A new method based on induced aggregation operator and distance measures for fuzzy decision-making

Zhihong Ma[1], Jianping Chen[2 *], Shouzhen Zeng[3]

[1] *Basic Science Department, Tianjin Agricultural University, Tianjin, 300384, China;*

[2] *School of Management, Ningbo Institute of Technology, Zhejiang University, Ningbo, 310018, China*

[3] *College of Computer and Information, Zhejiang Wanli University, Ningbo, 310015, China*

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Abstract

In this paper we introduce a new fuzzy decision making model that unifies induced aggregation operators and distance measures in the same formulation. We develop the fuzzy induced generalized ordered weighted averaging distance (FIGOWAD) operator. The main advantage of this operator is that it provides a parameterized family of aggregation operators between the minimum and the maximum and a wide range of special cases. Another advantage is that it is able to deal with the fuzzy environment where the information is very imprecise and can be assessed with interval numbers. Moreover, it uses induced aggregation operators that provide a more general representation of the attitudinal character of the decision-maker. We study some families of the FIGOWAD operators. We end the paper with an application of the new approach in a business decision-making problem about the selection of strategies.

Keywords: Fuzzy numbers, induced aggregation operator, distance measure, decision making

1 Introduction

Different types of aggregation operators are found in the literature for aggregating the information [1-3]. A very common aggregation method is the ordered weighted averaging (OWA) operator introduced by Yager [4], whose prominent characteristic is the reordering step. The OWA operator provides a parameterized family of aggregation operators that includes as special cases the maximum, the minimum and the average criteria. Since its appearance, the OWA operator has been used in a wide range of applications such as [5-24].

An interesting extension of the OWA operator is the induced OWA (IOWA) operator [25, 26]. The IOWA operator differs in that the reordering step is not developed with the values of the arguments but can be induced by another mechanism such that the ordered position of the arguments depends upon the values of their associated order-inducing variables. The IOWA operator has been studied by different authors in recent years [27-38].

A further interesting extension is the one that uses the OWA and the IOWA operator in distance measures. Recently, motivated by the idea of the OWA operator, Xu and Chen [39] defined the ordered weighted distance (OWD) measure whose prominent characteristic is that they can alleviate (or intensify) the influence of unduly large (or small) deviations on the aggregation results by assigning them low (or high) weights. Yager generalized Xu and Chen's distance measures and provided a variety of ordered weighted averaging norms, based on which he proposed several similarity measures between fuzzy sets. Merigó and Gil-Lafuente [40] introduced a new index for decision-making using the OWA operator to calculate Hamming distance called the ordered weighted averaging distance (OWAD) operator, and gave its application in the selection of financial products and sport management. Zeng and Su [41] and Zeng [42] extended Xu and Chen's result to intuitionistic fuzzy environment and presented the intuitionistic fuzzy ordered weighted distance (IFOWD) operator.

On the basis of the idea of the IOWA operator, Merigó and Casanovas [43] presented an induced ordered weighted averaging distance (IOWAD) operator that extends the OWA operator by using distance measures and a reordering of arguments that depends on order-inducing variables. The IOWAD generalizes the OWAD operator and provides a parameterized family of distance aggregation operators between the maximum and the minimum distance. Merigó and Casanovas presented an induced Euclidean ordered weighted averaging distance (IEOWAD) operator, which uses the IOWA operator and the Euclidean distance in the same formulation. Going a step further, Merigó and Casanovas introduced the induced generalized OWA distance (IGOWAD) (or induced Minkowski OWA distance (IMOWAD) operator), which generalizes the OWD measure, the OWAD operator, the IOWAD operator, the IEOWAD

* Corresponding author - Tel: +86013920939678; fax: +8602223781295;
E-mail: idmake@163.com

operator and a lot of other particular cases. It is very useful for decision-making problems because it can establish a comparison between an ideal, though unrealistic, alternative and available options in order to find the optimal choice. As such, the optimal choice is the alternative closest to the ideal one. The main advantage of the IGOWAD operator is that it is able to deal with complex attitudinal characters (or complex degrees of orness) in the aggregation process. Therefore, we are able to deal with more complex situations more close to the real world.

Usually, when using the IGOWAD operator and above distance measures, it is assumed that the available information is clearly known and can be assessed with exact numbers. However, this may not be the real-life situation found in the decision-making problems because often the available information is vague or imprecise, or it is not possible to analyse the situation with exact numbers. In this case, a better approach may be the use of fuzzy numbers (FNs). With the use of FNs, we are able to analyse the best and worst possible scenarios and the possibility that the internal values of the fuzzy interval will occur. The fuzzy numbers are highly useful in depicting uncertainty and vagueness of an object, and thus can be used as a powerful tool to express data information under various different uncertain environments, which has attracted great attentions. Thus it is necessary to extend the IGOWAD operator and above distance measures to accommodate these situations.

For doing so, in this paper we will develop the fuzzy induced generalized ordered weighted averaging distance (FIGOWAD) operator (or fuzzy induced Minkowski OWA distance (FIMOWAD) operator), which is a generalization of the OWA operator that uses fuzzy numbers, distance measures, order inducing variables and generalized means in order to provide a more general formulation. The FIGOWAD includes a wide range of distance operators such as the fuzzy maximum distance, the fuzzy minimum distance, the fuzzy normalized generalized distance (FNGD), the fuzzy weighted generalized distance (FWGD), the fuzzy generalized ordered weighted averaging distance (FGOWAD) operator, the fuzzy induced ordered weighted averaging distance (FIOWAD) operator and the fuzzy induced Euclidean ordered weighted averaging distance (FIEOWAD) operator. We study some families of the FIGOWAD operators. The main advantage of the FIGOWAD is that it is able to deal with complex reordering processes that represent a wide range of factors in an uncertain environment that can be assessed with fuzzy numbers. Then, the decision-making problem can be represented more completely because we now consider the best and worst possible scenarios. Another advantage is that it is able to deal with complex attitudinal characters (or complex degrees of orness) in the decision process by using order-inducing variables. Finally, develop an application of the new operator in a strategic group decision-making problem under fuzzy environment.

This paper is organized as follows. Section 2 presents some basic concepts. In Sect. 3, we present the FIGOWAD operator and Sect. 4 we develop an application in decision making. Finally, Sect. 5 summarizes the main conclusions of the paper.

2 Preliminaries

The distance measures are very useful techniques that have been used in a wide range of applications such as fuzzy set theory, decision-making, operational research, etc. The generalized (or Minkowski) distance is one of the most widely used distance measures which generalizes a wide range of other distances such as the Hamming distance, the Euclidean distance, etc. For two sets, $A=\{a_1,a_2,\ldots,a_n\}$ and $B=\{b_1,b_2,\ldots,b_n\}$, they can be described as follows.

Definition 1. A normalized generalized distance (NGD) of dimension n is a mapping NGD: $R^n \to R$, which has the following form:

$$NGD(A,B)=\left(\frac{1}{n}\sum_{i=1}^{n}|a_i-b_i|^{\lambda}\right)^{1/\lambda}, \tag{1}$$

where a_i and b_i is the i th arguments of the sets A and B and λ is a parameter such that $\lambda\in(-\infty,+\infty)$. If we give different values to the parameter λ, we can obtain a wide range of special cases. For example, if $\lambda=1$, we obtain the normalized Hamming distance. If $\lambda=2$, the normalized Euclidean distance.

Sometimes, when normalizing the generalized distance, we prefer to give different weights to each individual distance. In this case, the distances are known as the weighted generalized distance, which can be defined as follows, respectively:

Definition 2. A weighted generalized distance (WGD) of dimension n is a mapping WGD: $R^n \to R$ that has an associated weighting $w=(w_1,w_2,\ldots,w_n)$ with $w_j\in[0,1]$ and $\sum_{j=1}^{n} w_j=1$ such that:

$$WGD=\left(\sum_{i=1}^{n} w_i|a_i-b_i|^{\lambda}\right)^{1/\lambda}, \tag{2}$$

where a_i and b_i is the i th arguments of the sets A and B and λ is a parameter such that $\lambda\in(-\infty,+\infty)$. If $\lambda=1$, we obtain the weighted Hamming distance (WHD). If $\lambda=2$, the weighted Euclidean distance (WED).

The IOWA operator is an extension of the OWA operator. The main difference is that the reordering step is not carried out with the values of the argument a_i. In this case, the reordering step is developed with order-inducing variables that reflect a more complex reordering process. The IOWA operator also includes as particular cases maximum, minimum and average criteria. It can be defined as follows:

Definition 3. An IOWA operator of dimension n is a mapping $IOWA$: $R^n \times R^n \to R$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$IOWA(\langle u_1, a_1 \rangle, \ldots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j \ , \tag{3}$$

where b_i is a_i value of the IOWA pair $\langle u_1, a_1 \rangle$ having the j th largest u_i, u_i is the order inducing variable and a_i is the argument variable.

The IGOWAD (or IMOWAD) operator is a distance measure that uses the IOWA operator in the normalization process of the Minkowski distance. Then, the reordering of the individual distances is developed with order inducing variables. For two sets $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$, the IGOWAD operator can be defined as follows:

Definition 4. An IGOWAD operator of dimension n is a mapping $f: R^n \times R^n \times R^n \to R$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$f\left((u_1, a_1, b_1), \ldots, (u_n, a_n, b_n)\right) = \left(\sum_{j=1}^{n} w_j d_j^{\lambda} \right)^{1/\lambda} , \tag{4}$$

where d_j is the $|a_i - b_i|$ value of the IGOWAD triplet (u_i, a_i, b_i) having the j largest u_i, u_i is the order inducing variable, $|a_i - b_i|$ is the argument variable represented in the form of individual distances and λ is a parameter such that $\lambda \in (-\infty, +\infty)$. Especially, if $\lambda = 1$, then the IGOWAD is called the induced ordered weighted averaging distance (IOWAD) operator [30], and if $\lambda = 2$, then the induced Euclidean ordered weighted averaging distance (IEOWAD) operator [32].

When using the IGOWAD operator, it is assumed, that the available information is represented in the form of exact numbers. However, this may not be the real situation found in the decision-making problem. Sometimes the available information is vague or imprecise and it is not possible to analyse it with exact numbers. In this case, it is more suitable to use linguistic variables to assess the uncertainty. In the following, we shall develop the linguistic induced generalized ordered weighted averaging distance (LIGOWAD) operator.

3 Fuzzy induced generalized ordered weighted averaging distance (LIGOWAD) operator

The fuzzy numbers are highly useful in depicting uncertainty and vagueness of an object, and thus can be used as a powerful tool to express data information under various different fuzzy environments, which has attracted great attentions there are many papers concerning the fuzzy number arithmetic. For practical reasons we use, however, the notation introduced by Van Laarhoven and Pedrycz [44]. According to this notation, a triangular fuzzy number $\hat{a}$ may be expressed as following:

Definition 5. Let $\hat{a} = [a^L, a^M, a^U]$, where $a^L \le a^M \le a^U$, then $\hat{a}$ is called a triangular fuzzy number (TFN), where a^L and a^U stand for the lower and upper values of $\hat{a}$, and a^M stands for the modal value. Especially, if $a^L = a^M = a^U$, then $\hat{a}$ is reduced to a real number.

Let $\hat{a} = [a^L, a^M, a^U]$ and $\hat{b} = [b^L, b^M, b^U]$ be two triangular fuzzy numbers, below we first introduce some operational laws of triangular fuzzy numbers as follows:

(1) $\hat{a} + \hat{b} = [a^L + b^L, a^M + b^M, a^U + b^U]$;

(2) $\lambda\hat{a} = [\lambda a^L, \lambda a^M, \lambda a^U]$, where $\lambda \ge 0$. Especially, _ˆa = 0, if _ = 0.

Definition 6. Let $\hat{a} = [a^L, a^M, a^U]$ and $\hat{b} = [b^L, b^M, b^U]$ be two triangular fuzzy numbers, then:

$$d(\hat{a}, \hat{b}) = \sqrt{\frac{1}{3}\left(a^L - b^L\right)^2 + \left(a^M - b^M\right)^2 + \left(a^U - b^U\right)^2} \ , \tag{5}$$

is called a distance between $\hat{a}$ and $\hat{b}$.

The fuzzy induced generalized ordered weighted averaging distance (FIGOWAD) operator is an extension of the IGOWAD operator that uses uncertain information in the aggregation represented in the form of fuzzy numbers. The reason for using this operator is that sometimes, the uncertain factors that affect our decisions are not clearly known and in order to assess the problem we need to use fuzzy numbers in order to consider the different uncertain results that could happen in the future. Note that the FIGOWAD operator can also be seen as an aggregation operator that uses the main characteristics of the IOWA, distance measures and fuzzy numbers. Moreover, it also uses a complex reordering process by using order inducing variables. Let Ψ be the set of all triangular fuzzy numbers, for two collections of fuzzy numbers $A = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_n)$ and $B = (\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_n)$, it can be defined as follows.

Definition 7. A FIGOWAD operator of dimension n is a mapping FIGOWAD: $R^n \times \Psi^n \times \Psi^n \to R$ that has an associated weighting W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ such that:

$$FIGOWAD(\langle u_1, \hat{a}_1, \hat{b}_1 \rangle, ..., \langle u_n, \hat{a}_n, \hat{b}_n \rangle) = \left(\sum_{j=1}^{n} w_j d_j^{\lambda} \right)^{1/\lambda}, \quad (6)$$

where d_j is $d(\hat{a}_i, \hat{b}_i)$ value of the FIGOWAD pair $\langle u_1, \hat{a}_i, \hat{b}_i \rangle$ having the j th largest u_i, u_i is the order inducing variable and $d(\hat{a}_i, \hat{b}_i)$ is the argument variable represented in the form of individual distances and λ is a parameter such that $\lambda \in (-\infty, +\infty)$.

Example 1. Let $A = ([3,4,5],[6,7,9],[4,6,7],[2,4,5])$ and $B = ([4,6,7],[3,4,6],[2,5,7],[3,4,6])$ be two collections fuzzy numbers, then $d(\hat{a}_1, \hat{b}_1) = \sqrt{\frac{1}{3}(3-4)^2 + (4-6)^2 + (5-7)^2} = 2.16$.

Similarly, we have $d(\hat{a}_2, \hat{b}_2) = 3$, $d(\hat{a}_3, \hat{b}_3) = 1.29$, $d(\hat{a}_4, \hat{b}_4) = 0.82$.

Assume that both sets have the same order-inducing variables $U = (6,7,3,9)$. Assume the following weighting vector $W = (0.1, 0.2, 0.2, 0.5)$ and without loss of generality, let $\lambda = 2$, then we can calculate the distance between A and B by using the FIGOWAD operator: $FIGOWAD(A, B) = 1.98$.

From a generalized perspective of the reordering step, we can distinguish between the descending LIGOWAD (DLIGOWAD) operator and the ascending LIGOWAD (ALIGOWAD) operator by using $w_j = w^*_{n-j+1}$, where w_j is the j th weight of the DLIGOWAD and $w_j = w^*_{n-j+1}$ 1 is the j th weight of the ALIGOWAD operator.

Note that if the weighting vector is not normalized, i.e., $W = \sum_{j=1}^{n} w_j \neq 1$, then, the LIGOWAD operator can be expressed as:

$$FIGOWAD(\langle u_1, \hat{a}_1, \hat{b}_1 \rangle, ..., \langle u_n, \hat{a}_n, \hat{b}_n \rangle) = \left(\frac{1}{W} \sum_{j=1}^{n} w_j d_j^{\lambda} \right)^{1/\lambda}. \quad (7)$$

Similar to the IGOWAD operator, the FIGOWAD operator is commutative, monotonic, bounded and idempotent. Another interesting issue is the problem of ties in the order inducing variables. As it was explained by Yager and Filev, the easiest way to solve this problem consists in replacing each argument of the tied inducing variables by its linguistic normalized linguistic generalized distance.

Analysing the applicability of the FIGOWAD operator, we can see that it is applicable to similar situations already discussed in other types of induced aggregation operators where it is possible to use linguistic information. For example, we could use it in different decision making problems, etc.

The FIGOWAD operator provides a parameterized family of aggregation operators. Basically, we distinguish between the families found in the weighting vector W and those found in the parameter λ.

If we analyse the parameter λ, we can find a wide range of distance measures such as the FIOWAD, the FIEOWAD, the fuzzy induced ordered weighted geometric distance (FIOWGD) operator, the fuzzy induced ordered weighted harmonic averaging distance (FIOWHAD) operator and a lot of other cases.

Remark 1. If $\lambda = 1$, then, we get the FIOWAD operator.

$$FIGOWAD(\langle u_1, \hat{a}_1, \hat{b}_1 \rangle, ..., \langle u_n, \hat{a}_n, \hat{b}_n \rangle) = \sum_{j=1}^{n} w_j d_j^{\lambda}. \quad (8)$$

Note that if $w_j = 1/n$ for all j, we get the fuzzy normalized Hamming distance (FNHD). The fuzzy weighted Hamming distance (FWHD) is obtained if $u_i > u_{i+1}$ for all i, and the fuzzy ordered weighted averaging distance (FOWAD) is obtained if the ordered position of u_i is the same as the ordered position of d_j such that d_j is the j th largest of $d(\hat{a}_i, \hat{b}_i)$.

Remark 2. If $\lambda = 2$, then we get the FIEOWAD operator.

$$FIGOWAD(\langle u_1, \hat{a}_1, \hat{b}_1 \rangle, ..., \langle u_n, \hat{a}_n, \hat{b}_n \rangle) = \left(\sum_{j=1}^{n} w_j d_j^2 \right)^{1/2}. \quad (9)$$

Note that if $w_j = 1/n$ for all j, we get the fuzzy normalized Euclidean distance (FNED). The fuzzy weighted Euclidean distance (FWED) is obtained if $u_i > u_{i+1}$ for all i, and the fuzzy Euclidean ordered weighted averaging distance (FEOWAD) is obtained if the ordered position of u_i is the same as the ordered position of d_j such that d_j is the j th largest of $d(\hat{a}_i, \hat{b}_i)$.

Remark 3. When $\lambda = 0$, we get the FIOWGD operator.

$$FIGOWAD(\langle u_1, \hat{a}_1, \hat{b}_1 \rangle, ..., \langle u_n, \hat{a}_n, \hat{b}_n \rangle) = \prod_{j=1}^{n} d_j^{w_j}. \quad (10)$$

Remark 4. When $\lambda = -1$, we get the FIOWHAD operator.

$$FIGOWAD\left(\langle u_1, \hat{a}_1, \hat{b}_1\rangle, \ldots, \langle u_n, \hat{a}_n, \hat{b}_n\rangle\right) = \frac{1}{\sum_{j=1}^{n} \frac{w_j}{d_j}}. \quad (11)$$

By choosing a different manifestation of the weighting vector in the FIGOWAD operator, we are able to obtain different types of distance aggregation operators. For example, we can obtain the fuzzy maximum distance, the fuzzy minimum distance, the FNGD, the FWGD, the FGOWAD, the Step-FIGOWAD and the Olympic-FIGOWAD

If $w_j = 1/n$, we get the FNGD.

The fuzzy maximum distance is obtained if $w_p = 1$, $w_j = 0$, for all $j \neq p$, and $u_p = Max\{d(\hat{a}_i, \hat{b}_i)\}$.

The fuzzy minimum distance is obtained if $w_p = 1$, $w_j = 0$, for all $j \neq p$, and $u_p = Min\{d(\hat{a}_i, \hat{b}_i)\}$.

The FWGD is obtained if $u_i > u_{i+1}$ for all i.

The FGOWAD operator is obtained if the ordered position of u_i is the same as the ordered position of d_j such that d_j is the j^{th} largest of $d(\hat{a}_i, \hat{b}_i)$.

Step-FIGOWA: If $w_k = 1$ and $w_j = 0$ for all $j \neq k$.

Olympic-FIGOWAD: If $w_1 = w_n = 0$ and for all others $w_j = 1/(n-2)$.

Remark 5. Using a similar methodology, we could develop numerous other families of FIGOWAD operators. For more information, refer to [30-32].

4 Illustrative Example

The FIGOWAD operator can be applied in a wide range of problems such as statistics, engineering, economics, decision theory and clustering under fuzzy environment. In this paper, we will consider a decision-making application in the selection of strategies by using a group analysis. The process to follow in the selection of investments with the FIGOWAD operator in group decision-making is similar to the process developed in Ref. [32], [33], with the difference that now we are considering an uncertain situation where the group of experts of the company needs to assess the available information with fuzzy numbers.

Assume a company that operates in North America and Europe is analysing the general policy for the next year and they consider five possible strategies to follow (adopted from Ref. [35]):

A_1 is a computer company. A_2 is a chemical company. A_3 is a food company. A_4 is a car company. A_5 is a TV company.

In order to evaluate these strategies, the group of experts considers that the key factor is the economic situation of the next year. Thus, depending on the situation, the expected benefits will be different. The experts have considered five possible situations for the next year:

S_1 = Negative-growth rate. S_2 = Growth rate near 0. S_3 = Low-growth rate. S_4 = Medium-growth rate. S_5 = High-growth rate.

The group of experts of the company is constituted by three persons that give its own opinion about the uncertain expected results that may occur in the future. The expected results depending on the situation S_i and the alternative A_k are shown in Tables 1-3. Note that the results are TFNs.

TABLE 1 Fuzzy Payoff Matrix-expert 1

	S_1	S_2	S_3	S_4	S_5
A_1	[60,70,80]	[30,40,50]	[50,60,70]	[70,80,90]	[30,40,50]
A_2	[50,60,70]	[70,80,90]	[20,30,40]	[50,60,70]	[40,50,60]
A_3	[10,20,30]	[30,40,50]	[40,50,60]	[60,70,80]	[70,80,90]
A_4	[20,30,40]	[40,50,60]	[60,70,80]	[80,90,100]	[70,80,90]
A_5	[30,40,50]	[40,50,60]	[70,80,90]	[20,30,40]	[60,70,80]

TABLE 2 Fuzzy Payoff Matrix-expert 2

	S_1	S_2	S_3	S_4	S_5
A_1	[50,60,70]	[70,80,90]	[80,90,100]	[20,30,40]	[70,80,90]
A_2	[60,70,80]	[20,30,40]	[50,60,70]	[30,40,50]	[40,50,60]
A_3	[60,70,80]	[50,60,70]	[20,30,40]	[70,80,90]	[60,70,80]
A_4	[70,80,90]	[10,20,30]	[40,50,60]	[70,80,90]	[20,30,40]
A_5	[30,40,50]	[50,60,70]	[40,50,60]	[50,60,70]	[30,40,50]

TABLE 3 Fuzzy Payoff Matrix-expert 3

	S_1	S_2	S_3	S_4	S_5
A_1	[20,30,40]	[60,70,80]	[40,50,60]	[70,80,90]	[40,50,60]
A_2	[40,50,60]	[10,20,30]	[70,80,90]	[50,60,70]	[50,60,70]
A_3	[40,50,60]	[70,80,90]	[80,90,100]	[30,40,50]	[50,60,70]
A_4	[70,80,90]	[80,90,100]	[20,30,40]	[50,60,70]	[60,70,80]
A_5	[60,70,80]	[50,60,70]	[60,70,80]	[30,40,50]	[60,70,80]

According to their objectives, the company establishes the following collective ideal investment shown in Table 4.

TABLE 4 Ideal Strategy

	S_1	S_2	S_3	S_4	S_5
I	[80,90,100]	[80,90,100]	[80,90,100]	[80,90,100]	[80,90,100]

In order to aggregate the information, the group of experts calculates the attitudinal character of the candidate. Due to the fact that the attitudinal character depends upon the opinion of several members of the board of directors, it is very complex. Therefore, they need to use order-inducing variables in the reordering process. The results are shown in Table 5.

TABLE 5 Order-inducing Variables

	S_1	S_2	S_3	S_4	S_5
A_1	10	8	15	17	24
A_2	15	12	9	18	20
A_3	16	14	12	10	8
A_4	13	22	17	15	9
A_5	20	25	18	14	16

With this information, we can make an aggregation to make a decision. First, we aggregate the information of the three experts to obtain a unified payoff matrix. We use the FWA operator to obtain this matrix while assuming that $V = (0.3,0.3,0.4)$. The results are shown in Table 6.

TABLE 6 Collective Fuzzy Payoff Matrix

	S_1	S_2	S_3	S_4	S_5
A_1	[41,51,61]	[54,64,74]	[55,65,75]	[55,65,75]	[46,56,76]
A_2	[49,59,69]	[31,41,51]	[49,59,69]	[44,54,64]	[44,54,64]
A_3	[37,47,57]	[52,62,72]	[50,60,70]	[51,61,71]	[59,69,79]
A_4	[55,65,75]	[47,57,67]	[38,48,58]	[65,75,85]	[51,61,71]
A_5	[42,52,62]	[47,57,67]	[57,67,77]	[33,43,54]	[51,61,71]

With this information, it is possible to aggregate the available information in order to take a decision. The method consists in comparing the available investments with the ideal one by using the FIGOWAD operator and its particular cases. The optimal choice would be the alternative closest to the ideal. In this example, we consider the fuzzy maximum distance, the fuzzy minimum distance, the FNHD, the FWHD, the FOWAD, the FIOWAD, the FWED, the FEOWAD and the FIEOWAD operators. We assume the following weighting vector $W = (0.1,0.2,0.2,0.2,0.3)$. The results are shown in Table 7.

TABLE 7 Aggregated Results

	FND	FWHD	FOWAD	FIOWAD	FWED	FEOWAD	FIEOWAD
A_1	0.43	0.26	0.24	0.26	0.41	0.19	0.28
A_2	0.34	0.24	0.23	0.24	0.29	0.22	0.23
A_3	0.56	0.17	0.27	0.20	0.21	0.15	0.16
A_4	0.42	0.21	0.20	0.22	0.33	0.16	0.21
A_5	0.35	0.25	0.22	0.23	0.26	0.18	0.25

As we can see, depending on the aggregation operator used, the ordering of the strategies may be different. Therefore, the decision about which strategy select may be also different. If we establish an ordering of the investments, a typical situation if we want to consider more than one alternative, we will get the following orders shown in Table 8. Note that the first alternative in each ordering is the optimal choice.

TABLE 8 Ordering of the Strategies

	Ordering
FND	$A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$
FWHD	$A_3 \succ A_4 \succ A_2 \succ A_5 \succ A_4$
FOWAD	$A_4 \succ A_5 \succ A_2 \succ A_1 \succ A_3$
FIOWAD	$A_3 \succ A_4 \succ A_5 \succ A_1 \succ A_2$
FWED	$A_3 \succ A_5 \succ A_2 \succ A_4 \succ A_1$
FEOWAD	$A_3 \succ A_4 \succ A_5 \succ A_2 \succ A_1$
FIEOWAD	$A_3 \succ A_4 \succ A_2 \succ A_5 \succ A_1$

5 Conclusions

We have presented a wide range of fuzzy induced generalized aggregation distance operators. First, we have introduced the FIGOWAD operator. It is a generalization of the OWA operator that uses order-inducing variables in order to assess complex reordering processes, distance measure, fuzzy information and generalized means. We have analysed some of its main properties. We have seen that it generalizes a wide range of distance aggregation operators such as the FNGD, the FGOWAD and the FOWAD operator.

We have also developed an application of the new approach in a strategic decision-making problem. We have seen that the FIGOWAD is very useful because it represents very well the uncertain information by using IFNs. We have also seen that depending on the particular case of the FIGOWAD operator used the results may lead to different decisions.

In future research, we expect to develop further improvements by adding more characteristics in the model such as the use of other types of aggregation operators and apply it in other decision-making problems.

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Authors

Zhihong Ma, born in May, 1975, TianJin City, TianJin Province, P.R. China

Current position, grades: Associate Professor of the TianJin agricultural University
University studies: University Mathematics Teaching and Application of mathematical research.
Scientific interest: Study on mathematical model, data mining.
Publications: more than 20 papers published in various journals.
Experience: Graduated from Jilin University in 2000, has completed 6 scientific research projects; more than 20 papers published in various journals.

Jianping Chen, born in December, 1976, Ningbo City, Zhejiang Province, P.R. China

Current position, grades: the lecturer of the Ningbo Institute of Technology Zhejiang University.
University studies: Service Innovation, Aggregation Operators, Decision Making and Comprehensive
Scientific interest: Service Innovation, Aggregation Operators, Decision Making and Comprehensive
Publications: more than 6 papers published in various journals.
Experience: Jianping Chen has a PhD degree in management from Zhejiang Gongshang University, China. He is a full-time Lecture in Ningbo Institute of Technology Zhejiang University.

Shouzhen Zeng, born in September, 1981, Ningbo City, Zhejiang Province, P.R. China

Current position, grades: the lecturer in Zhejiang Wanli University.
University studies: Statistics Research, Knowledge-based Systems and Group decision and Negotiation
Scientific interest: Aggregation Operators, Decision Making, Comprehensive evaluation and Uncertainty
Publications: 40 papers in journals, books and conference proceedings including journals such as Statistics Research,
Experience: He graduated from Zhejiang Gongshang University and obtained the phD degree in applied statistics in 2013.