Seismic signals wavelet packet de-noising method based on improved threshold function and adaptive threshold

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Abstract

Wavelet analysis is one of the effective method to improve the signal to noise ratio and resolution of seismic data, a wavelet packet seismic signal denoising method based on a new threshold function and adaptive threshold was put forward according to the distortion problem of traditional threshold function denoising method, which make up the defects of traditional thresholding method. Wavelet packet decomposition techniques was used for seismic wave signal denoising processing, and the synthetic seismic signals and actual seismic data was done wavelet packet decomposition processing through MATLAB, better removing high frequency random noise to retain the useful signals. Experimental results showed that the method can effectively remove noise and improve seismic resolution, with better denoising effect.

Keywords: seismic signal, adaptive threshold, denoising, resolution

1 Introduction

In seismic exploration, the signals-noise-ratio of seismic data will directly affect the reliability of seismic data, the accuracy of parameter extraction and the resolution enhancement effect etc. Therefore, denoising research plays a very important role in seismic data processing. In the high-resolution seismic exploration, seismic records have very wide frequency band, in the frequency range of significant wave may contain surface wave, random noise interference noise, which makes effective separation of signal to noise more important. Currently in seismic data processing the de-noising method are more, but each de-noising method has its own applicable conditions. Wavelet transform technique as a new approach has been widely applied to the signal denoising because of its time-frequency analysis and multi-resolution and other characteristics will have a good research value and application prospect when applied to the seismic signal noise elimination treatment. Wavelet packet transform [3-4] is the promotion of the wavelet transform, is a more detailed analysis and reconstruction methods. It divides the band by multiple-levels, further divide the high frequency portion which wavelet analysis cannot be subdivided and can adaptively select the appropriate frequency band so as to match the signal spectrum according to the characteristics of the analysed signals, thereby improving time-frequency resolution.

A wavelet packet seismic signal denoising method based on improving threshold function and adaptive threshold was presented for a more sophisticated decomposition to the low-frequency and high-frequency of seismic signals, wavelet packet coefficients derived were processed by the improved refinement threshold function and self-adaptive thresholding, which could effectively remove the noises, reduce the useful signals removal. Synthetic seismogram denoising and real seismic signal de-noising was achieved by simulation. Studies show that the improved threshold function and adaptive threshold wavelet packet seismic signal denoising method is very effective in denoising, have got better denoising effect.

2 Wavelet packet transform

2.1 WAVELET PACKET DEFINITION

In the multi-resolution analysis, $L^2(R) = \bigoplus_{j\in Z} W_j$ show multi-resolution analysis is that the Hilbert space $L^2(R)$ is decomposed to subspaces $W_j (j \in Z)$ orthogonal according to a different scale factor j. In which, W_j is the closures for the wavelet function $\varphi(t)$ (wavelet subspace). The wavelet subspace W_j was done frequency subdivision as binary fractions, in order to achieve the purpose of improving the frequency resolution. The scale subspace V_j and wavelet subspace W_j characterize uniformly with a new subspace U_j^n, and order

$$\begin{cases} U_j^0 = V_j \\ U_j^1 = W_j \end{cases}, (j \in Z) . \tag{1}$$

The Hilbert space orthogonal decomposition $V_{j+1} = V_j \oplus W_j$ could unify using U_j^n decomposition:

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$$U_{j+1}^0 = U_j^0 \oplus U_j^1, j \in Z . \quad (2)$$

Define subspace U_j^n is the closure space of function $u_n(t)$, U_j^{2n} is the closure space of $u_{2n}(t)$ and order $u_n(t)$ satisfies the following two-scale equation:

$$\begin{cases} u_{2n}(t) = \sqrt{2}\sum_{k\in Z} h(k)u_n(2t-k) \\ u_{2n+1}(t) = \sqrt{2}\sum_{k\in Z} g(k)u_n(2t-k) \end{cases}, \quad (3)$$

where $g(k) = (-1)^k h(1-k)$, that is two coefficients have orthogonal relationships. When $n = 0$, the above equation becomes

$$\begin{cases} u_0(t) = \sum_{k\in Z} h_k u_0(2t-k) \\ u_1(t) = \sum_{k\in Z} g_k u_0(2t-k) \end{cases}, \quad (4)$$

$u_0(t)$ and $u_1(t)$ are respectively scale function $\phi(t)$ and wavelet function $\varphi(t)$, Equation (4) is the equivalent representation of Equation (2). This equivalent representation were extended to $n \in Z_+$ (non-negative integer), that is equivalent representation of Equation (3)

$$U_{j+1}^n = U_j^{2n} \oplus U_j^{2n+1} \qquad j \in Z, n \in Z_+ . \quad (5)$$

This allow the wavelet subspace further subdivided as binary:

$$\begin{aligned} & W_j = U_j^1 = U_{j-1}^2 \oplus U_{j-1}^3 \\ & U_{j-1}^2 = U_{j-2}^4 \oplus U_{j-2}^5, U_{j-2}^5 = U_{j-2}^6 \oplus U_{j-2}^7 \\ & \ldots \\ & W_j = U_{j-k}^{2^k} \oplus U_{j-k}^{2^k+1} \oplus \ldots \oplus U_{j-k}^{2^k+m} \oplus \ldots \oplus U_{j-k}^{2^{k+1}-1} \end{aligned}, \quad (6)$$

in which orthonormal basis $\{2^{(j-k)/2}\varphi_{2^k+m}(2^{j-k}t-l); l \in z\}$ are called wavelet packet corresponding to $k = 1,2,\ldots j; j = 1,2\ldots$ and subspaces $U_{j-k}^{2^k+m}$.

Assume $g_j^n(t) \in U_j^n$, then $g_j^n(t)$ can be expressed as $g_j^n(t) = \sum_l d_l^{j,n} u_n(2^j t - l)$. Wavelet packet decomposition are that $\{d_l^{j,2n}\}$ and $\{d_l^{j,2n+1}\}$ are obtained by $\{d_l^{j+1,n}\}$, that is

$$\begin{aligned} & d_l^{j,2n} = \sum_k a_{k-2l} d_k^{j+1,n} \\ & d_l^{j,2n+1} = \sum_k b_{k-2l} d_k^{j+1,n} \end{aligned}. \quad (7)$$

Wavelet packet reconstruction are that $\{d_l^{j+1,n}\}$ are obtain by $\{d_l^{j,2n}\}$ and $\{d_l^{j,2n+1}\}$, that is

$$d_l^{j+1,n} = \sum_k [h_{l-2k} d_k^{j,2n} + g_{l-2k} d_k^{j,2n+1}] . \quad (8)$$

2.2 DENOISING

In seismic exploration, the general wave interference include: surface waves, high frequency random noise, side wave, multiples, etc, in which the high-frequency surface wave and random interference waves have more serious impact on the effective wave, therefore it need to reduce noise processing to extract the useful signals. For non-stationary seismic signals, the wavelet packet decomposition technique is an effective noise reduction method.

In the wavelet analysis, the signal is decomposed into low frequency portion and high frequency detail rough. However, only the low-frequency portion is used for the second layer decompose while the high-frequency part without treatment, when the scale increases frequency resolution of wavelet analysis became lower and the resolution in high frequency became poor.

Wavelet packet analysis is further promotion of the wavelet analysis, which provides a more complex and flexible analysis tools. Wavelet packet analysis subdivide the low-frequency part and high-frequency portion of last layer while the high frequency part of the signal can be described more detailed, with a more precise local analysis capabilities. Typically, wavelet packet noise reduction steps [5, 6].

The first was wavelet packet decomposition of the signal: Choose a wavelet and determine a wavelet decomposition level *N*, then do *N*-layer wavelet packet decomposition on the seismic waves signal *S*. Described with a three-tier decomposition, the wavelet packet decomposition tree were shown in Figure 1. In Figure 1 *A* represented a low frequency, *D* represented high frequency, the end serial number represented wavelet packet decomposition layers, namely the scale number. The original signal *S* is equivalent to:

$$\begin{aligned} & S = AAA3 + DAA3 + ADA3 + DDA3 + \\ & AAD3 + DAD3 + ADD3 + DDD3 \end{aligned}. \quad (9)$$

The second was to calculate the optimal tree: compute the best tree for a given entropy criteria. Commonly used entropy criteria were Shannon threshold norm log energy sure and user and so on.

The third was thresholding quantization of wavelet packet decomposition coefficiences. Select a threshold and do coefficients thresholding quantization for each wavelet packet decomposition coefficients. Wavelet transform coefficient values are compared with a threshold, it is believed that the values smaller than the threshold value were generated by the noise and set to zero, the values greater than the threshold values were corresponding to the signal mutation point and retained in order to achieve the purpose of denoising. In the process of denoising there were usually three treatments: force denoising the default threshold denoising given soft (or hard) threshold denoising.

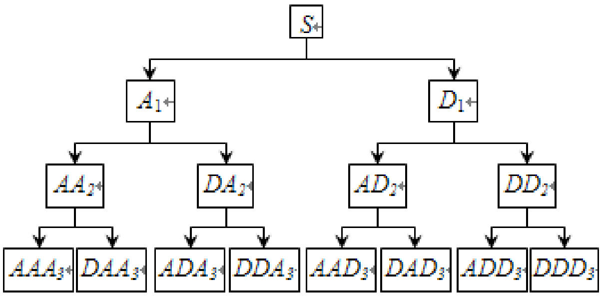


FIGURE 1 Wavelet packet decomposition technique

Soft (or hard) threshold denoising [7, 8] are most commonly used, but there is distortion.

The fourth was signal reconstruction: Do wavelet packet reconstruction on signals according to the L layer wavelet packet decomposition low-frequency coefficients of the original signal and high frequency coefficients after threshold quantization processing.

3 Selection of wavelet and threshold functions

3.1 DETERMINATION OF THE BEST WAVELET PACKET BASIS (CALCULATION OF OPTIMAL TREE)

A signal with length of $L = 2^N$ could have 2^N kinds of different signal decomposition method, while the number of full two forks trees with depth of N are 2^N. This number is too large to enumerate every situation, and the minimum entropy criterion can be obtained by an optimal signal decomposition method.

There are several types of traditional standards based on entropy: Shannon entropy, threshold entropy, norm entropy, logenergy entropy, sure entropy and user entropy and so on. Shannon entropy is defined as follows:

$$El(s_i) = -s_i^2 \log(s_i^2) \quad 0\log 0 = 0 . \tag{10}$$

The signals were decomposed layer by layer, the entropy of each decomposition node was calculated, entropy values of a node and its child nodes were compared, the base obtaining minimum entropy is the optimal wavelet packet basis.

3.2 SELECTION OF THRESHOLD FUNCTION

Wavelet thresholding denoising was threshold processing of high frequency coefficient according to the characteristics of noise manifests high-frequency signals, if the coefficients less than the threshold it was considered by the noise and set zero, if the coefficients greater than the threshold value it correspond to the useful signals and should be retained, thereby to achieve the purpose of de-noising.

The choice of traditional threshold is the hard threshold processing and soft threshold processing. Soft threshold processing is comparing the absolute value of signals and the threshold, when the absolute value of data is less than or equal to the threshold, set it to zero, when greater than the threshold, it becomes the difference between the point with a threshold value. The hard threshold is comparing the absolute value of signals and the threshold, if less than or equal to the threshold the point was set zero, greater than the threshold value it remains unchanged.

Hard thresholding:

$$\hat{W}_{j,k} = \begin{cases} W_{j,k}, |W_{j,k}| \geq \lambda \\ 0, |W_{j,k}| < \lambda \end{cases} \tag{11}$$

Soft thresholding:

$$\hat{W}_{j,k} = \begin{cases} \operatorname{sgn}(W_{j,k})(|W_{j,k}| - \lambda), |W_{j,k}| \geq \lambda \\ 0, |W_{j,k}| < \lambda \end{cases} . \tag{12}$$

In hard threshold, $\hat{W}_{j,k}$ were discontinuous on the points of $W_{j,k} = \lambda$, which brings oscillationto the reconstructed signal; Despite the $\hat{W}_{j,k}$ calculated in soft thresholding have overall good continuity, $\hat{W}_{j,k}$ and wavelet coefficients of noise signal have constant deviation, reconstructed signal appears to be too smooth that precision declined. Because of some flaws of hard and soft threshold function itself, there is a certain deviation in reconstructed signal, it need to improve the threshold function, to reduce the deviation of the wavelet coefficients, to make it continuous in wavelet space, with higher order derivatives. So an improved threshold function to combine hard and soft thresholding was proposed [9, 10]:

$$\hat{W}_{j,k} = \begin{cases} W_{j,k} - 0.5\dfrac{\lambda^n \cdot k}{(W_{j,k})^{n-1}} + (k-1)\lambda, W_{j,k} \rangle \lambda \\ 0.5\dfrac{|W_{j,k}|^m \cdot k}{(\lambda)^{m-1}} sign(W_{j,k}), |W_{j,k}| \leq \lambda \\ W_{j,k} + 0.5\dfrac{\lambda^n \cdot k}{(W_{j,k})^{n-1}} - (k-1)\lambda, W_{j,k} \langle -\lambda \end{cases} . \tag{13}$$

In Equation (13), *m*, *n*, *k* are the adjustment factor for improving threshold function, which enhance the flexibility of the threshold function. Parameters *m* and *n* determined the form of threshold function, the parameters *k* values between zero and one, if *k* = *0*, the threshold function is equivalent to the soft threshold function, if *k* = 1, then the threshold function is equivalent to the hard threshold function. Therefore, the adjustable parameter *k* can overcome the discontinuity of hard threshold functions and the constant deviation of soft threshold function when dealing with wavelet coefficients, but also retains the original advantages of the soft and hard threshold. Improved threshold function have infinite order continuous derivative, which provides the foundation for selecting wavelet adaptive threshold.

4 Selection of adaptive threshold

Traditional threshold function will produce the phenomenon of over-kill, perform poor in practical applications. Since the noise has a negative singularity, its magnitude and dense degree decreases with the increase of scale, but the signal is the opposite. With the increase of the scale, amplitude and dense modulus degree of maxima controlled by the noise quickly reduced and amplitude and dense modulus degree of maxima of the signal will be significantly increased. It can be seen that using the same scale thresholds on the same level are clearly inappropriate, because at a lower scale it will removes useful information and at the largest scale it will leave parts of the noise.

Adaptive threshold is a adaptive threshold selection algorithm using the wavelet transform coefficients of minimum risk mount as threshold. By the Baswarr theorem, the square of wavelet coefficients have energy dimension, therefore the square of decomposed wavelet coefficients are sorted, the risks are calculated according to the given threshold values, which will gets its likelihood estimation. The non-likelihood minimization is done and the selected threshold values are obtained, which is a soft threshold estimator. The specific algorithm is:

1) The square of the wavelet transform coefficient $w_{j,k}$ of each layer are arranged in ascending order, to get a vector $w=[w_1,w_2,...,w_n]$, $w_1 \le w_2 \le ... \le w_n$, wherein n is the number of wavelet coefficients.

2) Calculate the risk vector $R=[r_1,r_2,...,r_n]$, then $r_i = \left| n-2i+(n-i)w_i+\sum_{k=1}^{i} w_k \right| / n$, where r_i is the introduced risk vector elements.

The minimum r_i yielded by the above equation multiple iterations are denoted by r_0, and the corresponding w_i are denoted by w_0.

3) Calculate the threshold value

$$\lambda = \sigma(w_0)^{\frac{1}{2}}. \qquad (14)$$

5 Simulation

In order to compare the different threshold noise reduction methods, the original signals and the noisy signals are selected as the standard signal $f(n)$ and $s(n)$, the length of signals are L, the signal-noise-ratio (SNR) is defined as the Equation:

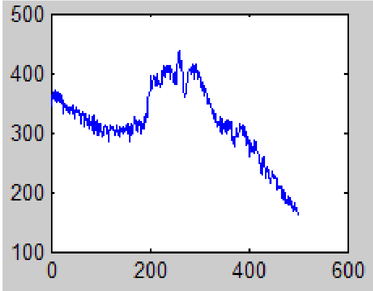
$$SNR = 10*\log\left[\frac{\sum_{i=1}^{L} f^2(i)}{\sum_{i=1}^{L}(s(i)-f(i))^2}\right](db). \qquad (15)$$

Root mean square error (RMSE) between the original signal and the estimated signals is defined as:

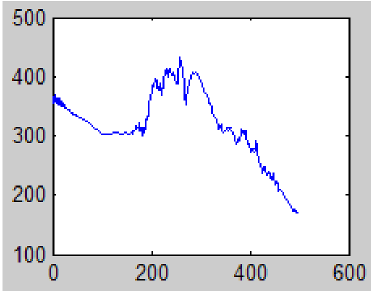
$$RMSE = \sqrt{\frac{1}{L}\sum_{i=1}^{L}(s(i)-f(i))^2}. \qquad (16)$$

5.1 DENOISING OF SIMULATION SIGNAL

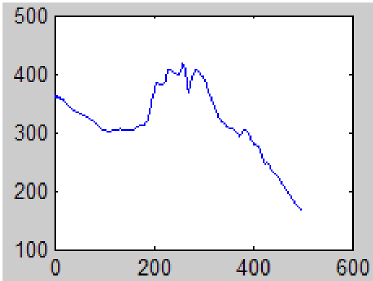
In order to verify the effect of improved threshold denoising method, leleccum signals in matlab are selected in simulation experiments, "sym5" wavelet basis are selected in wavelet packet decomposition, the maximum decomposition scale is three. Figure 2a are the original analog signals, Figure 2b are conventional wavelet thresholding waveforms, Figure 2c are the improved threshold de-noising method waveform.



a) The original signals



b) Signals of conventional wavelet thresholding denoising



c) signals of improved thresholding denoising

FIGURE 2 simulation signal denoising

As can be seen from Figure 2, the two denoising methods improve the signal to noise ratio. Most of the noise have been suppressed, but the use of the improved threshold denoising method get small distortion and better smoothness, high-frequency useful information of signals are relatively well preserved, the SNR is higher than the conventional wavelet threshold de-noising and has a good effect.

4.2 SYNTHETIC SEISMOGRAM DENOISING

Seismic data simulation was done by using MATLAB, common shot gather profiles are synthesized with a low-frequency ricker wavelet and a high-frequency ticker wavelet superimposed, specific parameters: a depth of one hundred meters, upper reflective layer velocity of two thousand meters every second, lower speed of three thousand meters every second, the minimum offset of two meters, the number of channels of sixty, a record length of one hundred and twenty millisecond. Figure 3 is the synthetic common shot gathers, Figure 4 is common shot gathers with 5db random noise, Figure 5 is the common shot gathers after the common wavelet packet denoising. Figure 6 is the common shot gathers after the improved threshold wavelet packet denoising.

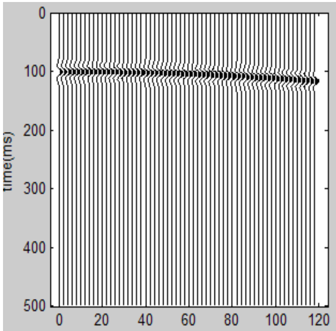


FIGURE 3 Synthetic common shot gathers

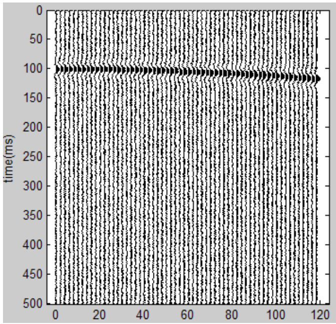


FIGURE 4 Common shot gathers with noise

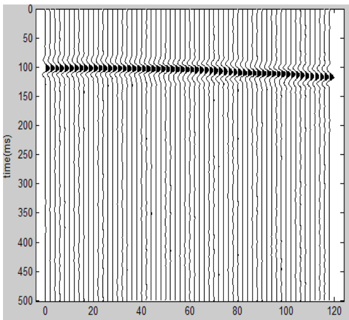


FIGURE 5 The common shot gathers after the common wavelet packet denoising

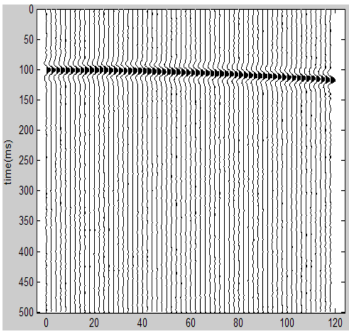


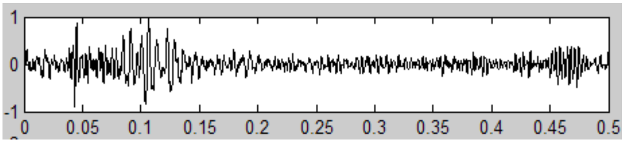
FIGURE 6 The common shot gathers after the improved threshold wavelet packet denoising

It can be seen from Figure 5 and Figure 6, when the valid signal contains high frequency components, the denoising visual effects of improved method is better than conventional wavelet thresholding denoising. The SNR of conventional wavelet thresholding and the new threshold wavelet packet denoising were 25.3018 db and 32.1148 db, SNR have also increased, SNR of improved threshold wavelet packet denoising are higher.

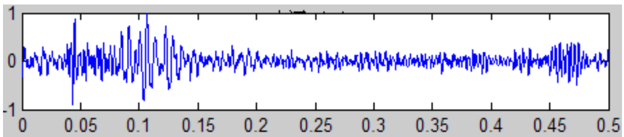
5 Seismic data denoising

The interference noise is inevitable in real seismic data acquisition and the collected data must be denoised. In the test Germany SUMMIT distributed seismograph were used for observation seismic data of a monitoring point, channels were twenty-six, channel spacing were one kilometers, offset distances (two short distances) were ten kilometers, gun dot distances were two kilometers, the detector arrays and source arrays were with one line, recording time was six seconds. Actual seismic data are generally stored in the format of segy, it need to convert, and to remove the channel header and volume header in real data processing. sym4 wavelet were selected for three times decomposition to the acquired seismic wave.

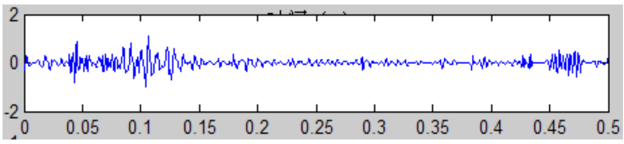
Figure 7a are part data of the original signal in one channel, Figure 7b are signals after conventional wavelet thresholding denoising, and Figure 7c are signals of improved threshold function denoising.



a) The original signals



b) The signals after conventional wavelet thresholding denoising



c) The signals of improved threshold function denoising

FIGURE 7 The common shot gathers after the improved threshold wavelet packet denoising

As can be seen from Figure 7 that most noises of seismic signals by hard threshold function denoising are filtered, which also weaken the details of the original signals, resulting in distortion, which is the results of the coefficients less than the threshold value are set to zero caused by hard threshold value function. The amplitude of soft threshold function denoising and the original seismic signals are of maximum distortion. Instead, the smoothness of seismic signals with the new threshold function denoising are best, the useful signals components are well reserved reducing the distortion. It can be seen visually seismic signals denoised by improved threshold function are closest to the original signals.

6 Conclusions

An denoising method based on improved wavelet threshold function and adaptive threshold wavelet packet seismic signals was presented. In wavelet packet decomposition process, the improved threshold function and a hierarchical adaptive threshold were selected to apply to the synthetic seismogram and real seismic records. It can be seen from the SNR improvement and simulation results, that this method can effectively remove noises from the sampled signals, keep the useful signal and improve signal to noise ratio of seismic data.

Acknowledgments

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