MEMS model order reduction method based on SPRIM

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Abstract

According to large scale MEMS united constraints equations, the paper investigates model order reduction (MOR) techniques based on Structure Preserving Reduced-order Interconnect Macro modelling (SPRIM) method, which can be used to generate computationally efficient solutions for multiphasic MEMS simulation. To united constrained model, the high dimensionality of the original system state space is mapped to a suitable low-dimensional subspace, obtained a low-dimensional state sub-space model. A improved algorithms(SPRIM) from Arnoldi algorithms are implemented to extract low dimensional Krylov subspaces from Unified state subspace models for model order reduction, reduced order electro thermal–mechanical models are generated for a MEMS micro beam using the developed programs. Developed programs automatically generate compact structure preserving models and can be used to significantly improve the computational efficiency without much loss in accuracy and model stability for coupled-field MEMS simulation.

Keywords: MEMS (Micro-Electrical Mechanical System), model order reduction, sub-space model, model reflection

1 Introduction

Micro-Electro-Mechanical Systems (MEMS) are the integration of mechanical elements, sensors, actuators, and electronics at the micron-scale or even at nano-scale through micro fabrication technology, which involves the mechanical, electronic, fluid, thermal, optical, magnetic and other disciplines [1]. Most micromechanical devices have some form of nonlinearity, either geometric nonlinearity due to large amplitude deflections or intrinsic nonlinearities due to governing equations, Multi-energy domain coupling effect of MEMS can hardly be described perfectly, because in a micron-scale or even in a nano-scale working room, physical quantities of energy among the different domains interact with each other. So, in MEMS's design process, the comprehensive effects which have been made by a variety of physical phenomena must be taken into consideration. Most MEMS devices are modelled using fully meshed finite-element based dynamic models, such models often require unacceptably long simulation time. As a result, fully meshed dynamical simulation can be computationally lunation time, and as a result, fully meshed dynamical simulation can be computationally infeasible in a typical workstation environment. Such models need to reduce the computation cost without comprising accuracy, thus allowing hundreds or thousands of dynamical simulations necessary to study how a MEMS device actually functions under various input excitations in a reasonable time.

The paper investigates model order reduction (MOR) techniques based on Structure Preserving Reduced-order Interconnect Macro modelling (SPRIM) method, which can be used to generate computationally efficient solutions for multi-physics MEMS simulation. We organized the paper as 4 parts:

1) the preview of MOR method;
2) the principle of SPRIM;
3) the detailed example of MOR based on SPRIM;
4) an application example.

2 Previous work

The basic idea of MOR method of system state space node variable mapping transform based on the PDES dynamics system is controlled by the spatial discretization, get a great degree of freedom of ODES, then using the coordinate transformation method transform the large-scale ODES system into a system with approximation the low dimension ODES, the dynamic characteristic of the approximate system can depict the original system without sacrificing too much precision condition, so as to achieve the state vector of the aim of reducing the number of degree of freedom. Many existing MOR algorithms are basically the reduction of a linear system model, for nonlinear systems, usually use MOR in the vicinity of the operating point linearization, there are three type of the MOR method in general [2-4]:

1) Based on Krylov approximation subspace or moment matching;

2) Based on balanced truncation method (TBR) with approximate Hankel norm;

3) Combined with singular value decomposition (SVD) and the Krylov subspace iterative method. The transient analysis approximate effect of Krylov subspace methods is good,

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and its operation is relatively simple, Krylov methods construct a reduced model based on either explicitly or implicitly approximating the transfer function of a full scale model. Asymptotic waveform evaluation (AWE) is an explicit method that has been used to create reduced models; however, AWE has been shown to be a numerically unstable process. The Arnoldi and Lanczos algorithms are implicit methods that generate Krylov subspaces and reduce models by projection onto these subspaces [5, 6]. The Arnoldi and Lanczos algorithms method, which have been widely used to do model order reduction for VLSI interconnects, is used to generate reduced-order models for a linearized fixed-fixed beam microstructure, the method has been proved to be very efficient to generate reduced-order models when the device is operated in the linear regime. However, when the device undergoes large amplitude deflection, the linearized model deviates from the original nonlinear model significantly, suggesting that nonlinear reduced-order models are needed [7, 8]. To solve this problem, the paper presents a new technique by combining the Taylor series expansion and an improve Arnoldi method - SPRIM method, to develop accurate reduced-order models for nonlinear MEMS devices. The main idea of the approach is to devise a state-variable transformation operation from the linearized device system, and use this state-variable transformation operation to project a high-dimensional device nonlinear state-space model onto a low-dimensional form. Simulation of a fixed-fixed electrostatic actuated beam device shows that the reduced-order nonlinear models can accurately capture the original device nonlinear behavior over a much larger range of device deformation than the conventional linearized model.

3 Structure preserving reduced-order interconnect macro modeling (SPRIM) method

Krylov subspace model order reduction method under the condition of certain can maintain the stability of the original system and passive, so to improve the traditional Krylov subspace methods, makes some important attributes order reduction system can keep the original system. Krylov subspace projection methods extract low dimensional Krylov subspaces from models described by ODEs with the desired model reduction achieved by projection of the models onto the subspaces while dynamic characteristics are maintained through a property called moment matching. To illustrate the MOR formulation procedure and moment matching we will take the first order system shown in Equation (1) as an example and rewrite it as follows:

$$\begin{cases} E\dfrac{dx(t)}{dt} = Ax(t) + Bu(t) \\ y(t) = B^T x(t) \end{cases}, \tag{1}$$

where $E = \begin{bmatrix} M & 0 \\ 0 & H \end{bmatrix} \in R^{n\times n}$, $A = \begin{bmatrix} N & F \\ -F^T & 0 \end{bmatrix} \in R^{n\times n}$, $B = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \in R^{n\times P}$, M, N and B_1 has the same number of rows, M, H and N is positive semi-definite matrix, at the same time, a given point, as well as the positive integer, specific algorithm is:

Step 1: let: $G = (S_0 E - A)^{-1}$, $Q = (s_0 E - B)^{-1}$, use Arnoldi block method compute matrix $\hat{V}$, which satisfies: colspan$\{\hat{V}\} = K_r(G:Q)$;

a) given matrix $A \in R^n, B \in R^{n\times p}$ and QR decomposition of matrix B, let $B = V_0 T$;

b) Compute: $\hat{V}_1 = AV_0 - V_0 H_{00}$, where, $H_{00} = V_0^T AV_0$, then QR decomposition $\hat{V}_1$, we can get: $\hat{V}_1 = V_1 H_{10}$;

c) Compute: $\hat{V}_2 = A_0 V_1 - V_1 H_{11} - V_0 H_{01}$, where: $H_{11} = V_1^T AV_1$, $H_{01} = V_0^T AV_1$, then QR decomposition $\hat{V}_2$, we can get $\hat{V}_2 = V_2 H_{21}$;

d) Then compute following parameter intron: $\hat{V}_r = AV_{r-1} - V_{r-1}H_{r-1,r-1} - \ldots - V_1 H_{1,r-1} - V_0 H_{0,r-1}$, where: $H_{r-1,r-1} = V_{r-1}^T AV_{r-1}, \ldots, H_{1,r-1} = V_1^T AV_{r-1}$, $H_{0,r-1} = V_0^T AV_{r-1}$ and QR decomposition $\hat{V}_r$, we can get $\hat{V}_r = V_r H_{r,r-1}$.

Step 2: According to the characteristics of block structure matrix E and A matrix to do division $\hat{V}$: $\hat{V} = [V_1^T, V_2^T]$, and constructing the corresponding matrix $V = diag\{V_1, V_2\}$.

Step 3: let $\tilde{M} = V_1^T MV_1$, $\tilde{H} = V_2^T HV_2$, $\tilde{N} = V_1^T NV_1$, $\tilde{F} = V_2^T FV_2$, $\tilde{B} = V_1^T B_1$, we get the reduced order system:

$$\begin{cases} \tilde{E}\dfrac{d\tilde{x}(t)}{dt} = \tilde{A}\tilde{x}(t) + \tilde{B}u(t) \\ \tilde{y}(t) = \tilde{B}^T \tilde{x}(t) \end{cases}, \tag{2}$$

where: $\tilde{E} = \begin{bmatrix} \tilde{M} & 0 \\ 0 & \tilde{H} \end{bmatrix} \in R^{n\times n}$, $\tilde{A} = \begin{bmatrix} \tilde{N} & \tilde{F} \\ -\tilde{F}^T & 0 \end{bmatrix} \in R^{n\times n}$, $\tilde{B} = \begin{bmatrix} \tilde{B}_1 \\ 0 \end{bmatrix} \in R^{n\times P}$.

4 Application example

In order to illustrate the model reduction technique, we give an example of the well-known beam structure in a electrical environment. Figure 1 shows the beam structure, When a voltage is applied, the top plate of the structure bends downward due to the resultant electrostatic force. Also when then beam bends, the pressure distribution of the ambient air under the beam increases. This pressure increase produces a backward pressure force that damps the beam motion. The beam structure has been used in many sensor applications.

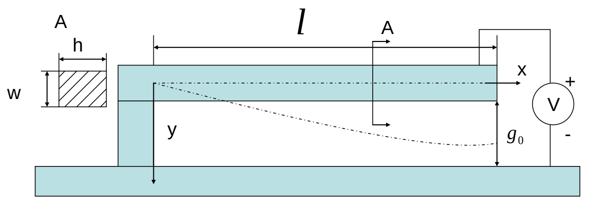

FIGURE 1 Single-ended fixed electrostatic actuating micro beam structure

The beam can be modeled by coupling the Euler beam equation with the electrostatic force and the Reynolds squeeze-film damping equation as:

$$\begin{cases} EI\dfrac{\partial^4 y}{\partial x^4}-s\dfrac{\partial^2 y}{\partial x^2}=-\dfrac{\xi_0\omega V^2}{2y^2}+\int_0^\infty\left(p-p_a\right)dy-\rho\dfrac{\partial^2 y}{\partial t^2} \\ \nabla\cdot\left(y^3 p\nabla p\right)=\dfrac{12\mu}{1+6k}\dfrac{\partial\left(py\right)}{\partial t}\end{cases}, \quad (3)$$

where: $y(x,t)$ is the immunity of the z direction, E is young's modulus, $I=\dfrac{wh^3}{12}$ is the moment, ρ is the density, ε_0 is dielectric constant, p is the air damping of the pressure, p_0 is the ambient pressure, μ is the air viscosity coefficient, g is the distance from the plate under initial state, λ is air mean free path (Table 1).

TABLE 1 The parameters of Single-ended fixed electrostatic actuating micro beam structure

Parameters	Value
Length	610 μm
Width	40μm
Thickness	2.2μm
Gap	2.3μm
Young's modulus	149GPa
Density	2330kg/m3
Air viscosity	1.82×10^{-5}kg/(m·s)
Mean-free path of air	0.0064

Let: $\tilde{y}=\dfrac{y(x,t)-g}{g}$, $\tilde{p}\left(x,t\right)=\dfrac{p\left(x,t\right)-p_0}{p_0}$, substitute them into Equation (3), and in g and p_0 using Taylor series expansion, and take position, speed and pressure as state variables, like: $x=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$, we can get the micro beam for original system state equation:

$$\dot{x}=\begin{pmatrix}\dot{y}\\ \ddot{y}\\ \dot{p}\end{pmatrix}=\begin{pmatrix} x_2 \\ \left(-\dfrac{EI}{\rho}\cdot\dfrac{\partial^4 y_i}{\partial x^4}+\dfrac{s}{\rho}\dfrac{\partial^2 y_i}{\partial x^2}+\dfrac{\varepsilon_0\omega V^2}{g^3\rho}\right)x_1+\dfrac{p_0}{g\rho}\int_0^\omega x_3dy-\dfrac{3\varepsilon_0\omega V^2}{2g^3\rho}x_1^2-\dfrac{\varepsilon_0\omega V^2}{2g^3\rho} \\ \left[\left(1+\dfrac{6\lambda}{g}\right)+\left(1+\dfrac{6\lambda}{g}\right)x^3+\left(2+\dfrac{6\lambda}{g}\right)x_1\right]\dfrac{g^2p_a}{12\mu}\nabla^2x^3-(1+x_3+x_1)\end{pmatrix}. \quad (4)$$

To solve the linearized equation numerically, the Equation (4) should be discretized in space, and then converted into state-space model. We use $(N+1)\times(M+1)$ mesh as shown in Figure 2, where N represents the number of inner grid points in the x direction and M is the number of inner grid points in the y direction. After the mesh is generated, we then project the unknowns $u\left(x,t\right)$ and $p\left(x,y,t\right)$ onto the mesh points and apply the trapezoidal rule to discretize the special integral operator and the central difference method to discretize the special derivative operators as follows:

$$\frac{\partial^2 y_i}{\partial x^2}=\frac{y_{i+1}-2y_i+y_{i-1}}{\Delta x^2}, \quad (5)$$

$$\frac{\partial^4 y_i}{\partial x^4}=\frac{y_{i+2}-4y_{i+1}+6y_i+4y_{i-1}+y_{i+2}}{\Delta x^4}. \quad (6)$$

Accelerometer beam clamped at one end, so the boundary conditions are: $y_0=0, y_{-1}=y_1$, substitute Equations (5) and (6) into Equation (4) to discrete, then map the n state vector: $X=\begin{bmatrix} y_1 & \dots & y_N & \dfrac{\partial y_i}{\partial t} & \dots & \dfrac{\partial y_N}{t} & p_{11} & \dots & p_{mn}\end{bmatrix}^T$ to the various nodes, we can convert discrete equations into n dimensional state space Equation (7), where: $n=2N+NM$.

$$\begin{cases} X(t)=Ax(t)+Bv(t) \\ y(t)=B^TX(T)\end{cases}, \quad (7)$$

where: $E=\begin{bmatrix}M & 0\\ 0 & H\end{bmatrix}\in R^{n\times n}$, $A=\begin{bmatrix}N & F\\ -F^T & 0\end{bmatrix}\in R^{n\times n}$, $B=\begin{bmatrix}B_1\\0\end{bmatrix}\in R^{n\times P}$.

$y(t)$ is the micro displacement of the beam end, $v(t)$ is the input voltage of the system.

According to the two-dimensional model establishment of beam parameters as shown in the Figure 2. The beam was divided by with 2D quadrilateral element PLANE82, and the air medium was divided with PLANE121, displacement constraints were applied to fix the micro beam anchors in ANSYS, each degree of freedom beam base and left side of beam were restricted, the finite element model as shown in the Figure 3 then applied effect of voltage on the beam structure. Figure 4 is the displacement of micro-beam.

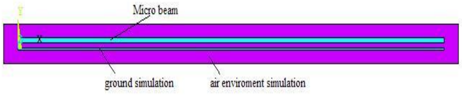

FIGURE 2 Structure Model of Micro Beam

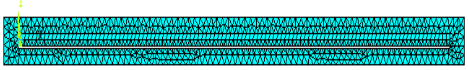

FIGURE 3 Finite Element Model of Micro Beam

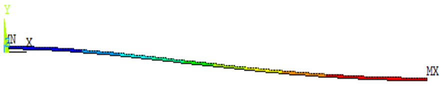

FIGURE 4 The displacement of Micro Beam

Applying the boundary conditions, the finite element model is solved can change the endpoint micro beam under different voltage conditions. The voltage load vector along with the mass and stiffness matrices were extracted and imported into Matlab. For problem simplification, viscous damping was not considered and Rayleigh damping was employed with both proportional damping constants equal to 0.25. Reduced models of size 10 and 30 were generated via the developed SPRIM reduction program. Reduced order models were numerically integrated in Matlab and reduced model solutions were expanded to full scale using Equation (8) to determine the micro beam displacement states. Algorithm stopping criteria were applied similar to those described in the electro structure results section. Figure shows the transient vertical displacement at the lower tip of the micro beam for the reduced and full models. As shown in [5], the simulation results of pull in voltages for the reduced and full models were both nearly in 8.7 V~8.8V, corresponding to relative percent errors of 2.2%.

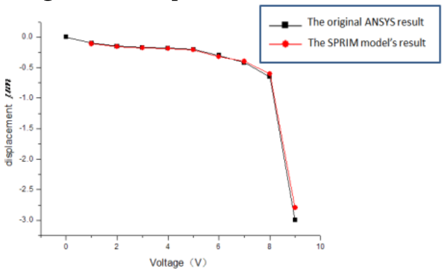

FIGURE 5 the transient vertical displacement of the micro beam for the reduced and full models

The reduced model is applied step signal under the condition of dynamic simulation, the original simulation model and reduced the response curve of the model, the simulation results are shown in Figure 6.

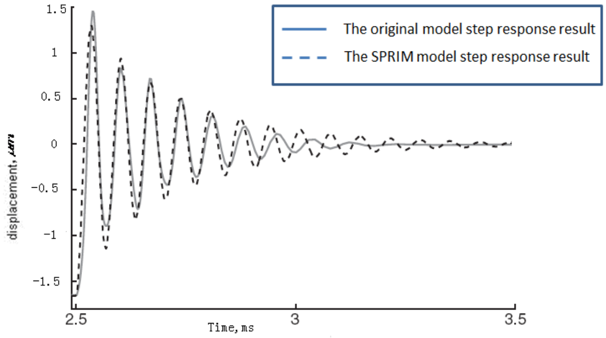

FIGURE 6 dynamic simulation comparison between the SPRIM model and original model

5 Conclusion

Model order reduction methods for coupled metaphysics FEA simulation of a common MEMS device are investigated in this paper. Simulation is performed for various sized reduced models of an electro structure actuated micro beam and compared with the full model simulation. Reduced model comparison illustrates that the reduction methods greatly increase the efficiency of FEA simulation with SPRIM models achieving simulation time reductions 84%, respectively, while SPRIM efficiency improvement was slower than PRIM model, but models experiencing relative errors of 2.2% is least. The relationship between accuracy and reduced model size is demonstrated by an increase in accuracy as model size increases.

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