H_∞ fault-tolerant control for nonlinear singular system via a fault diagnosis observer

Dongwen Zhang[1]*, Xinguo An[2]

[1]*School of information science & engineering, Hebei University of Science &Technology, Shijiazhuang, China*

[2]*School of Sciences, Hebei University of Science &Technology, Shijiazhuang, China*

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Abstract

An H_∞ fault-tolerant control scheme based on fault diagnosis observer was developed for a class of nonlinear singular systems with external disturbances and actuator faults. A fault diagnosis observer was designed to estimate the system states and the actuator faults and a sufficient condition for the existence of this observer was presented in the form of feasibility problem of a linear matrix inequality. Based on linear matrix inequality (LMI) technique and the estimates of the states and faults, an H_∞ fault-tolerant control scheme was worked out. The H_∞ fault-tolerant control system via a state feedback controller can be made solvable, impulse free, asymptotically stable, and the effect of external disturbances on the system was attenuated in terms of the prescribed H_∞ performance index. Finally, a simulation example was given to illustrate the procedure of designing the fault diagnosis observer and the state feedback controller, and the simulation result showed the effectiveness of the proposed method.

Keywords: singular systems, H_∞ control, fault-tolerant control, observer, linear matrix inequality (LMI)

1 Introduction

The safety, reliability and maintainability in actual systems and industrial process have motivated researchers to concentrate on the so-called fault-tolerant control (FTC) [1-5]. FTC is primarily meant to ensure safety, i.e., the stability of a system after the occurrence of a fault in the system. There are two approaches to synthesize controllers that are tolerant to system faults. One approach, known as passive FTC, aims at designing a controller which is a prior robust to some given expected faults. Another approach, known as active FTC, relies on the availability of a fault detection and diagnosis (FDD) block that gives, in real-time, information about the nature and intensity of the fault. This information is then used by a control reconfiguration block to adjust online the control effort in such a way to maintain stability and to optimize the performance of the faulty systems. Researches on FDD for systems have long been recognized as one of the important aspects in seeking effective solutions to an improved reliability of practical control systems. Accurate fault estimation can determine the size, location and dynamic behaviour of the fault, which automatically indicates FDD, and has thus attracted interests recently. Many methods have been proposed for FDD, e.g., parity relations approach [6], Kalman filters approach [7], parameter estimation approach [8] as well as observer-based approach [9-11]. Observer-based FDD is one of the most effective methods and has obtained much more attention. So far, various observer-based FDD approaches have been reported in the literatures. Based on Euler approximate discrete model observer, a fault estimation method was proposed in [9]. A novel augmented fault diagnosis observer design was well addressed in [10], which not only broaden application scopes of adaptive fault diagnosis observer, but also cope with system disturbances. In [11], the observers were designed for both linear and nonlinear systems considering both noise and uncertainties, and the main advantage of these observers is that they can handle both noise and uncertainties simultaneously. Overall, the basic idea behind the use of the observer for FDD is to estimate the state or/and output of the system from the measurement by using some type of observers, and then to construct a residual by a properly weighted the state or/and output error. The residual is then examined for the likelihood of faults by using a fixed or adaptive threshold.

However, only a few efforts were made to investigate FTC for nonlinear singular systems. Nonlinear singular system model characterizes a class of rather complex systems, which not only possesses nonlinearities, but also has singular nature of the algebraic constraints. Therefore, the investigation on this class of systems is more difficult and challenging [12-15]. Several works on FDD and FTC for nonlinear singular systems were reported in [16, 17]. For nonlinear singular systems, [18] designed an observer based on the new parameterization of the generalized Sylvester equations solutions, and the condition for the existence of the observer was given and the sufficient condition for its stability was derived using linear matrix inequality (LMI) formulation. By using the linear matrix inequality (LMI) technique, an interesting descriptor estimator was presented to simultaneously estimate system

* ***Corresponding author*** e-mail: zdwwtx@hebust.edu.cn

states, output noises and sensor faults for a class of Lipschitz nonlinear descriptor systems [19]. However, some restrictive equivalent transformations were needed for obtaining the state-space observer in the [19]. Obviously, the usages of restrictive equivalent transformations are not desirable from the viewpoint of computation. Moreover, it is not possible to totally decouple the fault effects from the perturbation effects on the system, and the H_∞ control theory has been proved to be an effective tool to tackle the issue. Consequently, this motivates us to investigate the topic of FTC and H_∞ control, which is very important in many practical systems.

Our objective in this paper is to propose an observer-based FTC and H_∞ control method for a class of nonlinear singular systems with input disturbances and actuator faults, a novel design method of fault diagnosis observer is presented, which can overcome the nonlinearity and precisely estimate the values of the states and actuator faults. By linear matrix inequality (LMI) technique and by using the obtained states and faults information, robust fault-tolerant state feedback control scheme is worked out. The solvability, asymptotic stability and H_∞ performance of the closed-loop system are guaranteed after the actuators occur faults.

Throughout this paper, $\mathbb{R}, \mathbb{C}$ denote real number set and complex number set respectively, $\mathbb{R}^n$ denotes the n dimensional Euclidean space and $\mathbb{R}^{n\times n}$ is the set of all $n\times n$ real matrices. I is the identity matrix with appropriate dimensions. $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ refer to the minimal and maximal eigenvalues of the matrix P respectively. The vector norm $\|x\|$ is defined as $\|x\| = \sqrt{x^T x}$. For a symmetric matrix, * denotes the matrix entries implied by symmetry.

2 Problem statement and preliminaries

Consider the following nonlinear singular system with actuator faults as well as external disturbances:

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t) + g(t,x) + Fu_f(t) + D_1 w(t) \\ y(t) = Cx(t) + D_2 w(t), \qquad x(0) = 0 \end{cases}, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ denote respectively the control input and the measurable output vectors, $u_f \in \mathbb{R}^h$ is the unknown actuator fault vector and $E \in \mathbb{R}^n$ is singular with $rank(E) = r < n$. A, B, C, D_1, D_2 and F are known constant real matrices with appropriate dimensions. $g(t,x) \in \mathbb{R}^n$ is a vector-valued time varying nonlinear perturbation with $g(t,0) = 0$ for all $t \ge 0$ and satisfies the following Lipschitz constraint:

$$\|g(t,x) - g(t,\tilde{x})\| \le \alpha_0 \| G(x-\tilde{x}) \| \le \alpha \| x - \tilde{x} \| \tag{2}$$

for all (t,x), $(t,\tilde{x}) \in \mathbb{R}\times\mathbb{R}^n$, and G is a known constant real matrix, α and α_0 are both known positive scalars and are called Lipschitz constants. $w(t) \in \mathbb{L}_2$ is the external disturbance on the system, and there exists a position constant f such that $|u_f(t)\| \le f$. In the paper, only actuator faults are investigated and it is assumed that, when no fault occurs, $u_f(t) = 0, \forall t \ge 0$.

In this paper, our first goal is to design a fault diagnosis observer to estimate the systems states $x(t)$ and the fault signal $u_f(t)$ simultaneously on the basis of the known input $u(t)$ and the measured output $y(t)$. The second goal is to work out a state feedback controller for FTC and H_∞ control by mean of the estimates of the system states and faults.

Now, one first recalls a lemma that will be used in the next sections.

Lemma 1 (Schur Complement Lemma) Given constant matrices Σ_1, Σ_2 and Σ_3 with appropriate dimensions, where $\Sigma_1^T = \Sigma_1 < 0$, $\Sigma_2^T = \Sigma_2 < 0$, then $\Sigma_1 - \Sigma^T{}_3 \Sigma^{-1}{}_2 \Sigma_3 < 0$ if and only if:

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & \Sigma_2 \end{bmatrix} < 0, \text{ or } \begin{bmatrix} \Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

3 Fault detection and fault diagnosis

3.1 FAULT DETECTION OBSERVER DESIGN

In this section, a state-space observer will be proposed for FDD, which can provide the information of states and faults. The information is sent to the controller to obtain the control law, which is sent to the actuator.

For the system to admit a feasible FTC solution, the following assumptions are made:

Assumption 1: The row vectors of the matrices E and C in the Equation (1) must be a basis of the n dimensional vector space, that is, $rank\begin{bmatrix} E^T & C^T \end{bmatrix}^T = n$.

Assumption 2: The linear part of the Equation (1) has to be observable, that is

$$rank\begin{bmatrix} (sE-A)^T & C^T \end{bmatrix}^T = n, \quad \forall s \in \mathbb{C}.$$

Under assumptions 1-2, a fault detection observer can be constructed as follows:

$$\begin{cases} \dot{z}(t) = \hat{A}z(t) + \hat{B}u(t) - L_P\left(\hat{y}(t) - Cx(t)\right) + \\ \hat{g}(t,\hat{x}) + \hat{F}\hat{u}_f(t) + \hat{D}_1 w(t) \\ \hat{x}(t) = z(t) + L_D y(t) \\ \hat{y}(t) = C\hat{x}(t) + \hat{D}_2 w(t) \end{cases}, \tag{3}$$

where $\hat{z}\in\mathbb{R}^n$ is the state vector of the detection observer, $\hat{x}\in\mathbb{R}^n$ denotes the observed state vector, $\hat{y}\in\mathbb{R}^p$ is the output vector of the observer, $\hat{u}_f\in\mathbb{R}^h$ is the estimate of the system faults, $\hat{A},\hat{B},\hat{D}_1,\hat{D}_2$ and $\hat{F}$ are known parameter matrices with appropriate dimensions.

It's easy to derive that the assumption 1 and the assumption 2 imply that the trio (E,A,C) is completely observable, so the matrix L_D can be selected to make matrix $E+L_DC$ be non-singular. To guarantee the asymptotical stability of the detection Equatuon (3), L_P to be designed and the Lipschtiz constant α should make the following inequality hold [19]:

$$\alpha\le\frac{\lambda_{\min}\left(\sqrt{1-\beta^2}C^TC+\eta L_D{}^T(-L_P)^{-1}\right)-d}{2\lambda_{\max}(C^TL_P\hat{F})},\tag{4}$$

where d is a pre-specified positive constant. β and η should be chosen properly such that inequality (4) can be satisfied.

Supposing the state error vector $e_m(t)=x(t)-\hat{x}(t)$, the residual signal vector $e_y(t)=y(t)-\hat{y}(t)$, the fault error vector $e_f(t)=u_f(t)-\hat{u}_f(t)$, respectively, then the detection error equation can be written as follows:

$$\begin{aligned}&\dot{e}_m(t)=\hat{A}e_m(t)+(A-\hat{A}E-C)x(t)+\\&(B-\hat{B})u(t)+g(t,x)-\hat{g}(t,x)-L_Pe_y(x)+(\hat{F}-F)e_f(x)\end{aligned}.\tag{5}$$

In next subsection, one will give the fault diagnosis rule for the Equation (1).

3.2 FAULT DIAGNOS

In this subsection, we will discuss the design method of the fault diagnosis observer. According to the detection error Equation (5), the following fault detection rule is introduced

1) If $\|e_y(t)\|=\|Ce_m(t)\|<\lambda$, then no fault occurs at time *t*,

2) If $\|e_y(t)\|=\|Ce_m(t)\|\ge\lambda$, then faults have occurred at time *t*,

where λ is a pre-specified threshold.

According to the above rule, the fault diagnosis observer is presented as follows:

$$\begin{cases}\dot{\tilde{e}}(t)=\hat{A}\tilde{e}(t)+\hat{B}u(t)+\hat{D}_1w(t)+\\L_P(\tilde{y}(t)-y)+g(t,\tilde{x})+\hat{F}\tilde{u}_f(t)\\\tilde{x}(t)=\tilde{e}(t)+L_Dy(t)\\\tilde{y}(t)=C\tilde{x}(t)+\hat{D}_2w(t)\end{cases},\tag{6}$$

where $\tilde{e}\in\mathbb{R}^n$ is the state vector of the diagnosis observer, $\tilde{x}\in\mathbb{R}^n$ denotes the observed state vector, $\tilde{y}\in\mathbb{R}^p$ is the output vector of the observer. $\tilde{u}_f\in\mathbb{R}^h$ is the estimates of the system faults.

Now, one will provide a sufficient condition for the existence of the fault diagnosis observer.

Theorem 1. For the Equation (1), there exists an asymptotical steady state-space observer in the form of Equation (6) to make the estimated error as small as any desired accuracy, if there exist a positive definite matrix $P\in\mathbb{R}^{n\times n}>0$ and a matrix $Q\in\mathbb{R}^{p\times n}$ such that

$$\begin{bmatrix}\Psi_{11}&PW\\W^TP&-I\end{bmatrix}<0,\tag{7}$$

where $\Psi_{11}=\hat{A}^TW^TP+PW\hat{A}-C^TQ^T-QC+D^TP+PD+\alpha^2I$, and $W=(E+L_DC)^{-1}$. Specifically, the gain L_D is selected such that $E+L_DC$ is non-singular, and L_P can be computed as $L_P=P^{-1}W^{-1}Q$. Obviously, Equation (7) is a LMI with respect to matrices P,Q.

Proof: For the Equation (1), the detection error Equation (5) and the fault diagnosis observer Equation (6), the error dynamic equation can be characterized as follows:

$$\begin{aligned}&\dot{e}(t)=(E+L_DC)^{-1}(\hat{A}-L_PC)e(t)+\\&Dw(t)+g(t,\tilde{x})-g(t,x)\end{aligned},\tag{8}$$

where $e(t)=\tilde{x}(t)-x(t)$.

Define a Lyapunov function as $V(e(t))=e^T(t)Pe(t)$ with $P>0$.

Letting $W=(E+L_DC)^{-1},\Theta=g(t,\tilde{x})-g(t,x)$ the derivative of $V(e(t))$ along Equation (6) can be obtained as follows:

$$\begin{aligned}&\dot{V}(e(t))=\\&e^T((W(\hat{A}-L_PC)+(D_1-L_PD_2))^TP+\\&P(W(\hat{A}-L_PC)+(D_1-L_PD_2)))e+\Theta^TW^TPe+ePW\Theta\le\\&e^T(\hat{A}^TW^TP+PW\hat{A}-(WL_PC)^TP-P(WL_PC)+\\&D_1^TP+PD_1-D_2^TL_P^TP-PD_2L_P)e+\Theta^T\Theta+e^TPWW^TPe\le\\&e^T(\hat{A}^TW^TP+PW\hat{A}-(WL_PC)^TP-P(WL_PC)+D_1^TP+\\&PD_1-D_2^TL_P^TP-PD_2L_P+\alpha^2I+PWW^TP)e\end{aligned}$$

Letting $Q=PWL_P$, thus Equation (7) can be recast to the following inequality by the Schur complement lemma

$$\begin{aligned}&\hat{A}^TW^TP+PW\hat{A}-C^TQ^T-QC+D^TP+\\&PD-R^TP-PR+\alpha^2I+PWW^TP<0\end{aligned}$$

If Equation (7) holds, one can derive that $\dot{V}(e(t))<0$. Furthermore, $e(t)$ converges towards 0 while t converges towards ∞. The proof is completed.

4 Fault-tolerant control and H∞ control

Consider the Equation (1) and the fault diagnosis observer Equation (6). One can construct the state -feedback fault tolerant controller as follows:

$$u(t) = K\tilde{x}(t), \quad (9)$$

Where $\tilde{x}(t)\in\mathbb{R}^n$ denotes the observed state vector in (6). In this section, one will discuss how to design the state-feedback gain K.

Appling (9) to the Equation (1), the closed-loop system can be written as follows:

$$\begin{cases} E\dot{\tilde{x}}(t) = A\tilde{x}(t) + BK\tilde{x}(t) + D_1 w(t) + \\ g(t,\tilde{x}) + BKe_f(t) \\ \tilde{y}(t) = C\tilde{x}(t) + D_2 w(t) \end{cases}. \quad (10)$$

Now, one will present the following result.

Theorem 2 For the closed-loop Equation (10) and the given scalar $\gamma<1$, the closed-loop Equation (10) is solvable, impulse free, asymptotical stable and $\|y(t)\|_\infty \le \gamma^2 \|w(t)\|_\infty$ if there exist a non-singular matrix $H\in\mathbb{R}^n$ and a controller gain $K\in\mathbb{R}^{m\times n}$ such that the following inequalities holds

$$E^T H = H^T E \ge 0, \quad (11)$$

$$\begin{aligned}&(A+BK)^T H + H^T(A+BK) + I + C^T C + \\ &D_2^T D_2 + \gamma^{-2} H^T D_1 D_1^T H + \alpha^2 H^T H < 0\end{aligned}. \quad (12)$$

Proof: Choosing a Lyapunov function:

$$\begin{aligned}&V(\tilde{x}(t)) = \left(E\tilde{x}^T(t)\right)^T H\left(E\tilde{x}(t)\right) = \\ &\left(E\tilde{x}(t)\right)H^T\left(E\tilde{x}^T(t)\right)\end{aligned},$$

where $E^T H = H^T E \ge 0$ and H is nonsingular.

The derivative of $V(\tilde{x}(t))$ along Equation (10) can be obtained as follows:

$$\begin{aligned}&\dot{V}(\tilde{x}(t)) = \tilde{x}(t)^T((A+BK)^T H + H^T(A+BK) + I + \\ &C^T C + H^T\Theta^T\Theta H)\tilde{x}(t) + 2\tilde{x}(t)^T H^T BKe_f(t) + \\ &2\tilde{x}(t)H^T D_1 w(t) + 2\tilde{x}(t)D_2 D_2^T w(t)\end{aligned}.$$

Letting

$$\Omega = (A+BK)^T H + H^T(A+BK) + I + C^T C + \alpha^2 H^T H,$$

so

$$\begin{aligned}&\dot{V}(\tilde{x}(t)) \le \tilde{x}^T(t)\Omega\tilde{x}(t) + 2\tilde{x}^T(t)H^T BKe_f(t) + \\ &2\tilde{x}^T(t)(H^T D_1 + D_2 D_2^T)w(t)\end{aligned}. \quad (13)$$

From Equation (2), it is easily derived that

$$\dot{V}(\tilde{x}(t)) \le -\varepsilon_1\|\tilde{x}(t)\|^2 + \varepsilon_2\|e_f(t)\|\|\tilde{x}(t)\| - \varepsilon_3\|e_f(t)\|^2,$$

where $\varepsilon_1 = \lambda_{\min}(-\Omega)$, $\varepsilon_2 = 2\|H\|\|BK\|$, $\varepsilon_3 = 2\|H\|\|D_1\| + \|D_2\|^2$.

Letting $\varepsilon = \min\left(\frac{\varepsilon_1}{2}, \frac{\varepsilon_2}{2\varepsilon_1}\right)$, one has $\dot{V}(\tilde{x}(t)) \le -\varepsilon\left\|\begin{matrix}\tilde{x}\\ e_f\end{matrix}\right\| < 0$.

By the Schur complement lemma, it is clear that (12) implies

$$\Omega = (A+BK)^T H + H^T(A+BK) + I + C^T C + \alpha^2 H^T H < 0$$

and further indicates

$$(A+BK)^T H + H^T(A+BK) < 0. \quad (14)$$

Moreover, Equations (11) and (14) and $\dot{V}(\tilde{x}(t)) < 0$ indicate that the Equation (10) is solvable, impulse free, asymptotically stable.

Next, one will discuss the H_∞ performance of the Equation (10).

Defining $Z(t) = \dot{V}(\tilde{x}(t)) + y^T y - \gamma^2 w^T w$, using the Equation (10) and inequality (13), one has

$$Z(t) \le \breve{x}\breve{\Omega}\breve{x} + \varepsilon_1\|\tilde{x}(t)\|\|e_f(t)\| - \varepsilon_3\|e_f(t)\|^2,$$

where $\breve{\Omega} = \begin{bmatrix}\Omega + C^T C & H^T D_1 + D_2^T \\ D_1 H^T + D_2 & -\gamma^2 I\end{bmatrix}$, $\breve{x} = \begin{bmatrix}\tilde{x}^T & w^T\end{bmatrix}^T$.

Applying the Schur complement lemma to inequality (12), it is clear that $\breve{\Omega} < 0$.

Letting $\varepsilon_4 = \lambda_{\min}(-\breve{\Omega})$, it is true that

$Z(t) \le -\varepsilon_4\|\breve{x}\|^2 + \varepsilon_1\|\breve{x}(t)\|\|e_f(t)\| - \varepsilon_3\|e_f(t)\|^2$ and $\sqrt{\varepsilon_4\varepsilon_3}\|\breve{x}(t)\|\|e_f(t)\| \le \frac{\varepsilon_4}{2}\|\breve{x}(t)\|^2 + \frac{\varepsilon_3}{2}\|e_f(t)\|^2$, so one could select $\bar{\varepsilon}$ such that $\bar{\varepsilon} > \varepsilon_1^2/\hat{\varepsilon}\varepsilon_4$, where $\hat{\varepsilon} = \min(\alpha, \varepsilon_3)$. Furthermore, $Z(t) \le -\frac{\varepsilon_4}{2}\|\breve{x}(t)\|^2 - \frac{\alpha\bar{\varepsilon}}{2}\|e_f(t)\|^2$.

Under zero initial conditions $x(0) = 0$ and the above discussion, it is known that:

$$\int_0^\infty (y^T(t)y(t) - \gamma^2 w^T(t)w(t))d\tau \le \int_0^\infty H d\tau \le 0,$$

is true, that is $\|y(t)\|_\infty \le \gamma^2\|w(t)\|_\infty$ the proof is completed.

Remark: Equation (12) with respect to matrices H, K is a nonlinear matrix inequality. One thus has a continuous interest to transform Equation (12) into the LMI form.

Theorem 3. For the closed-loop Equation (10) and the given scalar $\gamma<1$, the closed-loop Equation (10) is solvable, impulse free, asymptotical stable and $\| y(t)\|_\infty \le \gamma^2 \| w(t)\|_\infty$ if there exist a non-singular matrix $W \in \mathbb{R}^n$ and a matrix $T \in \mathbb{R}^{m\times n}$ such that

$$E^T W = W^T E \ge 0, \tag{15}$$

$$\begin{bmatrix} \Xi_{11} & W^T & W^T C^T & W^T D_1 & D_2 D_2 \\ * & -I & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{16}$$

where $\Xi_{11} = (AW)^T + AW + BT + (BT)^T + \alpha^2 I$.

Moreover, if there exists a feasible solution (T,W) for the above Equations (15) and (16), the state feedback controller gain matrix K can be signed as $K = TW^{-1}$. Obviously, Equations (15) and (16) are LMIs with respect to matrices T,W.

Proof: Pre-multiplying and post-multiplying inequality (12) by $diag\{H^{-T}, I, I, I\}$ respectively, and letting $H^{-1} = W$, $KH^{-1} = T$, then using the Schur complement lemma, Equations (15) and (16) can be obtained immediately. This completes the proof.

Now, one will describe the procedure of the fault diagnosis and H∞ fault-tolerant control for a class of the Equation (1).

Input: the Equation (1) and the H∞ performance index $\gamma<1$.

Output: the state feedback controller gain K.

Step 1 Choosing scalars β,η, and d such that Equation (4) is satisfied.

Step 2 Choosing suitable matrix L_D such that $E + L_D C$ is non-singular, then solving the Equation (7) by Matlab LMI control toolbox. If there is a feasible solution (P,Q) to the Equation (7), then one can compute $L_P = P^{-1}W^{-1}Q = P^{-1}(E + L_D C)^{-1} Q$.

If there is no feasible solution to the Equation (7), thus the step 2 will be repeated and another matrix L_D is choose until there is a feasible solution to the Equation (7).

Step 3 Solving the Equations (15) and (16) by Matlab LMI control toolbox, if there is a feasible solution (T,W) of the Equations (15) and (16) then one can compute the state feedback controller gain $K = TW^{-1}$.

5 Numerical example

Consider a nonlinear singular system in the form of Equation (1), where

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 0.9 & 3.1 & 1.5 \\ 0 & 0.5 & 0.6 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0.28 & 1.23 & 0.02 \\ 0 & 0.01 & 0.56 \\ 0.22 & 1.21 & 0.36 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.05 & 0 & 1.29 \\ 0.01 & 0.87 & 0 \\ 0.03 & 0.12 & 0.5 \end{bmatrix}, D_1 = \begin{bmatrix} 4 & 0.6 & 0.4 \\ 1.3 & 1.5 & 0.3 \\ 0.5 & 0.6 & 0.7 \end{bmatrix}, F = \begin{bmatrix} 0.4 & 1.6 \\ 4.2 & 0.2 \\ 1.6 & 9.2 \end{bmatrix}$$

The nonlinear function of Equation (1) is assumed to be

$$g(t,x) = \begin{bmatrix} 0.3\sin(x_1 + x_2) \\ 0.1 \\ 0.5\cos x_3 \end{bmatrix}, G = \begin{bmatrix} 0.2 & 0.1 & 0.4 \\ 0.6 & 0 & 0.7 \end{bmatrix}.$$

The Lipschitz constants are $\alpha_0 = 0.7$ and $\alpha = 0.8$. It is clear that the assumption 1 and the assumption 2 are easily satisfied. The parameter matrices of fault diagnosis observer Equation (6) are selected as follows:

$$\hat{A} = \begin{bmatrix} 0.2 & 2.1 & 0.5 \\ 0 & 0.1 & 0.4 \\ 0 & 0 & 1 \end{bmatrix}, \hat{B} = \begin{bmatrix} 0.4 & 0.8 & 0.1 \\ 0.1 & 0 & 0.7 \\ 0.1 & 0.5 & 0.9 \end{bmatrix}, \hat{D}_1 = \begin{bmatrix} 0.7 & 0.2 & 0.7 \\ 0.2 & 0.8 & 0.6 \\ 1.1 & 0.4 & 0.7 \end{bmatrix},$$

$$\hat{D}_2 = \begin{bmatrix} 0.01 & 0 & 0.03 \\ 0.04 & 1 & 0.01 \\ 0.07 & 0.06 & 0.41 \end{bmatrix}, \hat{F} = \begin{bmatrix} 0.5 & 1.1 \\ 0.2 & 1.2 \\ 0.7 & 3.1 \end{bmatrix}$$

The system faults have the following forms:

$$u_{f1} = \begin{cases} 0, & t \in [0,\ 2.5) \\ 0.002t^2 - 0.3t + 2, & t \in [2.5,\ 6) \\ 2\sin(0.3t) + 0.3, & t \in [6,\ 10) \\ 0.1t + 0.12, & t \in [10,\ \infty) \end{cases}$$

$$u_{f2} = \begin{cases} 0, & t \in [0,\ 3) \\ \cos t + 0.02, & t \in [3,\ 7) \\ \sin 2t - 0.3, & t \in [7,\ 11) \\ 0.2t + 1.3, & t \in [11,\ 14) \\ 4.1 & t \in [14,\ \infty) \end{cases}$$

Using Matlab LMI Control Toolbox to solve Equations (7), (11) and (12), the following results could be obtained:

a) fault diagnosis observer design.

Letting $L_D = \begin{bmatrix} 0 & -3 & 1 \\ 0 & -2 & 0 \end{bmatrix}^T$, $d = 0.35$, $\beta = 0.5$ and $\eta = 0.2$, one can acquire

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0.0196 & -1.3514 & -2.2837 \\ -0.0218 & 0 & 0.4376 \end{bmatrix}$$

$$L_P = \begin{bmatrix} 84.08 & -62.45 & -448.98 \\ 71.85 & -104.07 & -669.16 \\ -49.51 & 40.82 & 236.10 \end{bmatrix}.$$

The trajectories of the faults and their estimates are given in Figure 1 and Figure 2. One can see that the tracking performance is desired. In Figure 1, no fault occurs when $0 < t < 2s$, there is no failure false alarm for the designed fault detect observer, The fault f_1 occurs when $2s \le t < 3.2s$. Similar result can be seen from the Figure 2. The results can be summarized as follows: the method presented in this paper may cope with well the constant value faults, but there is some lag for the fluctuant value faults.

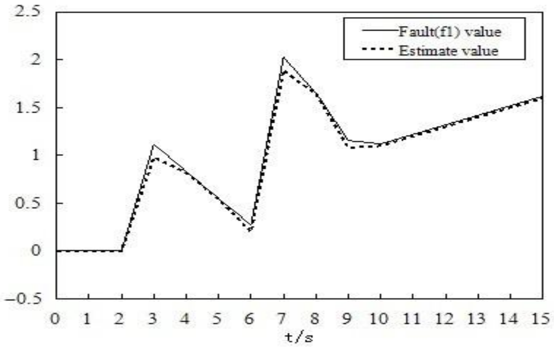

FIGURE 1 The value of fault f_1 and its estimate

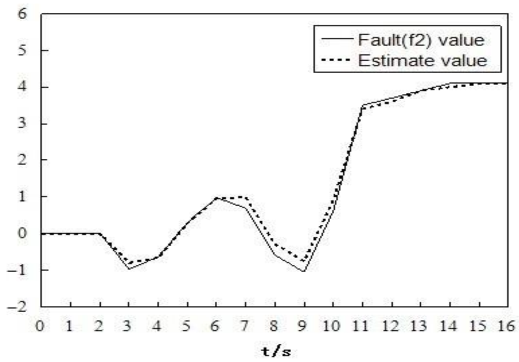

FIGURE 2 The value of fault f_2 and its estimate

b) State-feedback controller design. Letting $\gamma = 0.5$, the controller gain K is solved:

$$K = \begin{bmatrix} 1.1178 & 1.6750 & -0.3748 \\ 0.8438 & 1.3940 & -0.7596 \\ -3.7636 & -5.4994 & 1.6743 \end{bmatrix}$$

The fault-tolerant dynamic output responses are characterized by Figure 3, which show that the closed-loop system is ensured to be stable although the open-loop system is subject to impulsive modes and bounded faults.

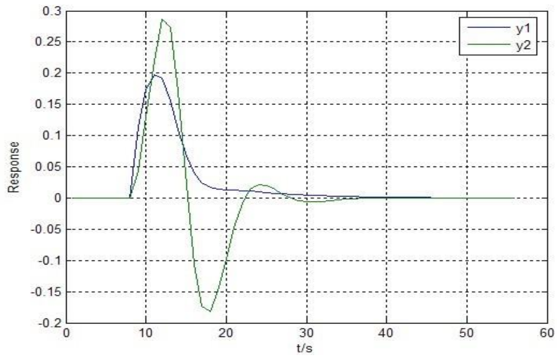

FIGURE 3 Output responses of the system via a fault-tolerant controller

From the above simulation results, it is proven that the fault diagnosis scheme is effective in estimating the states and faults of system, and the designed controller can guarantee the stability and H∞ performance of the close-loop system when the failures occur.

6 Conclusion

This paper proposed a novel observer-based H∞ and fault-tolerant control approach for Lipschitz singular systems with bounded perturbations and actuator faults. In terms of Lyapunov theory and linear matrix inequality (LMI) technique, a sufficient condition for the existence of the parameters of fault diagnosis observer was presented. Under assumptions 1–2, a kind of robust full-order observer was developed, which provided the information of both states and faults information. An H∞ fault-tolerant controller via state feedback was designed. Under this controller, the Lipschitz singular systems could be solvable, impulse free and asymptotically stable, and possess the prescribed H∞ performance. Moreover, the result obtained in this study is reliable in computation and preferable in application. Furthermore, a numerical example was provided to illustrate the effectiveness of the proposed approach. Future work will focus on the problem of designing the fault detection, the fault diagnosis and fault-tolerant control in an integrated manner for on-line application.

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Authors

Zhang Dongwen, born in December, 1964, Shijiazhuang County, Hebei Province, China

Current position, grades: the professor of school of information science & engineering, Hebei University of Science &Technology, China.
University studies: M. Eng. from Harbin Institute of Technology in China. D. Eng. from Beijing Institute of Technology in China.
Scientific interest: robust control, fault-tolerant control.
Publications: 50 papers.
Experience: teaching experience of 24 years, 10 scientific research projects.

An Xinguo, born in April, 1988, Shijiazhuang County, Hebei Province, China

Current position, grades: postgraduate student of school of sciences, Hebei University of Science &Technology, China.
University studies: B.Sc. in mathematics and applied mathematics from Hengshui University in China. M.Sc. in applied mathematics from Hebei University of Science and Technology in China.
Scientific interest: robust control, fault-tolerant control.
Publications: 4 papers.