Approximation reductions in an incomplete variable precision multigranulation rough set

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Abstract

This paper deals with approaches to the granular space reductions in the variable precision multigranulation rough set model. The main objective of this study is to extend four kinds of the granular space reductions called a tolerance relations optimistic multigranulation β ower approximation distribution reduction, a tolerance relations optimistic multigranulation β upper approximation distribution reduction, a tolerance relations pessimistic multigranulation β lower approximation distribution reduction and a tolerance relations pessimistic multigranulation β upper approximation distribution reduction ,which preserve the optimistic/pessimistic multigranulation β lower/upper approximation distribution of the decision classes. Some judgement theorems are investigated. The example proves that the new variable precision multigranulation rough set model can effectively deal with incomplete information, from which we can obtain approaches to the granular space reductions of incomplete decision systems in variable precision multigranulation rough theory.

Keywords: Approximation Reduction, Tolerance Relation, Variable Precision Rough Set, Multigranulation

1 Introduction

In the past twenty years, rough set theory [1],which was first proposed by Pawlak, has been widely applied in the field of data mining [2-5], feature selection [20], machine learning, decision support [6-8], pattern recognition [9, 10] and so on. One of the important aspect of this theory is searching for particular subsets of condition granulations, which have the same information for classification purposes as all the condition granulations.

From the viewpoint of the granular computing, an equivalence relation can be regarded as a granulation. So the classical rough set can be regarded as a single granulation rough set because it is constructed on only one equivalence relation. However, in fact, we often describe an obscure concept by multiple granulations not by one single granulation for satisfying the demand of users or problem specification. Based on this need, Qian et al. proposed multigranulation rough set theory [11], in which an obscure concept is described by multiple equivalence relations on one universe. Later, some extension to multigranulation rough set has been appeared rapidly. Qian [12] proposed a multigranulation rough set model based on tolerance relation, in which an incomplete decision table was researched under multiple tolerance relations. The topological properties of multigranulation rough sets was first researched by Raghavan and Tripathy [13]. Xu et al. [14-16] developed a variable multigranulation rough set model, a fuzzy multigranulation rough set model and an ordered multigranulation rough set model. Variable Precision multigranulation rough set was proposed and has been studied by many researchers.

In recent years, from the classical rough set model to the multigranulation rough set model, more attention has been paid to the granular space reduction. In a single granulation rough set model, many types of attribute reduction have been studied. β - reduct was researched in the variable precision rough sett proposed by Ziarko [17]. The heuristic method has been applied in the attribute reduction of all the kinds of single rough set model according to different requirements. In addition, discernibility matrix is also used to eliminate the redundant condition attributes. S Kowron [18] first proposed famous knowledge reduction algorithm based on discernibility matrix. Later, many researchers have different improvements on discernibility matrix for getting better attribute reduction [21].

At present, in many kinds of multigranulation rough set model, the heuristic approach is usually used to the granular space reduction. However, the discernibility matrix is still not be researched for the granular space reduction in the context of multigranulation, which limits the development of the granular space reduction in the multigranulation rough set.

The objective of this paper is to introduce four concepts of approximation reduction named as a tolerance relations optimistic multigranulation β lower

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approximation distribution reduction, a tolerance relations optimistic multigranulation β upper approximation distribution reduction, a tolerance relations pessimistic multigranulation β lower approximation distribution reduction and a tolerance relations pessimistic multigranulation β upper approximation distribution reduction ,which preserve the optimistic/pessimistic multigranulation β lower/upper approximation distribution of the decision classes. The rest is organized as follows. Some preliminary concepts such as rough set, incomplete decision system, variable precision rough set and multigranulation rough sets are briefly reviewed in Section 2. In Section 3, the notions of a tolerance relations optimistic multigranulation β lower approximation distribution reduction, a tolerance relations optimistic multigranulation β upper approximation distribution reduction, a tolerance relations pessimistic multigranulation β lower approximation distribution reduction and a tolerance relations pessimistic multigranulation β upper approximation distribution reduction are introduced to an incomplete decision system in the content of multigranulation, and their some important properties are also obtained. In Section 4, the approaches to the four approximation reductions are provided. An illustrate example is employed to examine their validity in Section 5. In Section 6, we also compare the performances between the granular space reduction based on heuristic approach and the proposed discernibility matrix approach through experiment. Finally, Section 7 concludes this paper by giving some discussions.

2 Preliminaries

In this section, we briefly review some basic concepts such as rough set, incomplete decision system, variable precision rough set and multigranulation rough sets.

2.1 PAWLAK ROUGHT SET

Definition 1. [1] Let $K=(U,R)$ be an approximation space, where U is a finite and nonempty set of objects called the universe, and $R\subseteq U\times U$ is an equivalence relation on U . U/R denote the partition of U induced by R, which is the set of equivalence classes generated by R .For any set $X\subseteq U$, the lower approximation $\underline{R}(X)$ and the upper approximation $\overline{R}(X)$ are defined by [12]:

$$\underline{R}(X)=\bigcup\{[x]\in U/R \mid [x]_R\subseteq X\}, \quad (1)$$

$$\overline{R}(X)=\bigcup\{[x]\in U/R \mid [x]_R\cap X\neq\varnothing\}. \quad (2)$$

2.2 TOLERANCE RELATION

Definition 2. [19] Let $I=(U,AT,V,f)$ be an incomplete information system, $B\subseteq AT$ an attribute set. We denote some null value by *, a tolerance relation on U is defined as follows:

$$TOL(B)=\left\{\begin{matrix}(x,y)\in U\times U:\forall a\in B,a(x)\\ =a(y)\vee a(x)=*\vee a(y)=*\end{matrix}\right\}. \quad (3)$$

2.3 VARIABLE PRECISION ROUGH SET

Definition 3. [17] Let U be a finite nonempty universal set, for two arbitrary nonempty sets X and Y, the relative degree of misclassification of the set X with respect to the set Y is defined as follows:

$$c(X,Y)=\begin{cases}1-card(X\cap Y)/card(X), & card(X)>0\\ 0, & otherwise\end{cases}, \quad (4)$$

where card denotes set cardinality.

2.4 MULTIGRANULATION ROUGH SET

2.4.1 Optimistic multigranulation rough set

Definiton 4. [11] Let $I=(U,AT,V,f)$ be an information system, in which $A_1,A_2,\cdots,A_m\subseteq AT$, and $X\subseteq U$. The optimistic multigranulation lower and upper approximations are denoted by $\underline{\sum_{i=1}^{m}A_i}^{O}(X)$ and $\overline{\sum_{i=1}^{m}A_i}^{O}(X)$, respectively,

$$\underline{\sum_{i=1}^{m}A_i}^{O}(X)=\left\{x\in U:[x]_{A_1}\subseteq X\vee[x]_{A_2}\subseteq X\vee\cdots\vee[x]_{A_m}\subseteq X\right\}, \quad (5)$$

$$\overline{\sum_{i=1}^{m}A_i}^{O}(X)=\sim\left(\underline{\sum_{i=1}^{m}A_i}^{O}(\sim X)\right), \quad (6)$$

where $[x]_{A_i}$ $(i=1,2,\cdots,m)$ is the equivalence class of x in terms of set of attributes A_i, and $\sim X$ is the complement of X .

2.4.2 Pessimistic multigranulation rough set

Definiton 5. [11] Let $I=(U,AT,V,f)$ be an information system, in which $A_1,A_2,\cdots,A_m\subseteq AT$, and $X\subseteq U$. The pessimistic multigranulation lower and upper approximations are denoted by $\underline{\sum_{i=1}^{m} A_i}^{P}(X)$ and $\overline{\sum_{i=1}^{m} A_i}^{P}(X)$, respectively,

$$\underline{\sum_{i=1}^{m} A_i}^{P}(X)=\left\{x\in U:[x]_{A_1}\subseteq X\vee[x]_{A_2}\subseteq X\vee\cdots\vee[x]_{A_m}\subseteq X\right\}, \tag{7}$$

$$\overline{\sum_{i=1}^{m} A_i}^{P}(X)=\sim\left(\underline{\sum_{i=1}^{m} A_i}^{P}(\sim X)\right), \tag{8}$$

where $[x]_{A_i}\ (i=1,2,\cdots,m)$ is the equivalence class of x in terms of set of attributes A_i, and $\sim X$ is the complement of X.

3 Approximation Distribution Reduction

Definition 6. Suppose $S=\langle U,C\bigcup D,f\rangle$ is an incomplete decision information system, where $C=\{c_1,c_2,\cdots,c_n\}$, $0\le\beta<0.5$, $U/D=\{D_1,D_2,\cdots,D_r\}$, then for any $B=\{b_1,b_2,\cdots,b_m\}\subseteq C$, the tolerance relations optimistic multigranulation β lower approximation distribution function with respect to B, the tolerance relations optimistic multigranulation β upper approximation distribution function with respect to B, the tolerance relations pessimistic multigranulation β lower approximation distribution function with respect to B and the tolerance relations pessimistic multigranulation β upper approximation distribution function with respect to B are denoted by $LB_\beta^{O,T}(D)$, $UB_\beta^{O,T}(D)$, $LB_\beta^{P,T}(D)$ and $UB_\beta^{P,T}(D)$, respectively, where

$$LB_\beta^{O,T}(D)=\left\{\underline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_1),\underline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_2),\cdots,\underline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_r)\right\}, \tag{9}$$

$$UB_\beta^{O,T}(D)=\left\{\overline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_1),\overline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_2),\cdots,\overline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_r)\right\}, \tag{10}$$

$$LB_\beta^{P,T}(D)=\left\{\underline{\sum_{i=1}^{m} b_i}_{\beta}^{P,T}(D_1),\underline{\sum_{i=1}^{m} b_i}_{\beta}^{P,T}(D_2),\cdots,\underline{\sum_{i=1}^{m} b_i}_{\beta}^{P,T}(D_r)\right\}, \tag{11}$$

$$UB_\beta^{P,T}(D)=\left\{\overline{\sum_{i=1}^{m} b_i}_{\beta}^{P,T}(D_1),\overline{\sum_{i=1}^{m} b_i}_{\beta}^{P,T}(D_2),\cdots,\overline{\sum_{i=1}^{m} b_i}_{\beta}^{P,T}(D_r)\right\}. \tag{12}$$

By the four approximation distribution functions, we have the following definition of the four corresponding approximation consistent sets and the four corresponding approximation reduct in incomplete decision information system.

Definition 2. Suppose $S=\langle U,C\bigcup D,f\rangle$ is an incomplete decision information system, where $C=\{c_1,c_2,\cdots,c_n\}$, $0\le\beta<0.5$, $U/D=\{D_1,D_2,\cdots,D_r\}$, then for any $B=\{b_1,b_2,\cdots,b_m\}\subseteq C$.

(1) If $LB_\beta^{O,T}(D)=LC_\beta^{O,T}(D)$, we say that B is a tolerance relations optimistic multigranulation β lower approximation distribution consistent attributes set of S. If B is a tolerance relations optimistic multigranulation β lower approximation distribution consistent attributes set, and no proper subset of B is tolerance relations optimistic multigranulation β lower approximation distribution consistent, then B is called a tolerance relations optimistic multigranulation lower approximation distribution reduct of C.

(2) If $UB_\beta^{O,T}(D)=UC_\beta^{O,T}(D)$, we say that B is a tolerance relations optimistic multigranulation β upper approximation distribution consistent attributes set of S. If B is a tolerance relations optimistic multigranulation β upper approximation distribution consistent attribute set ,and no proper subset of B is tolerance relations optimistic multigranulation β upper approximation distribution consistent, then B is called a tolerance relations optimistic multigranulation upper approximation distribution reduct of C.

(3) If $LB_\beta^{P,T}(D)=LC_\beta^{P,T}(D)$, we say that B is a tolerance relations pessimistic multigranulation β lower approximation distribution consistent attributes set of S. If B is a tolerance relations pessimistic multigranulation β lower approximation distribution consistent attribute set, and no proper subset of B is tolerance relations pessimistic multigranulation β lower approximation distribution consistent, then B is called a tolerance relations pessimistic multigranulation lower approximation distribution reduct of C.

(4) If $UB_\beta^{P,T}(D)=UC_\beta^{P,T}(D)$, we say that B is a tolerance relations pessimistic multigranulation β upper approximation distribution consistent attributes set of S. If B is a tolerance relations pessimistic multigranulation β upper approximation distribution consistent attribute set, and no proper subset of B is tolerance relations pessimistic multigranulation β upper approximation distribution consistent, then B is called a tolerance relations pessimistic multigranulation upper approximation distribution reduct of C.

From Definition 2, we obtain the following interpretations:

(1) A tolerance relations optimistic multigranulation β lower approximation distribution consistent attributes set of S is a subset of the attributes, which preserves the optimistic multigranulation β lower approximations of each class; a tolerance relations optimistic multigranulation β lower approximation distribution reduct of S is a minimal subset of the attributes ,which preserves the optimistic multigranulation β lower approximations of each decision class.

(2) A tolerance relations optimistic multigranulation β upper approximation distribution consistent attributes set of S is a subset of the attributes, which preserves the optimistic multigranulation β upper approximations of each class; a tolerance relations optimistic multigranulation β upper approximation distribution reduct of S is a minimal subset of the attributes ,which preserves the optimistic multigranulation β upper approximations of each decision class.

(3) A tolerance relations pessimistic multigranulation β lower approximation distribution consistent attributes set of S is a subset of the attributes ,which preserves the pessimistic multigranulation β lower approximations of each class; a tolerance relations pessimistic multigranulation β lower approximation distribution reduct of S is a minimal subset of the attributes ,which preserves the pessimistic multigranulation β lower approximations of each decision class.

(4) A tolerance relations pessimistic multigranulation β upper approximation distribution consistent attributes set of S is a subset of the attributes, which preserves the pessimistic multigranulation β upper approximations of each class; a tolerance relations pessimistic multigranulation β upper approximation distribution reduct of S is a minimal subset of the attributes ,which preserves the pessimistic multigranulation β upper approximations of each decision class.

For convenience of discussions, we first give some equivalent characterizations for four kinds of approximation distribution consistent attributes sets.

Let $S=\langle U,C\cup D,f\rangle$ be an incomplete decision information system, where $C=\{c_1,c_2,\cdots,c_n\}$, $0\le\beta<0.5$, $U/D=\{D_1,D_2,\cdots,D_r\}$ then for any $B=\{b_1,b_2,\cdots,b_m\}\subseteq C$, denote

$$OL_{\beta}^{C,T}(x)=\left\{D_j\in U/IND(\{d\}):x\in\underline{\sum_{i=1}^{n}c_i}_{\beta}^{O,T}(D_j)\right\},\tag{13}$$

$$OU_{\beta}^{C,T}(x)=\left\{D_j\in U/IND(\{d\}):x\in\overline{\sum_{i=1}^{n}c_i}_{\beta}^{O,T}(D_j)\right\},\tag{14}$$

$$PL_{\beta}^{C,T}(x)=\left\{D_j\in U/IND(\{d\}):x\in\underline{\sum_{i=1}^{n}c_i}_{\beta}^{P,T}(D_j)\right\},\tag{15}$$

$$PU_{\beta}^{C,T}(x)=\left\{D_j\in U/IND(\{d\}):x\in\overline{\sum_{i=1}^{n}c_i}_{\beta}^{P,T}(D_j)\right\}.\tag{16}$$

Theorem 1. Let $S=\langle U,C\cup D,f\rangle$ be an incomplete decision information system, $0\le\beta<0.5$, $U/D=\{D_1,D_2,\cdots,D_r\}$, $\forall B\subseteq C$, the following properties hold.

$$\underline{\sum_{i=1}^{n}c_i}_{\beta}^{O,T}(x)=\{D_j:x\in U,D_j\in OL_{\beta}^{C,T}(x),\mathrm{j}=1,2,\cdots \mathrm{r}\},\tag{17}$$

$$\overline{\sum_{i=1}^{n}c_i}_{\beta}^{O,T}(x)=\{D_j:x\in U,D_j\in OU_{\beta}^{C,T}(x),\mathrm{j}=1,2,\cdots,\mathrm{r}\},\tag{18}$$

$$\underline{\sum_{i=1}^{n}c_i}_{\beta}^{P,T}(x)=\{D_j:x\in U,D_j\in PL_{\beta}^{C,T}(x),\mathrm{j}=1,2,\cdots \mathrm{r}\},\tag{19}$$

$$\overline{\sum_{i=1}^{n}c_i}_{\beta}^{P,T}(x)=\{D_j:x\in U,D_j\in PU_{\beta}^{C,T}(x),\mathrm{j}=1,2,\cdots,\mathrm{r}\}.\tag{20}$$

Proof. From the above equivalent characterizations, these are straightforward.

In what follows, we study some judgement methods of the four kinds of approximation consistent set in an incomplete decision.

Theorem 2 (Judgement theorem of consistent set I) Suppose $S=\langle U,C\cup D,f\rangle$ is an incomplete decision information system, where $C=\{c_1,c_2,\cdots,c_n\}$, $0\le\beta<0.5$, $U/D=\{D_1,D_2,\cdots,D_r\}$, then for any $B=\{b_1,b_2,\cdots,b_m\}\subseteq C$, we have:

(1) B is a tolerance relations optimistic multigranulation β lower approximation distribution consistent attributes set of S iff $\forall x\in U,OL_{\beta}^{B,T}(x)=OL_{\beta}^{C,T}(x)$;

(2) B is a tolerance relations optimistic multigranulation β upper approximation distribution consistent attributes set of S iff $\forall x \in U, OU_{\beta}^{B,T}(x) = OU_{\beta}^{C,T}(x)$;

(3) B is a tolerance relations pessimistic multigranulation β lower approximation distribution consistent attributes set of S iff $\forall x \in U, PL_{\beta}^{B,T}(x) = PL_{\beta}^{C,T}(x)$;

(4) B is a tolerance relations pessimistic multigranulation β upper approximation distribution consistent attributes set of S iff $\forall x \in U, PU_{\beta}^{B,T}(x) = PU_{\beta}^{C,T}(x)$.

Proof. We only prove (1), others can be proved analogously.

(1) Sufficiency: The assumption that B is a tolerance relations optimistic multigranulation β lower approximation distribution attributes set of S implies $LB_{\beta}^{O,T}(D) = LC_{\beta}^{O,T}(D)$. Then by equation (7), we have $\underline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_j) = \underline{\sum_{i=1}^{n} c_i}_{\beta}^{O,T}(D_j)$ for each $D_j \in IND(\{d\})$, it follows that $\forall x \in U$, $x \in \underline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_j) \Leftrightarrow x \in \underline{\sum_{i=1}^{n} c_i}_{\beta}^{O,T}(D_j)$ and $x \notin \underline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_j) \Leftrightarrow x \notin \underline{\sum_{i=1}^{n} c_i}_{\beta}^{O,T}(D_j)$, i.e. $\forall x \in U, OL_{\beta}^{B,T}(x) = OL_{\beta}^{C,T}(x)$.

Necessary: Since $\forall x \in U, OL_{\beta}^{B,T}(x) = OL_{\beta}^{C,T}(x)$, then $\forall x \in U, x \in \underline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_j) \Leftrightarrow x \in \underline{\sum_{i=1}^{n} c_i}_{\beta}^{O,T}(D_j)$ and $x \notin \underline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_j) \Leftrightarrow x \notin \underline{\sum_{i=1}^{n} c_i}_{\beta}^{O,T}(D_j)$, it follows that $\underline{\sum_{i=1}^{m} b_i}_{\beta}^{O,T}(D_j) = \underline{\sum_{i=1}^{n} c_i}_{\beta}^{O,T}(D_j)$, for each $D_j \in IND(\{d\})$, i.e. $LB_{\beta}^{O,T}(D) = LC_{\beta}^{O,T}(D)$. Consequently, B is a tolerance relations optimistic multigranulation β lower approximation distribution consistent attributes set of S.

Theorem 2 provides an approach to judge whether a subset of condition attributes is a tolerance relations optimistic/pessimistic multigranulation β lower/upper approximation distribution consistent attributes set of S.

4 Approaches to approximation Distribution Reduction

In this section, we further provide practical approaches to approximation distribution reductions in an incomplete decision information system.

First, we give the following notions:

Definition 7. Let $S = \langle U, C \cup D, f \rangle$ be an incomplete decision information system, where $C = \{c_1, c_2, \cdots, c_n\}$, $0 \le \beta < 0.5$, $U/D = \{D_1, D_2, \cdots, D_r\}$, then for each $B = \{b_1, b_2, \cdots, b_m\} \subseteq C$, we denote

$$D_{OL}^{*\beta} = \{(x,y) : D_j \in OL_{\beta}^{C,T}(x), y \notin D_j, (x,y) \in U \times U \wedge (x,y) \notin T^{\beta}(C)\}, \quad (21)$$

$$D_{OU}^{*\beta} = \{(x,y) : x \in D_j, D_j \notin OU_{\beta}^{C,T}(y), (x,y) \in U \times U \wedge (x,y) \notin T^{\beta}(C)\}, \quad (22)$$

$$D_{PL}^{*\beta} = \{(x,y) : D_j \in PL_{\beta}^{C,T}(x), y \notin D_j, (x,y) \in U \times U \wedge (x,y) \notin T^{\beta}(C)\}, \quad (23)$$

$$D_{PU}^{*\beta} = \{(x,y) : x \in D_j, D_j \notin PU_{\beta}^{C,T}(y), (x,y) \in U \times U \wedge (x,y) \notin T^{\beta}(C)\}. \quad (24)$$

Defined by $D_l^{\beta}(x,y) = \begin{cases} a \in C : f_a(x) \ne f_a(y) \wedge f_a(x) \ne * \wedge f_a(y) \ne *, & (x,y) \in D_l^{*\beta} \\ \phi, & (x,y) \notin D_l^{*\beta} \end{cases}$ $(l \in \{OL, OU, PL, PU\})$, then $D_l^{\beta}(x,y), l \in \{OL, OU, PL, PU\}$ are called the tolerance relations optimistic multigranulation β lower approximation distribution discernibility attribute set, the tolerance relations optimistic multigranulation β upper approximation distribution discernibility attribute set, the tolerance relations pessimistic multigranulation β lower approximation distribution discernibility attribute set and the tolerance relations pessimistic multigranulation β upper approximation distribution discernibility attribute set, respectively.

According to Definition 7, the following judgement theorem of a consistent set can be obtained.

Theorem 3 (Judgement theorem of consistent set II)

Suppose $S = \langle U, C \cup D, f \rangle$ is an incomplete decision information system, where $C = \{c_1, c_2, \cdots, c_n\}$, $B = \{b_1, b_2, \cdots, b_m\} \subseteq C$, $0 \le \beta < 0.5$, $U/D = \{D_1, D_2, \cdots, D_r\}$, then

(1) B is a tolerance relations optimistic multigranulation β lower approximation distribution consistent attributes set of S iff $B \cap D_{OL}^{\beta}(x,y) \ne \phi$ for each $(x,y) \in D_{OL}^{*\beta}$;

(2) B is a tolerance relations optimistic multigranulation β upper approximation distribution consistent attributes set of S iff $B \cap D_{OU}^{\beta}(x,y) \ne \phi$ for each $(x,y) \in D_{OU}^{*\beta}$;

(3) B is a tolerance relations pessimistic multigranulation β lower approximation distribution consistent attributes set of S iff $B \cap D_{PL}^{\beta}(x,y) \ne \phi$ for each $(x,y) \in D_{PL}^{*\beta}$;

(4) B is a tolerance relations pessimistic multigranulation β upper approximation distribution

consistent attributes set of S iff $B\bigcap D_{PU}^{\beta}(x,y)\neq\phi$ for each $(x,y)\in D_{PU}^{*\ \beta}$.

Proof. We only prove (1), others can be proved analogously.

(2) Sufficiency: Suppose $\exists(x,y)\in D_{OL}^{*\ \beta}$ such that $B\bigcap D_{OL}^{\beta}(x,y)=\phi$, and then we have $(x,y)\in T^{\beta}(B)$, i.e. $y\in T_B^{\beta}(x)$. Since $(x,y)\in D_{OL}^{*\ \beta}$, then there must be $D_j\in U/D$ such that $x\in \underline{\sum_{i=1}^{n}c_i}_{\beta}^{O,T}(D_j)$ and $y\notin D_j$.

By condition we know that B is a tolerance relations optimistic multigranulation β lower approximate distribution consistent attributes set of S, then $x\in \underline{\sum_{i=1}^{m}b_i}_{\beta}^{O,T}(D_j)$, i.e. there exist $b_k\in B$ such that $T_{\{b_k\}}^{\beta}(x)\subseteq D_j$. By the basic property of the tolerance relation, we have $T_B^{\beta}(x)\subseteq T_{\{b_k\}}^{\beta}(x)\subseteq D_j$, from which we can conclude that $y\in D_j$, which is contradictive to the assumption $(x,y)\in D_{OL}^{*\ \beta}$ because $(x,y)\in D_{OL}^{*\ \beta}\Rightarrow y\notin D_j$.

Necessary: Since $B\subseteq C$, then we have $OL_{\beta}^{B,T}(x)\subseteq OL_{\beta}^{C,T}(x)$ for each $x\in U$ obviously. Therefore, it must be proved that $OL_{\beta}^{C,T}(x)\subseteq OL_{\beta}^{B,T}(x)$ for each $x\in U$.

If $\exists(x,y)\in D_{OL}^{*\ \beta}\left(D_j\in OL_{\beta}^{C,T}(x),y\notin D_j\right)$ such that $B\bigcap D_{OL}^{\beta}(x,y)=\phi$, then we have $(x,y)\in T^{\beta}(B)$, i.e. $y\in T_B^{\beta}(x)$. By the basic property of the tolerance relation, we have $y\in T_{\{b_k\}}^{\beta}(x)$ for each $b_k\in B$, i.e. $T_{\{b_k\}}^{\beta}(x)\not\subset D_j$ for each $b_k\in B$, it follows that $x\notin \underline{\sum_{i=1}^{n}c_i}_{\beta}^{O,T}(D_j)$, $D_j\in OL_{\beta}^{B,T}(x)$. From discussions above, we can conclude that if $B\bigcap D_{OL}^{\beta}(x,y)\neq\phi$ for each $(x,y)\in D_{OL}^{*\ \beta}$, then we have $x\in \underline{\sum_{i=1}^{n}c_i}_{\beta}^{O,T}(D_j)\left(D_j\in OL_{\beta}^{B,T}(x)\right)$, where $D_j\in OL_{\beta}^{C,T}(x)$, i.e. $OL_{\beta}^{C,T}(x)\subseteq OL_{\beta}^{B,T}(x)$.

Definition 4. Let $S=\langle U,C\bigcup D,f\rangle$ be an incomplete decision information system, in which $C=\{c_1,c_2,\cdots,c_n\}$, $B=\{b_1,b_2,\cdots,b_m\}\subseteq C$, $0\leq\beta<0.5$, $D_l^{\beta}=\{D_l^{\beta}(x,y):x,y\in U\}$, $l=\{OL,OU,PL,PU\}$ are called the tolerance relations optimistic multigranulation β lower approximation distribution discernibility matrix of S, the tolerance relations optimistic multigranulation β upper approximation distribution discernibility matrix of S, the tolerance relations pessimistic multigranulation β lower approximation distribution discernibility matrix of S, and the tolerance relations pessimistic multigranulation β upper approximation distribution discernibility matrix of S, respectively. Let $\Delta_l^{\beta}=\bigwedge_{D_l^{\beta}(x,y)\in D_l^{\beta}}\left(\vee D_l^{\beta}(x,y)\right),l\in\{OL,OU,PL,PU\}$.

Then, $\Delta_l^{\beta},l\in\{OL,OU,PL,PU\}$ are, respectively, referred to as the tolerance relations optimistic multigranulation β lower approximation distribution discernibility function, the tolerance relations optimistic multigranulation β upper approximation distribution discernibility function, the tolerance relations pessimistic multigranulation β lower approximation distribution discernibility function, and the tolerance relations pessimistic multigranulation β upper approximation distribution discernibility function.

Using Boolean Reasoning technique, it is easy to obtain the following theorem by Theorem 3.

Theorem 4 (Judgement theorem of consistent set III) Suppose $S=\langle U,C\bigcup D,f\rangle$ is an incomplete decision information system, where $C=\{c_1,c_2,\cdots,c_n\}$, $B=\{b_1,b_2,\cdots,b_m\}\subseteq C$, $0\leq\beta<0.5$, $U/D=\{D_1,D_2,\cdots,D_r\}$. The minimal disjunctive normal form of each discernibility function $\Delta_l^{\beta}\left(l\in\{OL,OU,PL,PU\}\right)$ is $\Delta_l^{\beta}=\bigvee_{k=1}^{t}\left(\bigwedge_{s=1}^{q_k}a_{i_s}\right),l\in\{OL,OU,PL,PU\}$, let $B_{lk}=\{a_{i_s}:s=1,2,\cdots,q_k\}$ and then $\{B_{lk}:k=1,2,\cdots,t\}\left(l\in\{OL,OU,PL,PU\}\right)$ are, respectively, the set of the tolerance relations optimistic multigranulation β lower approximation distribution reduction, the set of the tolerance relations optimistic multigranulation β upper approximation distribution reduction, the set of the tolerance relations pessimistic multigranulation β lower approximation distribution reduction, the set of the tolerance relations pessimistic multigranulation β upper approximation distribution reduction.

Proof. For any $k\leq t$ and $(x,y)\in D_l^{*\beta}$, by the definition of the minimal disjunctive normal form, we have $B_{lk}\bigcap D_l^{\beta}(x,y)\neq\phi$, and then from Theorem 3, we obtain the conclusion that B_{lk} is the corresponding approximation distribution consistent attributes set.

Simultaneously, $\Delta_l^{\beta}=\bigvee_{k=1}^{t}B_{lk}$, if $B_{lk}^{'}$ is formed by deleting an element from B_{lk}, then there exist

$(x,y)\in D_l^{*\beta}$ such that $B_{lk}^{'}\cap D_l^{\beta}(x,y)=\phi$, so $B_{lk}^{'}$ is not an corresponding approximation distribution consistent attributes set. Hence, B_{lk} is the corresponding approximation distribution reduction.

Since all $D_l^{\beta}(x,y)$ are included in the corresponding approximation distribution discernibility function, so there does not exist other corresponding approximation distribution reduction.

Theorem 4 provides practical approaches to some attribute reductions in incomplete information decision system.

5 Example Analysis

In this section, an illustrate example is employed to explain the approach introduced in Section 3.Suppose the following incomplete information decision table is given in Table 1.

TABLE 1 An incomplete information decision table

U	a_1	a_2	a_3	a_4	d
x_1	3	2	1	0	Φ
x_2	2	3	2	0	Φ
x_3	2	3	2	0	Ψ
x_4	*	2	*	1	Φ
x_5	*	2	*	1	Ψ
x_6	2	3	2	1	Ψ
x_7	3	*	*	3	Φ
x_8	*	0	0	*	Ψ
x_9	3	2	1	3	Ψ
x_{10}	1	*	*	*	Φ
x_{11}	*	2	*	*	Ψ
x_{12}	3	2	1	*	Φ

The decision classes of objects are $D_\Phi=\{x_1,x_2,x_4,x_7,x_{10},x_{12}\}$ and $D_\Psi=\{x_3,x_5,x_6,x_8,x_9,x_{11}\}$.

It can easily be calculated that $T_{R_C}(x_1)=\{x_1,x_{11},x_{12}\}$, $T_{R_C}(x_2)=\{x_2,x_3\}$, $T_{R_C}(x_3)=\{x_2,x_3\}$.

$T_{R_C}(x_4)=\{x_4,x_5,x_{10},x_{11},x_{12}\}$,

$T_{R_C}(x_5)=\{x_4,x_5,x_{10},x_{11},x_{12}\}$,

$T_{R_C}(x_6)=\{x_6\}$,

$T_{R_C}(x_7)=\{x_7,x_8,x_9,x_{11},x_{12}\}$,

$T_{R_C}(x_8)=\{x_7,x_8,x_{10}\}$,

$T_{R_C}(x_9)=\{x_7,x_9,x_{11},x_{12}\}$,

$T_{R_C}(x_{10})=\{x_4,x_5,x_8,x_{10},x_{11}\}$,

$T_{R_C}(x_{11})=\{x_1,x_4,x_5,x_7,x_9,x_{10},x_{11},x_{12}\}$,

$T_{R_C}(x_{12})=\{x_1,x_4,x_5,x_7,x_9,x_{11},x_{12}\}$.

Thus, for $\beta=0.3$,

$$D_{OL}^{*\,0.3}=\{(x_2,x_1),(x_2,x_4),(x_2,x_7),(x_2,x_{10}),(x_2,x_{12}),(x_3,x_1),(x_3,x_4),(x_3,x_7),(x_3,x_{10}),(x_3,x_{12}),(x_6,x_1),(x_6,x_2),(x_6,x_4),(x_6,x_7),(x_6,x_{10}),(x_6,x_{12})\}$$

Hence, we obtain the tolerance relations optimistic multigranulation 0.3 lower approximation distribution discernibility matrix as follows:

$$\begin{bmatrix}
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
a_1a_2a_3 & \phi & \phi & a_2a_4 & \phi & \phi & a_1a_4 & \phi & \phi & a_1 & \phi & a_1a_2a_3 \\
a_1a_2a_3 & \phi & \phi & a_2a_4 & \phi & \phi & a_1a_4 & \phi & \phi & a_1 & \phi & a_1a_2a_3 \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
a_1a_2a_3a_4 & a_4 & \phi & a_2 & \phi & \phi & a_1a_4 & \phi & \phi & a_1 & \phi & a_1a_2a_3 \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi
\end{bmatrix}$$

Then the tolerance relations optimistic multigranulation 0.3lower approximation distribution discernibility function is

$$\Delta_{OL}^{0.3}=(a_1\vee a_2\vee a_3)\vee(a_2\vee a_4)\vee(a_1\vee a_4)\vee a_1\vee(a_1\vee a_2\vee a_3)\vee(a_1\vee a_2\vee a_3)\vee(a_2\vee a_4)\vee(a_1\vee a_4)\vee a_1\vee(a_1\vee a_2\vee a_3)\vee(a_1\vee a_2\vee a_3\vee a_4)\vee a_4\vee a_2\vee(a_1\vee a_4)\vee a_1\vee(a_1\vee a_2\vee a_3)=a_1\wedge a_2\wedge a_4.$$

In addition, we also obtain the tolerance relations optimistic multigranulation 0.3 upper approximation distribution discernibility matrix as follows:

$$\begin{bmatrix}
\phi & a_1a_2a_3 & a_1a_2a_3 & \phi & \phi & a_1a_2a_3a_4 & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & a_4 & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & a_2a_4 & a_2a_4 & \phi & \phi & a_2 & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & a_1a_4 & a_1a_4 & \phi & \phi & a_1a_4 & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & a_1 & \phi & \phi & a_1 & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\
\phi & a_1a_2a_3 & a_1a_2a_3 & \phi & \phi & a_1a_2a_3 & \phi & \phi & \phi & \phi & \phi & \phi
\end{bmatrix}$$

Then the tolerance relations optimistic multigranulation 0.3 lower approximation distribution discernibility function is

$\Delta_{OL}^{0.3}=(a_1\vee a_2\vee a_3)\vee(a_2\vee a_4)\vee(a_1\vee a_4)\vee a_1\vee(a_1\vee a_2\vee a_3)$
$\vee(a_1\vee a_2\vee a_3)\vee(a_2\vee a_4)\vee(a_1\vee a_4)\vee a_1\vee(a_1\vee a_2\vee a_3)\vee$
$(a_1\vee a_2\vee a_3\vee a_4)\vee a_4\vee a_2\vee(a_1\vee a_4)\vee a_1$
$\vee(a_1\vee a_2\vee a_3)=a_1\wedge a_2\wedge a_4$.

By theorem 4, $\{a_1,a_2,a_4\}$ is the tolerance relations optimistic multigranulation 0.3 lower approximation distribution reduction.

Similarly, it is not difficult to obtain that the tolerance relations pessimistic multigranulation 0.3 lower approximation distribution reduction and the tolerance relations pessimistic multigranulation 0.3 upper approximation distribution reduction in Table 1 is the set of the attributes $C=\{a_1,a_2,a_3,a_4\}$.

6 Experimental Comparison

In what follows, through experimental analysis, in the multigranulation environment, we will show the power of our proposed method by comparing it with the granular space reduction based on Heuristic approach proposed in literature [12]. Six public data sets have been downloaded from UCI Repository of Machine Learning databases. The property of these seven data sets is shown in Table 2.

TABLE 2 Data Sets Description

ID	Data Sets	Samples	Features	Decision Classes
1	Monk’s Problems	432	7	2
2	Abscisic Acid Signaling Network	300	43	2
3	University	285	17	2
4	Pittburgh Bridges	108	13	2
5	Mammographic Mass	961	6	2
6	Led17	2000	22	2
7	SkillCraft1 Master Table	3395	20	2

TABLE 3 The tolerance relations optimistic multigranulation 0.2 lower approximation distribution reductions

Data ID	Reduction numbers	Granules of the shortest reduction	Granules of the longest reduction	The shortest reduction efficiency	The longest reduction efficiency
1	1	5	5	28.6%	28.6%
2	373	12	17	72.1%	60.5%
3	285	11	13	35.3%	23.5%
4	38	6	8	53.8%	38.5%
5	6	6	6	0	0
6	117	18	18	18.2%	18.2%
7	536	13	14	53.8%	0.3%

TABLE 4 Time comparisons between the two algorithms

Data ID	Time consuming of AGSR(/s)	Time consuming of MAGSR(/s)
1	45.1025	2.7641
2	37.7021	3.2001
3	24.8925	1.7541
4	0.2572	0.0852
5	384.1362	4.2531
6	1015.5264	1009.4907
7	3426.1726	3426.8823

Table 3 displays the tolerance relations optimistic multigranulation 0.2 lower approximation distribution reductions on seven public data sets. The time consuming of the two algorithms are shown in Table 4.In table 4, the granular space reduction based on Heuristic approach is denoted by HAGSR, and the proposed discernibility matrix approach is denoted by MAGSR.

It can be seen from Table 3 that the Monk’s Problem data set has one reduction only, the Abscisic Acid Signaling Network data set has 373 reductions, the University data set has 285 reductions, the data set has 38 reductions, the data set has 6 reductions, the Led17 data set has 117 reductions, and the SkillCraft1 Master Table data set has These results show that the proposed discernibility matrix approach can obtain all the tolerance relations optimistic/pessisitic multigranulation 0.2 lower/upper approximation distribution reductions, but not a single corresponding reduction.

From Table 4, we can see that the time consuming of the proposed discernibility matrix approach is far lower than the time consuming of the granular space reduction based on Heuristic approach, especially to small-scale data sets. We also find that from the first data set to the fifth data set, the difference in time consuming between the two algorithms is greatly obvious, but in Led17 data set and in SkillCraft1 Master Table Data set, the time consuming of the proposed discernibility matrix approach is almost the same as the time consuming of the granular space reduction based on Heuristic approach. With the increase of the size of the data set, the difference of the time consuming between the two algorithms is smaller and smaller. This phenomenon show that the proposed discernibility matrix approach is more suitable for small-scale data set, and the granular space reduction based on Heuristic approach is more suitable for large-scale data set.

7 Conclusions

To access some brief and effective decision rules with a threshold from incomplete information decision system under multigranulation environment, good and convenient granular space reduction is needed. This paper has introduced four kinds of the granular space reduction called a tolerance relations optimistic/pessimistic multigranulation β lower/upper approximation distribution reduction, which preserve the optimistic/pessimistic multigranulation β lower/upper approximation distribution of the decision classes. The judgement theorems and discernibility matrices associated with the four reductions have been obtained. Finally, an illustrative example has been applied to explain the mechanism of the proposed method. The proposed discernibility matrix approach renders a set of simpler discernibility functions for finding all approximation reductions of an incomplete decision system under multigranulation environment. Through experiments, the advantage of the proposed method get a better reflection.

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