Dual-hop variable gain relaying in mixed multipath/shadowing fading channels

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Received 15 June 2014, www.cmnt.lv

Abstract

In this paper, we investigated the end-to-end performance of a dual-hop variable gain relaying system over mixed fading environment. In such environment, the wireless links of relaying system undergo different fading conditions, where one link is subject to the Nakagami-m fading, the other link is subject to the composite Nakagami-lognormal fading which is approximated by using mixture gamma fading model. Based on the cumulative distribution function of the end-to-end signal-to-noise ratio (SNR), some novel closed-form expressions of the average end-to-end SNR, the outage probability, the symbol error rate and the ergodic capacity for the dual-hop variable gain system are derived, respectively. Then, some approximate analysis and the diversity order are found based on the above new expressions in high SNR region. Finally, numerical and simulation results are shown to verify the accuracy of the theoretical analysis.

Keywords: dual-hop relaying, mixed fading channels, mixture Gamma distribution, performance analysis

1 Introduction

Cooperative relaying transmission has emerged as a promising technique for extending coverage, enhancing connectivity, and saving transmitter power in wireless communications networks. In a cooperative relaying network, a source communicates with the destination via one or several intermediate terminals called relays. In the past few years, a great deal of attention has been devoted to study the performance analysis of cooperative relaying systems in term of outage probability (OP), average bit/symbol error rate (ABER/ASER) and ergodic capacity over different fading environments, such as Rayleigh, Nakagami-m, Rician, Weibull, Lognormal, Generalized Gamma and so on, e.g., [1-5] and the references therein. The common characteristic of these works is that they consider only multipath or shadowing fading channels.

In a practical scenario, relaying nodes (R) are usually located in different geographical locations and at different distances with respect to the source node (S) and the destination node (D). The signal in one link may be in line of sight (LOS) situation and other links may be in NLOS situation, even in shadowing or composite multipath/shadowing situation. In the published literature, such scenario has been regarded as asymmetric or mixed fading models [6, 7]. On the contrary, the channel situation in [1-5] is regarded as symmetric fading models in which all the single-hop links experience the same fading conditions. So far, most previous works have considered the latter, only a few works have evaluated the former [5-16]. Recently, there is an increasing research interest on the former.

In [8], we studied the performance of two-hop and multihop relaying links in random (i.e., mixed Rayleigh/ Nakagami-m) fading channels. The authors in [6] and [7] first studied the end-to-end performance of dual-hop AF relaying with both channel state information (CSI) based and fixed gain over mixed Rayleigh and Rician fading channels, respectively. After that, more cooperative relaying models are studied in mixed Rayleigh and Rician fading channels, for example, the dual-hop decode-and-forward (DF) cooperative model in [9], the dual-hop AF cooperative model in [10], the repetition-based and opportunistic amplify-and-forward (AF) model in [11], the two-hop AF networks with beamforming in [12], and so on. In [13], the authors analysed the performance of dual-hop AF relaying in mixed Nakagami-m and Rician fading channels. In [14,15], the authors studied the performance of a dual-hop DF system and an AF cooperative system under mixed Rayleigh and generalized Gamma fading channels, respectively.

Despite these recent studies related to the analysis of AF or DF relaying over asymmetric fading channels, their fading condition is limited to the mixture of various multipath fading, i.e., Rayleigh/Rician, Rician/Nakagami-m and Rayleigh/generalized Gamma fading. The performance of the cooperative networks has not as yet been widely studied under mixed multipath/shadowing fading conditions except [16, 17]. The authors in [16,17] investigated the performance of the dual-hop fixed gain relaying system over Nakagami/Generalized-K (KG) fading channels. Nevertheless, since the probability density function (PDF) of average signal-to-noise ratio (SNR) over KG fading channel includes modified Bessel functions, the outage probability and average SER in [16]

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include Meijer's G functions and some infinite-series representations. The end-to-end Moment generating function (MGF) in [17] includes Lommel function. Some expressions still keep complicated and intractable.

Recently, the authors in [18] developed an alternative approach to approximate the Nakagami-lognormal (NL) distribution by using the mixture gamma (MG) distribution. This distribution avoids the above-mentioned problems, and some exact results obtained are possible by adjusting some parameters. To the best of our knowledge, there are few papers in performance analysis of cooperative system over mixed fading channels by using MG fading model.

In this paper, we consider an asymmetric scenario of a dual-hop AF relaying system in a wireless propagation environment where multipath fading, shadowing and the propagation path loss occur simultaneously. The S−R (first-hop) and the R−D (second-hop) links experience Nakagami-m or NL fading, where the NL fading model is approximated by using MG fading model. The primary contributions of this paper are as follows: Firstly, some exact closed-form expressions of the average end-to-end SNR, the OP, the ASER and the ergodic capacity for the dual-hop system over mixed Nakagami/MG fading channels are derived, respectively. And then, some approximate analysis of the above performance is discussed and the diversity order is obtained in high SNR region. Finally, the numerical and simulation results are given to show the accuracy of the theoretical analysis.

2 System and channel model

We consider a classical wireless dual-hop variable gain relaying system consisting of S, D and R. The whole transmission is divided into two phase. In the first phase, S only transmits its signals to R, and in the second phase, R amplifies the received signal by a gain factor β and then forwards their amplified versions to D. Thus, the instantaneous end-to-end SNR, γ_{SRD}, at the destination can be expressed as in [1, 2]:

$$\gamma_{SRD} = \frac{(P_1|h_1|^2/N_0)(P_2|h_2|^2/N_0)}{[(P_2|h_2|^2/N_0)+(1/N_0\beta^2)]}, \tag{1}$$

where P_1 and P_2 are the transmitted power at S and R respectively, $|h_i|$ is the fading amplitude of the i^{th}-hop link, $i\in\{1,2\}$, N_0 is the power of the additive white Gaussian noise component. If β is selected according to the instantaneous CSI assisted relay gain, then γ_{SRD} in Equation (1) can be re-expressed as in [1]:

$$\gamma_{SRD} = \frac{\gamma_1\gamma_2}{(\gamma_1+\gamma_2+c)}, \tag{2}$$

where $\gamma_i = \rho_i|h_i|^2$ is the instantaneous SNR of the i^{th}-hop link, $\rho_i=P_i/N_0$ denotes the un-faded SNR. In addition, exact γ_{SRD} is given by substituting $c=1$ when $\beta_2= 1/(P_1|h_1|^2+N_0)$, and well approximated at medium and high SNR by substituting $c=0$ when $\beta_2=1/(P_1|h_1|^2)$.

Note that due to the symmetry of γ_{SRD} in Equation (2) with respect to γ_1 and γ_2, the statistics of γ_{SRD} will be identical despite that the i^{th}-hop link is subject to Nakagami-m or NL fading. In a practical scenario, it is possible that a mobile station at the edge of the cell communicates with the base station via one or more mobile/fixed relaying stations over mixed multipath/shadowing situation.

If the i^{th}-hop link experiences Nakagami-m fading, γ_i is a Gamma distributed variable with the PDF given in [19]:

$$f_{\gamma_i}(\gamma) = (m_i/\bar{\gamma}_i)^{m_i}\gamma^{m_i-1}\exp[-m_i\gamma/\bar{\gamma}_i]/\Gamma(m_i), \tag{3}$$

where $\bar{\gamma}_i = \rho_i\mathbf{E}[|h_i|^2] = \rho_i\Omega_i$ is the average SNR of the i^{th}-hop link, m_i is Nakagami-m fading parameter, $\mathbf{E}(*)$ denotes the statistical expectation, $\Gamma(*)$ is the standard Gamma function. Due to capture the path-loss effect, we use the local mean power $\Omega_i = (d_0/d_i)^{\varepsilon}$, d_0 denotes the distance between S and D, d_i is the distance of the i^{th}-hop link, and ε is the path-loss exponent.

The cumulative distribution function (CDF) of γ_i, defined as $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(x)dx$ in [19], can be obtained as

$$F_{\gamma_i}(\gamma) = 1-\Gamma(m_i, m_i\gamma/\bar{\gamma}_i)/\Gamma(m_i), \tag{4}$$

where $\Gamma(*;*)$ is the upper incomplete gamma function defined in [20].

When the i^{th}-hop link experiences NL fading, γ_i is a composite Gamma-lognormal distribution variable with the PDF given by [19]:

$$f_{\gamma_i}(\gamma) = \int_0^\infty \frac{m_i^{m_i}\gamma^{m_i-1}\exp(-m_i\gamma/\rho_i y)}{\Gamma(m_i)(\rho_i y)^{m_2}}\frac{1}{\sqrt{2\pi}\lambda_i y} \exp\left[-\frac{(\ln y-\mu_i)^2}{2\lambda_i^2}\right]dy, \tag{5}$$

where μ_i and λ_i are the mean and the standard deviation of lognormal shadowing, respectively, λ_i=(ln10/10)σ, μ_i= lnΩ_i, σ denotes the standard deviation in dB.

Since a closed-form expression of Equation (5) is not available in the published literature, the performance metrics of digital communication systems over composite NL distribution is intractable or difficult, some simple forms or approximations of Equation (5) have been given great attention recently, such as, KG distribution and MG distribution. Due to that KG distribution includes modified Bessel functions, some expressions of the performance metrics still keep mathematical complications, and further approximations have to be adopted. In order to avoid the above problems, we use MG distribution proposed by [18] to approximate the composite NL distribution in this paper. Thus, the PDF of γ_i can be expressed as:

$$f_{\gamma_i}(\gamma)=\sum_{j=1}^{N}(Ca_j/2\rho_i^{m_i})\gamma^{m_i-1}\exp(-b_j\gamma/\rho_i)\,, \tag{6}$$

where $a_j=2m_i^{m_i}w_j\exp[-m_i(\sqrt{2}\lambda_i t_j+\mu_i)]/\sqrt{\pi}\Gamma(m_i)$, $b_j=m_i\exp[-(\sqrt{2}\lambda_i t_j+\mu_i)]$, C is the normalization factor, defined as $C=\sqrt{\pi}/\sum_{j=1}^{N}w_j$, w_j and t_j are abscissas and weight factors for Gaussian-Hermite integration. w_j and t_j for different N values are available in [21, table (25.10)].

Based on the definition of the CDF, The CDF of γ_i over MG fading can be obtained as:

$$F_{\gamma_i}(\gamma)=1-\sum_{j=1}^{N}(Ca_j/2b_j^{m_i})\Gamma(m_i,b_j\gamma/\rho_i)\,. \tag{7}$$

3 Performance analysis

In this section, first we find the closed-form CDF of the end-to-end SNR for the dual-hop system, then derived the closed-form expressions of the average end-to-end SNR, the OP, the ASER and the ergodic capacity over mixed Nakagami-m/MG fading channels, respectively. Finally, some approximate analysis of the OP and the ASER is discussed and the diversity order is obtained in high SNR region.

3.1 CDF OF END-TO-END SNR

For the dual-hop system, assuming that the first-hop link is subject to Nakagami-m fading and the second-hop link is subject to NL fading, by using Equation (2), the CDF of γ_{SRD} can be expressed as in [6]:

$$F_{\gamma_{SRD}}(x)=\Pr(\gamma_{SRD}\le x)=\Pr[\gamma_1\gamma_2/(\gamma_1+\gamma_2+c)\le x]\,, \tag{8}$$

After applying some algebraic manipulations, Equation (8) can be rewritten as:

$$F_{\gamma_{SRD}}(x)=1-\int_0^\infty \bar{F}_{\gamma_1}[x+(x^2+cx)/y]f_{\gamma_2}(x+y)dy\,, \tag{9}$$

where $\bar{F}_{\gamma_1}(*)$ is the complementary CDF of γ_1, which is defined as $\bar{F}_{\gamma_1}(*)=1-F_{\gamma_1}(*)$. According to Nakagami-m fading of the first-hop link, $\bar{F}_{\gamma_1}(*)$ can be expressed by using Equation (4) as:

$$\bar{F}_{\gamma_1}(x)=\Gamma[m_1,M(x+(x^2+cx)/y)]/\Gamma(m_1)\,, \tag{10}$$

where $M=m_1/\bar{\gamma}_1$.

By substituting Equations (6) and (10) into Equation (9), and with the help of the series expression of $\Gamma(*;*)$ defined in [20, eq. (8.352.2)] when m_i is an integer, and the binomial expansion defined in [20, eq. (1.111)], we can obtain the CDF of γ_{SRD} as:

$$F_{\gamma_{SRD}}(x)=1-\sum_{i=1}^{N}\sum_{k=0}^{m_1-1}\sum_{s=0}^{m_2-1}\sum_{j=0}^{k}\Xi(i,k,s,j)x^{k+(m_2+s-j)/2}\times \\ (x+c)^{(m_2-s+j)/2}\exp\left[-\Phi(i)x\right]K_v\left[\Theta(i)\sqrt{x^2+cx}\right], \tag{11}$$

where $\Phi(i)=M+M_i$, $\Theta(i)=2\sqrt{MM_i}$, $M_i=b_i/\rho_2$, $v=m_2-s-j$, $\Xi(i,k,s,j)=\dfrac{C_{m_2-1}^{s}C_k^jCa_iM^{k+v/2}}{\left(b_i^{v/2}\rho_2^{(m_2+s+j)/2}k!\right)}$, $C_j^i=j!/[(j-i)!i!]$ is the binomial coefficients, $K_\alpha(*)$ is the second kind modified Bessel function of order α defined in [20, eq.(8.407.1)].

3.2 OUTAGE PROBABILITY

The OP is an important performance measure that is commonly used to characterize a wireless communication system. It is defined as the probability that the instantaneous end-to-end SNR falls below a given threshold (γ_{th}), this is $P_{out}=\int_0^{\gamma_{th}}f_\gamma(\gamma)d\gamma$, where $f_\gamma(\gamma)$ is the PDF of the instantaneous SNR. Using (11), the OP of the dual-hop system over mixed fading channels can be obtained as:

$$P_{out-1}=F_{\gamma_{SRD}}(\gamma_{th})\,. \tag{12}$$

3.3 AVERAGE END-TO-END SNR

The average end-to-end SNR is a useful performance measure serving as an excellent indicator of the overall system's fidelity. The q^{th} moment of the end-to-end SNR can be derived by using CDF as:

$$\mu(\gamma^q)=\int_0^\infty \gamma^q f_{\gamma_{SRD}}(\gamma)d\gamma=q\int_0^\infty \gamma^{q-1}[1-F_{\gamma_{SRD}}(\gamma)]d\gamma\,. \tag{13}$$

By using Equations (11) with c=0, which is analytically more tractable and with the help of [20, eq. (6.621.3)], Equation (13) can be re-expressed as:

$$\mu(\gamma^q)=\sum_{i=1}^{N}\sum_{k=0}^{m_1-1}\sum_{s=0}^{m_2-1}\sum_{j=0}^{k}\Psi(i,k,s,j)\times \\ {}_2F_1[u_1+v,v+0.5;\; u_1+0.5;\; \frac{\Phi(i)-\Theta(i)}{\Phi(i)+\Theta(i)}], \tag{14}$$

where

$$\Psi(i,k,s,j)=\frac{4^v\sqrt{\pi}C_{m_2-1}^{s}C_k^jCa_iqM^{k+v}\Gamma(u_1+v)\Gamma(u_1-v)}{k!\rho_2^{m_2}\Gamma(u_1+0.5)[\Phi(i)+\Theta(i)]^{u_1+v}},$$

$u_1=m_2+k+q$, ${}_2F_1(a,b;c;z)$ is the hypergeometric function defined in [20, eq. (9.100)].

Therefore, the average end-to-end SNR can be obtained by setting q=1 in Equation (14).

3.4 AVERAGE SYMBOL ERROR RATE

The ASER is a useful measurement for investigating the performance of wireless communication systems. For several modulations with Gray bit mapped constellations, a uniform expression of the ASER can be written as equation [19]:

$$P_s = E[aQ(\sqrt{2b\gamma})] = \int_0^\infty aQ(\sqrt{2b\gamma}) f_\gamma(\gamma) d\gamma \,, \tag{15}$$

where $Q(*)$ is the Gaussion Q-funtion defined by $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-t^2/2) dt$, the parameters a and b change by specific modulation scheme. Equation (15) provides exact SER results for binary PSK (a=1, b=1), binary frequency shift keying (BFSK) (a=1, b=0.5) and M-ary pulse amplitude modulation (M-PAM) (a=2(M-1)/M, b=3/(M^2-1)). Furthermore, Equation (15) also provides approximate SER results for other modulations such as M-PSK (a=2, b=sin2(π/M)). After integration by parts Equation (15) can be rewritten using the CDF of γ_{SRD}, as:

$$P_s = (a\sqrt{b}/2\sqrt{\pi}) \int_0^\infty x^{-1/2} \exp(-bx) F_{\gamma_{SRD}}(x) dx \,. \tag{16}$$

Therefore, similar to Equation (14) one can get the analytical expression of ASER for the dual-hop system as:

$$P_{s-1} = \frac{a}{2} - \sum_{i=1}^{N} \sum_{k=0}^{m_1-1} \sum_{s=0}^{m_2-1} \sum_{j=0}^{k} \Upsilon(i,k,s,j) \times {}_2F_1\left[u_2+v, v+0.5; u_2+0.5; \frac{\Phi(i)+b-\Theta(i)}{\Phi(i)+b+\Theta(i)}\right], \tag{17}$$

where:

$$\Upsilon(i,k,s,j) = \frac{C_{m_2-1}^s C_k^j 4^{v-0.5} a\sqrt{b} Ca_i M^{k+v} \Gamma(u_2+v)\Gamma(u_2-v)}{k!\rho_2^{m_2}\Gamma(u_2+0.5)[\Phi(i)+b+\Theta(i)]^{u_2+v}},$$

$$u_2 = m_2 + k + 0.5 \,.$$

3.5 ERGODIC CAPACITY

For a dual-hop variable gain system with the single relay, the ergodic capacity can be obtained as in [22]:

$$\overline{C} = \Delta \mathbf{E}\left[\ln(1+\gamma_{SRD})\right], \tag{18}$$

where $\Delta = 1/2\ln 2$.

Since an exact closed-form expression in Equation (18) over mixed Nakagami-m/MG fading channels is not mathematically tractable by directly using a traditional approach (i.e., finding the PDF of γ_{SRD} with c=1), we thus restructure Equation (18) as in[22]:

$$\overline{C} = \Delta\{\underbrace{\mathbf{E}[\ln(1+\gamma_1)]}_{\overline{C}_1} + \underbrace{\mathbf{E}[\ln(1+\gamma_2)]}_{\overline{C}_2} - \underbrace{\mathbf{E}[\ln(1+\gamma_1+\gamma_2)]}_{\overline{C}_3}\} \,. \tag{19}$$

Note that Equation (19) provides an interesting information. Theoretic result that states that the ergodic capacity of the dual-hop system is equal to the sum of the ergodic capacities of the source–relay link $\overline{C_1}$ and relay–destination link $\overline{C_2}$ minus the ergodic capacity $\overline{C_3}$ of the single input multiple output system, in which the source acts as a transmitter, and the relay and destination are the receivers. The advantage of Equation (19) is now clear, because the methods of finding closed-form expressions for the ergodic capacities are already available in the open literature, i.e., using the PDF of γ_i. In the following, we will now derive new closed-form expressions for the ergodic capacities of the dual-hop variable gain relay network over mixed Nakagami-m/MG fading channels by using Equation (19).

First, we find the closed-form expressions of $\overline{C_1}$ and $\overline{C_2}$. By using Equation (3) and the integral expression in [19, eq.(15B. 7)] as:

$$\Im_n(\mu) = \int_0^\infty t^{n-1}\ln(1+t)\exp(-\mu t)\, dt = (n-1)!\exp(\mu)\sum_{k=1}^{n}\Gamma(-n+k,\mu)/\mu^k, \mu>0, n=1,2,...,$$

then the closed-form expression of $\overline{C_1}$ can be written as

$$\overline{C_1} = \int_0^\infty \ln(1+x) f_{\gamma_1}(x) dx = M^{m_1} \Im_{m_1}(M)/\Gamma(m_1) \,. \tag{20}$$

Similarly, the closed-form expression of $\overline{C_2}$ can be written as:

$$\overline{C_2} = \int_0^\infty \ln(1+x) f_{\gamma_2}(x) dx = \sum_{i=1}^{N} Ca_i \Im_{m_2}(M_i)/2\rho_2^{m_2} \,. \tag{21}$$

Then, we find the closed-form expression of $\overline{C_3}$. Here, we let $z = \gamma_1 + \gamma_2$. By using Equation (3) and (6), the PDF of variable z can be obtained as:

$$f_z(z) = \sum_{i=1}^{N} [Ca_i M^{m_1}/2\rho_2^{m_2}\Gamma(m_1)] \exp(-M_i z) \times \int_0^z x^{m_1-1}(z-x)^{m_2-1} \exp[-x(M-M_i)] dx \,. \tag{22}$$

When $M \neq M_i$, by using the binomial expansion defined in [20, eq.(1.111)] and eq.(3.381) in [20], after applying some algebraic manipulations, Equation (22) can be rewritten as:

$$f_z(z) = \sum_{i=1}^{N} \sum_{j=0}^{m_2-1} \frac{Ca_i M^{m_1} (-1)^j C_{m_2-1}^j}{2\rho_2^{m_2}\Gamma(m_1)(M-M_i)^{m_1+j}} \times z^{m_2-j-1} \exp(-M_i z) \gamma[m_1+j, (M-M_i)z] \,, \tag{23}$$

where $\gamma(*,*)$ is the lower incomplete gamma function defined in [20, eq.(8.350.1)].

With the help of the series expression of $\gamma(*,*)$ defined in [20, eq. (8.352.1)], and similar to the Equation (20), after applying some algebraic manipulations, the closed-form expression of $\overline{C_3}$ when $M \neq M_i$ can be written as:

$$\overline{C_3} = \sum_{i=1}^{N}\sum_{j=0}^{m_2-1}\frac{Ca_i M^{m_1}(-1)^j C_{m_2-1}^{j}\Gamma(m_1+k)}{2\rho_2^{m_2}\Gamma(m_1)(M-M_i)^{m_1+j}}\times \left(\Im_{m_2-j}(M_i)-\sum_{k=0}^{m_1+j}\frac{(M-M_i)^k}{k!}\Im_{m_2+k-j}(M)\right). \quad (24)$$

When $M = M_i$, by using equation (3.191.1) in [20], (22) can be rewritten as:

$$f_z(z) = \sum_{i=1}^{N}\frac{Ca_i M^{m_1}\Gamma(m_2)}{2\rho_2^{m_2}\Gamma(m_1+m_2)}z^{m_1+m_2-1}\exp(-M_i z). \quad (25)$$

Similarly, the closed-form expression of $\overline{C_3}$ when $M = M_i$ can be written as:

$$\overline{C_3} = \sum_{i=1}^{N}\frac{Ca_i M^{m_1}\Gamma(m_2)}{2\rho_2^{m_2}\Gamma(m_1+m_2)}\Im_{m_1+m_2}(M_i). \quad (26)$$

Finally, by substituting Equations (20), (21) and (24) or Equation (26) into Equation (19), we can obtain the exact closed-form expression for the ergodic capacity of the dual-hop system over mixed Nakagami-m/MG fading channels.

3.6 DIVERSITY ORDER ANALYSIS

Performance results obtained for OP and SER expressions in Equations (12) and (17) do not reveal any information about the diversity order and array gain of the relaying system. Therefore, we first try to find the upper bound of the OP and SER performance in this section, and then obtain their approximate performance in the high SNR region. Recently, in order to simplify the performance analysis of Equation (2) over Nakagami-m, Weibull and KG fading, its looser upper bound is often adopted in many recent literatures as [3]

$$\gamma_{SRD} < \gamma_b = \min(\gamma_1,\gamma_2). \quad (27)$$

The physical interpretation of the upper bound SNR in (27) is that at high SNR region, the hop with the weaker SNR determinates the end-to-end system performance. This upper bound has been shown to be accurate enough at high SNR region. Based on (27), the OP of the dual-hop system can be expressed as:

$$P_{out-\gamma_b} = F_{\gamma_1}(\gamma_{th}) + F_{\gamma_2}(\gamma_{th}) - F_{\gamma_1}(\gamma_{th})F_{\gamma_2}(\gamma_{th}), \quad (28)$$

where $F_{\gamma_i}(*)$ (i=1,2) is the CDF of the i^{th}-hop link, and can be found using Equations (4) and (7). Substituting them into Equation (28) the following equation is received:

$$P_{out-2} = 1-\sum_{i=1}^{N}\frac{Ca_i}{2b_i^{m_2}\Gamma(m_1)}\Gamma(m_1, M\gamma_{th})\Gamma(m_2, M_i\gamma_{th}). \quad (29)$$

Although Equation (12) is valid and accurate and Equation (29) is its upper bound, they are too complicated to provide a clear insight about the diversity order and array gain of the system. Thus, in what follows, we derive the approximate OP expression at high SNR to reveal them for the dual-hop relaying system.

Since the values of $F_{\gamma_i}(*)$ in Equation (17) range between 0 and 1, the product of these two CDFs will be much less than their addition when $\rho_i\to\infty$. Hence, by neglecting the product term in Equation (28) and using the series expression of $\Gamma(*;*)$ defined in [20, eq. (8.354.2)] in high SNR region, we can derive an approximating OP when $\rho_1=\rho_2=\rho\to\infty$, as:

$$P_{out-3} \approx F_{\gamma_1}(\gamma_{th}) + F_{\gamma_2}(\gamma_{th}) \approx A(\gamma_{th}/\rho)^t + O[(\gamma_{th}/\rho)^{t+1}], \quad (30)$$

where $O[|x|^{t+1})]$ represents the terms of order higher than t, $t=\min(m_1, m_2)$,

$$A = \begin{cases} \sum_{i=1}^{N}\frac{Ca_i}{2m_2} & m_1 > m_2 \\ \frac{(m/\Omega_1)^m}{m\Gamma(m)} + \sum_{i=1}^{N}\frac{Ca_i}{2m} & m_1 = m_2 = m \\ \frac{(m_1/\Omega_1)^{m_1}}{m_1\Gamma(m_1)} & m_1 < m_2 \end{cases}.$$

Similarly, by setting $\gamma_{th}=x$ in Equation (29) and substituting Equation (29) into Equation (16), the upper bound expression of SER can be obtained as:

$$P_{s-2} = \frac{a}{2} - \sum_{i=1}^{N}\sum_{j=0}^{m_1-1}\sum_{k=0}^{m_2-1}\frac{a\sqrt{b}Ca_i\Gamma(m_2)\Gamma(0.5+j+k)M^j}{4\sqrt{\pi}\,j!k!\rho_2^k(b_i)^{m_2-k}\xi(i)^{0.5+j+k}}. \quad (31)$$

To find the asymptotic analysis of the SER, P_s can be expressed as:

$$P_{s-3} = (G_a\rho)^{-G_d}, \quad (32)$$

where G_a and G_d are the array gain and diversity order, respectively. By substituting Equation (30) into Equation (16), the diversity order of the dual-hop system can be given as $G_d = \min(m_1, m_2)$ when $\rho_1=\rho_2=\rho\to\infty$, the array gain can be expressed as:

$$G_a = \begin{cases} \left(\frac{a\Gamma(m_1+1)(m_1/\Omega_1)^{m_1}}{2m_1\sqrt{\pi}\Gamma(m_1)b^{m_1}}\right)^{-1/m_1} & m_1 < m_2 \\ \left(\frac{a\Gamma(m+1)(m/\Omega_1)^m}{2m\sqrt{\pi}\Gamma(m)b^m} + \sum_{i=1}^{N}\frac{aCa_i\Gamma(m+0.5)}{4m\sqrt{\pi}b^m}\right)^{-1/m} & m_1 = m_2 = m \\ \left(\sum_{i=1}^{N}\frac{aCa_i\Gamma(m_2+0.5)}{4m_2\sqrt{\pi}b^{m_2}}\right)^{-1/m_2} & m_1 > m_2 \end{cases}. \quad (33)$$

For Equations (30) and (32), we can obtain the diversity order of the dual-hop system is min(m_1,m_2), which implies that the weaker hop dominates the system performance.

4 Numerical and simulation results

In this section, we present some numerical and simulation results to evaluate the performance of the dual-hop system in mixed fading channels, where the S-R link is subject to the Nakagami-m fading, R-D link is subject to the MG fading, vice versa.

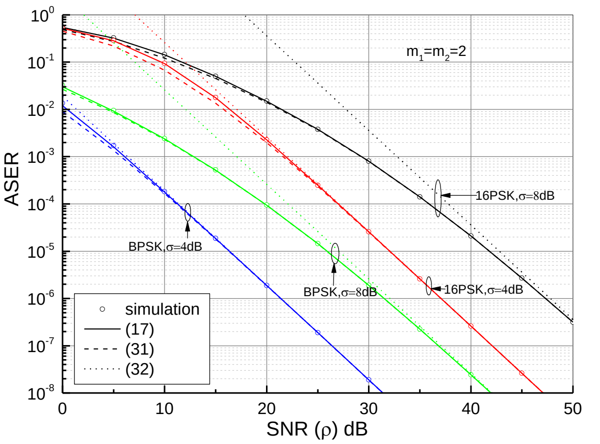

FIGURE 1 ASER of BPSK and 16PSK for the dual-hop system versus the unfaded SNR (ρ)

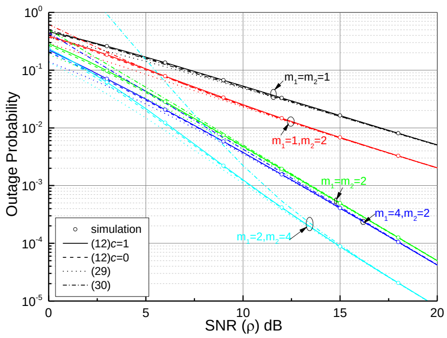

FIGURE 2 Outage probability for the dual-hop system versus the unfaded SNR (ρ)

Figure 1 illustrates the ASER of BPSK and 16PSK of the dual-hop variable gain system. The analytical results in Equations (17), (31), (32) and the simulation results (c=1) are given, respectively. In this case, a symmetric network geometry is assumed, this is, $d_0=1$, $d_1=d_2=0.5$, $\varepsilon=4$, $\rho_1=\rho_2=\rho$. Each hop has the same fading parameters (m_1=m_2=2), N =10 for MG distribution. It can be seen from Figure 1 that the analytical results in Equation (17) has almost the same as the ones in Equation (31) in high SNR region, only a small gap in low SNR region. At the same time, the analytical results of Equation (17) coincide perfectly with the simulation results. The slops of approximate performance in Equation (32) show agreement with Equation (17) in high SNR region. As expected, the system performance is degraded when the shadow deviation increases, and the performance of BPSK outperforms that of 16PSK at the same channel conditions.

The analysis and simulation results of OP are shown in Figure 2, where d_1=d_2=0.5, σ=4dB, ρ_1=ρ_2=ρ and $N=10$ for MG distribution. The analytical results in (12), (29), (30) and the simulation results are also given, respectively. As expected, these results are similar as that in Figure 1. Moreover, for discussing their diversity orders, it can be seen the diversity order of the dual-hop is determined by the minimum value between m_1 and m_2. It can be also seen that the fading parameter value of the second-hop increases, the array gain of the dual-hop system is improved. This is due to the fact that the system performance is determined by the weaker hop (i.e., the second-hop, σ=4dB).

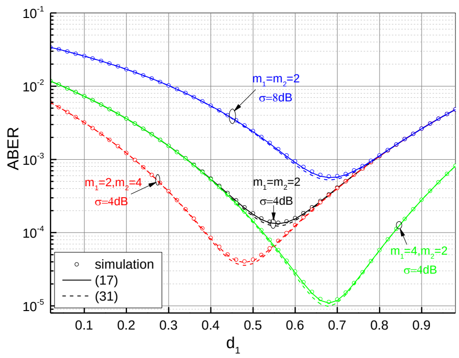

FIGURE 3 ABER of BPSK for the dual-hop system versus d_1

We show the effect of the relay location on the ABER of BPSK for the dual-hop system in Figure 3. In this section, the asymmetric network geometry is examined where R is moved on a straight line between S and D, d_1 denotes the distance between S and R. Each hop has different fading parameters, ρ_1=ρ_2=10dB, N =10 for MG distribution. It can be seen from Figure 3 that the optimum performance of the dual-hop system moves with the channel parameters of the weaker hop. When R is closer to S, the system performance is determined by the channel condition of the second hop, for example, if the fading parameter increases (m_2=2→m_2=4), the performance is improved and the location of the optimum performance moves toward S, and if the shadow deviation increases (σ=4dB→σ=8dB), the performance is degraded. When R is closer to D, the system performance is determined by the channel condition of the first hop. If the fading parameter increases (m_1=2→m_1=4), the performance is improved and the location of the optimum performance moves toward D. It can also be explained that they show the same performance when R is closer to D if only the channel conditions of the second-hop change. These results are helpful to the selection of relaying nodes in cooperative networks. Moreover, we also show the comparisons among Equations (17), (31) and the simulation results. It is clear that the difference between Equations (17) and (31) is small except the nearby region of the optimum performance. At the same time, the simulation results show agreement with Equation (17).

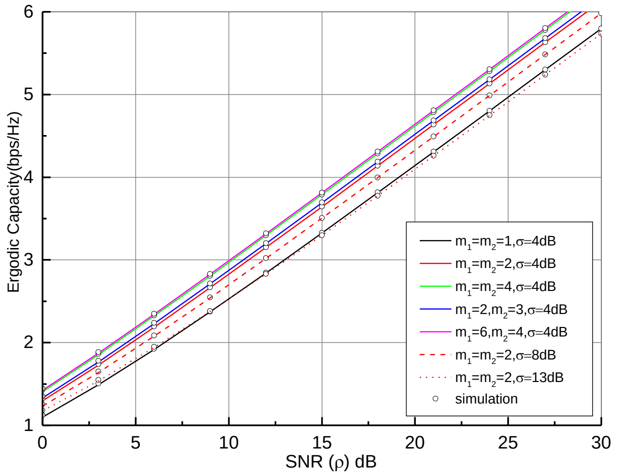

FIGURE 4 Ergodic capacity for the dual-hop system versus the unfaded SNR (ρ) under different fading parameters

5 Conclusion

In this paper, we investigated the end-to-end performance of a dual-hop AF relaying system over mixed multipath/ shadowing fading environment, where the composite NL distribution is approximated by using mixture gamma distribution. Based on the CDF of the end-to-end SNR, some novel closed-form expressions of the average end-to-end SNR, the OP, the ASER and the ergodic capacity for the dual-hop AF system are derived, respectively. And then, some approximate analysis and the diversity order are found in high SNR region. Finally, we showed numerical and simulation results to verify the accuracy of the analytical results, and discussed the effect of the location of relaying node on the performance of the dual-hop system. These results are helpful to the selection of relaying nodes in cooperative networks. Furthermore, these works in this paper can be helpful to analyse the performance of cooperative relaying systems with co-channel interference over composite fading channels by using MG fading model in the future.

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