The two-step motion compensation combined squint wavenumber domain algorithm based on fractional Fourier transform

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Abstract

Fractional Fourier transform (FrFT) is a kind of generalized Fourier transform, which processes signals in the unified time-frequency domain and the linear frequency modulation signal can be well focused after FrFT. Motion error is an important factor affecting the SAR resolution, conventional wavenumber domain (CWD) algorithm is an ideal solution of SAR focusing problem as long as nominal straight flight track is given, especially in the case of high squint angles and long synthetic apertures, but it has certain limitation in processing airborne SAR data affected by motion error, so extended wavenumber domain algorithm (EWD) is presented. Pointing to the problem that the effect of error elimination is not obvious in processing non-stationary motion error using EWD algorithm, FrFT based the two-step motion compensation combined squint wavenumber domain algorithm is put forward in this paper, which is expected to eliminate the influence of motion error more effectively, so as to obtain high quality SAR images. The simulation results and the imaging results of real SAR data show that the proposed algorithm can eliminate the influence of motion error effectively. (the real SAR data provided by Institute of Electronics, Chinese Academy of Sciences).

Keywords: fractional Fourier transform, motion compensation, wavenumber domain algorithm, extended wavenumber domain algorithm, high resolution

1 Introduction

The system theory and related technology for SAR are established on keeping the flight path of radar platform in a straight line, but the atmospheric turbulence or other natural factors often make the aircraft deviated from the nominal track and generating motion error of antenna phase center(APC), which will cause the amplitude modulation and phase modulation of radar echo signal, result in the image blurring and geometric distortions. In order to obtain high quality SAR images, the motion error must be compensated [1,2].

Conventional wavenumber domain algorithm is an ideal solution of SAR focusing problem as long as nominal straight flight track is given, especially in the case of high squint angles and long synthetic apertures, but it has certain limitation in processing airborne SAR data affected by motion error, so EWD algorithm is presented, which can integrate with two-step motion compensation, but the compensation effect is not obvious when motion errors are non-stationary.

Fractional Fourier transform (FrFT) is proposed by Namias in 1980 [3], which is a new time-frequency analysis tool. Compared to the Fourier transform, FrFT has incomparable superiority in processing non-stationary signals. Chirp signal can be well focused after FrFT of specific rotation angle which provides a possibility to achieve high resolution and high accuracy in SAR imaging. Especially when signal and interference source are coupled to each other, good separation effect in fractional domain can be obtained [4-6].

There has been literatures putting forward SAR imaging algorithm combining with FrFT. Refences [7,8] proposed an improving CS imaging algorithm based on FrFT. References [9,10] also proposed a combination of FrFT and RD imaging algorithm. These research results show that FrFT can contribute to the improvement of SAR resolution.

In order to eliminate the influence of motion error more effectively and improve the resolution, in the paper, FrFT is combined with the two step motion compensation technology and SAR imaging algorithm, through utilizing the superiority of FrFT in processing chirp signal and non-stationary signal, so as to obtain high quality SAR images.

2 Fractional Fourier transform

The fractional Fourier transform and its inverse transform for signal $x(t)$ is defined as:

$$X(u) = F_p\left[x(t)\right](u) = \int_{-\infty}^{+\infty} x(t) K_p(t,u)\, dt\,, \tag{1}$$

$$x(t) = F_{-p}\left[X(u)\right](t) = \int_{-\infty}^{+\infty} X(u) K_{-p}(t,u)\, du\,, \tag{2}$$

where:

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$$K_p(t,u)=\begin{cases}\sqrt{\dfrac{1-j\cot\alpha}{2\pi}}\exp\left\{j\pi\left(\left(t^2+u^2\right)\cot\alpha-2ut\csc\alpha\right)\right\} & \alpha\notin n\pi\\ \delta(t-u) & \alpha\in n\cdot 2\pi\\ \delta(t+u) & \alpha+\pi\in n\cdot 2\pi\end{cases} \quad (3)$$

and $p=\frac{2}{\pi}\alpha$, which is the order of FrFT when $p=1$, FrFT is the Fourier transform.

The fractional Fourier transform for chirp signal $x(t)=e^{j2\pi kt^2}$ is as follows:

$$X(u)=F_p\left[x(t)\right](u)=\int_{-\infty}^{+\infty}x(t)K_p(t,u)dt= Ae^{j\pi u^2\cot\alpha}\int_{-\infty}^{+\infty}e^{j2\pi kt^2}e^{j\pi t^2\cot\alpha-j2\pi ut\csc\alpha}dt,$$

where $A=\sqrt{\frac{1-j\cot\alpha}{2\pi}}$, when $\cot\alpha=-2k$, $p=-\frac{2}{\pi}\operatorname{arccot}(2k)=p_{opt}$ is the optimal order of FrFT. Thus:

$$X(u)=F_p\left[e^{j2\pi kt^2}\right]=Ae^{-j2\pi ku^2}\int_{-\infty}^{+\infty}e^{-j2\pi ut\csc\alpha}dt= 2\pi Ae^{-j2\pi ku^2}\delta(2\pi\csc\alpha u)=\frac{A}{\csc\alpha}\delta(u)=C\delta(u) \quad (4)$$

So the optimal order FrFT for chirp signal is an impulse signal.

3 The motion error model of squint mode and SAR echo signal with motion error

Figures 1 and 2 are geometry models of SAR system under squint mode, in which: θ_{sq} is squint angle of antenna beam; r_{s0} is the range of antenna beam center without trajectory deviation; r_s is the range of antenna beam center with trajectory deviation; x_c is the APC azimuth coordinate; x_0 is the azimuth position of target; θ_s is antenna beam angle.

Based on law of cosines, there is:

$$R_0(x,x_0,r_0)=\sqrt{r_{s0}^2+\left(x+x_c-x_0\right)^2+2r_{s0}\left(x+x_c-x_0\right)\sin\theta_{sq}},$$

$$R(x,x_0,r_0)=\sqrt{r_s^2+\left(x+x_c-x_0\right)^2+2r_s\left(x+x_c-x_0\right)\sin\theta_{sq}},$$

where, $r_{s0}=\frac{r_0}{\cos\theta_{sq}}$, $r_s=\frac{r}{\cos\theta_{sq}}$, $r=r_0+\delta r_0$, $r_s=r_{s0}+\delta r_s$,

$$\delta R(x,x_0,r_0)=R(x,x_0,r_0)-R_0(x,x_0,r_0)\approx \delta r_s-\frac{\left(x+x_c-x_0\right)^2}{2r_{s0}^2}\delta r_s,$$

if $\left(x+x_c-x_0\right)\ll r_{s0}$,

$$\delta R\approx\delta r_s=\frac{\delta r_0(x,r_0)}{\cos\theta_{sq}}. \quad (5)$$

So motion compensation is determined by the APC position, the closest approach of target and squint angle, and has no relation with target position, therefore, the motion errors of all targets in the same range can be compensated together. Motion error can be divided into range-independent and range-dependent components [11], as following:

$$\delta R(x,r_0)=\delta R_I(x,r_m)+\delta R_{II}(x,r_x).$$

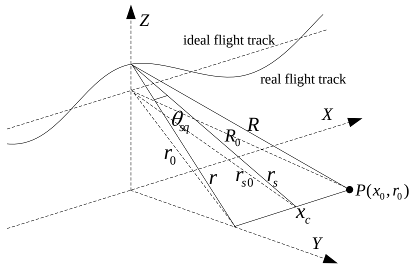


FIGURE1 Geometry model of SAR system under squint mode

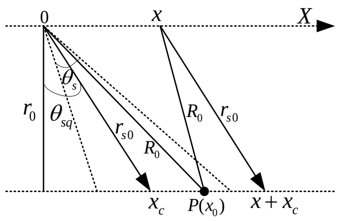


FIGURE2 Geometry relation in range plane

Suppose that SAR transmit the chirps to an observed scene, their echoes after the demodulation are:

$$sd\left(t,\tau;r_0\right)=\sigma\left(x_0,r_0\right)rect\left(\frac{\tau-\frac{2R}{c}}{T_p}\right)\exp\left\{-j\pi k\left(\tau-\frac{2R}{c}\right)^2\right\}rect\left(\frac{t-t_0}{T_s}\right)\exp\left\{-j\frac{4\pi R}{\lambda}\right\}, \tag{6}$$

where τ is fast time in the slant range direction, t is slow time along the radar flight path, c and λ are the speed of light and the radar wavelength respectively, and k is the chirp rate. *rect*( ) is the rectangle function, in which T_p and T_s are the pulse duration and synthetic aperture time. R is the instant range from the radar to a point target in the observed scene, which includes motion error.

Ignore the azimuth motion error, the Fourier transform in the range direction first is performed by the stationary phase point, the result is:

$$sD\left(t,f_r;r_0\right)=C_1\sigma\left(x_0,r_0\right)rect\left(\frac{f_r}{B_r}\right)\exp\left\{j\pi\frac{f_r^{\,2}}{k}\right\}\exp\left\{-j\frac{4\pi R_0}{c}f_r\right\}\exp-j\frac{4\pi\left(\delta R_I+\delta R_{II}\right)}{c}f_r\cdot$$
$$rect\left(\frac{t-t_0}{T_s}\right)\exp\left\{-j\frac{4\pi R_0}{\lambda}\right\}\exp\left\{-j\frac{4\pi\left(\delta R_I+\delta R_{II}\right)}{\lambda}\right\} \tag{7}$$

Range-independent motion errors cause the echo envelope delay errors and azimuth phase errors, which can be compensated along with range compress. Range-dependent motion errors is related to slant range. In real SAR data, all point target are spread out in range and azimuth direction, so compensation to such error must be implemented after range compression and the range migration correction is completed and before azimuth compression.

4 Two-step motion compensation combined squint wavenumber domain algorithm based on the FrFT

Stolt interpolation is a key step in CWD algorithm, which not only increases the amount of calculation, but also brings additional error. Fractional Fourier transform can replace the interpolation operation and can transform signal into range-Doppler domain at the same time, thus improving the calculation efficiency and imaging resolution.

Processing flow for the two-step motion compensation combined squint wavenumber domain algorithm based on FrFT is as shown in Figure 3.

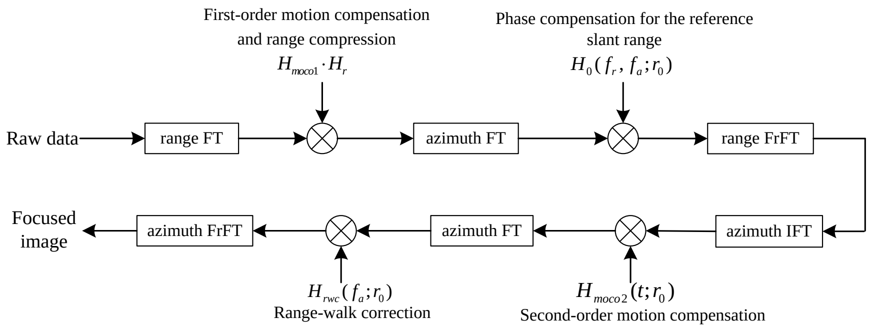


FIGURE 3 Processing flow for the two-step motion compensation combined squint wavenumber domain algorithm based on FrFT

First raw data is transformed into range frequency domain by Fourier transform on the range (shown as 7), then multiplying H_{moco1} and H_r to complete range focusing and first-order motion compensation.

$$H_{moco1}H_r=\exp\left\{j\frac{4\pi\delta R_I}{c}\left(f_r+f_c\right)\right\}\exp\left\{-\pi\frac{f_r^{\,2}}{K_r}\right\}. \tag{8}$$

Next performing azimuth FT, in the two-dimensional frequency domain, the signal is:

$$SD\left(f_a,f_r;r_0\right)=$$
$$C_2\sigma rect\left[\frac{f_r}{B_r}\right]rect\left[\frac{f_a}{B_d}\right]\exp\left\{j\psi\left(f_a,f_r;r_0\right)\right\}, \tag{9}$$

where $\psi\left(f_a,f_r;r_0\right)=\psi_0\left(f_a,f_r;r_m\right)+\psi_1\left(f_a,f_r;r_0\right)$,

$$\psi_0\left(f_a,f_r;r_m\right)=-\frac{4\pi r_m}{\lambda}\sqrt{\left(1+\frac{f_r}{f_c}\right)^2-\left(\frac{\lambda f_a}{2v}\right)^2},$$
$$\psi_1\left(f_a,f_r;r_m\right)=-\frac{4\pi\delta r}{\lambda}\sqrt{\left(1+\frac{f_r}{f_c}\right)^2-\left(\frac{\lambda f_a}{2v}\right)^2}, \tag{10}$$

where $r_0=r_m+\delta r$, $\delta r\in\left[-\frac{w_r}{2},\frac{w_r}{2}\right]$, r_m is the slant range of swath center, w_r is the swath width.

The Taylor series expansion around the Doppler frequency f_a for ψ_0 is:

$$\psi_0(f_a,f_r;r_m)=-\frac{4\pi r_m}{c}\left[f_c\gamma(f_a)+\frac{f_r}{\gamma(f_a)}-\frac{1-\gamma(f_a)^2}{2f_c\gamma(f_a)^3}f_r^2+\frac{1-\gamma(f_a)^2}{2f_c\gamma(f_a)^5}f_r^3+...\right] \quad (11)$$

First item of 11 corresponds to the azimuth compression, second to range cell migration, third to second range compression, the fourth is high-order coupling item of range and azimuth.

In squint mode, motion error is projected in the direction of antenna beam centre, the minimum approximate error of 5 is in the direction of range-walk (squint angle), so the second order motion compensation should be performed after correction of range-curve and before correction of range-walk [12], so Equation (10) is changed as:

$$\psi_0(f_a,f_r;r_m)=-\frac{4\pi r_m}{\lambda}\left[\sqrt{\left(1+\frac{f_r}{f_0}\right)^2-\left(\frac{\lambda f_a}{2v}\right)^2}-\sqrt{1-\left(\frac{\lambda f_a}{2v}\right)^2}-D(f_a-f_{DC})f_r\right],$$

$$\psi_1(f_a,f_r;r_m)=-\frac{4\pi\delta r}{\lambda}\sqrt{\left(1+\frac{f_r}{f_0}\right)^2-\left(\frac{\lambda f_a}{2v}\right)^2}\frac{4\pi r_m}{\lambda}\left[\sqrt{1-\left(\frac{\lambda f_a}{2v}\right)^2}+D(f_a-f_{DC})f_r\right],$$

where $D=\dfrac{\frac{\lambda}{2v}\sin\theta_{sq}}{f_0\cos^3\theta_{sq}}$, f_{DC} is Doppler centroid, $D(f_a-f_{DC})f_r$ is range-walk, which is obtained from the RCM expression in frequency domain [13]. Multiplying 9 with:

$$H_0(f_a,f_r;r_m)=\exp\{-j\psi_0(f_a,f_r;r_m)\},$$

can complete range-curve correction, second range compression and phase compensation for high-order coupling item of range and azimuth for the reference range. Then performing FrFT on the range so that completing residual range-curve correction and change the signal into range-Doppler domain:

$$\begin{aligned}
Sd(t,\tau;r_0)&=\int_{-\infty}^{+\infty}\left[SD(f_a,f_r;r_0)H_0(f_a,f_r;r_m)\right]K_p(f_r,\tau)df_r=\\
&C_3rect\left[\frac{f_a}{B_d}\right]\int_{-\infty}^{+\infty}rect\left[\frac{f_r}{B_r}\right]\exp\{j\psi_1(f_a,f_r;r_0)\}\exp\left\{j\pi\left(\left(f_r^2+\tau^2\right)\cot\alpha-2f_r\tau\csc\alpha\right)\right\}df_r=\\
&C_4\sigma rect\left[\frac{f_a}{B_d}\right]\exp\left\{-j\frac{4\pi r_0}{\lambda}\gamma(f_a)\right\}\exp\left\{-j\frac{4\pi r_0}{\lambda}D(f_a-f_{DC})\right\}\sin c\left(\tau-\frac{2r_0}{c}\right),
\end{aligned} \quad (12)$$

where $\alpha=-\cot^{-1}\left(\dfrac{2\delta r}{c}\dfrac{1-\gamma(f_a)^2}{f_c\gamma(f_a)^3}\right)$, the optimal order of FrFT is $p_{opt}=\dfrac{2}{\pi}\alpha$.

Next, performing azimuth IFT into two-dimensional time-domain, implementing second-order motion compensation, the corresponding phase compensation function is:

$$H_{moco2}(t,r_0)=\exp\left\{-j\frac{4\pi}{\lambda}\Delta R_{II}\right\}, \quad (13)$$

In range-Doppler domain, completing range-walk correction, and the compensation function is $H_{rwc}(f_a,r_0)=\exp\left\{j\dfrac{4\pi r_0}{\lambda}D(f_a-f_{DC})\right\}$. In order to obtain azimuth chirp signal, operating the Taylor series expansion around the Doppler frequency f_a f_a for $\gamma(f_a)$,

$$\gamma(f_a)=\sqrt{1-\left(\frac{\lambda f_a}{2v}\right)^2}\approx1-\frac{1}{2}\left(\frac{\lambda f_a}{2v}\right)^2.$$

Finally, performing FrFT on the azimuth by the optimal order $p_{opt}=\dfrac{2}{\pi}\beta=-\dfrac{2}{\pi}\operatorname{arccot}\left(\dfrac{\lambda r_0}{2v^2}\right)$, the result is:

$$sd\left(t,\tau;r_0\right)=\int_{-\infty}^{+\infty}\left[Sd\left(f_a,f_r;r_0\right)H_{rwc}\left(f_a,r_0\right)\right]K_p\left(f_a,\tau\right)df_a=$$
$$C_4\sigma\sin c\left(\tau-\frac{2r_0}{c}\right)\int_{-\infty}^{+\infty}rect\left[\frac{f_a}{B_d}\right]\exp\left\{-j\frac{4\pi r_0}{\lambda}\gamma\left(f_a\right)\right\}\exp\left\{j\pi\left(\left(f_a^2+t^2\right)\cot\beta-2f_at\csc\beta\right)\right\}df_a= \tag{14}$$
$$C_5\sigma\sin c\left(\frac{t}{\sin\beta}\right)\sin c\left(\tau-\frac{2r_0}{c}\right).$$

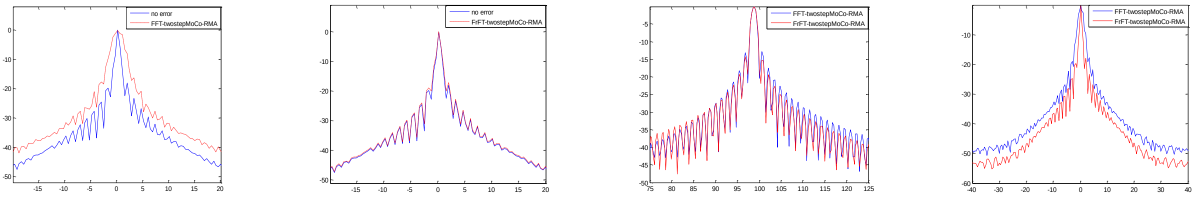
4 Simulations and SAR imaging results based on real data for the algorithm

Simulations of point target with motion error for the algorithm put forward in this paper is as follow, the simulation parameters as shown in Table 1.

The total error measured is non-stationary random motion error whose mean value is exponential function, so range-variant motion error is $\delta R_{\Pi}(x,r_x)=\delta R(x,r_0)-\delta R_{\mathrm{I}}(x,r_m)$.

TABLE 1 Simulation parameters

Parameter	value	Parameter	value
Carrier frequency	1.5*GHz*	Forward velocity	180m/s
Bandwidth for transmit signal	150*MHz*	Length of synthetic aperture	320m
Pulse duration of transmit signal	1.5μs	The number of azimuth sampling points	512
The number of range sampling points	1024	Range-invariant motion error	t^2



a) b) c) d)

FIGURE 4 Comparing of azimuth impulse response between no error and with error after compensation
a) Comparison of azimuth impulse response with no error and after motion compensation using EWD algorithm; b) Comparison of azimuth impulse response with no error and after motion compensation using the proposed algorithm; c) Comparison of range impulse response after motion compensation by the two algorithm; d)Comparison of azimuth impulse response after motion compensation by the two algorithm.

Simulation to point target with motion error using FFT based the EWD algorithm and FrFT based the two-step motion compensation combined squint wavenumber domain algorithm are shown in Figure 4. Figure 4b and d) shows that the main lobe of impulse response in azimuth direction is narrower and the influence of quadratic phase error is eliminated virtually. The wider mainlobe of Figure 4a indicates the influence of quadratic phase error still exists, which proves the processing effect of non-stationary motion error using FrFT is obvious than FFT, at the same time proves that the focusing effect of chirp signal using FrFT is better than FFT. The performance comparison of FFT-EWD and FrFT-EWD algorithm is shown in Table 2.

TABLE 2 performance comparison of impulse response in azimuth direction

Condition and processing algorithm	Main lobe	ISLR	PSLR
No error	1.10m	−26.012 dB	−31.432dB
FFT-EWD algorithm processing under motion error	4.864m	−24.681dB	−24.218dB
FrFT-EWD algorithm processing under motion error	1.125m	−25.589dB	−30.353dB

Figure 5a is the imaging results of the real SAR data with motion error, the blurring image shows there are obvious quadratic phase errors. Processing result for such SAR data using the traditional FFT based EWD algorithm is shown in Figure 5b, due to the elimination of most motion error, image resolution is improving significantly, but in some places with more details (such as part of the circle line), the image is not clear and image resolution deteriorates because of the residual phase error. The processing result with the proposed FrFT based two step motion compensation squint wavenumber domain algorithm is shown in Figure 5c, where details information increases, the image resolution is further improved.

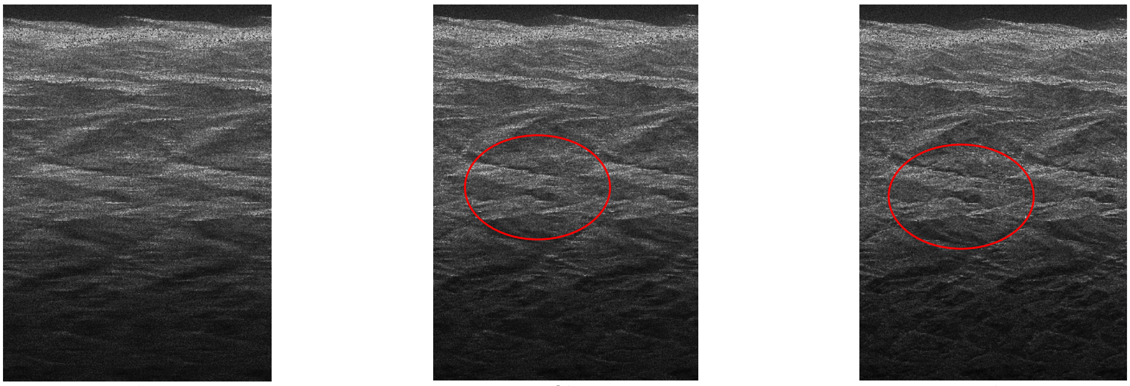


FIGURE 5 comparison of imaging results before and after motion compensation (resolution: $3m\times3m$)
a) SAR image with motion error; b) SAR image after FFT based the EWD algorithm processing; c) SAR image after FrFT based the two step motion compensation combined squint wavenumber domain algorithm processing

5 Conclusions

Motion error is a crucial factor to limit airborne SAR resolution improving. As modern SAR systems are continuously developing into the direction of higher spatial resolution, how to overcome the motion error caused by air turbulence is an urgent problem to radar workers. The two-step motion compensation combined squint wavenumber domain algorithm based on fractional Fourier transform proposed in this paper can solve various motion errors effectively, especially can eliminate the image blurring caused by non-stationary motion errors, the research provides an effective solution scheme for squint SAR data with motion error.

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