New information multivariable optimization MGM(1,*n*) model with non equidistance and based on background value optimization

Youxin Luo*, Qiyuan Liu, Xiaoyi Che

[1]*College of Mechanical Engineering, Hunan University of Arts and Science, Changde, 415000, P.R. China*

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Abstract

The function with non-homogeneous exponential law, based on index characteristic and integral characteristic of grey model GM(1,1), was used to fit the one-time accumulated sequence, and the formula of background value was given, aiming at the problem of lower precision as well as lower adaptability in non-equidistant multivariable model MGM(1,*n*). A new information optimization model MGM(1,*n*) with non equidistance and multi variable based on background optimization was put forward, took the *m-th* component of the original sequence as initial condition, the mean relative error as objective function, and the modified one of initial value and the parameters of background value as design variables. This proposed MGM(1,*n*) model can be used in equidistance & non-equidistance modelling with higher precision as well as stronger adaptability. Examples have validated the practicability and reliability.

Keywords: multivariable, background value; optimizing, new information, non-equidistance sequence, non-equidistant MGM(1,*n*) model, least square method

1 Introduction

MGM(1,*n*) model is extended from GM(1,1) model in the case of *n* variables, and the parameters of MGM(1,*n*) model can reflect the relationships of mutual influence and restriction among multiple variables. The MGM(1,*n*) model was established [1] and the optimizing model of MGM (1,*n*) was set up by taking the first component of the sequence $\mathbf{x}^{(1)}$ as the initial condition of the grey differential equation and modifying [2]. The multivariable new information MGM(1,*n*) model taking the nth component of $\mathbf{x}^{(1)}$ as initial condition was established [3]. Take the *n*th component of $\mathbf{x}^{(1)}$ as initial condition and optimize the modified initial value and the coefficient of background value *q* where the form is $z_i^{(1)} = qx_i^{(1)}(k+1)+(1-q)x_i^{(1)}(k) \quad (q \in [0,1])$, and the new information MGM(1,*n*) model with multivariable was established [4]. These MGM(1,*n*) models are equidistant, and the non-equidistance multivariable MGM(1,*n*) model, with homogeneous exponent function fitting background value, was established [5]. However, non-homogeneous exponent function is more widespread, there are inherent defects in the modelling mechanism of this model. The model MGM(1,*n*) with non-equidistance and multivariable was established [6], and its background value is generated by mean value so as to bring lower accuracy. The non-equidistant and multi-variable model GM(1,n), based on non-homogeneous exponent function fitting background value, was established [7] and improves the accuracy of the model. The constructing method for background value is a key factor affecting the prediction accuracy and the adaptability. In order to improve the accuracy of GM(1,1); some constructing methods for background value were proposed, and some non-equidistance GM(1,1) model were established [8-11]. How it is of great significance establishing non-equidistance of MGM(1,*n*) model with high precision to extend GM(1,1) model to MGM(1,*n*) model. In this study, the method constructing background value in [9] and the optimizing modelling method in [2] were absorbed, and the function with non-homogeneous exponential law was used to fit the one-time accumulated sequence. According to the new information priority principle in the grey system, A new information optimization model MGM(1,*n*) with non equidistance and multi variable based on background optimization was put forward. took the *m-th* component of the original sequence as initial condition, the mean relative error as objective function, and the modified one of initial value and the parameters of background value as design variables. This model, with higher precision, better theoretical and practical value, can be used in equidistance and non-equidistance model and extend the application range of the grey model.

****Corresponding author*** e-mail: llyx123@126.com

2 New information optimization model MGM(1,*n*) with non-equidistance and multi-variable based on background value optimization

Definition 1: Supposed the sequence $\mathbf{X}_i^{(0)}=[x_i^{(0)}(t_1),\dots,x_i^{(0)}(t_j),\dots,x_i^{(0)}(t_m)]$, if $\Delta t_j=t_j-t_{j-1}\neq const$, where $i=1,2,\dots,n, j=2,\dots,m$, n is the number of variables, m is the sequence number of each variable, $\mathbf{X}_i^{(0)}$ is called as non-equidistant sequence.

Definition 2: Supposed the sequence $\mathbf{X}_i^{(1)}=\{x_i^{(1)}(t_1),x_i^{(1)}(t_2),\dots,x_{t_j}^{(1)}(t_j),\dots,x_i^{(1)}(t_m)\}$, if $x_i^{(1)}(t_1)=x_i^{(0)}(t_1)$ and $x_i^{(1)}(t_j)=x_i^{(1)}(t_{j-1})+x_i^{(0)}(t_j)\cdot\Delta t_j$ where $j=2,\dots,m,\ i=1,2,\dots,n,$ and $\Delta t_j=t_j-t_{j-1}$, $\mathbf{X}_i^{(1)}$ is one-time accumulated generation of non-equidistant sequence $\mathbf{X}_i^{(0)}$, and it is denoted by 1-AG0.

Suppose the original data matrix:

$$\mathbf{X}^{(0)}=\{\mathbf{X}_1^{(0)},\mathbf{X}_2^{(0)},\dots,\mathbf{X}_n^{(0)}\}^{\mathbf{T}}=\begin{bmatrix}x_1^{(0)}(t_1) & x_1^{(0)}(t_2) & \dots & x_1^{(0)}(t_m)\\ x_2^{(0)}(t_1) & x_2^{(0)}(t_2) & \dots & x_2^{(0)}(t_m)\\ \dots & \dots & \dots & \dots\\ x_n^{(0)}(t_1) & x_n^{(0)}(t_2) & \dots & x_n^{(0)}(t_m)\end{bmatrix}, \tag{1}$$

where, $\mathbf{X}^{(0)}(t_j)=[x_1^{(0)}(t_j),x_2^{(0)}(t_j),\dots,x_n^{(0)}(t_j)]$ $(j=1,2,\dots,m)$ is the observation value of each variable at t_j, and the sequence $[x_i^{(0)}(t_1),x_i^{(0)}(t_2),\dots,x_i^{(0)}(t_j),\dots,x_i^{(0)}(t_m)]$ $(i=1,2,\dots,n,\ j=1,2,\dots,m)$ is non-equidistant, that is, the distance t_j-t_{j-1} is not constant.

In order to establish the model, firstly the original data is accumulated one time to generate a new matrix:

$$\mathbf{X}^{(1)}=\{\mathbf{X}_1^{(1)},\mathbf{X}_2^{(1)},\cdots,\mathbf{X}_n^{(1)}\}^{\mathbf{T}}=\begin{bmatrix}x_1^{(1)}(t_1) & x_1^{(1)}(t_2) & \cdots & x_1^{(1)}(t_m)\\ x_2^{(1)}(t_1) & x_2^{(1)}(t_2) & \cdots & x_2^{(1)}(t_m)\\ \cdots & \cdots & \cdots & \cdots\\ x_n^{(1)}(t_1) & x_n^{(1)}(t_2) & \cdots & x_n^{(1)}(t_m)\end{bmatrix}, \tag{2}$$

where, $x^{(1)}(t_j)(j=1,2,\dots,m)$ meets the conditions in the definition 2, that is,

$$x_i^{(1)}(t_j)=\begin{cases}\sum_{j=1}^{k}x_i^{(0)}(t_j)(t_j-t_{j-1}) & (k=2,\dots,m)\\ x_i^{(0)}(t_1) & (k=1)\end{cases}. \tag{3}$$

Non-equidistant and multi-variable MGM(1,*n*) model can be expressed as first-order differential equations with *n* variables:

$$\begin{cases}\dfrac{dx_1^{(1)}}{dt}=a_{11}x_1^{(1)}+a_{12}x_2^{(1)}+\dots+a_{1n}x_n^{(1)}+b_1\\ \dfrac{dx_2^{(1)}}{dt}=a_{21}x_1^{(1)}+a_{22}x_2^{(1)}+\dots+a_{2n}x_n^{(1)}+b_2\\ \dots\\ \dfrac{dx_n^{(1)}}{dt}=a_{n1}x_1^{(1)}+a_{n2}x_2^{(1)}+\dots+a_{nn}x_n^{(1)}+b_n\end{cases}. \tag{4}$$

Assume $\mathbf{A}=\begin{bmatrix}a_{11} & a_{12} & \dots & a_{1n}\\ a_{21} & a_{22} & \dots & a_{2n}\\ \dots & \dots & \dots & \dots\\ a_{n1} & a_{n2} & \dots & a_{nn}\end{bmatrix}$, $\mathbf{B}=\begin{bmatrix}b_1\\ b_2\\ \dots\\ b_n\end{bmatrix}$,

Equation (4) can be expressed as:

$$\frac{dX^{(1)}(t)}{dt}=AX^{(1)}(t)+B. \tag{5}$$

According to new information priority principle in the grey system, it is inadequate for utilizing new information when the first component of the sequence $\mathbf{x}_i^{(1)}(t_j)$ $(j=1,2,\cdots,m)$ is taken as the initial condition of grey differential equation. Regard the *m-th* component as the initial conditions of the grey differential equation, the continuous time response of Equation (5) is:

$$\mathbf{X}^{(1)}(t)=e^{At}\mathbf{X}^{(1)}(t_m)+\mathbf{A}^{-1}(e^{\mathbf{A}t}-\mathbf{I})\mathbf{B}. \tag{6}$$

The *m-th* component of data in Equation (6) is taken as the initial value of the solution, and then $X_i^{(0)}(t_m)$ is substituted by $X_i^{(0)}(t_m)+\beta_i$, where the dimension of $\beta=[\beta_1,\beta_2,\dots,\beta_n]^T$ is the same as one of $X^{(0)}(t_m)$. After restoring, the fitting value of the original data is:

$$\hat{\mathbf{X}}_i^{(0)}(t_j)=\begin{cases}\lim\limits_{\Delta t\to 0}\dfrac{\mathbf{X}_i^{(1)}(t_1)-\mathbf{X}_i^{(1)}(t_1-\Delta t)}{\Delta t}, & j=1\\ (\hat{\mathbf{X}}_i^{(1)}(t_j)-\hat{\mathbf{X}}i_{t_{j-1}}^{(1)}(t_{j-1}))/(t_j-t_{j-1}), & j=2,3,\dots,m\end{cases}, \tag{7}$$

where, $e^{\mathbf{A}t}=\mathbf{I}+\sum_{k=1}^{\infty}\frac{\mathbf{A}^k}{k!}t^k$, $\mathbf{I}$ is a unit matrix.

In order to identify $\mathbf{A}$ and $\mathbf{B}$, Equation (4) is made the integration in $[t_{j-1},t_j]$ and we can obtain:

$$x_i^{(0)}(t_j)\Delta t_j=\sum_{l=1}^{n}a_{il}\int_{t_{j-1}}^{t_j}x_i^{(1)}(t_j)dt+b_i\Delta t_j. \quad (i=1,2,\dots,n;\ j=2,3,\dots,m) \tag{8}$$

Note $z_i^{(1)}(t_j)=\int_{t_{j-1}}^{t_j}x_i^{(1)}(t_j)dt$, and the common formula for background value, actually based on the trapezoidal area $z_i^{(1)}(t_j)\Delta t_j$, is appropriate when the time interval is small, that is, the change of sequence data is slow. However, when this change is sudden, the background value using the common formula often brings out the larger error, so it is more suitable for Equation (4) that parameter matrix $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ estimated by the background value in $[t_{j-1}, t_j]$ are obtained by $z_i^{(1)}(t_j) = \int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt$ substituting for $x_i^{(1)}(t_j)$. Based on quasi-exponentially law of the grey model and the modelling principles and methods in [9], we set that $x_i^{(1)}(t) = A_i e^{B_i t} + C_i$, where A_i, B_i, C_i are the undetermined coefficients.

Assume that the curve $x_i^{(1)}(t) = A_i e^{B_i t} + C_i$ passes through three points ($t_j, x_i^{(1)}(t_j)$), ($t_{j-1}, x_i^{(1)}(t_{j-1})$) and ($t_{j-2}, x_i^{(1)}(t_{j-2})$), we can obtain:

$$x_i^{(1)}(t_j) = A_i e^{B_i t_j} + C_i \,,\; x_i^{(1)}(t_{j+1}) = A_i e^{B_i t_{j+1}} + C_i \,. \tag{9}$$

Take C_i as the parameter, obtain the undetermined coefficients A_i, B_i, C_i in Equation (9):

$$A_i = \frac{[x_i^{(1)}(t_j) - C_i]^{\frac{t_{j+1}}{\Delta t_{j+1}}}}{[x_i^{(1)}(t_{j+1}) - C_i]^{\frac{t_j}{\Delta t_{j+1}}}}$$
$$B_i = \frac{\ln(x_i^{(1)}(t_{j+1}) - C_i) - \ln(x_i^{(1)}(t_j) - C_i)}{\Delta t_{j+1}} \,. \tag{10}$$

That Equation (10) substituting for the formula for background value $\int_{t_{j-1}}^{t_j} x_i^{(1)}(t_j)dt$ can be obtained.

$$z_i^{(1)}(t_{j+1}) = \int_{t_j}^{t_{j+1}} x_i^{(1)}dt = \int_{t_j}^{t_{j+1}} (A_i e^{B_i t} + C_i)dt =$$
$$\frac{(\Delta t_{j+1})^2 x_i^{(0)}(t_{j+1})}{\ln(x_i^{(0)}(t_{j+1}) - C_i) - \ln(x_i^{(0)}(t_j) - C_i)} + C_i \Delta t_{j+1}. \tag{11}$$

Note $\mathbf{a}_i = (a_{i1}, a_{i2}, \ldots, a_{in}, b_i)^{\mathbf{T}}$ $(i = 1,2,\ldots,n)$ and the identified value $\hat{\mathbf{a}}_i$ of a_i can be obtained by using the least square method as follows:

$$\hat{\mathbf{a}}_i = [\hat{a}_{i1}, \hat{a}_{i2}, \ldots, \hat{a}_{in}, \hat{b}_i]^{\mathbf{T}} =$$
$$(\mathbf{L}^T\mathbf{L})^{-1}\mathbf{L}^T\mathbf{Y}_i, i = 1,2,\ldots,n, \tag{12}$$

where:

$$\mathbf{L} = \begin{bmatrix} z_1^{(1)}(t_2) & z_2^{(1)}(t_2) & \ldots & z_n^{(1)}(t_2) & \Delta t_2 \\ z_1^{(1)}(t_3) & z_2^{(1)}(t_3) & \ldots & z_n^{(1)}(t_3) & \Delta t_3 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ z_1^{(1)}(t_m) & z_2^{(1)}(t_m) & \ldots & z_n^{(1)}(t_m) & \Delta t_m \end{bmatrix}, \tag{13}$$

$$\mathbf{Y}_i = [x_i^{(0)}(t_2)\Delta t_2, x_i^{(0)}(t_3)\Delta t_3, \cdots, x_i^{(0)}(t_m)\Delta t_m]^{\mathbf{T}} \,. \tag{14}$$

Then the identified values of A and B can be obtained.

$$\hat{\mathbf{A}} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \ldots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \ldots & \hat{a}_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \hat{a}_{n1} & \hat{a}_{n2} & \ldots & \hat{a}_{nn} \end{bmatrix}, \; \hat{\mathbf{B}} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \ldots \\ \hat{b}_n \end{bmatrix}. \tag{15}$$

The calculated value in new information MGM(1,n) model is:

$$\hat{\mathbf{X}}_i^{(1)}(t_j) = e^{\hat{\mathbf{A}}(t_j - t_m)}\mathbf{X}_i^{(1)}(t_m) + \hat{\mathbf{A}}^{-1}(e^{\hat{\mathbf{A}}(t_j - t_m)} - \mathbf{I})\hat{\mathbf{B}} \,. \tag{16}$$
$(j = 1,2,\ldots,m)$

After restoring, the fitting value of the original data is:

$$\hat{\mathbf{X}}_i^{(0)}(t_j) = \begin{cases} \lim\limits_{\Delta t \to 0} \dfrac{\mathbf{X}_i^{(1)}(t_1) - \mathbf{X}_i^{(1)}(t_1 - \Delta t)}{\Delta t}, j = 1 \\ (\hat{\mathbf{X}}_i^{(1)}(t_j) - \hat{\mathbf{X}}_i^{(1)}(t_{j-1}))/(t_j - t_{j-1}), j = 2,3,\ldots,m \end{cases} . \tag{17}$$

The absolute error of the *i-th* variable is $\hat{x}_i^{(0)}(t_j) - x_i^{(0)}(t_j)$. The relative error of the *i-th* variable is: $e_i(t_j) = \dfrac{\hat{x}_i^{(0)}(t_j) - x_i^{(0)}(t_j)}{x_i^{(0)}(t_j)}100$. The mean of the relative error of the *i-th* variable is $\frac{1}{m}\sum_{j=1}^{m}|e_i(t_j)|$. The average error of the whole data is $f = \frac{1}{nm}\sum_{i=1}^{n}\left(\sum_{j=1}^{m}|e_i(t_j)|\right)$.

After taking the average error f as the objective function and β and $C = [C_1, C_2, \ldots, C_n]^T$ as the design variables, and the optimization function *fmincon* in Matlab 7.5 or other optimization methods [14] was used. If $C = [C_1, C_2, \ldots, C_n]^T$ was not optimized, $C = [0,0,\ldots,0]^T$, and at this time $x_{t_j}^{(1)}(i) = A_i e^{B_i t} + C_i$ is the homogeneous form as $x_{t_j}^{(1)}(i) = A_i e^{B_i t}$, and the precision of the model constructing background value is low.

3 Example

Example 1: In the calculation on contact strength, the coefficients m_a and m_b among the principal curvature function $F(\rho)$, the radius of the major axis a and the minor b in the ellipse with the point contact is generally obtained by looking-up, and these data are extracted in Table 1 [12]:

TABLE 1 Values of $F(\rho)$, m_a and m_b

$F(\rho)$	0.9995	0.9990	0.9980	0.9970	0.9960	0.9950
m_a	23.95	18.53	14.25	12.26	11.02	10.15
m_b	0.163	0.185	0.212	0.228	0.241	0.251
$F(\rho)$	0.9880	0.9870	0.9860	0.9850	0.9840	0.9830
m_a	7.25	7.02	6.84	6.64	6.47	6.33
m_b	0.297	0.301	0.305	0.310	0.314	0.317
$F(\rho)$	0.9940	0.9930	0.9920	0.9910	0.9900	0.9890
m_a	9.46	8.92	8.47	8.10	7.76	7.49
m_b	0.260	0.268	0.275	0.281	0.287	0.292
$F(\rho)$	0.9820	0.9810	0.9800	0.9790	0.9780	0.9770
m_a	6.19	6.06	5.95	5.83	5.72	5.63
m_b	0.321	0.325	0.328	0.332	0.335	0.338

Assume m_b is as t_k, $F(\rho)$ as x_1 and m_a as x_2, non-equidistant new information optimizing MGM(1,2) model was established by using the proposed method in this study. The parameters of this model are as follows:

$$\mathbf{A}=\begin{vmatrix}-0.0501 & 0.0003\\ -60.1400 & 0.1394\end{vmatrix},\ \mathbf{B}=\begin{vmatrix}1.0421\\ 76.9607\end{vmatrix},\ C=\begin{vmatrix}-89.9368\\ -14.6408\end{vmatrix},$$

$$\beta=\begin{vmatrix}0.260702\\ 11.7602\end{vmatrix}.$$

The fitting value of $F(\rho)$ is:

$\hat{F}(\rho)$ = [0.99686, 0.99339, 0.99228, 0.9913, 0.99063, 0.9901, 0.98966, 0.98927, 0.98892, 0.98861, .98833, .98808, 0.98784, 0.98763, 0.98744, 0.98723, 0.98701, 0.98685, 0.98668, 0.98649, 0.98633, 0.98616, 0.98599, 0.98585]

The absolute error of $F(\rho)$:

$q=$ [0.0026385, 0.0056107, 0.0057196, 0.0057004, 0.0053663, 0.004897, 0.0043371, 0.0037321, .0030816, 0.0023854, 0.0016663, 0.00092441, 0.00015948, –0.00062861, –0.00144, –0.0022274, –0.0030145, –0.0038487, –0.0046827, –0.0054927, –0.0063262, –0.0071595, –0.0079927, –0.0088495]

The relative error of $F(\rho)$ (%):

$e=$[–0.26398, –0.56164, –0.5731, –0.57175, –0.53879, –0.49216, –0.43632, –0.37584, –0.31065, –0.2407, –0.16832, –0.093469, –0.016142, 0.063689, 0.14604, 0.22613, 0.30635, 0.39152, 0.47685, 0.5599, .64553, 0.73131, 0.81725, 0.90578].

The mean of the relative error of $F(\rho)$ is 0.04822%, and one of this model is 0.49941%, therefore, this model has higher precision. In the model without optimization, the mean of the relative error of $F(\rho)$ is 0.065315%, and the one of the model is 0.79897%.

Example 2: In the conditions of the load 600N and the relative sliding speed 0.314m/s, 0.417m/s, 0.628m/s, 0.942m/s and 1.046m/s, the test data of the thin film with TiN coat are shown in Table 2 [13].

TABLE 2 Test data of the thin film with TiN coat

No.	1	2	3	4	5
Sliding speed (m/s)	0.314	0.471	0.628	0.942	1.046
Friction coefficient μ	0.251	0.258	0.265	0.273	0.288
Wear rate $\omega\times10^{-5}$ (mg/m)	7.5	8	8.5	9.5	11

Assume sliding speed t_j, friction coefficient $X_1^{(0)}$ and wear rate $X_2^{(0)}$, non-equidistant new information optimizing MGM(1,2) model was established by using the proposed method in this study. The parameters of this model are as follows:

$$\mathbf{A}=\begin{vmatrix}-0.1947 & 0.0102\\ -16.2105 & 0.9536\end{vmatrix},\ \mathbf{B}=\begin{vmatrix}0.2249\\ 4.3908\end{vmatrix},\ C=\begin{vmatrix}-1.1263\\ 7.5\end{vmatrix}$$

and $\beta=\begin{vmatrix}0.085424\\ 2.2022\end{vmatrix}$.

The fitting value of $X_1^{(0)}$:

$\hat{X}_1^{(0)}$ = [0.25103, 0.2597, 0.265, 0.27462, 0.28471].

The absolute error of $X_1^{(0)}$:

$q=$[2.9151e–005, 0.0016991, –1.607e–009, 0.0016164, –0.0032885].

The relative error of $X_1^{(0)}$ (%):

$e=$[0.011614, 0.65857, –6.0642e–007, 0.5921, –1.1418].

The mean of the relative error of $X_1^{(0)}$ is 0.48082%, and one of this model is 1.3706%, thus, this model has higher precision. In the model without the optimization of β and C, the relative error mean of $X_1^{(0)}$ is 4.1851%, and the one of the model is 5.9398%. When equidistant MGM(1,3) model was used in [13], the relative error mean of $X_1^{(0)}$ is 1.6225%.

4 Conclusions

Aiming at non-equidistant multivariable sequence with mutual influence and restriction among multiple variables, the function with non-homogeneous exponential law, based on index characteristic and integral characteristic of grey model, was used to fit the one-time accumulated sequence. A new information optimization model MGM(1,*n*) with non equidistance and multi variable based on background optimization was put forward, took the *m-th* component of the original sequence as initial condition, the mean relative error as objective function, and the modified one of initial value and the parameters of background value as design variables. The proposed MGM(1,*n*) model can be used in equidistance and non-equidistance and extent the application scope of grey model. New model is with high precision and easy to use.

Examples have validated the practicability and reliability of the proposed model. This model is of important practice and theoretical significance and is worthy of promotion.

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Authors

Youxin Luo, born in December, 1966, Xinhua, Hunan, China

Current position, grades: professor at the College of Mechanical Engineering, Hunan University of Arts and Science, Changde, China.
University study: B.S and M.S degrees in mechanical design & manufacturing, material engineering at Chongqing University, Huazhong University of Science and Technology, China, in 1988 and 2003 respectively.
Scientific interests: information science, grey system, mechanics, and optimizing.
Publications: 180 papers and 2 monographs.
Experience: teaching and scientific research work since 2000.

Qiyuan Liu, born in May, 1981, Changde, Hunan, China

Current position, grades: teacher at the College of Mechanical Engineering, Hunan University of Arts and Science, Changde, China.
University study: B.S and M.S degrees in mechanical manufacturing, material engineering at Xiangtan University, China, in 2002 and 2005, respectively.
Scientific interests: information science, grey system and material engineering.
Publications: 15 papers.
Experience: teaching and scientific research work since 2006.

Xiaoyi Che, born in January, 15, 1966, Changde, Hunan, China

Current position, grades: professor at the College of Mechanical Engineering, Hunan University of Arts and Science, Changde, China.
University study: B.S and M.S degrees in mechanical manufacturing, material engineering at Shengyan Institute of Technology, Huazhong University of Science and Technology, China, in 1988 and 2003, respectively.
Scientific interest: Information science, grey system, CNC and optimum design.
Publications: 30 papers.
Experience: teaching and scientific research work since 1988.