{j=1}^{3} p_j^{\alpha_j} w^{\beta}$$ \hspace{1cm} (18)

That is demand model for $i$ th market.

1) Demand model of main Market

a. Demand model of Shanghai main market

First differential model of Shanghai main market: $d \ln x_{Shanghai} = 1.1289d \ln w$  \hspace{1cm} (22.0965)

$$R^2_{Shanghai} = 0.6296$$

Demand model: $x_{Shanghai} = w^{1.1290}$

From the above first difference model, we can see that $R^2_{Shanghai} = 0.6296$ indicates that the extent of the first differential model of Shanghai Stock Exchange Main Board fit well, the equation is significant, and only the total wealth variable coefficients is significant, that is to
say changes in investors’ demand for the Shanghai Stock Exchange main board market is only the case with investors owned total wealth, and shows the same relationship to changes.

\[ d \ln x_{Shenchen} = -2.4655 d \ln P_{Shanghai} + 6.0191 d \ln P_{Shenchen} - 3.4643 d \ln P_{Smallboard} - 0.8143 d \ln P_{GEM} + 0.9695 d \ln w \]

\( \bar{R}^2 = 0.9564 \)

Demand model:

\[ x_{Shenchen} = P_{Shanghai} P_{Shenchen} P_{Smallboard} P_{GEM} w \]

Seen, \( \bar{R}^2 = 0.9564 \) indicates the model fits well, and the coefficients of prices and total wealth variables of Shanghai Stock Exchange main market, Shenzhen main market small board and the GEM market are significant, namely changes in investors’ demand for the Shenzhen main market are related to the changes of price of Shanghai main market, Shenzhen main market, small board market, GEM market and investors’ total wealth. And changes in investor demand of Shenzhen main market with its own price changes and changes in total wealth is in the same direction of changes, in relationships with the price changes of other alternative market such as Shanghai Stock Exchange main market, small board and the GEM market inversely to changes in market relations.

2) Demand model of small board market
First difference model of Small board:

\[ d \ln x_{Smallboard} = 3.1144 d \ln P_{Smallboard} + 0.8891 d \ln w \]

\( \bar{R}^2 = 0.9077 \)

Demand model:

\[ x_{Smallboard} = P_{Smallboard} w^{0.8891} \]

Seen, \( \bar{R}^2 = 0.9077 \) indicates the model fits well, while the coefficients of variables of small board market price and total wealth are significant, that is, changes in investors’ demand for small board market are related to its own price and investors’ total wealth. And changes in investors’ demand for small board market with its own price changes and changes in total wealth was the relationship in the same direction.

3) Demand model of GEM market
First differential model of GEM:

\[ d \ln x_{GEM} = -7.7118 d \ln P_{Shenchen} + 9.6736 d \ln P_{GEM} + 0.784 d \ln w \]

\( \bar{R}^2 = 0.5275 \)

Demand model:

\[ x_{GEM} = P_{Shenchen} P_{GEM} w^{0.784} \]

Seen, \( \bar{R}^2 = 0.5275 \) indicates the model fits better, the coefficients of variables of prices of Shenzhen main market and GEM market and total wealth are significant, changes in investors’ demand for the GEM market are related to its own market price changes, price movements of Shenzhen main market and investors’ total wealth. And changes in investors’ demand for the GEM market with its own price changes and changes in total wealth had a positive relationship, with the price changes of its alternative market Shenzhen main market inversely to changes in the relations.

Through the above analysis, it shows that changes in investors’ demand for \( i \) th market are related to its own price in the same movement direction (except Shanghai Stock Exchange Main Market), which reflects in the actual market with the increasing price the amount of investors to buy the stock increases, with the decline in stock prices the amount reduces, namely "chase sell"; reverse changes in the relationship with the price of other alternative market (except Shanghai Stock Exchange Main Market), that is, after the prices of alternative markets increasing investors will reduce the demand for changes the original market, in the situation of wealth unchanged, will increase the wealth of assigned to the alternative markets, which is an increase of the purchase of the alternative markets, and conversely it is also be established, which once again confirms the "chase sell" phenomenon; with the same movement relationship to the investors’ own overall wealth, that the total wealth increasing investors will increase their demand for the market, the total wealth reducing the investors’ demand for the market will be reduced.

4.4 UTILITY FUNCTION AND DISTRIBUTION SET

1) Utility function of each market
According to the definition of the utility function of the foregoing, the utility function can be obtained in each market:

**a. Utility function of Main Market**

Utility function of Shanghai Main Market:

\[ U_{Shanghai} = \int_{0}^{P_{Shanghai}} W^{1.1290} dP_{Shanghai} \]
Among them, \( p_{Shanghai}^0 \) is the base price of the Shanghai main market, \( p_{Shanghai}^1 \) is the reporting period price of Shanghai main market.

When we get the utility function of Shanghai main market, but also shows that this layer of meaning: when investors in Shanghai main market randomly choose whether to enter the market, or increase or decrease the market demand, actually investors follow their own implied the selecting behavior when the utility function maximized in Shanghai main market, that is, investors’ demand for Shanghai main market is guided to conduct under the above utility function essentially.

Utility function of Shenzhen main market:

\[
u_{Shenzhen} = \int_{0}^{1} \left( p_{Shenzhen}^1 - p_{Shenzhen}^0 \right) p_{Shenzhen}^0 p_{Smallboard}^1 \cdot p_{GEM}^{0.8143} \cdot w^{0.9695} dp_{Shenzhen}
\]

Among them, \( p_{Shenzhen}^0 \) is the base price of Shenzhen main market, \( p_{Shenzhen}^1 \) is the reporting period prices of Shenzhen main market.

Similarly we can see that the utility function of Shenzhen main market is the basis to guide investors’ demand choice behavior in Shenzhen main market, and its conducting code is to maximize the utility function of Shenzhen main market.

Obviously, although the current main market is constitute by Shanghai main market and Shenzhen main market together, there is a big difference for the investors following the conducting code in the two main market, that is, investors follow the two different conducting codes in Shanghai and Shenzhen main markets.

**b. Utility function of small board**

Utility function of Small board:

\[
u_{Smallboard} = \int_{0}^{1} \left( p_{Smallboard}^1 - p_{Smallboard}^0 \right) p_{Smallboard}^0 \cdot p_{GEM}^{0.8891} dp_{Smallboard}
\]

Among them, \( p_{Smallboard}^0 \) is the base market price of small board, \( p_{Smallboard}^1 \) is the reporting period price of small board market.

After the above analysis, investors’ demand for small board market is completed under the guidance of the utility function of small board.

**c. Utility function of GEM**

Utility function of GEM:

\[
u_{GEM} = \int_{0}^{1} \left( p_{GEM}^1 - p_{GEM}^0 \right) p_{GEM}^0 \cdot w^{0.7874} dp_{GEM}
\]

Among them, \( p_{GEM}^0 \) is the base price of GEM market, \( p_{GEM}^1 \) is the reporting period price of GEM market.

Similarly, investors get the guidelines of demand of GEM, that is, actually investors’ behavior is the performance when the utility function maximize in the GEM.

2) Distribution set of utility function of each market

Based on the above utility function to estimate the utility level in different markets, due to the distribution of the utility level sequence for each market is unknown, the paper carries out the empirical distribution test. The paper uses the Watson test, Cramer-von Mises test, Kolmogorov testing and other testing methods to test the four markets utility level sequences goodness of fit with the theoretical distribution (normal, chi-square distribution, exponential distribution, extreme value distribution, logistic distribution, the Pareto distribution, uniform distribution, etc.) in Shanghai main market, Shenzhen main market, small board and GEM, the results are shown in table 2:

<table>
<thead>
<tr>
<th>sequence</th>
<th>distribution</th>
<th>Test method</th>
<th>Asymptotic distribution statistics</th>
<th>Limited sample adjustment statistics</th>
<th>( p )-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_{Shanghai} )</td>
<td>exponential</td>
<td>Watson</td>
<td>0.1139</td>
<td>0.1141</td>
<td>0.226</td>
</tr>
<tr>
<td>( u_{Shenzhen} )</td>
<td>normal</td>
<td>Watson</td>
<td>0.1179</td>
<td>0.1181</td>
<td>0.239</td>
</tr>
<tr>
<td>( u_{Smallboard} )</td>
<td>exponential</td>
<td>Cramer-von Mises</td>
<td>0.1051</td>
<td>0.1051</td>
<td>0.2985</td>
</tr>
<tr>
<td>( u_{GEM} )</td>
<td>uniform</td>
<td>Kolmogorov</td>
<td>0.0033</td>
<td>0.0581</td>
<td>0.9933</td>
</tr>
</tbody>
</table>
Table 2 shows, the utility level sequences for Shanghai main market obeys to the exponential distribution by Watson test, because of \( p - value = 0.226 > \alpha = 0.05 \); similarly we can see that the utility level sequences for Shenzhen main market obeys to the normal distribution by Watson test, the utility level sequences for small board obeys to the exponential distribution by Cramer-von Mises test, and the utility level sequence GEM obeys to the exponential distribution by Kolmogorov test.

Using the maximum likelihood method to estimate the parameters of each distribution density function, the results are shown in table 3:

<table>
<thead>
<tr>
<th>sequence</th>
<th>Density functional form</th>
<th>Associated parameter estimates and test values</th>
</tr>
</thead>
</table>
| \( U_{Shanghai} \) | \( f(u) = \frac{1}{\mu} \exp(-\frac{u-a}{\mu}) \) | parameter 1 \( \alpha = 21041391 \)  
|                 |                          | parameter 2 \( \mu = 6.10E + 09 \)  
|                 |                          | \( p - value \) 0.0296  
| \( U_{Shenzhen} \) | \( f(u) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{1}{2\sigma^2}(u-\mu)^2) \) | parameter 1 \( \mu = 0.0052 \)  
|                 |                          | parameter 2 \( \sigma = 0.3331 \)  
|                 |                          | \( p - value \) 0.0485  
| \( U_{Smallboard} \) | \( f(u) = \frac{1}{\mu} \exp(-\frac{u-a}{\mu}) \) | parameter 1 \( \alpha = 3.06E + 14 \)  
|                 |                          | parameter 2 \( \mu = 2.96E + 17 \)  
|                 |                          | \( p - value \) 0.0425  
| \( U_{GEM} \) | \( f(u) = \frac{1}{b-a} \) | parameter 1 \( \alpha = 1.52E + 08 \)  
|                 |                          | parameter 2 \( b = 8.72E + 10 \)  
|                 |                          | \( p - value \) 0.0399  

In table 3, the second column shows the form of density distribution function of different markets utility level sequence obedience, third and fourth columns are given the estimate of parameters of density function and test conditions. And by determining \( p - value \) and the significant level, it shows the estimate of parameters in density functions are significant ( \( p - value < \alpha \) ).

4.5 MEASUREMENT OF RISK APPETITE

Based on the each density function we can calculate the risk and gains constructed before, select the minimal risk \( E_{\min}(U_i) \) and minimal gains \( E_{\max}(U_i) \) and bring them into risk appetite measurement model, because the risk \( E_{\bar{R}}(U_i) \) is negative, and measure the degree of risk appetite in different markets, thus identify them. In risk appetite measurement models, this article defines the one-year bank deposit rates 3.5% as the risk free interest rate, and the measurement unit of "Fortune" indicators is million, so to keep the data caliber consistency here, we adjust the risk free rate to 350 (per million). According to the calculation we can separately get \( E_{\min}(U_i) \), \( E_{\bar{R}}(U_i) \) and \( \eta_{\max} \), and the results are listed in table 4:

<table>
<thead>
<tr>
<th>Markets</th>
<th>risk free interest rate ( r )</th>
<th>( E_{\min}(U_i) )</th>
<th>( E_{\max}(U_i) )</th>
<th>( \eta_{\max} )</th>
<th>risk appetite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shanghai main market</td>
<td>350</td>
<td>1079370.6029</td>
<td>-238860113539527</td>
<td>238860112460506</td>
<td>risk lovers</td>
</tr>
<tr>
<td>Shenzhen main market</td>
<td>350</td>
<td>0.0001</td>
<td>-67184.2497</td>
<td>67534.2496</td>
<td>risk lovers</td>
</tr>
<tr>
<td>Small market</td>
<td>350</td>
<td>25458830575936</td>
<td>-26770641238137800</td>
<td>267680953550803000</td>
<td>risk lovers</td>
</tr>
<tr>
<td>GEM</td>
<td>350</td>
<td>20301.4482</td>
<td>-14424635394</td>
<td>14424615442.5215</td>
<td>risk lovers</td>
</tr>
</tbody>
</table>
According to table 4, in China current multi-layer capital market, no matter in which capital market, investors are expressed as risk lovers, only the degree of risk appetite is different. By comparison, the degree of risk pursuit of Shenzhen main market is the weakest, followed by the GEM market, the Shanghai main market, the highest level of risk pursuit is small board market, that is to say, \( \eta_{\text{Smallboard}} > \eta_{\text{Shanghai}} > \eta_{\text{GEM}} > \eta_{\text{Shenzhen}} \). Although the Shanghai main market and Shenzhen main market together constitute the current overall main market, at the point of view of the degree of investors’ risk loving, these two markets and can not be explained as a whole, there is a clear difference between them. The original intention of establishing multi-layer capital market is to distinguish the different levels of the capital market among them, meanwhile the performance of the internal layer of risk should be roughly the same, the performance should be the difference among the risk layer, and the investors’ attitude to risk shows a progressive trend, that is, the weakest degree of market risk hobby should be the main market, the degree of small board risk hobby is in the middle, the degree of GEM risk hobby is the strongest. Through the comparison of current realities and theoretical purposes, it is not difficult to find that there is a clear departure between them, not only on the level of the layers the risk away from the hobby, but also on the level of internal layer it clearly does not conform the intention of the theory. The risk appetite of main market is not the weakest, but the situation of Shanghai Stock Exchange main market also exceeds GEM which should have been the strongest risk-loving, and small board shows the strongest degree of risk-loving. This shows that the current multi-layer capital market is not reasonable division for investors with different risk preferences, which shows the current multi-layer capital market in China is non-Pareto efficient on risk allocation, Pareto improvement be needed.

5 Policy and recommendations

We can say that it is a state of disorder China current multi-layer capital market in the risk allocation, the root cause resulting in this non-valid and disorder may be the trading mechanism of the current market. Because in the multi-layer capital market, there is difference on the basis in each market, and there is a hierarchy among market distinction, these indicate that each market has its own characteristics, and in China the basically same transaction mode is still in the use of multi-layer capital market, that is, in the use of uniform, symmetric price limits, which is obviously inconsistent with the characteristics of multi-layer capital market. To change this inconsistent situation, we must distinguish the various markets, with different trading patterns, according to the characteristics of each market to develop and implement the different price limits, such as appropriately rising the price limits of small board and GEM in accordance with their level of risk should bear, or introducing the asymmetric price limits for each market, in order to improve the Pareto efficiency, so that China multi-layer capital market can be improved.

References

[1] Fangsheng Zhou 2003 Constructed pyramid capital market system China Youth College for Political Sciences 22(1)83-6
[4] Liping Xu, Huang Xiaoq 2009 Listed companies to introduce strategic investor behavior Statistics and Decision Research 288 (12) 144-6
[6] Zongqian Liu, Fu Weniang, Feng Sufen 2008 Another proof of expected utility function theorem Capital Normal University (Natural Science) 2(2) 6-17
[8] Lijun Song, Yang Yongyu 2008 Investment portfolio based utility function Beijing University of Chemical Technology 9 (2)119-21
<table>
<thead>
<tr>
<th>Author</th>
<th>Details</th>
</tr>
</thead>
</table>
| **Xiaoyuan Geng**, born on October 22, 1981, Heilongjiang province of China | Current position, grades: lecturer in Heilongjiang Bayi Agricultural University; PH.D. candidate in statistics in Tianjin University of Finance and Economics  
University studies: Bachelor degree in statistics from Tianjin University of Finance and Economics in 2004; masters degree in statistics from Yunnan University of Finance and Economics in 2007; now PH.D. in statistics candidate in Tianjin University of Finance and Economics  
Scientific interest: Mathematical Statistics; Technical analysis of financial risk  
Publications: She has published more than 10 papers. |
| **Yongde Wang**, born in 1964, Heilongjiang province of China | Current position, grades: Professor; PH.D.  
University studies: Bachelor degree in accounting from Heilongjiang Bayi Agricultural University in 1986; PH.D. in management in Northeast Forestry University  
Scientific interest: Agricultural trade; Accounting Theory and Methods  
Publications: He has published more than 30 papers |