A method of relative Grey relation degree combined with combination weights for materials selection

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Abstract

The quality and cost of a product rely heavily on suitable material selection, and therefore the ability to select the most appropriate material for a given application is the fundamental challenges faced by the design engineer. The general grey relational analysis (GRA) has three weaknesses, (i) the weight determination depends only on expert judgments, (ii) the qualitative indexes are simply quantified with exact numbers, and (iii) the general GRA only takes into account the relationship between the imaginarily best material and the candidate materials. Weights were determined by combining subjective and objective weights based on maximum deviation, the qualitative indexes were fuzzily quantified through trapezoidal fuzzy numbers (TFNs), and then ranked alternatives according to relative grey relation grade. The illustrative example showed that the results matched well with that using WAA and TOPSIS, proved the proposed method reasonable and trustworthy. And therefore the proposed method possesses important application values.

Keywords: material selection, combination weights (CW), relative Grey relation analysis (RGRA), fuzzy numbers, maximum deviation

1 Introduction

It is well known that materials play an important role in engineering designs. After the conceptual design stage, designers always need to select materials with specific properties which can guarantee optimum system performance by satisfying all existing constraints [1]. And it is an important step in engineering designs, since an inappropriate choice of material(s) can adversely affect the productivity, profitability, and reputation of a manufacturing organization as well [2]. When selecting materials for engineering designs, a clear understanding of the functional requirements for each individual component is required and various important criteria or attributes need to be simultaneously considered. These attributes include not only the traditional ones such as usability, machinability, and cost, but also material impact on environment, recycling, and even cultural aspects. They contradict and even conflict with each other, and furthermore Deng and Edwards [3] emphasized that the process of materials selection should be combined with structural optimazation. And therefore the ability to select the most appropriate material for a given application is the fundamental challenges faced by the design engineers. There have been much literature dealing with the material selection, and so great progress has been made in this field. Zhou et al. [4] proposed an integration of artificial neural networks (ANN) with genetic algorithms (GA) to optimize the multi-objectives of material selection, and applied it to selecting proper materials for drink containers. [5] proposed fuzzy inference method and applied it to material selection for a liquid nitrogen storage tank and spar of an aircraft wing. [6] also proposed fuzzy inference method for material substitution selection in electric industry, while combined with fuzzy weight average to extend fuzzy inference to uncertain environment. The advantage of fuzzy inference is that it does not require normalizations to ratings, but identifying membership function for each attribute is strongly subjective, depending entirely on expert's experiences. [7] presented an intelligent method to deal with the materials selection problems where the design configurations, working conditions, as well as the design-relevant information are not precisely known, and applied it to selecting optimal materials for robotic components at early stage of design.. [8] presented digital tools for material selection in product design, where about three hundred software, database and website references were collected, and 87 were selected to try to answer a few important questions to help designers, engineer students, and all kinds of professionals perform materials selection for product design.

The grey system theory proposed by Deng in 1982 [9] has been proven to be useful for dealing with problems with poor, insufficient, and uncertain information. According to it, systems can be divided into three classes: white systems, which have completely clear information, black systems, which have completely unknown information, and grey systems, which lie between white systems and black systems [10]. The grey relational analysis based on this theory can further be effectively adopted for solving the complicated interrelationships among the designated performance characteristics. Through this analysis, a grey

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relational grade (GRD) is favourably defined as an indicator of multiple performance characteristics for evaluation. In recent years, grey relational analysis has become a powerful tool to multi-attribute decision making (MADM). In [11] used GRA to compute the weights of each decision maker: a larger weight is assignned to the expert whose preferences are more similar to the others' preferences, while a less weight is assigned to the expert whose preferences are less similar to the others' preferences. [12] adopted grey relation entropy analysis to evaluate information technology impact on business performance of biotechnology industry. There is a great deal of literature on the applications of GRA to material selection. In [13] presented an integrated methodology of performing an order pair of materials and end-of-life strategy for the purpose of material selection, where the GRA is employed to calculate the grey relational grade between the imaginarily best material and candidate materials. The material with the greatest grey relational grade is the best choice. [14] also presented GRA to select handbag materials for a leading handbag manufacturer in Guangdong Province, while combined with binary dominance matrix to specify the weights. However, the general GRA has three weaknesses, (i) the weight determination depends only on expert judgments, (ii) the qualitative indexes are simply quantified with exact number, and (iii) the general GRA only takes the relationship between the imaginarily best material and the candidate materials into account, but takes into no account the relationship between the imaginarily worst material and the candidate materials. Therefore the general GRA in a sense has limitations. The methodology of relative GRA (RGRA) is proposed, not only taking into the account the relationship between the imaginarily best material and the candidate materials but also into the account the relationship between the imaginarily worst material and the candidate materials. It is predicted that the results of RGRA are more reliable than that of general GRA. Weights are determined by combining subjective and objective weights based on maximum deviation, and the qualitative indexes are fuzzily quantified through trapezoidal fuzzy number (TFN).

The paper is structured as follows. Section 2 enunciates the mechanism of maximum deviation and gives the corresponding formulations to calculate combination weights. Section 3 explains the concept of trapezoidal fuzzy numbers and their operation rules. The calculation procedure for relative grey relation grade is enunciated in Section 4. Section 5, taking the bearing material selection for example, details the decision making process, and each ways to determine weights. Section 6 closes the paper with a short discussion of the issues raised and pointing the way to future research direction.

2 Combination weights based on maximum deviation

The way of identifying weights largely includes subjective weighting and objective weighting. The former identifies weights depending only on the subjective preferences or experiences of an expert, such as Delphi method, analytic hierarchy process (AHP) etc., while the latter does depending only on the information of a matrix of decision making and the mathematical model based on it, like entropy weight, principal component analysis, multi-objective optimization, etc [15]. Subjective weights fully reflect decision makers' empirical judgments, not violating common sense in identifying the relative importance of attributes, but with much more arbitrariness and poorer accuracy and reliability. However, objective weights enjoy objective criteria, but neglect subjective preferences or experiences of an expert, sometimes resulting in irrational practice. To make the selection more scientific, the subjective weights and objective ones are combined to obtain the better ones, combination weights (CW), which reflect not only subjective information but also objective one. Supposing a MADM problem, there are m alternatives, expressed as $\mathbf{s}=\{s_1,s_2,\cdots,s_m\}$, and n attributes, expressed as $\mathbf{p}=\{p_1,p_2,\cdots,p_n\}$. Letting a_{ij} be the ratings of s_i with respect to p_j, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$ and $A=(a_{ij})_{m\times n}$ be decision making matrix, ratings may be in different units (e.g. material cost expressed in dollars, yield strength expressed in MPa), resulting in incommensurability, which hence entails normalizations. Suppose the normalized decision making matrix is $\mathbf{b}=(b_{ij})_{m\times n}$ with any element within [0, 1], and the larger the b_{ij}, the better the performance.

If the ratings of the j-th attribute are nearly same to all alternatives, then it contribute little or nothing to the alternative rank, and zero should be assigned to the weight of the attribute. Otherwise, if the ratings of the j-th attribute vary greatly to all alternatives, then it contributes greater to the alternative rank, and greater weight should be assigned to it. Deviation is the index in statistics reflecting the differences to all alternatives under an attribute, and as such, the weighting vector $\boldsymbol{\omega}_c$ should be selected, which enables the total deviation sum of all n attributes to reach the maximum, called maximum deviation principle.

Suppose there are l methods of weighting and the weight vector according to k-th method is:

$$\boldsymbol{\omega}_k=(\omega_{1k},\omega_{2k},\cdots,\omega_{nk})^T\ k=1,2,\cdots,l, \tag{1}$$

where $\omega_{jk}\geq 0$ and $\sum_{j=1}^{n}\omega_{jk}=1$. Combination weight vector can be taken as $\boldsymbol{\omega}_c=(\omega_{c1},\omega_{c2},\cdots,\omega_{cn})$, where $\boldsymbol{\omega}_c=\theta_1\boldsymbol{\omega}_1+\theta_2\boldsymbol{\omega}_2+\cdots+\theta_l\boldsymbol{\omega}_l$ and $\theta_1,\theta_2,\cdots,\theta_l$ represent linear coefficient respectively of $\boldsymbol{\omega}_1,\boldsymbol{\omega}_2,\cdots,\boldsymbol{\omega}_l$ satisfying $\theta_k\geq 0, k=1,2,\cdots,l$, and $\sum_{k=1}^{l}\theta_k^2=1$. According to [16], the value of θ_k based on maximum deviation principle can be taken as:

$$\theta_k = \mathbf{C\omega_k} / \sqrt{\sum_{k=1}^{l} (\mathbf{C\omega_k})^2}\,, \tag{2}$$

where $\mathbf{C}$ is a row vector with n dimensions and can be calculated as:

$$\mathbf{C} = (c_1, c_2, \cdots, c_j, \cdots, c_n) = \left(\sum_{i=1}^{m}\sum_{i_1=1}^{m} \left| b_{i1} - b_{i_1 1} \right|, \sum_{i=1}^{m}\sum_{i_1=1}^{m} \left| b_{i2} - b_{i_1 2} \right|, \cdots, \sum_{i=1}^{m}\sum_{i_1=1}^{m} \left| b_{in} - b_{i_1 n} \right| \right). \tag{3}$$

3 Fuzzy quantification for linguistic information

3.1 TRAPEZOIDAL FUZZY NUMBERS

Suppose $\tilde{A}$ is a bounded and convex fuzzy subset in the domain of R $(R \in [0,1])$, and possesses the continuous membership function $u_{\tilde{A}}(x)$ as:

$$u_{\tilde{A}}(x) = \begin{cases} u^L_{\tilde{A}}(x) & a < x < m \\ 1 & m \le x \le n \\ u^R_{\tilde{A}}(x) & n < x < \beta \\ 0 & others \end{cases}, \tag{4}$$

where $u^L_{\tilde{A}}(x)$ is strictly increasing function in the domain of $[a,m]$, called the left membership function, while $u^R_{\tilde{A}}(x)$, strictly decreasing function in the domain of $[n,\beta]$, called the right membership function. Trapezoidal fuzzy numbers (TFNs) are the widely used fuzzy numbers, whose membership function is shown in Figure 1, and can be expressed as (a,m,n,β), also as $(m,n;\gamma,\delta)$ where $\gamma = m - a, \delta = \beta - n$. If $m = n$, then trapezoidal fuzzy numbers reduce to triangle fuzzy numbers, and it can be said that triangle fuzzy numbers are the special case of trapezoidal fuzzy numbers.

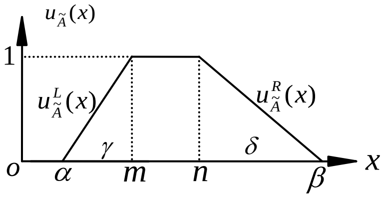

FIGURE 1 Membership function of TFN

For qualitative attributes, they are usually expressed in linguistic information. Table 1 shows their correspondding TFNs, while Figure 2 shows their membership function.

TABLE 1 Representations of linguistic information in trapezoidal fuzzy numbers

Linguistic information	In the form of (α, m, n, β)	In the form of $(m, n; \gamma, \delta)$
Very poor (VP)	(0.0, 0.0, 0.0, 0.2)	(0.0, 0.0; 0.0, 0.2)
Poor (P)	(0.0, 0.0, 0.1, 0.3)	(0.0, 0.1; 0.0, 0.2)
Medium poor (MP)	(0.0, 0.2, 0.2, 0.4)	(0.2, 0.2; 0.2, 0.2)
Fair (F)	(0.3, 0.5, 0.5, 0.7)	(0.5, 0.5; 0.2, 0.2)
Medium good (MG)	(0.6, 0.8, 0.8, 1.0)	(0.8, 0.8; 0.2,0.2)
Good (G)	(0.7, 0.9, 1.0, 1.0)	(0.9, 1.0; 0.2, 0.0)
Very good (VG)	(0.8,1.0,1.0,1.0)	(1.0, 1.0; 0.2, 0.0)

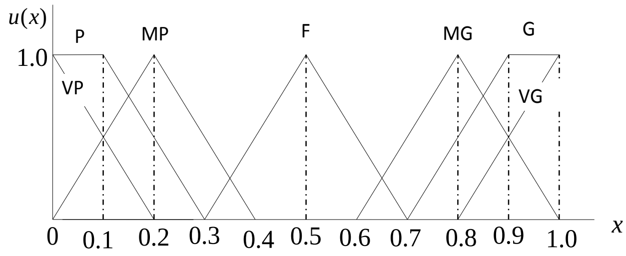

FIGURE 2 Representations of linguistic information in TFNs

3.2 OPERATIONS FOR TFNS

Suppose the two trapezoidal fuzzy numbers $M_1 = (m_1, n_1; \gamma_1, \delta_1)$ and $M_2 = (m_2, n_2; \gamma_2, \delta_2)$, then according to [17] the operators of multiplication and division can be respectively defined as:

$$M_1 \cdot M_2 \approx (m_1 m_2,\, n_1 n_2;\, m_1 \cdot \gamma_2 + m_2 \cdot \gamma_1 - \gamma_1 \cdot \gamma_2,\; n_1 \cdot \delta_2 + n_2 \cdot \delta_1 - \delta_1 \cdot \delta_2), \tag{5}$$

$$M_1 / M_2 \approx [m_1 / n_2,\, n_1 / m_2;\, (m_1 \cdot \delta_2 + n_2 \cdot \gamma_1) / n_2 (n_2 + \delta_2),\; (n_1 \cdot \gamma_2 + m_2 \cdot \delta_1) / m_2 (m_2 + \gamma_2)]. \tag{6}$$

By means of the approximate calculation, the operation of trapezoidal fuzzy number can be dramatically simplified, and the calculation precision in most cases meets the project requirements, but it cannot be repeatedly used, otherwise, the cumulative error would lead to erroneous results.

3.3 TOTAL EXPECTATIONS OF TFNS

Supposing $\tilde{A}$ is a fuzzy number, and $g^L_{\tilde{A}}(y)$, $g^R_{\tilde{A}}(y)$ respectively the inverse function of $u^L_{\tilde{A}}(x)$, $u^R_{\tilde{A}}(x)$. Letting $I_L(\tilde{A}) = \int_0^1 g^L_{\tilde{A}}(y)dy$ and $I_R(\tilde{A}) = \int_0^1 g^R_{\tilde{A}}(y)dy$, then $I(\tilde{A}) = [I_L(\tilde{A}) + I_R(\tilde{A})]/2$ is called as the total expectation of fuzzy number $\tilde{A}$. As for the TFN $\tilde{A} = (m,n;\gamma,\delta)$, the total expectation can be calculated as:

$$I(\tilde{A}) = (2m + 2n + \delta - \gamma)/4\,. \tag{7}$$

4 Relative Grey relation analysis (RGRA)

4.1 NORMALIZATION

The purpose of normalization is to obtain dimensionless values of the different criteria so that all of them can be compared with each other, which, in grey system theory, is also called grey relation generation.

1) For benefit type attributes

$$b_{ij} = \frac{a_{ij}}{\max_i a_{ij}} . \tag{8}$$

2) For cost type attributes

$$b_{ij} = \frac{\min_i a_{ij}}{a_{ij}} . \tag{9}$$

3) For fixation type attributes

$$b_{ij} = \begin{cases} \dfrac{\min_i \left|a_{ij} - g^j\right|}{\left|a_{ij} - g^j\right|} & a_{ij} \neq g^j \\ 1 & a_{ij} = g^j \end{cases} , \tag{10}$$

where, g^j is the optimum value to fixation attribute *j*.

4.2 GREY RELATION COEFFICIENT

Suppose comparison sequences are $\mathbf{b_i}(i=1,2,\cdots,m)$, $\mathbf{b_i} = (b_{i1}, b_{i2}, \cdots, b_{ij}, \cdots, b_{in})$, namely, the ratings of the each alternative material. Suppose the positive reference sequence is $\mathbf{b_o^+}$, $\mathbf{b_o^+} = (b_{o1}^+, b_{o2}^+, \cdots, b_{oj}^+, \cdots b_{on}^+)$, $b_{oj}^+ =$ $\max\{b_{1j}, b_{2j}, \cdots, b_{mj}\}$, namely, the ratings of the hypothetically optimal material, and the negative reference sequence is $\mathbf{b_o^-}$, $\mathbf{b_o^-} = (b_{o1}^-, b_{o2}^-, \cdots, b_{oj}^-, \cdots b_{on}^-)$, $b_{oj}^- = \min\{b_{1j}, b_{2j}, \cdots, b_{mj}\}$, namely, the ratings of the hypothetically worst material. Then grey relational coefficient between the hypothetically optimal material $\mathbf{b_o^+}$ and the alternative material $\mathbf{b_i}(i=1,2,\cdots,m)$ in the *j*-th attribute is defined as:

$$r_{io}^+(j) = \frac{\min_i \min_j \left|b_{oj}^+ - b_{ij}\right| + \rho \max_i \max_j \left|b_{oj}^+ - b_{ij}\right|}{\left|b_{oj}^+ - b_{ij}\right| + \rho \max_i \max_j \left|b_{oj}^+ - b_{ij}\right|} . \tag{11}$$

The grey relational coefficient between the hypothetically worst material $\mathbf{b_o^-}$ and the alternatives $\mathbf{b_i}(i=1,2,\cdots,m)$ in the *j*-th attribute is defined as follows:

$$r_{io}^-(j) = \frac{\min_i \min_j \left|b_{oj}^- - b_{ij}\right| + \rho \max_i \max_j \left|b_{oj}^- - b_{ij}\right|}{\left|b_{oj}^- - b_{ij}\right| + \rho \max_i \max_j \left|b_{oj}^- - b_{ij}\right|} , \tag{12}$$

where ρ is the distinguishing coefficient with $\rho \in [0,1]$, usually set as 0.5 in this study. Suppose ω_{cj} is the CW of the *j*-th attribute, and then grey relational grade (GRD), the weighted sum of the grey relational coefficient, can be defined as:

$$\gamma_{io}^+ = \sum_{j=1}^{n} \omega_{cj} \gamma_{io}^+(j) , \tag{13}$$

$$\gamma_{io}^- = \sum_{j=1}^{n} \omega_{cj} \gamma_{io}^-(j) . \tag{14}$$

4.3 RELATIVE GREY RELATION GRADE

The relative grey relation grade can be calculated as:

$$\gamma_{io} = \frac{\gamma_{io}^+}{\gamma_{io}^+ + \gamma_{io}^-} \quad (i=1,2,\cdots,m) \tag{15}$$

The larger the value γ_{io}, the better the performance of the alternative. It should be noted that the greater γ_{io}^+ does not always lead to the less γ_{io}^-, and therefore selecting materials only depending on γ_{io}^+ can be biased. It can be inferred from Equation (14) that the greater γ_{io}^+ and the less γ_{io}^- must result in the greater γ_{io}.

5 Case Study

Suppose a bearing works under the conditions of higher speed with stable, light load, and the task is to select the best material for the bearing. According to the rigid attributes, which, if a material to be accepted, must be fully satisfied, the materials not satisfying any of the requirements of rigid attributes are firstly eliminated. The materials initially screened out and their ratings are shown in Table 2. The procedure for the material selection is as follows.

5.1 NORMALIZATIONS

For the qualitative attributes, such as fatigue durability, corrosion durability, abrasion durability, and anti-seizing, they were expressed in TFNs. Benefit type, like fatigue durability, corrosion durability, abrasion durability, and anti-seizing, was normalized by Equations (6) and (8) and then converted into total expectations by Equation (7), fixation type, like hardness with optimum value of hardness specified as 45HBS normalized using Equation (10), and cost type by Equation (9). The normalized ratings are shown in Table 3.

TABLE 2 Raw ratings of the candidate materials

No.	Materials	Cost	Hardness (HBS)	Fatigue durability	Corrosion durability	Abrasion durability	Anti-seizing
1	*Zchsn3*	*15*	*24*	MP	VG	MP	VG
2	*Zchpb1*	*100*	*30*	MP	G	MG	VG
3	*ZQpb30*	*70*	*25*	VG	MG	MG	MP
4	*ZznM7-5*	*80*	*100*	G	VG	VG	MP

TABLE 3 Normalized ratings

No.	Materials	Cost	Hardness (HBS)	Fatigue durability	Corrosion durability	Abrasion durability	Anti-seizing
1	*Zchsn3*	1.0000	*0.7143*	0.1933	1.0000	0.1933	1.0000
2	*Zchpb1*	*0.1500*	1.0000	0.1933	0.9496	0.8319	1.0000
3	*ZQpb30*	*0.2143*	*0.7500*	1.0000	0.8319	0.8319	0.1933
4	*ZznM7-5*	*0.1875*	*0.2727*	0.9496	1.0000	1.0000	0.1933

5.2 COMBINATION WEIGHTS

5.2.1 Objective weights

1) Entropy method: according to Shannon's entropy method [18], if the ratings of each alternative under an attribute have more obvious differences, such an attribute plays a more important role in choosing the best alternative, and a greater weight should be assigned to it. The entropy value for the attribute j can be defined as:

$$H_j = -k\sum_{i=1}^{m} f_{ij} \ln f_{ij} \,, \tag{16}$$

where $k = 1/\ln m$ (m denoting the number of alternatives), $f_{ij} = b_{ij} / \sum_{i=1}^{m} b_{ij}$. The weight of attribute j can be defined as:

$$\omega_j = \frac{1-H_j}{\sum_{j=1}^{n}(1-H_j)} . \tag{17}$$

In the special case where under an attribute, say attribute j, the ratings of all alternatives are the same, it can be calculated $H_j = 1$ and $\omega_j = 0$. According to Table 3, the entropy weight vector $\boldsymbol{\omega}_1$ can be calculated as

$\boldsymbol{\omega}_1 = (0.3332,0.0795,0.2334,0.0027,0.1127,0.2386)^T$.

2) Principal component analysis (PCA). Its main advantage is significantly alleviating loading and complexion of information by simplifying several correlated variables into fewer uncorrelated and independent principal components, at the same time preserving as much original information as possible using linear combination. In recent time, PCA has gradually become an analytical tool for the optimization of a system with multiple performance characteristics. Firstly calculate the correlation coefficient matrix to the normalized decision making matrix $\mathbf{b} = (b_{ij})_{m\times n}$ using the function $\mathbf{S} = corrcoef(\mathbf{b})$ in Matlab, then calculate eigenvalues and corresponding eigenvectors, and finally select the eigenvector corresponding to the maximum eigenvalue, after normalization, as the weight vector $\boldsymbol{\omega}_2$, and the resultant weight vector is

$\boldsymbol{\omega}_2 = (0.1747,0.1100,0.2080,0.1067,0.1923,0.2084)^T$

5.2.2 Subjective weights

1) Analytic Hierarchy Process (AHP) [19-20], developed first by Satty as a popular tool for MADM, has been increasingly widely used in more and more domains. Its main steps include: (i) Against total goal, the six criteria are compared in pair, consequently constituting a judgment matrix; (ii) Calculating eigenvalues and corresponding eigenvectors to the judgment matrix; and finally (iii) selecting the eigenvector corresponding to the maximum eigenvalue, after normalization, as the weight vector $\boldsymbol{\omega}_3$. The resultant weight vector is

$\boldsymbol{\omega}_3 = (0.2856,0.2380,0.1904,0.0476,0.0952,0.1428)^T$.

2) Delphi method: suppose the weight vector is

$\boldsymbol{\omega}_4 = (0.1700, 0.1600, 0.1900, 0.1000, 0.1800, 0.2000)^T$.

5.2.3 Combination weights

The row vector $\mathbf{C}$ according to Equation (3) can be calculated as $\mathbf{C} = (5.1400, 4.4352, 6.300, 1.1002, 4.8000$ $6.4000)$, then the linear coefficient according to Equation (2) as $\theta_1 = 0.5371, \theta_2 = 0.4857, \theta_3 = 0.4929,$ $\theta_4 = 0.4823$, and finally the combination weight as

$\boldsymbol{\omega}_c = (0.2436, 0.1455, 0.2062, 0.0623,0.1440, 0.1984)^T$.

[21] provides a method to determine θ_l according to the consistency degree between any two ranking vectors generated by the two weight ones. Supposing the two ranking vectors as $p^{(t)} = (p_1^{(t)}, p_2^{(t)}, \cdots, p_1^{(t)})$, $p^{(k)} = (p_1^{(k)}, p_2^{(k)}, \cdots, p_1^{(k)})$ according respectively to the t-th and k-th weighting methods, the consistency degree in the light of Speaman rank correlation coefficient can be defined as:

$$\rho_{kt} = 1 - \frac{6}{n(n^2-1)}\sum_{j=1}^{n}(p_j^{(k)} - p_j^{(t)})^2 \,, \tag{18}$$

where n is the number of attributes.

And then the consistency degree of the k-th weighting method can be averaged as:

$$\rho_k = \frac{1}{l-1}\sum_{t=1,t\neq k}^{l} \rho_{kt}\,, \tag{19}$$

θ_k can be calculated as:

$$\theta_k = \rho_k / \sum_{k=1}^{l} \rho_k\,. \tag{20}$$

With the method in [21], the combination weight

$\hat{\boldsymbol{\omega}}_c = (0.2364, 0.1307, 0.2086, 0.0650, 0.1520, 0.2072)^T$

can be obtained. The Speaman rank correlation coefficient between $\hat{\boldsymbol{\omega}}_c$ and $\boldsymbol{\omega}_c$ is 0.9429, implying strong correlation and consistency degree between the two methods.

According to Table 3, the positive reference sequence is $\mathbf{b}_{\mathbf{o}}^{+}$, $\mathbf{b}_{\mathbf{o}}^{+} = (1.0000, 1.0000, 1.0000, 1.0000, 1.0000, 1.0000)$, and the negative reference sequence, $\mathbf{b}_{\mathbf{o}}^{-}$, $\mathbf{b}_{\mathbf{o}}^{-} = (0.1500, 0.2727, 0.1933, 0.8319, 0.1933, 0.1933)$.

According to Equations (11)-(15), the $\gamma_{oi}^{+}, \gamma_{oi}^{-}, and\ \gamma_{oi}$ are respectively calculated shown in Table 4. It can be concluded from Table 4 that in the light of $\gamma_{oi}^{+}, or\ \gamma_{oi}$ the material 1 is the best choice, followed by material 2, material 3, and material 4. The methods, weighted arithmetic averaging (WAA) and technique for order preference by similarity to ideal solution (TOPSIS), are widely used two ones in multi-attributes decision making, the results of which are also tabulated in Table 4. It can be seen that whether using the WAA or the TOPSIS the ranking results is the same, which proves selecting the *Zchsn3* for the bearing is trustworthy and reliable.

5.3 RELATIVE GREY RELATION GRADE

TABLE 4 Ranking results

No.	Materials	γ_{io}^{+}	Rank	γ_{io}^{-}	Rank	γ_{io}	Rank	WAA	Rank	TOPSIS	Rank
1	Zchsn3	0.7121	1	0.6159	1	0.5363	1	0.6759	1	0.5179	1
2	Zchpb1	0.6511	2	0.6783	3	0.4913	2	0.5993	2	0.5164	2
3	ZQpb30	0.5996	3	0.6695	2	0.4725	3	0.5775	3	0.4910	3
4	ZznM7-5	0.5964	4	0.7363	4	0.4475	4	0.5258	4	0.4519	4

6 Conclusions

From the performed research work using relative grey relation degree combined with combination weights, the main conclusions and directions for future research can be summarized as follows:

1) The proposed method expresses qualitative attributes with trapezoidal fuzzy number, more in conformity with actual situations than the traditional method with exact numbers.
2) Weights play a very significant role in the ranking results of the materials, Combination weights based on maximum deviation both considering subjective and objective weights are more appropriate for determining criteria weights than the subjective or objecttive weights alone. In comparison with the results by the method in literature [21], the speaman rank correlation coefficient is 0.9429, further implying the proposed method of computing combination weights is feasible.
3) The result using the proposed method in this paper matches well with the other methods such as WAA, TOPSIS, further indicating its feasibility and validity.

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