Multi-objective hub location problem in hub-and-spoke network

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Abstract

Through observations from the construction of Chinese national emergency material reserve system, we introduce the multi-objective hub location problem. We provide a mathematical model for finding the optimal hub locations to minimize the total transportation cost and maximize the coverage of the hubs simultaneously in the whole network. Then, a procedure for solving this model is proposed. By using a numerical example, we discuss the efficiency of the tabu-search-based algorithm compared to the complete enumeration method and the impact of cost discount factor on the performance of hub-and-spoke network. The results show that the heuristic algorithm based on tabu search may be better than the complete enumeration research method for big size multi-objective hub location problem and as the cost discount factor is increased, the cost savings in the hub-and-spoke network compared to the direct connect network would decrease while the covering rate remains the same unless the cost discount factor is close to 1. Finally, we set future research directions on the multi-objective hub location problem.

Keywords: hub location problem, multi-objective programming, hub-and-spoke network, tabu search

1 Introduction

Our research is motivated by the practices of the construction of Chinese national emergency material reserve system. In order to achieve quick response to the urgent need of emergency materials in affected areas right after disasters, China began to build a national emergency material reserve system in 1998. Until now, there have been 18 central-level warehouses in the whole country, and each province has established a provincial-level warehouse, as well as 92 percent of cities and 60 percent of towns own town-level warehouses. In the next ten years, China plans to build more emergency material warehouses with different levels to form a perfect-served emergency material reserve system.

In China, once a natural or man-made disaster occurs, the local town-level and provincial-level warehouses would transport the emergency materials to the affected areas as quickly as possible. If the amount of the materials required exceeds the available stock in the local town-level and provincial-level warehouses, the nearest central-level warehouses would be involved in supplying the materials to the local lower-level warehouses. The other central-level warehouses would gather the materials, receive the donations and transfer them to the disaster-affected area if needed. To a certain extent, Chinese emergency material reserve system works like a hub-and-spoke network since a certain portion of materials flowing among provincial-level warehouses and town-level warehouses is transferred via the central-level warehouses which can be regarded as hubs.

The one basic difficulty for associating the hub-and-spoke network with emergency logistics is that in the emergency logistics system the central-level warehouses need to cover as many emergency material flows as possible if the disaster occurs and, at the same time, the transportation cost should be acceptable when considering that the flows via central-level warehouses tend to generate many detours. As a result, the construction of emergency logistics system has multiple objectives, including the total transportation cost and the coverage of the warehouses. Consequently, the location of central-level warehouses in the emergency material reserve system is a *multi-objective hub location problem* (MOHLP) in the hub-and-spoke network.

Hub-and-spoke network is widely used in many transportation networks where hubs usually act as sorting, transhipment, and consolidation terminals. Instead of sending flows directly between all origin–destination pairs of nodes, hub facilities consolidate flows in order to take advantage of the economies of scale. Hub location problem (HLP) is an important issue arising in the design of hub-and-spoke network [1]. Much research has focused on presenting discrete hub median and related models to better capture behaviour observed in practice. However, to our knowledge there is very limited research that studies multi-objective multiple allocation hub location problem. Farahani et al. [2] reviewed all variants of HLP and discussed the mathematical models, solution methods, main specifications, and applications of HLPs, which including multi-objective HLP. Wang et al. [3] developed a fuzzy bi-objective programming model for emergency logistics systems by considering fuzzy demand of relief materials, timeliness and limited resources. The goal of their model is to minimize the total cost and the relief time of system. However, they focused

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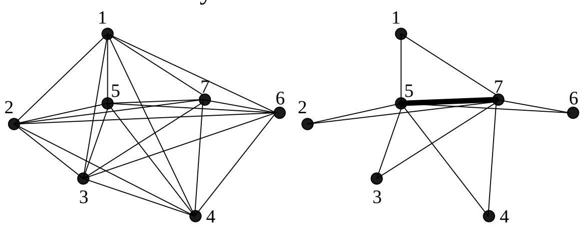
on location-routing problem (LRP) instead of HLP. Mirzaei and Bashiri [4] used multiple objective approach for hub location to minimize total cost of the networks and minimize maximum travel time between nodes, whereas they only presented a brief comparison between their formulation and the other existing models in terms of the number of variables and constraints of the models. Tavakkoli-Moghaddam et al. [5] presented a multi-objective mathematical model for a capacitated single-allocation hub location problem. The multiple objectives are to minimize the total cost of the networks and minimize the maximum travel time between nodes. They solved the model by a multi-objective imperialist competitive algorithm (MOICA). However, the algorithm they proposed need more time to solve the models due to introducing many comparison metrics. Barzinpour et al. [6] developed a mathematical programming formulation for the bi-objective non-strict single allocation hub location problem. The objectives include minimizing the total system-wide cost and minimizing the maximum commodity routing distance between pairs of nodes in the network. They proposed a heuristic algorithm based on tabu search approach to solve the model. Geramianfar et al. [7] considered a multi-objective hub covering location problem under congestion. The first objective is to minimize total transportation cost and the second one is to minimize total waiting time for all hubs. They used simulated annealing (SA) to solve the model and compared the performance of the model against two other alternative methods, that is, particle sward optimization and NSGA-II. Tajbakhsh et al. [8] proposed a hub location model with both qualitative and quantitative objectives. To achieve better solutions, infeasible regions were also taken into account and a graded penalty term was used to penalize infeasible solutions. However, they did not present any numerical examples to prove the effectiveness of the algorithm they suggested. Costa et al. [9] proposed a multi-objective HLP in which the first objective is to minimize the total transportation cost, and the second one is to minimize the maximum service time of the hub nodes, whereas they did not provide any heuristic algorithm.

The purpose of this study is to develop a mathematical mode in the context of MOHLP in hub-and-spoke network and also present an effective algorithm to work out the best hub locations. The rest of this paper is organized as follows. In Section 2 we build a mathematical model for finding the optimal hub locations to minimize the total transportation time and to maximize the coverage of the hub(s). Section 3 proposes a heuristic algorithm to find the optimal solutions of the model. The results of a numerical example are presented in Section 4. Section 5 discusses the impact of the cost discount factor on the performance of the hub-and-spoke network and provides some managerial insights. Finally, Section 6 presents summary comments and discusses promising areas for future research.

2 Model formulation

We consider a hub-and-spoke network consisting of several nodes. Traditionally, each origin-destination pair of nodes can be connected directly, which is so-called direct connect network (see Figure 1a). However, in the context of a hub-and-spoke network, each origin–destination flow must be routed via the hubs (see Figure 1b). The main problem in this network is to decide on the location of the hubs and the allocation of the non-hub nodes to these hubs. Our objectives are to minimize the total transportation cost and maximize the coverage of the hubs simultaneously.



(a) Direct connect network (b) Hub-and-spoke network

FIGURE 1 Direct connect network and hub-and-spoke network

2.1 NOTATIONS

Consider a complete graph $G(V,A)$ with node set $V = 1,2,\ldots,N$ where nodes correspond to origins and destinations as well as potential hub locations. The notations we used are as follows.

Index and parameters:

A : set of all arcs;

N : the number of nodes;

p : the number of hubs;

i , j : index for nodes ($i, j = 1,\cdots,N$);

k , m : index for potential hub locations;

c_{ij} : standard cost per unit from origin i to j ;

h_{ij} : the amount of flows between nodes i and j ;

J : set of origin-destination pairs of nodes, $J = (i,j) | h_{ij} > 0, i, j \in V$;

C_{ij}^{km} : the transportation cost of a unit of flow from origin i to destination j via hubs k and m on path $i-k-m-j$. Note that c_{ij}^{km} is composed of three parts: a cost of communication from the source node to its respective hub, a cost of communication to a sink node from its respective hub, and the cost of communication between the two hubs. Consequently, we have

$$C_{ij}^{km} = c_{ik} + \alpha c_{km} + c_{mj} \,, \tag{1}$$

where α is the cost discount factor for the inter-hub transportation due to heavy traffic [10].

V_{ij}^{km} : the binary variable. V_{ij}^{km} is equal to 1 if hubs k and m cover origin-destination pair (i, j), and zero otherwise. Let β_{ij} be the maximum cost for origin-destination pair (i, j). The interpretation of coverage is that origin-destination pair (i, j) would be covered by hubs k and m if the transportation cost from i to j via k and m does not exceed a specified value, that is,

$$V_{ij}^{km} = \begin{cases} 1, & if\ C_{ij}^{km} \le \beta_{ij} \\ 0, & otherwise, \end{cases} \tag{2}$$

Decision variables:

X_{ij}^{km} : the binary variable. X_{ij}^{km} is equal to 1 if shipments from origin i to destination j are assigned to hubs k and m on path $i-k-m-j$, and zero otherwise.

Y_k : the binary variable. Y_k is equal to 1 if node k is chosen to be a hub, and zero otherwise.

2.2 ASSUMPTIONS

The assumptions of our model are as follows:

(1) The transportation between any two nodes in the network is performed only via hubs.

(2) The network has enough arcs with sufficient capacity to enable all the flows generated at the origin nodes to reach all the destination nodes regardless of the amount of flows received by the hubs, that is, we do not take capacity constraints into account.

(3) There are two objectives under consideration in this study. The first one is to minimize the total cost in the hub-and-spoke network, and the second one is to maximize the coverage of the hubs. These two objectives are of roughly comparable importance. As a result, the MOHLP is a non-preemptive objective programming.

2.3 MATHEMATICAL MODEL

In our model, the decision involves choosing X_{ij}^{km}, Y_k to minimize the total transportation cost and maximize the coverage of the hubs. The mathematical formulation for this MOHLP is as follows:

$$\min\ z_1 = \sum_i \sum_j \sum_k \sum_m h_{ij} X_{ij}^{km} C_{ij}^{km}, \tag{3}$$

$$\max\ z_2 = \sum_i \sum_j \sum_k \sum_m h_{ij} X_{ij}^{km} V_{ij}^{km} \tag{4}$$

subject to

$$\sum_k \sum_m X_{ij}^{km} = 1,\ \forall (i, j) \in J, \tag{5}$$

$$X_{ij}^{km} \le Y_k,\ \forall i, j, k, m, \tag{6}$$

$$X_{ij}^{km} \le Y_m,\ \forall i, j, k, m, \tag{7}$$

$$\sum_k Y_k = p, \tag{8}$$

$$X_{ij}^{km} \in \{0,1\},\ \forall i, j, k, m, \tag{9}$$

$$Y_k \in \{0,1\},\ \forall k. \tag{10}$$

In the above formulation, z_1 is the total transportation cost in the network and z_2 is the coverage of the hubs. Constraint (5) ensures that each origin–destination flow is sent via some hub pair (possibly a single hub as in X_{ij}^{kk}). Constraints (6) and (7) ensure that nonhub nodes can only be allocated to the hubs which work as transfer terminals. Constraint (8) requires exactly p hubs are selected. Constraints (9) and (10) define the decision variables to be binary.

3 Solution procedure for the model

3.1 SOLUTION PROCEDURE FOR SHLPOMIC

Before giving the algorithms for solving MOHLP, we define a single hub location problem of minimizing cost (SHLPOMIC) as follows: $\min\ z_1 = \sum_i \sum_j \sum_k \sum_m h_{ij} X_{ij}^{km} C_{ij}^{km}$ subject to constraints (5)-(10).

SHLPOMIC is NP-hard. A heuristic algorithm based on tabu search is widely used to solve this kind of problem [11, 12]. The elements of tabu search include initial solution, neighbourhood structure, tabu lists, and so on. Remember that V represents the set of all nodes. Let T be the set of hubs, $V-T$ be the set of nonhub nodes, $N(T)$ be the set of neighbouring solutions for T. For simplicity, we adopt a 'single location exchange' rule to generate a neighbourhood solution, denoted by T^i, for a given T, that is, replacing exactly one hub in T with one nonhub node in $V-T$. During the process of 'single location exchange', the node leaving T is denoted by V^i and the node leaving $V-T$ and entering T is denoted by W^i. Under this rule, there are $p(n-p)$ possible neighbouring solutions for a given T, i.e., $N(T) = \{T^i, i = 1, 2, \dots, p(n-p)\}$. The node leaving T and generating a new T^i at the current iteration is recorded in the tabu list, named as tabu status. In order to forbid the reversal of this replacement unless the move leads to a solution better than the best found so far (this is the so-called aspiration criterion), the tabu status at the current iteration cannot be selected to enter T again in a number (tabu list size) of future iterations. For instance, setting $tabu\_tag(i)=t$ means that node i acts as a tabu status and cannot be an element of T in the next t iterations. The value of the objective function is expressed in terms of just the current T as

$$Z_1(T)=\sum_i\sum_j h_{ij}\min_{k,m\in T} C_{ij}^{km} \quad . \tag{11}$$

In order to improve the efficiency of tabu search, we would obtain the initial solution where the search starts by employing a local search method. The solution procedure can be summarized as follows.

Step 1. Choose arbitrarily p nodes as an initial solution T. Designate this initial solution as the optimal solution, that is, set $T^0=T$.

Step 2. Generate $N(T)$ for current T and calculate $Z_1(T^i)$ via (11) for all $T^i\in N(T)$. Find the smallest one and let $T*$ denote corresponding T^i.

Step 3. If $Z_1(T^*)<Z_1(T^0)$, set $T^0=T^*$, $T=T^*$, and then go to Step 2. Otherwise, go to Step 4 with the initial solution T.

Step 4. Initialize the tabu lists and the number of iterations, that is, set a value for the maximal number of iterations denoted by $\max\_itm$ and set $t=0$, $tabu\_tag(i)=0$ for all $i\in V$. Update the current optimal solution T^0 and set $T^0=T$.

Step 5. Generate $N(T)$ for current T. Calculate $Z_1(T^i)$ via (11) for all $T^i\in N(T)$.

Step 6. Choose index l such that $Z_1(T^l)=\min_{i=1,\dots,p(n-p)} Z_1(T^i)$. If $tabu\_tag(W^l)=0$ or $Z_1(T^l)<Z_1(T^0)$ (the aspiration criterion), then set $T=T^l$, record V^l in the tabu list, i.e., set $tabu\_tag(V^l)$ equal to a uniform random number over the interval $[\sqrt{n},2\sqrt{n}]$, and go to Step 7. Otherwise, delete T^l from $N(T)$ (that is, set $N(T)=N(T)-T^l$), and return to Step 6.

Step 7. Set $t=t+1$. If $Z_1(T)<Z_1(T^0)$, then update the current optimal solution (that is, set $T^0=T$).

Step 8. If $t<\max\_itm$, then update the tabu list, i.e., set $tabu\_tag(i)=tabu\_tag(i)-1$ for all i such that $tabu\_tag(i)>0$, and return to Step 5. Otherwise, the best solution of SHLPOMIC is T^0.

3.2 SOLUTION PROCEDURE FOR SHLPOMAC

Similarly, we define a single hub location problem of maximizing covering (SHLPOMAC) as follows: $\max\ z_2=\sum_i\sum_j\sum_k\sum_m h_{ij}X_{ij}^{km}V_{ij}^{km}$ subject to constraints (5)-(10).

The value of the objective function is expressed in terms of just the current T as

$$Z_2(T)=\sum_i\sum_j h_{ij}\min\left(1,\sum_{k\in T}\sum_{m\in T}V_{ij}^{km}\right). \tag{12}$$

The solution procedure can be summarized as follows.

Step 1. Set the initial solution $T=\varphi$, and also set $k=0$.

Step 2. Choose a node $l*$ from $V-T$ by which the amount of the new origin-destination flows covered is maximal, i.e., $V_{l*}=\max_{l\in N-T}\left[Z_2\ T+l\ -Z_2\ T\ \right]$. Set $T=T+l$ and $k=k+1$.

Step 3. If $k<p$, then go to Step 2. Otherwise, go to Step 4 with the initial solution T.

Step 4. Initialize the tabu lists and the number of iterations, that is, set a value for the maximal number of iterations denoted by $\max\_itm$ and set $t=0$, $tabu\_tag(i)=0$ for all $i\in V$. Update the current optimal solution T^0 and set $T^0=T$.

Step 5. Generate $N(T)$ for current T. Calculate $Z_2(T^i)$ via (12) for all $T^i\in N(T)$.

Step 6. Choose index l such that $Z_2(T^l)=\max_{i=1,\dots,p(n-p)} Z_2(T^i)$. If $tabu\_tag(W^l)=0$ or $Z_2(T^l)>Z_2(T^0)$, then set $T=T^l$, record V^l in the tabu list, i.e., set $tabu\_tag(V^l)$ equal to a uniform random number over the interval $[\sqrt{n/2},\ \sqrt{n}]$, and go to Step 7. Otherwise, delete T^l from $N(T)$ (that is, set $N(T)=N(T)-T^l$), and return to Step 6.

Step 7. Set $t=t+1$. If $Z_2(T)>Z_2(T^0)$, then update the current optimal solution (that is, set $T^0=T$).

Step 8. If $t<\max\_itm$, then update the tabu list, that is, set $tabu\_tag(i)=tabu\_tag(i)-1$ for all i such that $tabu\_tag(i)>0$, and return to Step 5. Otherwise, the best solution of SHLPOMAC is T^0.

3.3 SOLUTION PROCEDURE FOR MOHLP

MOHLP will be harder to solve to optimality with the addition of multiple objectives. Thus, there will be a need to develop efficient heuristic algorithms for it.

One of the most common approaches to multi-objective optimization is the goal programming method [13]. Let $X=\ X_{ij}^{km},Y_k$ be the vector of decision variables and F represent the feasible set of decision vectors for which all the constraints are satisfied (that is, $X \in F$). Also let $z_l(X)$ denote the l th objective function, X_l^* represent the optimum of the l th single-objective function subject to the constraints in the multi-objective problem, z_l^* be the corresponding objective function values, and b_l be the target value for the l th objective function $z_l(X)$. In the absence of any other information, we can set $b_l = z_l^*$. Then, according to the theory of goal programming method, we should minimize the total deviation from the goals $\sum_l |d_l|$, where d_l is the deviation from the goal b_l for the l th objective. To model the absolute values, d_l is split into positive and negative parts such that $d_l = d_l^+ - d_l^-$, with $d_l^+ \geq 0$, $d_l^- \geq 0$, $d_l^+ d_l^- \geq 0$. We have $|d_l| = d_l^+ + d_l^-$. d_l^+ and d_l^- represent underachievement and overachievement, respectively, where achievement implies that a goal has been reached. In the case of our model, we have the following parameters: $l \in \{1,2\}$, F is determined by constraints (5) ~(10), $z_1(X)$ and $z_2(X)$ are given by (3) and (4) (note that $X = (X_{ij}^{km}, Y_k)$), z_1^* and z_2^* are the optimal objective function values for SHLPOMIC and SHLPOMAC, respectively. Consequently, the optimization problem, named as Problem (MOP), is formulated as follows:

Problem (MOP):

$$\min z' = \sum_{l=1}^{2} (P_l \lambda_l d_l^+ + P_l \lambda_l d_l^-) \tag{13}$$

subject to

$$z_l(X) - d_l^+ + d_l^- = z_l^*, \; l = 1,2, \tag{14}$$

$$X \in F, \tag{15}$$

$$d_l^+, d_l^- \geq 0, \tag{16}$$

where P_l is the weighting coefficients and λ_l is the normalization constant for the l th objective. According to the assumption that all the objectives have the same priority level, we set the weighting coefficients $P_l = 1$ for $l = 1,2$.

A major difficulty of adopting the goal programming method lies in the incommensurability, which occurs when deviational variables measured in different units are summed up directly. This simple summation will cause an unintentional bias towards the objectives with a larger magnitude, which may lead to erroneous or misleading results. To overcome the incommensurability, we suggest using the percentage normalization method where the normalization constant is hundred divided by the target value: $\lambda_l = 100 / b_l$ for $l = 1,2$ [14]. This ensures that all deviations are measured on a percentage scale.

However, (14) is a nonlinear equality constraint, which makes it not easy to find the optimal solution with larger problems. Here, we propose applying the tabu search method again to solve (MOP) approximately. The solution procedure can be summarized as follows:

Step1. Solve SHLPOMIC and SHLPOMAC by means of the algorithms mentioned earlier in this section and denote the best achieved objective function values by z_1^* and z_2^*, respectively. Set $\lambda_l = 100 / z_l^*$ for $l = 1,2$.

Step 2. Choose arbitrarily p nodes as an initial solution T. Designate this initial solution as the optimal solution, that is, set $T^0 = T$.

Step 3. Calculate $Z_l(T^0)$ via (11) and (12) for each $l = 1,2$, respectively. Substitute $Z_l(T^0)$ for $z_l(X)$ in (MOP). Note that once $Z_l(T^0)$ is given, (MOP) becomes a linear programming problem. Solve (MOP) and denote the optimal objective function value by $z'(T_0)$.

Step 4. Generate $N(T)$ for current T. For all $T^i \in N(T)$, Calculate $Z_1(T^i)$ and $Z_2(T^i)$ via (11) and (12), Substitute $Z_1(T^i)$ and $Z_2(T^i)$ for $z_1(X)$ and $z_2(X)$ in (MOP). Solve (MOP) to obtain the objective function value $z'(T^i)$. Find the smallest one and let $T*$ denote corresponding T^l.

Step 5. If $z'(T^*) < z'(T^0)$, set $T^0 = T^*$, $T = T^*$, and then go to Step 4. Otherwise, go to Step 6 with the initial solution T.

Step 6. Initialize the tabu lists and the number of iterations, that is, set a value for the maximal number of iterations denoted by $\max\_itm$ and set $t = 0$, $tabu\_tag(i) = 0$ for all $i \in V$. Update the current optimal solution T^0 and set $T^0 = T$.

Step 7. Generate $N(T)$ for current T. For all $T^i \in N(T)$, Calculate $Z_1(T^i)$ and $Z_2(T^i)$ via (11) and (12), Substitute $Z_1(T^i)$ and $Z_2(T^i)$ for $z_1(X)$ and $z_2(X)$ in (MOP). Solve (MOP) to obtain the objective function value $z'(T^i)$.

Step 8. Choose index l such that $z'(T^l) = \min_{i=1,\dots,p(n-p)} z'(T^i)$. If $tabu\_tag(W^l) = 0$ or $z'(T^l) < z'(T^0)$ (the aspiration criterion), then set $T = T^l$, record V^l in the tabu list, i.e., set $tabu\_tag(V^l)$ equal to a uniform random number over the interval $[\sqrt{n}, 2\sqrt{n}]$, and go to Step 9. Otherwise, delete T^l from $N(T)$ (that is, set $N(T) = N(T) - T^l$), and return to Step 8.

Step 9. Set $t = t+1$. If $z'(T) < z'(T^0)$, then update the current optimal solution (that is, set $T^0 = T$).

Step 10. If $t < \max\_itm$, then update the tabu list, that is, set $tabu\_tag(i) = tabu\_tag(i) - 1$ for all i such that $tabu\_tag(i) > 0$, and go to Step 7. Otherwise, the best solution of the MOHLP is T^0.

4 Numerical examples

In this section we adopt the AP date set that is used in various hub location studies and is available from OR Library (http://mscmga.ms.ic.ac.uk/info.html). AP data set consists of 200 nodes. Without loss of generality, we only consider the first 7 nodes in the data set corresponding to 7 cities (that is, $n = 7$). All the 7 cities are the candidates of hub locations. Set the transportation cost per kilometre per unit of flow to be 3. Consequently, c_{ij} is taken to be equal to 3 times the distance between i and j in AP set, as shown in Table 1. The flows in and out of these cities are shown in Table 2. We set the flows within the same node to be zero, i.e., $h_{ii} = 0$, to avoid the unreasonable detour between nodes and hubs. We also set $\max\_itm = 100$, $\alpha = 0.4$, $p = 2$, and $\beta_{ij} = 1.2c_{ij}$.

TABLE 1 Standard cost per unit between pairs of cities c_{ij} in thousand dollars

Origin i	Destination j						
	2	3	4	5	6	7	8
2	0	5.21	0.43	5.35	10.03	1.73	5.62
3	5.21	0	5.19	0.87	15.16	5.35	1.74
4	0.43	5.19	0	5.26	9.98	1.30	5.47
5	5.35	0.87	5.26	0	15.13	5.21	0.87
6	10.03	15.16	9.98	15.13	0	9.95	15.16
7	1.73	5.35	1.30	5.21	9.95	0	5.21
8	5.62	1.74	5.47	0.87	15.16	5.21	0

TABLE 2 The amount of flows between pairs of cities h_{ij}

Origin i	Destination j						
	2	3	4	5	6	7	8
2	0	0.01	0.01	0.01	0.01	0.01	0.01
3	0.01	0	0.20	0.01	0.05	0.26	0.16
4	0.01	0.09	0	0.01	0.05	0.26	0.16
5	0.01	0.01	0.01	0	0.01	0.02	0.01
6	0.01	0.03	0.07	0.01	0	0.10	0.06
7	0.01	0.14	0.31	0.01	0.07	0	0.25
8	0.01	0.05	0.12	0.01	0.03	0.15	0

We first program the algorithm described in Section 3 with MATLAB 7.1 for SHLPOMIC and SHLPOMAC. Under SHLPOMIC, the nodes 5 and 7 are designated as hubs and the corresponding optimal total transportation cost in this hub-and-spoke network z_1^* is 12.185. Under SHLPOMAC, nodes 4 and 5 are designated as hubs and the amount of flows that are covered by the hubs is 2.8535.

After obtaining the z_1^* and z_2^*, we then program the algorithm for MOHLP described in Section 3 with MATLAB 7.1. The results show that nodes 5 and 7 are designated as hubs. The corresponding objective function value z' is 0.701. In this situation, the total transportation cost z_1 is 12.185 and the coverage of hubs z_2 is 2.8335. The route that starting from origin i to destination j via two hubs are given below in Table 3. Here the symbol | indicates the pair of hubs. For instance, according to Table 3, the flow from origin 3 to destination 6 is sent on path 3-5-7-6.

TABLE 3 The routes for origin-destination flows

Origin i	Destination j						
	2	3	4	5	6	7	8
2	-	7|5	7|7	7|5	7|7	7|7	7|5
3	5|7	-	5|7	5|5	5|7	5|7	5|5
4	7|7	7|5	-	7|5	7|7	7|7	7|5
5	5|7	5|5	5|7	-	5|7	5|7	5|5
6	7|7	7|5	7|7	7|5	-	7|7	7|5
7	7|7	7|5	7|7	7|5	7|7	-	7|5
8	5|7	5|5	5|7	5|5	5|7	5|7	-

By multiplying each of the individual terms in Table 1 by the corresponding term in Table 2, and summing up these individual products, we also calculate the total transportation cost in the situation where the flows of materials are sent directly through the arcs linking origin–destination nodes, denoted by z^d, as follows.

$$z^d = \sum_i \sum_j c_{ij} h_{ij} , \tag{17}$$

let $\Delta z' = z^d - z_1$ be the cost savings in the hub-and-spoke network compared to direct connect network. We have $z^d = 15.07$ and $\Delta z' = 2.885$. This result shows that compared to direct connect network, the hub-and-spoke network can save 2.885 thousand dollars.

Meanwhile, we also calculate the covering rate, denoted by β, as follows

$$\beta = z_2 / \sum_i \sum_j h_{ij} . \tag{18}$$

The resulting value of β is 99%, which demonstrates that the majority of flows is covered by the hubs.

In order to observe the performance of the heuristic algorithm on a different data set, we generate a set of test instances with different parameters. Let t_{tabu} and t_{enum} denote the respective CPU time requirements of the heuristic algorithm presented in Section 3 and the complete enumeration method. Table 4 shows the CPU times in different methods.

TABLE 4 Performance of the heuristic algorithm on the network

p	**Hubs**	z'	$\Delta z'$	β	t_{tabu}	t_{enum}
2	5,7	0.701	2.885	99.3%	20.014	7.83
3	6,5,7	0.701	5.885	99.2%	21.221	9.865
4	6,7,4,5	0	7.412	1	34.563	10.074
5	7,3,4,8,6	0	8.901	1	24.233	11.072

From Table 4, we can see that, generally, the CPU time requirement of the heuristic algorithm for the MOHLP increases with the increase of the candidate number of hub locations. However, this outcome seriously depends on the selected initial solution.

It can also be seen that the solutions provided by the algorithm we proposed and the complete enumeration research method are the same as well as the complete enumeration research method seems more effective than the algorithm presented in Section 3 when the number of the nodes is small. However, due to the fact that the number of hub arc combinations increases faster than linearly, we recommend using the heuristic algorithm based on tabu search instead of the complete enumeration research method for big problems.

5 The impact of cost discount factor on the performance of hub-and-spoke network

Intuitively, the cost discount factor may have the effect on the performance of hub-and-spoke network. Keeping the other parameters presented in Section 4 unchanged, for different values of α, we calculate the corresponding $\Delta z'$, β, z_1^*, z_1, z_2^*, z_2. Figure 2 and Figure 3 display $\Delta z'$ and β with different values of α, respectively. Figure 4 shows the corresponding z_1^* and z_1. Figure 5 shows the corresponding z_2^* and z_2.

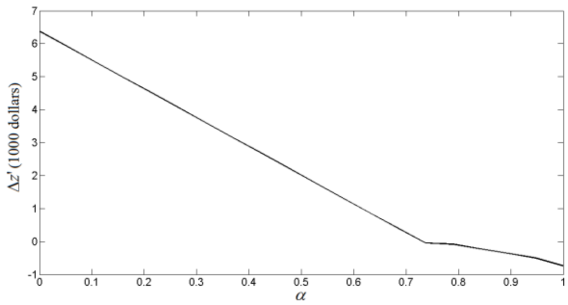


FIGURE 2 The relation between the cost savings in the hub-and-spoke network and the cost discount factor

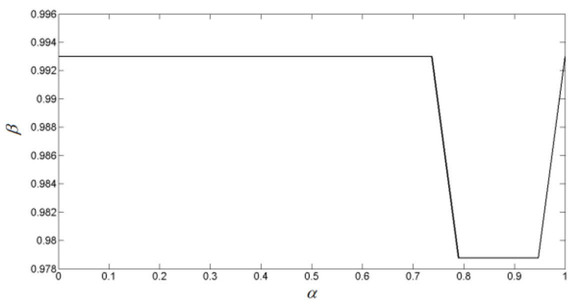


FIGURE 3 The relation between the covering rate and the cost discount factor

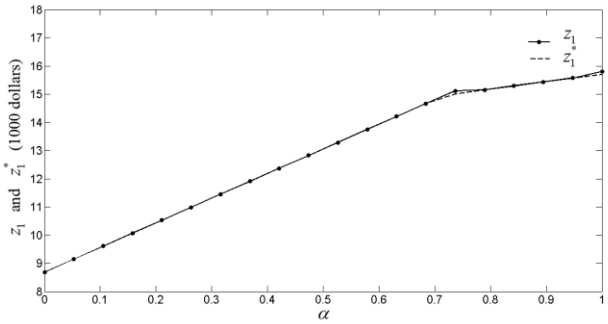


FIGURE 4 The total transportation costs in MOHLP and SHLPOMIC with changes in the transportation cost discount factor

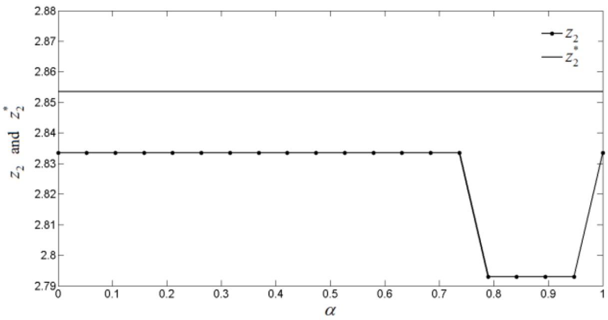


FIGURE 5 The total transportation costs in MOHLP and SHLPOMAC with changes in the transportation cost discount factor

From Figures 2 and 3, it can be seen that, as the cost discount factor is increased, the cost savings in the hub-and-spoke network compared to direct connect network would decrease, which means that the cost discount factor has negative effects on the performance of hub-and-spoke network. Meanwhile, the cost discount factor seems likely to have no significant effect on the covering rate because the latter depends much more upon the maximum cost for origin-destination pair (that is β_{ij}) than upon the cost discount factor. Note that when the cost discount factor is close to 1, the covering rate would sharply decrease to a certain value and then quickly increase to the value which is the same as that with a small α. The reason is that in a situation where α is large, there is no improvement in the total transportation cost in the hub-and-spoke network compared to the direct connect network so that few origin-destination flows would be sent via hubs.

From Figures 4 and 5, we can see that in the context of our example, generally, the goal of minimizing the total transportation cost can be achieved in MOHLP, whereas there is a difference between the coverage of the hubs achieved in MOHLP and that in SHLPOMAC. Note that when the cost discount lies in the interval between 0.7 and 0.8, the total transportation cost in MOHLP would be larger than that in SHLPOMIC. This result indicates that when there are multiple objectives in the hub-and-spoke network, one objective may be achieved better than the others. If we want to obtain the cost objective, the cost discount factor should be taken on a small value near zero.

6 Conclusion

In this paper we introduce a multi-objective hub location problem in the hub-and-spoke network. We provide a mathematical model for minimizing the total transportation cost and maximizing the coverage of the hubs. We then propose a heuristic algorithm based on tabu search for finding the optimal hub locations. Additionally, we compare our algorithm with the complete enumeration method and investigate the impact of cost discount factor on the performance of hub-and-spoke network. The results of a numerical example show that our heuristic algorithm based on tabu search may be better than the complete enumeration research method for big size MOHLP, and as the cost discount factor is increased, the cost savings in the hub-and-spoke network compared to direct connect network would decrease while the covering rate remains the same unless the cost discount factor is close to 1. Another interesting finding is that when there are multiple objective in the hub-and-spoke network, one objective may be achieved better than the others.

There are several possibilities for furthering our research on this topic area. First, one could research the situation where the hub facility has limited capacity and the amount of the flows in arcs of the hub-and-spoke network is limited. Naturally, the problem will be harder to solve to optimality with the addition of these capacity restrictions. Second, one could consider the other objectives, such as time objective, capacity objective and commodity routing distance objective. However, after adapted to the specific situations, our algorithm is available in many cases. A third extension would be to include demand uncertainty, which means that the demands for materials from origin nodes to destination nodes are stochastic. An efficient solution algorithms need to be developed for dealing with such a problem.

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References

[1] Campbell J F 1994 Integer programming formulations of discrete hub location problems *European Journal of Operational Research* **72**(2) 387–405

[2] Farahani R Z, Hekmatfar M, Arabani A B, Nikbakhsh E 2013 Hub location problems: A review of models, classification, solution techniques, and applications *Computers & Industrial Engineering* **64**(4) 1096–109

[3] Wang S, Ma Z, Zhuang B 2014 Fuzzy location-routing problem for emergency logistics systems *Computer Modelling & New Technologies* **18**(2) 265-73

[4] Mirzaei M, Bashiri M 2010 Multiple objective multiple allocation hub location problem *Proc. Int. Conf. Comput. Ind. Eng.: Soft Comput. Tech. Adv. Manuf. Serv. Syst., CIE40* (Awaji) IEEE Computer Society: Piscataway, NJ, United States pp 1-4

[5] Tavakkoli-Moghaddam R, Gholipour-Kanani Y, Shahramifar M 2013 A multi-objective imperialist competitive algorithm for a capacitated single-allocation hub location problem *International Journal of Engineering* **26**(6) 605-20

[6] Barzinpour F, Ghaffari-Nasa N, Saboury A 2011 Bi-objective non-strict single allocation hub location problem: mathematical programming model and a solution heuristic *Proc. Int. Conf. Comput. Ind. Eng., CIE41 (Los Angeles)* pp 86-91

[7] Geramianfar R, Pakzad M, Golhashem H, Moghaddam R T 2013 A multi-objective hub covering location problem under congestion using simulated annealing algorithm *Uncertain Supply Chain Management* **1**(3) 153-64

[8] Tajbakhsh A, Haleh H, Razmi J 2013 A multi-objective model to single-allocation ordered hub location problems by genetic algorithm *International Journal of Academic Research in Business and Social Sciences* **3**(2) 374-9

[9] Costa M G, Captivo M E, Climaco J 2008 Capacitated single allocation hub location problem – A bi-criteria approach *Computers and Operations Research* **35**(11) 3671-95

[10] Limbourg S, Jourquin B 2009 Optimal rail-road container terminal locations on the European network *Transportation Research Part E* **45**(4) 551-63

[11] Skorin-Kapov D, Skorin-Kapov J 1994 On tabu search for the location of interacting hub facilities *European Journal of Operational Research* **73**(3) 502–9

[12] Saboury A, Ghaffari-Nasab N, Barzinpour F, Jabalameli M S 2013 Applying two efficient hybrid heuristics for hub location problem with fully interconnected backbone and access networks *Computers & Operations Research* **40**(10) 2493–507

[13] Marler R T, Arora J S 2004 Survey of multi-objective optimization methods for engineering *Structural and Multidisciplinary Optimization* **26**(6) 369-95

[14] Zhao J 2012 Multi-objective location-routing problem in emergency logistics as time varies *Journal of Highway and Transportation Research and Development* **29**(4) 137-42 *(in Chinese)*

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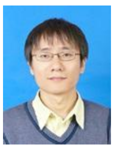

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