

Survey of research directions in fuzzy cognitive map

Nan Ma^{1*}, Yun Zhai², Bingru Yang³

¹College of Information Technology, Beijing Union University, 100101, Beijing, China

²E-Government Research Center, Chinese Academy of Governance, Beijing, 100089, China

³School of Computer and Communication Engineering, University of Science and Technology Beijing, Beijing 100083, China

Received 1 March 2014, www.cmnt.lv

Abstract

Fuzzy Cognitive Map as a generally recognized intelligent tool has been widely used in the data mining and the machine learning fields. This paper reviewed the present research situation of Fuzzy Cognitive Map at home and abroad in recent years. Firstly, it reviewed the development progress of Fuzzy Cognitive Map with its advantage over other fuzzy learning methods, followed by the research papers indexed by EI database, ACM database and the SCI database. Then it summarized the classification and the learning methods of Fuzzy Cognitive Map. Furthermore, the applications of Fuzzy Cognitive Map are analysed finally. We hope our work help the people to have a general understanding and push the Fuzzy Cognitive Map forward.

Keywords: fuzzy cognitive map, development, classification, learning methods

1 Introduction

1.1 DEVELOPMENT PROGRESS

In 1976, Axelord proposed a three-valued cognitive map [1]. It consists of two different arcs, one positive, and one negative. The positive indicates the causation have a same changing direction, negative contrast, and the arcs represent the relationship between the different concept nodes. Concept nodes specified action, causes, consequences, purpose, feelings, tendencies and trends of the system, which reflect the properties, performance, quality and status of the system. The relationship of the concept nodes indicates the relevance and effect of them, which expressed as arcs with arrows. In addition to the positive and negative, the strength of the relevance is expressed in numerical, who we called the weight of arc [2]. It can be applied to decision-making model of the application system.

In 1986, Kosko and others, who on the basis of Axelord, combining the fuzzy set theory, proposed a fuzzy cognitive map (FCM) theory, which extends the three-valued logic relationship between nodes to fuzzy relationship on the interval $[-1, 1]$ [3]. This theory carries more information than Axelord's three-valued logic cognitive maps, and points out that the limited input state of FCM can open up a path in the virtual space, and simple FCM path may terminate at a fixed point or limit cycle, and in the complex feedback FCM this path may be terminated in the "chaos" strange attractors [4].

In 1992, Hagiwara, on the basis of FCM, proposed the extended fuzzy cognitive map (EFCM) [5]. This cognitive map extends the linear relationship between concept nodes to the non-linear relationship, introduced the delay of time

and causation and the weight of conditions. In 1994, Wellman proposed a qualitative probabilistic network [6], which regards the cognitive map as a network with an unknown probability, but this cannot quantify the change degree of causation between concept nodes. In 1999, Carvalho and others proposed a rule-based cognitive map model [7], which can solve the problem of non-monotonic reasoning and non-causal representation. But this model has too many rules, cannot adapt to changes in the external environment, and also has a great amount of computation. In 2001, Yuan and others proposed a dynamic cognitive network [8], which is a continuation of the FCM and has a strong ability to adapt to the environment. In 2003, Luo Xiangfeng, who discussed the probabilistic fuzzy cognitive map [9, 10], by introducing fuzzy measure to the traditional cognitive map model to quantify the impact of causal relationship between concept nodes, a probabilistic model of cognitive maps.

1.2 BASIC CONCEPTS OF FCM

For two different nodes c_i and c_j in the FCM, if the value x_i of the node c_i changes, the value x_j of the node c_j changes consequently, we say that the nodes c_i and c_j have the causality relationship. And we call the arc from the node c_i to c_j a directed arc. Then the node c_i is call the reason node, the node c_j is call the result node. A basic model of FCM is demonstrated in the Figure 1.

*Corresponding author e-mail: xxtmanan@buu.edu.cn

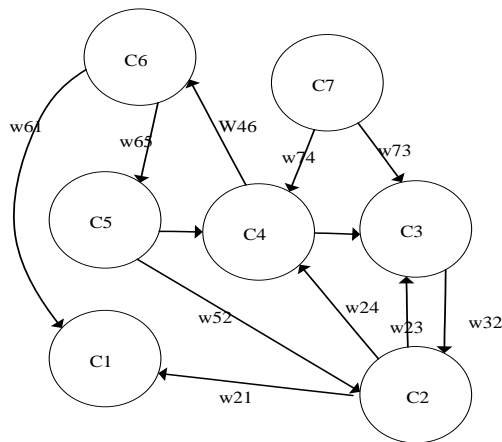


FIGURE 1 Basic model of FCM

Let $C = \{c_1, c_2, \dots, c_N\}$ be a finite set of vertices in FCM, where N is the number of nodes, for two any nodes c_i and c_j and the finite set:

$$E = \{e_{11}, e_{12}, \dots, e_{1N}, e_{21}, e_{22}, \dots, e_{2N}, \dots, e_{N1}, e_{N2}, \dots, e_{NN}\},$$

in the FCM, each arc has a corresponding weight w_{ij} indicating the influence of node c_i to c_j as shown in Equation (1):

$$W = \begin{bmatrix} w_{11} & w_{12} & \dots & w_{1N} \\ w_{21} & w_{22} & \dots & w_{2N} \\ \dots & \dots & \dots & \dots \\ w_{N1} & w_{N2} & \dots & w_{NN} \end{bmatrix}. \tag{1}$$

For any nodes c_i and c_j in FCM, if the two nodes exist a directed relation, the interval $[-1,1]$ can be used to describe the influence degree, i.e., $w_{ij} \in E$ and w_{ij} is the weight of c_i to c_j .

1.3 ADVANTAGES OF FCM

Compared with the neural network model, the advantages of fuzzy cognitive map model is as following:

- (i) Each node and arc in fuzzy cognitive map model has a very robust semantics, so that the entire diagram showing a robust semantic, and the results of reasoning is easy to understand. Neural network is a numerical framework, cannot directly represent the structure of knowledge.
- (ii) Fuzzy cognitive map model can apply expert knowledge to compensate for the lack of learning data to a certain extent. Because its reasoning is based on matrix operations, so it corresponds to the development of AI that intelligent behaviour driven by data. Neural network does not use expert knowledge of the system itself, and requires a lot of training data, but for those are not easy to get the training data, neural networks will be very limited.
- (iii) Fuzzy cognitive map model cannot only represent a semantic network, but also can handle the distribution of knowledge. For any number of knowledge sources it can construct their cognitive maps and can be superimposed on each other arbitrarily, resulting in an overall knowledge

sources joint distribution of knowledge. Neural network does not have the overlay, and it has poor learning ability for system.

(iv) The large-scale fuzzy cognitive map cannot only be utilized to predict, but also can be used to the sensitivity analysis, causal relationship, strategic planning of factor changes. Neural networks are black-box model; the whole system cannot achieve structural analysis.

Compared with traditional methods of knowledge representation and reasoning based on first-order logic, FCM has the following characteristics:

- (i) FCM is very convenient to establish, intuitive performance issues, and is suitable for knowledge engineers to interact with experts in the field. Related concepts are connected by arcs together, and the facts, features, and relationships associated with concepts can be deduced through the arc, easy to achieve the explanation of system by way of association.
- (ii) FCM can be deduced by digital matrix calculations. Owing to its derivation process is digital, it has more flexibility in terms of calculation and the derivation, the computer can give full play to the advantages of numerical computing.
- (iii) FCM supports feedback mechanism, so the result may be a limit cycle, unlike traditional expert system that will only come to a conclusion constant. This makes FCM's ability of knowledge representation and reasoning expanded over traditional expert systems.

Compared with HMM (Hidden Markov Model, HMMs), the differences between them are as following.

- (i) Each node in the HMMs represents a specific system state, and therefore, this technique only applies to simulate a finite state system.
- (ii) The transfers between states in HMMs is controlled by a set of probabilities, while in FCM the next state is calculated by the conversion function.

1.4 RESEARCH RESULTS

By searching the worldwide ACM, SCI and EI three authoritative database, we found that, from 2004 to 2013, more and more scholars and experts conducted in-depth study in fuzzy cognitive map and achieved fruitful results. Specifically is as showed in Figure 2-4:

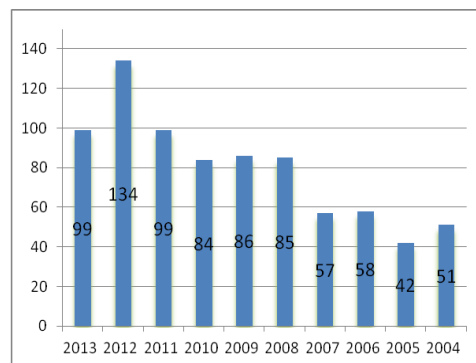


FIGURE 2 EI Database

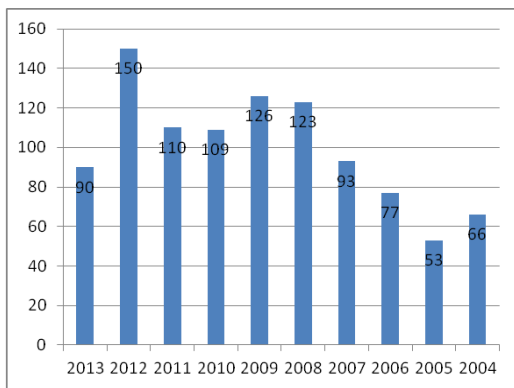


FIGURE 3 SCI Database

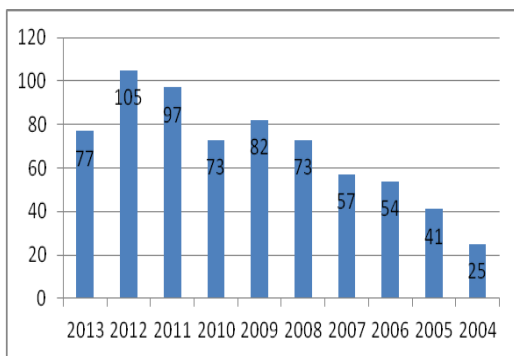


FIGURE 4 ACM Database

By searching the worldwide ACM, SCI and EI three authoritative database, we found that, from 2004 to 2013, more and more scholars and experts conducted in-depth study in fuzzy cognitive map and achieved fruitful results. Specifically is as showed above.

1.5 APPLICATION AREA

As can be seen from the development of FCM, researches in fuzzy cognitive map constantly goes deeper. It is also expanding its application areas, for example Cole and others proposed how to apply FCM to teaching and research [11]. Stylios and others proposed the FCM is applied to supervisory control system model [12]. Lee and others applied FCM to the web mining [13]. Min and others applied the concept of FCM to intelligent decision support [14]. Kardaras and others applied the FCM for commercial modelling [15].

2 Classification of fuzzy cognitive map

2.1 PROBABILISTIC FUZZY COGNITIVE MAP

Probabilistic Fuzzy Cognitive Map (PFCM) was first proposed by the Luo XiangFeng and others. PFCM introduced conditional probability measure in a causal relationship between concept node, the model can represent a qualitative and fuzzy causal relationship between concept nodes, but also represent conditional probability causation between concept nodes, and can degenerate into fuzzy cognitive map.

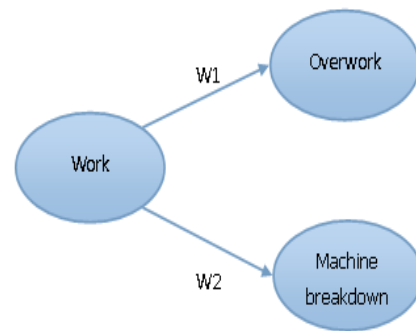


FIGURE 5 Causality of concept nodes

As shown in Figure 5, it is uncertain that the measure of causation between “work” and “overworked” or “machine breakdown” concept nodes. w_1 , w_2 measure is not only associated with "labor" concept node's own state values (intensity of labor and labor time). While w_1 is also associated with the values of "worker's body" concept node's state (health, weakness), w_2 is also associated with the values of concept nodes “quality of the machine” (or inferior quality, texture condition) and "machine" texture condition (manufacturing materials).

As FCM cannot represent the dynamic dependencies of concept node's status value on the measure of causation between concept nodes, PFCM first introduced the conditional probabilistic relationship in FCM. In order to introduce the probabilistic relationship to FCM, Stylios proposed the following formula to solve the dependencies of probabilistic between concept nodes in cognitive map feedback cycle, which form a FCM model with time and memory status:

$$Vc_j(t) = f \left(\sum_{\substack{i=1 \\ i \neq j}}^N Vc_i(t)w_{ij} + rVc_j(t-1) \right), \quad (2)$$

where the $Vc_i(t)$ of the formula c_j node means the state value at time t ; γ works as a factor of the last time.

Reference [12] renewed the previous formula with the probability of getting a memory function and the dynamic characteristics of fuzzy cognitive map model:

$$Vc_j(t) = f \left(\sum_{\substack{i=1 \\ i \neq j}}^N Vc_i(t)P(w_{ij}(t)|Vc_i(t), Vc_m(t), \dots, Vc_n(t)) + rVc_j(t-1) \right). \quad (3)$$

2.2 RANDOM FUZZY COGNITIVE MAP

In 1943 psychologist W. McCulloch and mathematical logician W. Pittsin on the basis of analysis and summary of the basic characteristics of neurons, first proposed a mathematical model of neuron. This model is still in use, and directly impacted the research progress in this area. Artificial neural networks have a preliminary adaptive and self-organizing ability. Changing the synaptic weights in the learning or training process in order to meet the

demands of the surrounding environment. The same network can have different functions because of different learning ways and content. Neural network provides a good tool to Fuzzy Cognitive Map.

References [16, 17] proposed a Random Fuzzy Cognitive Map (RFCM) based on random neural network, and demonstrate its application in process modeling. Based on the activation probability of network neurons, this model achieves reasoning process by numerical calculation instead of symbolic derivation. In RFCM, the arc defines the relationship between the concept nodes, and describes the causal process. The experiment demonstrated that the random fuzzy cognitive map and early fuzzy cognitive map produced approximation results, and less iteration. On this basis, the reference [18] proposed a RFCM based Adaptive Random Fuzzy Cognitive Map (ARFCM). ARFCM transform causal ambiguity into a causal relationship between the network mode conversion, with the use of experts to enhance the causal knowledge. Meanwhile, the document also describes how to use ARFCM interpretation model constituted by a dynamic process.

2.3 RULE-BASED FUZZY COGNITIVE MAP

Reference [19] proposed a Rule-Based Fuzzy Cognitive Maps (RBFCM). RBFCM, based on the traditional fuzzy network with feedback, can be used for modeling describe real-world dynamic system with feedback chain. Thus, proposed RBFCM makes FCM label cognition more completely. Based RBFCM, reference [9] regards time as an important qualitative entity in cognitive map, that is, implicit represent time in RBFCM through an association program. Time in a dynamic system is undoubtedly crucial, which also reveals an important role of RBFCM. As an extension of RBFCM, reference [18] raised a number of questions about stability around the qualitative dynamic system of real-world modeling. Meanwhile, it proposed the inherent stability as a required attributes concept node in the qualitative system. Reference [19] discussed how to achieve RBFCM causality itself. As the traditional fuzzy operation is defined by causal mapping, and therefore a causal relationship cannot be achieved, which is the literature [20] the main basis. The literature use specific methods to achieve fuzzy causation. This approach allows not both add, delete the concept of nodes and links between the nodes of the concept, but also a great degree of flexibility with a novel fuzzy operation, which can mimic causation "accumulate" attribute, namely Fuzzy Carry Accumulation (FCA).

2.4 COMPLEX FUZZY COGNITIVE MAP

Because of the large size and complexity of the internal links of complex systems to study the "big" FCM become one of the most arduous of the most pressing research.

Reference [21] proposed a structure thought polymerization fuzzy cognitive map of the complex

systems are often broken down into subsystems, each subsystem are represented by fuzzy cognitive map, then put each fuzzy cognitive map by aggregating them part of the same get the whole complex system of fuzzy cognitive map, also known as polymerization fuzzy cognitive map. The easiest way is to put the various subsystems of polymerization of the corresponding concept of a causal relationship between nodes by simple arithmetic sum and then make vague standardization. But the weight of each subsystem are not equal weight under normal circumstances, and even may have a different symbol, then how to construct an adjacency matrix has the consistency of a complex system, so that the system can achieve mutual cooperation and coordination is a very difficult thing. Reference [22] proposed a method by means of expert intervention. Mutual cooperation and coordination subsystem is affected by the right weight to the role of concepts, therefore, generally considered by summing the weighting factor, preferably the right or to the importance of trust for Subsystem weight coefficient.

If complex systems concepts or things are hierarchical, can be layered deal, each level corresponds to a FCM. FCM simple hierarchy has the following features: All nodes are hierarchical concept, but it results in each node and its lower-level node, the node is the reason for its parent node. FCM simple hierarchical structure is more like a tree, it stressed that it would have a close association between the nodes in the same layer, produced only a causal association between the different layers and parent nodes and child nodes, there is no causal relationship between grandparent and grandchild nodes. Reference [23] with FCM construct integrated two-stage hierarchy process modeling for complex radiotherapy proposed algorithm and the interaction between the two levels of monitoring the implementation of decision support , but did not begin a general study of the levels of FCM, only to specific examples of doing some research. In reference [24], the cognitive map model is applied to the field of corporate credit risk assessment established based on statistical learning and expert knowledge combined with multilayer fuzzy cognitive map. Model of multi- fuzzy cognitive map and its application had a very good try, but their discussion multilayer fuzzy cognitive map is to require anti- reflexive, symmetric and recovery, therefore, has the particularity.

Literature [25] put forward the theory of complex systems and packet decomposition algorithm FCM. That is the first node of the original group cognitive map, and then on the set of nodes construct fuzzy cognitive map providers. Such a decomposition of an original supplier FCM and several meaningful sub FCM. FCM on the original analysis of the cognitive map into the analysis and sub-suppliers of FCM. Causal reasoning quotient FCM provides overall information on the original.

3 Learning methods of fuzzy cognitive map

There are two constructor FCM: artificial construct and calculate structure. Artificially constructed using expert knowledge and experience to build applications FCM model; computing structure is to learn from the historical data to automatically establish FCM model.

Artificial construct ways of modelling process consists of three steps: identify the problem domain key concepts; identify the presence or absence of a causal relationship between these concepts; estimated value of relationships. Because of the way too dependent on artificial construct subjective consciousness and expert domain knowledge makes it difficult to build. Thus, in order to produce a calculation based FCM learning methods.

Currently, the two main types of computing structure FCM branches: one is based on Hebbian learning styles; second is based on the theory of evolution of learning. As shown in Table 1, the former including DHL (Differential Hebbian Learning), BDA (Balanced Differential

Algorithm) NHL (Nonlinear Hebbian Learning), DD-NHL (Data-Driven Nonlinear Hebbian Learning) and AHL (Active Hebbian Learning). The latter has GS (Genetic Strategy), PSO (Particle Swarm Optimization), SA (Simulated Annealing), RCGA (Real Coded Genetic Algorithm).

In addition to these methods are selected BDA continuous conversion function; except GS initial vector is obtained, other learning algorithms are FCM link target matrix; PSO method needs only a plurality of GS and the state vector, and this is not available in some areas. Based Hebbian algorithm uses an unsupervised learning mechanism, based on the weight associated with a single sample of the initial system state data FCM's training so as to reach a steady state (fixed-point state). Different algorithms that adjust each side of the weights way, because it does not require a lot of computing training faster, but they are simple systems for, FCM produced only node includes several concepts.

TABLE 1 Comparison of FCM learning methods.

Algorithm	Learning goal	Human intervention	Type of data used	Transformation function	Learning type
DHL	Connection matrix	No	Single	N/A	Hebbian
BDA	Connection matrix	No	Single	Binary	Modified Hebbian
NHL	Connection matrix	Yes&No	Single	Continuous	Modified Hebbian
DD-NHL	Connection matrix	Yes&No	Single	Continuous	Modified Hebbian
AHL	Connection matrix	Yes&No	Single	Continuous	Modified Hebbian
GS	Initial vector	No	Multiple	Continuous	Genetic
PSO	Connection matrix	No	Multiple	Continuous	Swarm
SA	Connection matrix	No	Single	Continuous	Simulated annealing
RCGA	Connection matrix	No	Single	Continuous	Genetic

4 The applications of fuzzy cognitive map

The existing learning algorithms based on FCM in data mining are mainly used in classification, where each attribute of the target is regarded as a node in the FCM, the initial weight of every edge is given by experts, and experts need to clear the input node and the output node. A typical classification model is as demonstrated in the Figure 6.

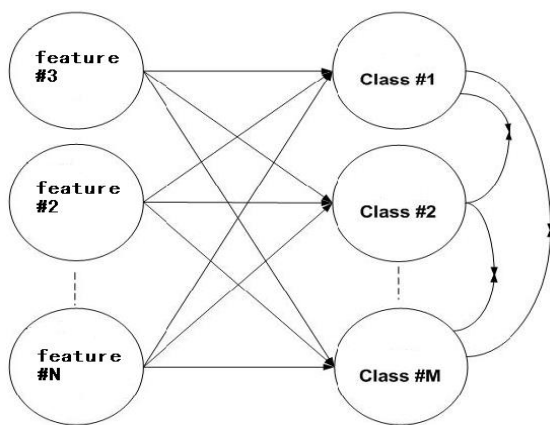


FIGURE 6 A typical classification model

The learning algorithm for fuzzy cognitive map is based on the genetic algorithm with real coding method, such that this learning algorithm tend to over-fit with

different initial values. Therefore, they implemented two criteria to reduce the over - fitting probability, i.e., the average error of one input value in all cycles and the average error of different input values in all cycles. The former calculation method is computed using Equation (4) and the latter using Equation (5):

$$error_behavior = \frac{1}{P(K-1)N} \sum_{p=1}^P \sum_{t=1}^{K-1} \sum_{n=1}^N |A_n^{act}(t) - A_n^{des}(t)|, \tag{4}$$

$$error_initial = \frac{1}{(K-1)N} \sum_{t=1}^{K-1} \sum_{n=1}^N |A_n^{act}(t) - A_n^{des}(t)|. \tag{5}$$

The references [28, 29] realized a FCM-GT based on the bladder tumour classification, the weighted matrix of FCM-GT is as shown in Figure 7.

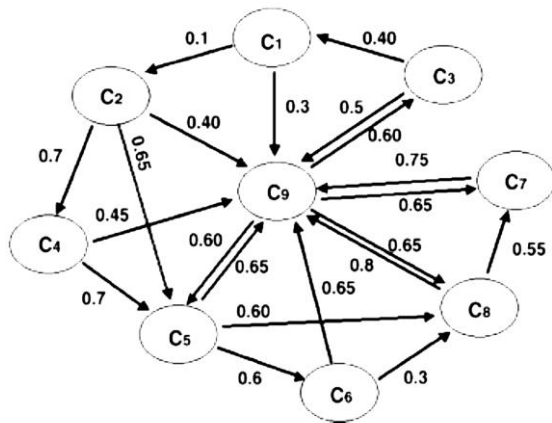


FIGURE 7 The weighted matrix of FCM-GT

5 Conclusions

As an important method of soft computing, the research of FCM has attracted more and more attentions, and just as analysed in this paper, the study of FCM has made great progress. In this field, however, there are still many problems to be solved, the focus will focus on the following aspects:

1) The importance of time in FCM. Time scale is crucial for the dynamic system, the issue of node segmentation and polymerization based on the time scale is worthy of more attention to concept of [28].

References

- [1] Axelrod R 1976 Structure of Decision:the Cognitive Maps of Political Elites Princeton University Press New Jersey
- [2] Fantacci R, Nannicini S 2000 *IEEE Journal on Selected Areas in Communications* 18(8) 1441-54
- [3] Kosko B 1986 *International Journal of Man-Machine Studies* 24 65-75
- [4] Chun M L 2006 Model method and application *Study of Fuzzy Cognitive Map Donghua University: Shanghai Chapter 2 (in Chinese)*
- [5] Hagiwara M 1992 *Proceedings of the International Conference on Fuzzy Systems IEEE Press San Diego* 795-801
- [6] Marchant T 1999 *European Journal of Operational Research* 114(3) 626-37
- [7] Carvalho J P, Tome J A 1999 *Evolutionary Computation & Fuzzy Logic for Intelligent Control Knowledge Acquisition & Information Retrieval IOS Press USA* 276-81
- [8] Carvalho J P, Tome J A 2000 *Proceedings of the International Conference of the North American Fuzzy Information Processing Society Atlanta IOS Press USA* 407-11
- [9] Feng L X, Jun GAO 2003 *Journal of Computer Research and Development* 40(7) 925-33 (in Chinese)
- [10] Feng L X, Gao J 2003 *Journal of University of Science and Technology of China* 33(1) 26-33 (in Chinese)
- [11] Cole R J, Persichitte K A 2000 *International Journal of Intelligent Systems* 15 1-25
- [12] Stylios C D, Groumpos P P 1999 *Computers in Industry* 39(3) 229-38
- [13] Lee K C, Kim J S, Chung N H, Kwon S J 2002 *Expert Systems with Application* 22(3) 197-211
- [14] Min, H Q, Hui J X, Lu Y S, Jia Z J 2006 *Proceedings of the International Conference on Intelligent Agent Technology Hong Kong, China IOS Press USA* 321-5
- [15] Kardaras D, Mentzas G 1997 *Advances in Industrial Engineering Applications and Practice II* 63-68
- [16] Aguilar J 2001 *Lecture Notes in Artificial Intelligence* 2070 333-8
- [17] Aguilar J 2001 *Proceedings of the International Conference of the International Joint Conference on Neural Networks Washington IOS Press USA* 1380-5
- [18] Aguilar J 2003 *International Journal of Computational Cognition* 1(4) 91-107
- [19] Groumpos P P, Stylios C D 2000 *Chaos Solitons and Fractals* 11(1-3) 329-36
- [20] Stylios C D, Groumpos P P 1999 *Journal of Intelligent and Robotic Systems* 26(3-4) 389-403
- [21] Ndousse T D, Okuda T 1996 *Proceedings of the IEEE International Conference on Communications* 1558-1562
- [22] Feng L X 2003 *Dissertation Submitted to Hefei University of Technology for the Degree of PhD* 1-67 (in Chinese)
- [23] Elpiniki I P, Stylios C D, Groumpos P P 2003 *IEEE Transactions on Biomedical Engineering* 50(12) 326-39
- [24] Stach W, Pedrycz W, Kurgan L 2004 *NAFIPS* 28-33
- [25] Yun Z G, Rong M X, Ru Y B 2007 *Computer Science in China* 34(4) 129-32 (in Chinese)
- [26] Papageorgiou E I, Spyridonos P P, Glotsos D Th, Stylios C D 2008 *Applied Soft Computing* 8 820-8
- [27] Papageorgiou E I, Georgoulas G, Stylios C D, Nikiforidis G N, Groumpos P P 2006 *Proceedings of the International Conference of International Conference on Knowledge-Based & Intelligent Information & Engineering Systems* 515-23
- [28] Wojciech F, Alicja W D 2009 *Proceedings of the International Conference on Human System Interactions IEEE Press Catania* 13-20

2) FCM integration strategy. Based on the existing ensemble theory, the ensemble theory is introduced into the FCM integrated framework in the progress of FCM.


3) To expand the application field of FCM. FCM has been applied to some simple environment, how to put it into more complex environment such as the virtual world with many experts need researchers pay more attentions, when the FCM perhaps becomes a powerful tool for intelligent decision support.

4) So far, the research of FCM is limited to the algorithms, but the theoretical basis is still lack and the research fruits are not enough rich to people, most research is on the basis of the experiment method lack of mature theoretical support, so it is very important to study further.

5) The mechanism, model theory of FCM may produce a breakthrough point, i.e., FCM will integrated with the nonlinear dynamics, streaming media dynamics, multi relational mining, manifold learning and directed hypergraph model.

Acknowledgments

This research is partially supported by the National Natural Science Foundation of China under grants 61300078 and 61175048, and the project of new starting point of Beijing Union University under grants ZK10201312(Research on fuzzy cognitive map ensemble classifiers and its application).

Authors	
	<p>Nan Ma, born in December, 1978, China</p> <p>Current position, grades: PhD, Beijing Union University. Scientific interest: knowledge discovery. Publications: 5 books and more than 20 papers.</p>
	<p>Yun Zhai, born in June, 1979, China</p> <p>Current position, grades: PhD, Chinese Academy of Governance. Scientific interests: knowledge discovery. Publications: more than 30 papers.</p>
	<p>Bingru Yang, born in March, 1941, China</p> <p>Current position, grades: University of Science and Technology Beijing. Scientific interests: knowledge discovery. Publications: 10 books and more than 500 papers.</p>