Study on improved elasto-plastic model for unsaturated soils based on Barcelona Basic Model

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Abstract

The strength and deformation behaviours of unsaturated soil can be approximately described by elasto-plastic constitutive model that was proved by abundance academic and test researches. The Barcelona elastic-plastic model is an excellent model that can simulate the strength and deformation of unsaturated soil. But their calculated result of shear strength is low. So an improved BBM model is settled by using drop-shaped shear yield surface and hardening theory of dual stress. The results show that the improved model can more accurately predict the strength and deformation behaviours of unsaturated soil under suction-controlled triaxial compression stress states.

Keywords: shear yield surface, drop-shaped, hardening theory, dual stress, unsaturated soil, elasto-plastic

1 Introduction

The constitutive relation is very complex for unsaturated soil, a three-phase material that consists of solids, water, and air, and very little, if any, promising models have been reported. Some representative models on constitutive relation include nonlinear elastic model, elasto-plastic model and structural model [1-3]. Of those models, Fredlund et al. [4] proposed non-linear elastic model for unsaturated soil; Alonso et al. [5-9] proposed the elasto-plastic model, namely, the Barcelona Basic Model (BBM). These proposed models reflect the strength and deformation properties of unsaturated soil in varying degrees. Theoretical and experimental results also show that the strength and deformation characteristics of unsaturated soil can be approximately described by elasto-plastic model. Therefore, BBM can be utilized for applications in engineering practice.

However, experimental studies showed that the generally BBM underestimates the shear strength of unsaturated soils in [6, 10], attributed to the elliptical yield surface of modified Cam-clay model. To address this issue, the drop-shaped yield surface was introduced as the shear yield surface, and the associated flow rule and dual stress hardening theory are used in the framework of the original model. Comparative studies show that the improved model can predict the properties of strength and deformation for unsaturated soil more accurately, under the tri-axial compression test of controlled suction.

2 Barcelona basic model

Alonso et al. [5-9] proposed the Barcelona Basic Model based on the classic modified Cam-clay model. The suction component was introduced and the unified elasto-plastic model was proposed in the generalized stress space (q, p, s) according to the concept of saturated critical state for non-expansive soil and weak expansive soil, in which q, p, s, represent the shear stress, the average net stress p and the suction s, respectively. When the soil is saturated, the model degenerates to the modified Cam-clay model.

2.1 STRESS YIELD SURFACE

According to the following Stress variables, two types of yield functions are defined.

$$p=(\sigma_1+2\sigma_3)/3-u_a\,, \quad (1)$$

$$q=\sigma_1-\sigma_3\,, \quad (2)$$

$$s=u_a-u_w\,. \quad (3)$$

In Equation (3), u_a and u_w are the pore air pressure and pore water pressure.

The first type of yield function is expressed as:

$$f_1(p,q,s,p_0)=q^2-M^2(p+p_s)(p_0-p)=0\,, \quad (4)$$

$$p_s=ks\,, \quad (5)$$

$$\frac{p_0}{p^c}=\left(\frac{p_0^*}{p^c}\right)^{[\lambda(0)-k]/[\lambda(s)-k]}\,, \quad (6)$$

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$$\lambda(s) = \lambda(0)[(1-r)\exp(-\beta s) + r]. \quad (7)$$

Equation (6) is the LC (after loading-collapse) yield surface equation, p_0 and p_0^* are pre-consolidation stresses, corresponding to suction and saturated conditions.

The second type of yield function is defined as

$$f_2(s, s_0) = s - s_0 = 0. \quad (8)$$

Equation (8) represents the SI yield curve (after suction increase), where s0 is the maximum suction in the history of soil.

2.2 FLOW RULE

Corresponding to the yield surfaces of f_1 and f_2, the plastic potential function are assumed as

$$g_1 = f_1, \quad (9)$$

$$g_2 = p. \quad (10)$$

The f_1 and f_2 are used on the associated and non-associated flow rule. The plastic strain increment is $(d\varepsilon_{vp}^p, d\varepsilon_q^p)$ and the following formulas are obtained

$$d\varepsilon_{vp}^p = \mu_1 n_p, \quad (11)$$

$$d\varepsilon_q^p = \mu_1 n_q, \quad (12)$$

$$n_p = 1, \quad (13)$$

$$n_q = [2q\alpha / M^2(2p + p_s - p_0)]. \quad (14)$$

In Equation (14), α is a constant that can be obtained by calculating the plastic strain increment in the direction of no lateral deformation, as

$$\frac{d\varepsilon_q^p}{d\varepsilon_{vp}^p} = (2/3)\{1/[1 - k/\lambda(0)]\}. \quad (15)$$

In order to simplify the formula, $d\varepsilon_q^e$ has been assumed to be 0. The stress state to meet K_0 conditions can be determined using Equation (16).

$$(q/p + p_s) = 3(1 - K_0)/(1 + 2K_0). \quad (16)$$

Thereafter, α can be expressed as

$$\alpha = \frac{M(M-9)(M-3)}{9(6-M)}\{1/[1 - k/\lambda(0)]\}. \quad (17)$$

To estimate yield surface of f_2, the associated plastic strain vector is given as $(d\varepsilon_{vs}^p, 0)$, where

$$d\varepsilon_{vs}^p = \mu_2. \quad (18)$$

μ_1 and μ_2 can be obtained through the plastic consistency condition, namely

$$\mu_1 = \frac{\frac{\partial f_1}{\partial p}dp + \frac{\partial f_1}{\partial q}dq + \frac{\partial f_1}{\partial s}ds}{\frac{\partial f_1}{\partial p_0^*}\frac{\partial p_0^*}{\partial \varepsilon_v^p}}, \quad (19)$$

$$\mu_2 = \left(\frac{\partial f_2}{\partial s}ds\right) \Big/ \left(\frac{\partial f_2}{\partial s_0}\frac{\partial s_0}{\partial \varepsilon_v^p}\right). \quad (20)$$

2.3 HARDENING LAW

The development of yield surface is controlled by hardening parameters p_0^* and s_0, which further depend on the total plastic volume strain increment $d\varepsilon_v^p$.

$$\frac{dp_0^*}{p_0^*} = \frac{v}{\lambda(0) - \kappa}d\varepsilon_v^p, \quad (21)$$

$$\frac{ds_0}{(s_0 + p_{at})} = \frac{v}{\lambda_s - \kappa_s}d\varepsilon_v^p. \quad (22)$$

2.4 ELASTIC STRAIN

The volumetric elastic strain and shear elastic strain are constituted by the following formula.

$$d\varepsilon_v^e = \frac{k}{v}\frac{dp}{p} + \frac{ks}{v}\frac{ds}{(s + p_{at})}, \quad (23)$$

$$d\varepsilon_q^e = (1/3G)dq. \quad (24)$$

3 Improved model

3.1 SHEAR YIELD SURFACE

BBM with elliptical shear yield surface underestimated the strength of unsaturated soils. The shear yield surface of drop-shaped for saturated soil [1, 11, 12] can be extended to the elasto-plastic model for unsaturated soils Figure 1, with the associated flow rule to improve the BBM.

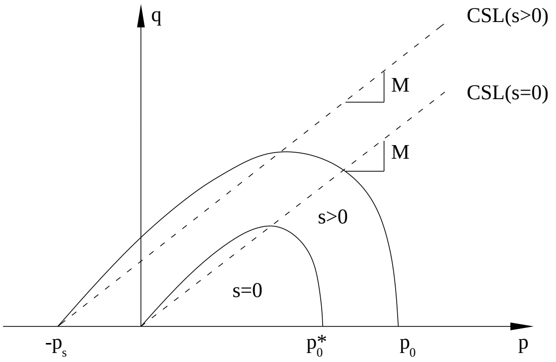


FIGURE 1 Yield surfaces of *p-q* space

Equation of drop-shaped yield surface is

$$f_1(p,q,s,p_0)=(p_0+p_s)q^2+2M^2\cdot (p+p_s)^2\cdot(p+p_0+2p_s)\cdot(p-p_0)=0 . \quad (25)$$

According to

$$\frac{\partial f_1}{\partial p}=\frac{16\left[(p+p_s)^2-\frac{1}{2}(p_0+p_s)^2\right](p+p_s)}{(p_0+p_s)^4}, \quad (26)$$

$$\frac{\partial f_1}{\partial q}=\frac{4q}{M^2(p_0+p_s)^2} . \quad (27)$$

BBM uses non-associated flow rule, while the improved model employs associated flow rule to simplify the calculation. The shear shrinkage rate is

$$\frac{\Delta\varepsilon_q^p}{\Delta\varepsilon_{vp}^p}=\frac{\partial f}{\partial q}\Big/\frac{\partial f}{\partial p} =\frac{q(p_0+p_s)^2}{4M^2\left[(p+p_s)^2-\frac{1}{2}(p_0+p_s)^2\right](p+p_s)} . \quad (28)$$

3.2 HARDENING THEORY

With drop-shaped shear yield surface, the predicted shear strength has been significantly improved, compared with the original model, but the results tend to be hard, and the deformation is small. To overcome the deficiency, the dual stress hardening theory is used in instead of the iso-surface hardening theory for the original model [1, 13, 14]. Initially, the dual stress theory appeared in the bounding surface model, and its method was to find the dual stress q_{CSL} that corresponded to the existing stress q on the boundary surface. Here, δ is the distance between q and q_{CSL}, hardening modulus H was assumed to be the function of δ.

$$H=H_{\mathrm{b}}+H_0\frac{\delta}{\delta_0-\delta} . \quad (29)$$

Here $H=\frac{1}{A}$ and A is the plasticity coefficient. Equation (29) shows that $\delta=0$ and $H=H_{\mathrm{b}}$ when the stress point reaches the boundary surface that is equal to the hardening modulus of the boundary surface. When $\delta=\delta_0$ and $H=\infty$, then no plastic strain is produced. If the boundary surface is replaced with the failure surface, the Equation (29) can also be extended to the isotropic hardening model, as

$$H=H_0\frac{\delta}{\delta_0} . \quad (30)$$

Equation (30) shows that $H=0$ when reaching the failure state. H_0 is the hardening modulus under the isotropic consolidation (Figure 2).

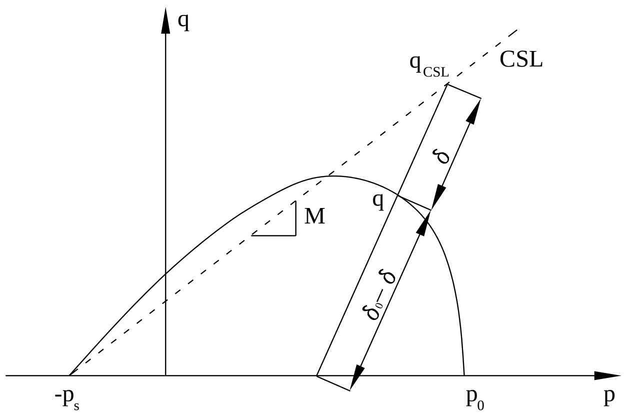


FIGURE 2 Dual Stress of *p-q* space

3.3 MODEL VALIDATION

Both the improved model and BBM are implemented to simulate the tri-axial compression tests under controlled suction conditions for the silt sand [10], and the simulation is compared with experimental data. The impact and difference of the two elasto-plastic constitutive models for unsaturated soils are compared and the results indicated that our new model is superior to the conventional BBM.

3.4 MODEL PARAMETERS

The parameters of numerical model are shown in Table 1.

The parameters of improved model and BBM include p^c, p_0^*, $\lambda(0)$, κ, r, β, s_0, λ_s, κ_s, G, M and k. All parameters can be obtained through three methods of following:

(1) The isotropic drainage compression test (loading-unloading) under different suction conditions can determine p^c, p_0^*, $\lambda(0)$, κ, r and β;

(2) The cycle test of wet and dry under average net stress can determine s_0, λ_s and κ_s;

(3) The test of shear strength under different suction conditions can provide data to define G, M and k.

TABLE 1 Parameters of model [10]

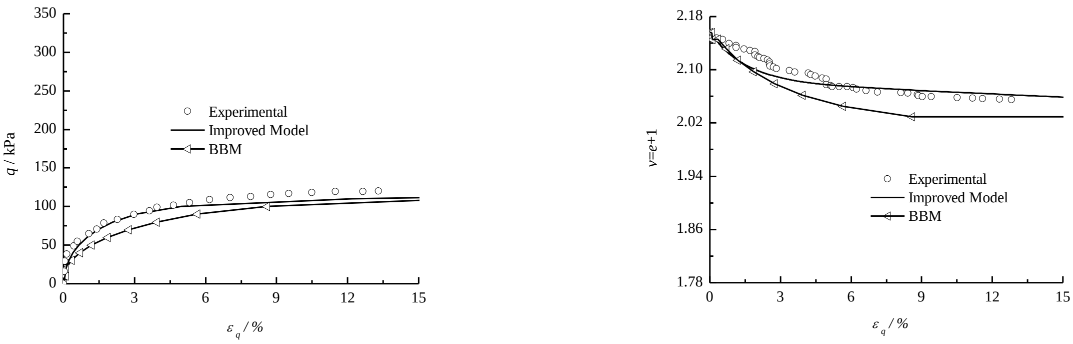
Parameter name	Parameter values	Unit
$\lambda(0)$	**0.220**	**-**
κ	**0.011**	**-**
β	**17.89**	**MPa-1**
r	**0.210**	**-**
p^c	**36**	**kPa**
G	**8.800**	**MPa**
M	**0.982**	**-**
k	**1.324**	**-**
p_0^*	**41**	**kPa**

3.5 RESULTS AND ANALYSIS

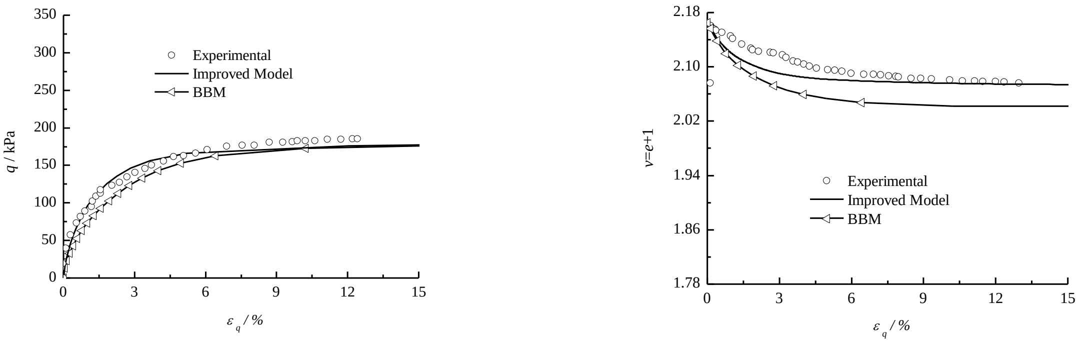
The numerical results of the two types of model, and the comparation with the experimental studies are shown in Figure 3.

The predicted curves show that two predicted models suggest consistent changes and trends with the experimental studies. For the stress-strain curves, the tendency of the improved model is harder than BBM and agrees better with most of the experimental results. However, the theoretical value of final calculated failure stress for two yield surface is equal. For the deformation curve, that is, the specific volume and total shear strain curve, the results of two yield surfaces achieve the same prediction accuracy.

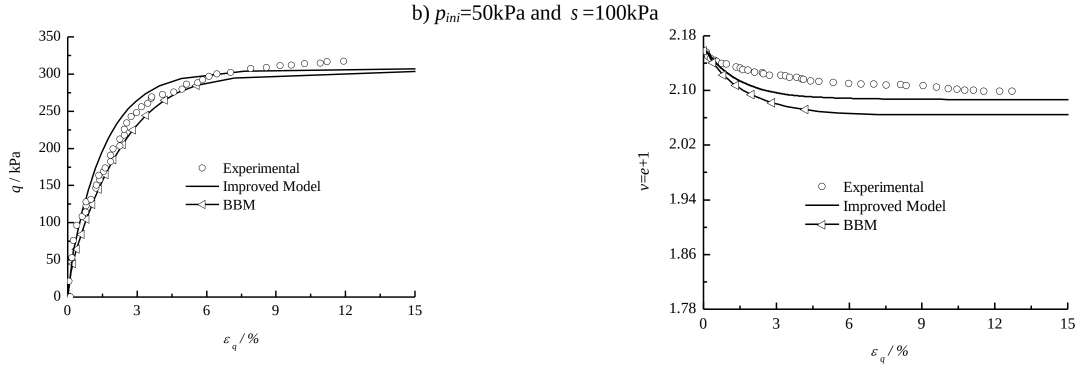
The application of drop-shaped yield surface and dual stress hardening theory in elasto-plastic model for unsaturated soils can provide higher simulation accuracy of the deformation characteristics of unsaturated soil, than the elliptical yield surface-based model



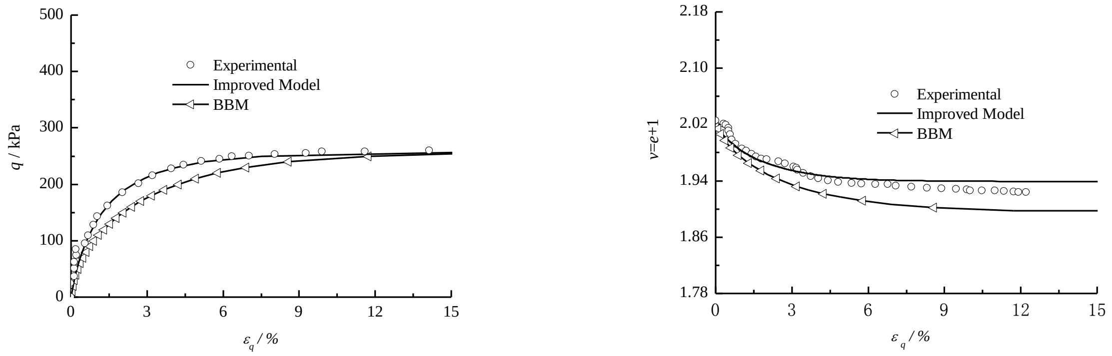
a) p_{ini}=50kPa and s =50kPa



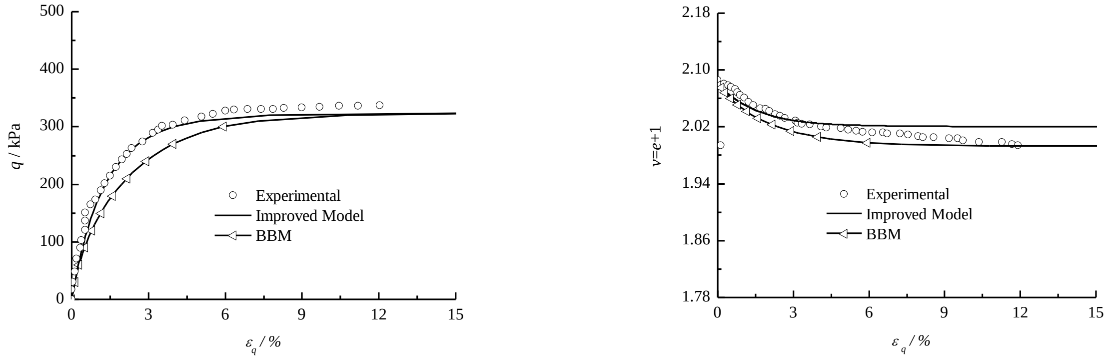
b) p_{ini}=50kPa and s =100kPa



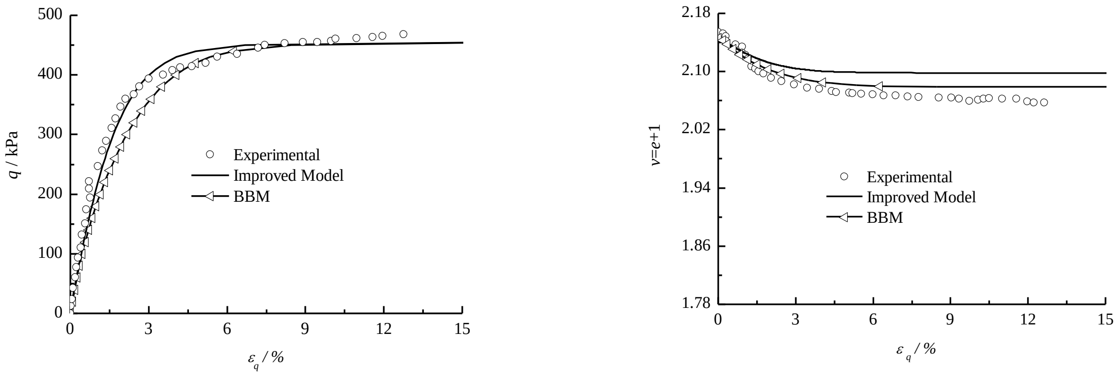
c) p_{ini} =50kPa and s =200kPa



d) p_{ini} =200kPa and s =50kPa



e) p_{ini} =200kPa and s =100kPa



f) p_{ini} =200kPa and s =200kPa

FIGURE 3 Experimental and predicted curve of $q - \varepsilon_q$ and $v = 1 + e - \varepsilon_q$

4 Conclusions

In this paper, we extended the drop-shaped yield surface to unsaturated soil and proposed the function of drop-shaped shear yield surface, with the associated flow rule to improve BBM. The dual stress hardening theory was then used in instead of the iso-surface hardening theory in the original model for the controlling of deformation in the improved model. The comparation of simulation models and the experimental results show that the improved model can be used to simulate the tri-axial compression test of controlled suction for unsaturated soil, and the results are consistent with the experimental studies. Comparison of two types of calculated results further show that the elasto-plastic constitutive model of shear yield surface of drop-shaped can provide higher-accuracy prediction than the elliptical yield surface for unsaturated soils. The only disadvantage we observed in our model is that the calculated stress-strain curves of BBM for unsaturated soil was softer.

Acknowledgments

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