Solution of compressed sensing for wide angle EM scattering analysis based on MFIE

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Abstract

Fast analysis of electromagnetic scattering problems over a wide incident angle is always a difficult problem in computational electromagnetics. Up to the present, almost all of the traditional numerical methods need to solve one discrete angle after another to finish calculating this kind of problem. In this paper, we propose a new method, which can fix it effectively by applying compressed sensing into method of moments for magnetic field integral equation. The theory and calculation process of the solution are described in detail in the paper, and by numerical experiments of different three dimensional objects, the accuracy and the efficiency of the algorithm are also discussed.

Keywords: compressed sensing, method of moments, magnetic field integral equation

1 Introduction

As an important research field of computational electromagnetics (CEM), up to the present, electromagnetic (EM) scattering problems can already be computed by a lot of solutions [1-3]. However, fast analysis of wide angle EM scattering problems is still a difficult task for all traditional numerical methods, almost all of them need to solve one discrete angle after another to finish calculating this kind of problem [4-6]. It means iterative operations are adopted as incident angle changes, which must result in low efficiency. Aiming at this difficulty, we propose a new method, which can solve it effectively by applying compressed sensing (CS) into method of moments (MoM) for magnetic field integral equation (MFIE).

CS is called as ‘a big idea’ in the field of signal processing [7]. One of the most interesting advantages in CS is that it breaks the restriction of Nyquist-Shannon sampling theorem [8] - it can capture and represent compressible signals at a rate significantly below the Nyquist rate. MoM is a classical numerical method of electromagnetic field integral equation, which is applied extensively in solving EM scattering problems [9]. MFIE, as is well known to all, is considered to have some advantages by its smaller condition number and faster iterative convergence speed [10], so we choose it as the basic integral equation. In this paper, the theory and implementation of the new solution are elaborated, and numerical simulation for different three dimensional objects is presented and discussed - it is shown that the new method can obtain accurate results by only several measurements and the efficiency can be improved greatly.

2 Traditional MoM for MFIE

For a closure perfect electric conductor (PEC) whose surface is S radiated by electromagnetic waves, MFIE could be represented as:

$$\mathbf{J}(\mathbf{r}) = 2\hat{\mathbf{n}}\times\mathbf{H}(\mathbf{r}) + 2\hat{\mathbf{n}}\times P.V.\int_{S'}\mathbf{J}(\mathbf{r}')\times[\nabla' G(\mathbf{r},\mathbf{r}')]\mathrm{d}S', \quad (1)$$

where $\mathbf{J}$ represents induced current density on S, $\hat{\mathbf{n}}$ represents unit normal vector of the surface of object, $\mathbf{H}(\mathbf{r})$ represents incident magnetic field, $G(\mathbf{r},\mathbf{r}')$ $=\mathrm{e}^{-\mathrm{j}kR}/4\pi R$, $P.V.\int$ means principal value integral.

Calculation process of traditional MoM for MFIE is as follows:

Step 1: Use RWG vector basis functions $\mathbf{f}_n^S$

$$\mathbf{f}_n^S(\mathbf{r}) = \begin{cases} \dfrac{l_n}{2A_n^{\pm}}\boldsymbol{\rho}_n^{\pm S} & \mathbf{r}\in \mathrm{T}_n^{\pm} \\ 0 & \text{else} \end{cases}, \quad (2)$$

to expand $\mathbf{J}(\mathbf{r})$.

Step 2: Make inner product operation by Galerkin's method:

$$<\mathbf{f}_m^S(\mathbf{r}),\hat{\mathbf{n}}\times\mathbf{H}>=\sum_{n=1}^{N} I_n[<\mathbf{f}_m^S(\mathbf{r}),\frac{\mathbf{f}_n^S(\mathbf{r}')}{2}>-<\mathbf{f}_m^S(\mathbf{r}), \hat{\mathbf{n}}\times\int_{S'}(\mathbf{f}_n^S(\mathbf{r}')\times\nabla' G(\mathbf{r},\mathbf{r}'))\mathrm{d}S'>], \quad (3)$$

that is:

$$\mathbf{Z}_{n\times n}\mathbf{I}_n = \mathbf{V}_n, \quad (4)$$

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where:

$$Z_{mn} = \frac{1}{2}\int_S \mathbf{f}_m^S(\mathbf{r})\cdot\mathbf{f}_n^S(\mathbf{r}')\mathrm{d}S - \int_S \mathbf{f}_m^S(\mathbf{r})\cdot\hat{\mathbf{n}}\times \int_{S'} \mathbf{f}_n^S(\mathbf{r}')\times\nabla' G(\mathbf{r},\mathbf{r}')\mathrm{d}S'\mathrm{d}S, \tag{5}$$

$$V_m = \int_S \mathbf{f}_m^S(\mathbf{r})\cdot[\hat{\mathbf{n}}\times\mathbf{H}(\mathbf{r})]\mathrm{d}S. \tag{6}$$

Step 3: Solve the matrix equation with gauss integration, mean value theorem and treatment of singularities.
Step 4: Numerical result of $\mathbf{J}(\mathbf{r})$ will be finally calculated as:

$$\mathbf{J}(\mathbf{r}) = \sum_n I_n \mathbf{f}_n^S(\mathbf{r}). \tag{7}$$

While the angle of incident wave is not certain but in a wide range, traditional MoM for MFIE has to compute the matrix equations at each small discrete angle repetitively, so the efficiency is low.

3 Basic CS theory

CS is based on sparse representation of signals, its mathematical model [11-13] can be formulated as follows:

$$\mathbf{\Phi}_{M\times N}\mathbf{X}_{N\times 1} = \mathbf{\Phi}_{M\times N}\mathbf{\Psi}_{N\times N}\boldsymbol{\alpha}_{N\times 1} = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_M \end{pmatrix} = \mathbf{s}_{M\times 1} \quad (M \ll N), \tag{8}$$

where $\mathbf{X}$ stands for original signal, $\boldsymbol{\alpha}$ is the sparse projection of $\mathbf{X}$, $\mathbf{\Psi}$ stands for sparse basis, $\mathbf{\Phi}$ stands for measurement matrix and it is incoherent with $\mathbf{\Psi}$. From the M-dimensional measurement $\mathbf{s}$, the approximation of $\boldsymbol{\alpha}$ can be calculated from a L-minimization problem as:

$$\hat{\boldsymbol{\alpha}} = \min \|\boldsymbol{\alpha}\|_L \quad s.t.\ \mathbf{\Phi\Psi}\boldsymbol{\alpha} = \mathbf{s}, \tag{9}$$

finally, the original signal can be approximated as:

$$\hat{\mathbf{X}} = \mathbf{\Psi}\hat{\boldsymbol{\alpha}}. \tag{10}$$

4 Solution of CS introduced to MFIE

We introduce CS into traditional MoM for MFIE to fasten calculating wide angle EM scattering problems, the procedure is as follows:
Step 1: Assume the discrete angles of incident waves are $\theta_1,\theta_2...\theta_n$, accordingly, the excitations could be denoted as $\mathbf{V}(\theta_1)$, $\mathbf{V}(\theta_2)$, ..., $\mathbf{V}(\theta_n)$, construct a new group of excitations as:

$$\mathbf{V}_i^{\mathrm{CS}} = \alpha_{i1}\mathbf{V}(\theta_1) + \alpha_{i2}\mathbf{V}(\theta_2) + \cdots + \alpha_{in}\mathbf{V}(\theta_n), \tag{11}$$

where $i = 1, 2, ...m$ and $m \ll n$.

Step 2: Substitute these new excitations into the matrix equation of MFIE, that is,

$$\mathbf{Z}\mathbf{I}_i^{\mathrm{CS}} = \mathbf{V}_i^{\mathrm{CS}}, \tag{12}$$

solve these matrix equations as the way of traditional MoM, then $\mathbf{I}_1^{\mathrm{CS}}, \mathbf{I}_2^{\mathrm{CS}}, ..., \mathbf{I}_m^{\mathrm{CS}}$ can be obtained.
Step 3: Since the impedance matrix $\mathbf{Z}$ does not vary with the angle of incidence, $\mathbf{I}_i^{\mathrm{CS}}$ can be expanded as

$$\mathbf{I}_i^{\mathrm{CS}} = \alpha_{i1}\mathbf{I}(\theta_1) + \alpha_{i2}\mathbf{I}(\theta_2) + \cdots + \alpha_{in}\mathbf{I}(\theta_n)\ (i = 1, 2, ...m), \tag{13}$$

thus $\mathbf{I}_1^{\mathrm{CS}}, \mathbf{I}_2^{\mathrm{CS}} ... \mathbf{I}_m^{\mathrm{CS}}$ are m measured values of $\mathbf{I}(\theta_1), \mathbf{I}(\theta_2) ... \mathbf{I}(\theta_n)$.
Step 4: According to the theory of CS, from these measured values, current coefficient vectors over the wide angle can be reconstructed accurately by the recovery algorithm and sparse basis, so we get the relation:

$$\begin{bmatrix} \mathbf{I}_1^{\mathrm{CS}} \\ \vdots \\ \mathbf{I}_m^{\mathrm{CS}} \end{bmatrix} = \mathbf{\Phi}_{m\times n} \begin{bmatrix} \mathbf{I}(\theta_1) \\ \mathbf{I}(\theta_2) \\ \vdots \\ \mathbf{I}(\theta_n) \end{bmatrix} = \mathbf{\Phi}_{m\times n}\mathbf{\Psi}_{n\times n} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_n \end{bmatrix}, \tag{14}$$

where $\mathbf{\Phi}_{m\times n}$ represents measurement matrix, $\mathbf{\Psi}_{n\times n}$ represents sparse basis, $[\mathbf{s}_1\ \mathbf{s}_2\ ...\ \mathbf{s}_n]^{\mathrm{T}}$ is sparse projection of $[\mathbf{I}(\theta_1)\ \mathbf{I}(\theta_2)\ ...\ \mathbf{I}(\theta_n)]^{\mathrm{T}}$.
Step 5: Calculate the optimization problem:

$$\begin{bmatrix} \hat{\mathbf{s}}_1 \\ \hat{\mathbf{s}}_2 \\ \vdots \\ \hat{\mathbf{s}}_n \end{bmatrix} = \min \left\| \begin{matrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_n \end{matrix} \right\|_L \quad s.t.\ (\mathbf{\Phi\Psi}) \begin{bmatrix} \hat{\mathbf{s}}_1 \\ \hat{\mathbf{s}}_2 \\ \vdots \\ \hat{\mathbf{s}}_n \end{bmatrix} = \begin{bmatrix} \mathbf{I}_1^{\mathrm{CS}} \\ \vdots \\ \mathbf{I}_m^{\mathrm{CS}} \end{bmatrix}. \tag{15}$$

Step 6: Finally, current coefficient vectors over the wide angle can be reconstructed as:

$$\begin{bmatrix} \hat{\mathbf{I}}(\theta_1) \\ \hat{\mathbf{I}}(\theta_2) \\ \vdots \\ \hat{\mathbf{I}}(\theta_n) \end{bmatrix} = \mathbf{\Psi}_{n\times n} \begin{bmatrix} \hat{\mathbf{s}}_1 \\ \hat{\mathbf{s}}_2 \\ \vdots \\ \hat{\mathbf{s}}_n \end{bmatrix} \tag{16}$$

and the induced current density $\mathbf{J}$ will be solved by Equation (7).

Above all, based on $m(m \ll n)$ times of calculation of MoM, $\mathbf{I}(\theta_1), \mathbf{I}(\theta_2)...\mathbf{I}(\theta_n)$ are solved. Compared with n times of calculation of matrix equations needed by traditional MoM, the amount of computation is reduced.

5 Numerical results

Take different three dimensional objects as numerical examples. Experimental electromagnetic parameters are set as follows: the frequency of incident wave f=1.35×10^{9}Hz, permittivity ε=1/(4π×9×10^9)F/m, permeability μ=4π×10^{-7}H/m. Consider 360 angles of incidence (from 1° to 360°, the angle of the wave which is propagating along *y* axis and *E*-polarized in the *x* direction is defined as 0°). Choose Gauss random matrix [14] as the measurement matrix, Hermite basis [15] as the sparse basis and orthogonal matching pursuit (OMP) [16,17] as the recovery algorithm, take 35 times of measurement as example. Compare the calculation results of our method with the ones of traditional MoM.

5.1 NUMERICAL EXAMPLE 1

Consider a PEC cuboid with size of 0.2×0.1×0.05m, as shown in Figure 1.

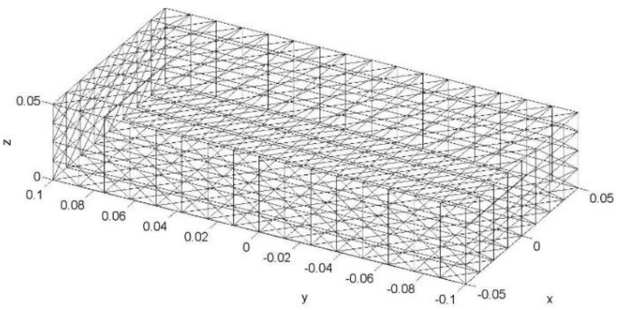


FIGURE 1 The PEC cuboid model

Calculate the current coefficients over 1°, 2°, ..., 360° by both traditional MoM and our solution, and compare the results of two methods. Figure 2 shows the comparison of the current coefficients over the wide angle on an arbitrary RWG basis (take basis number 327 which is centered at (0.025,-0.056,0) as example).

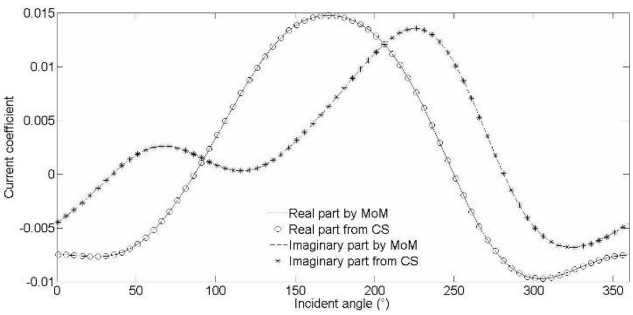


FIGURE 2 Comparison of the calculation results of the current coefficients over the wide angle on basis 327 of the cuboid

From Figure 2, we can see that the results of our method are completely consistent with the ones of traditional MoM.

5.2 NUMERICAL EXAMPLE 2

Consider a multi-objective model which consists of a PEC sphere, a PEC cube and a PEC rectangular pyramid, as shown in Figure 3 (assume the radius of the sphere is 0.05m, the edge-length of the cube is 0.1m and the size of the rectangular pyramid is 0.1×0.1×0.1m.).

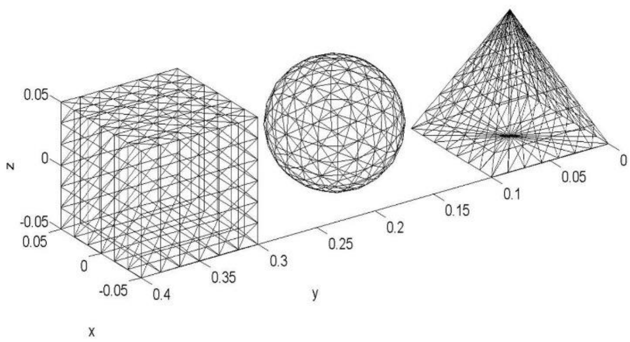
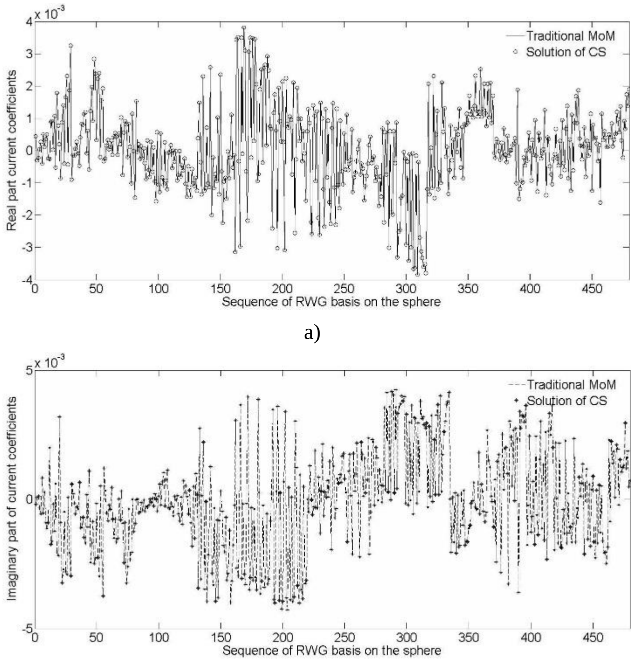


FIGURE 3 The multi-objective model

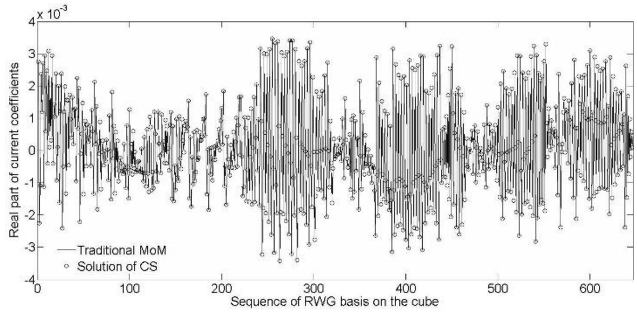
Figures 4-6 show the comparison between the current coefficients of all RWG basis on the sphere, the cube and the rectangular pyramid over an arbitrary incident angle (take 100°, 200°, 300° as examples) of the wide range calculated by our method and the results of traditional MoM respectively.



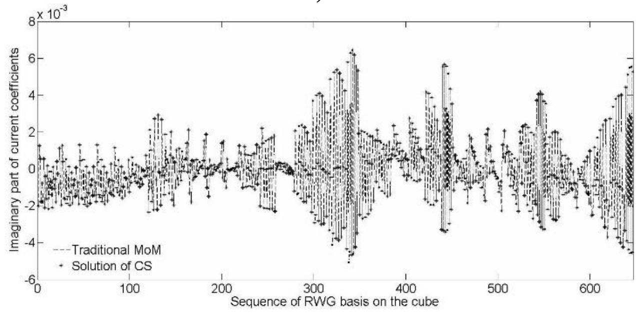
a)

b)

FIGURE 4 Comparison of the calculation results of the current coefficients on the sphere (incident angle=100°): a) Real part, b) Imaginary part

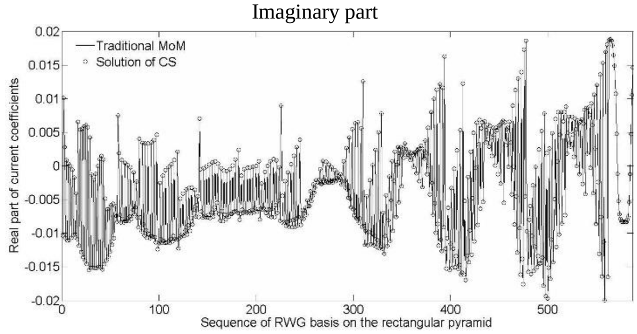


a)

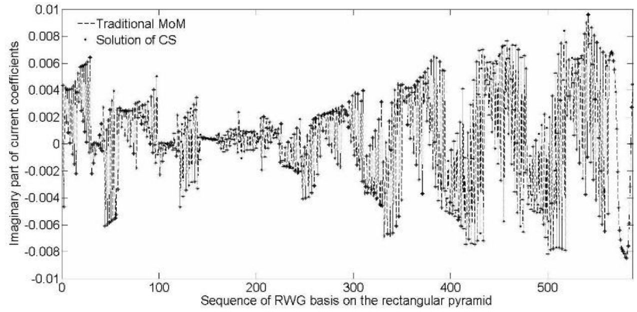


b)

FIGURE 5 Comparison of the calculation results of the current coefficients on the cube (incident angle=200°): a) Real part, b) Imaginary part



a)



b)

FIGURE 6 Comparison of the calculation results of the current coefficients on the rectangular pyramid (incident angle=300°): a) Real part, b) Imaginary part

From Figures 4-6, one can see that the results of our method are also accurate for the multi-objective model.

5.3 NUMERICAL EXAMPLE 3

Consider a simple missile-like model, as shown in Figure 7.

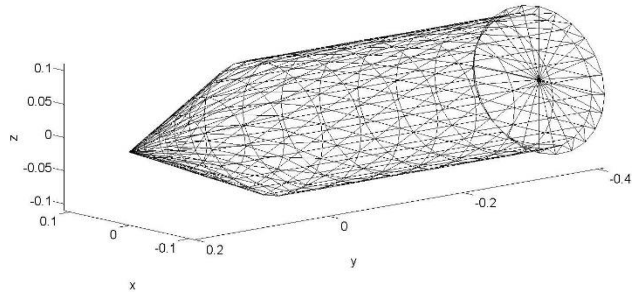
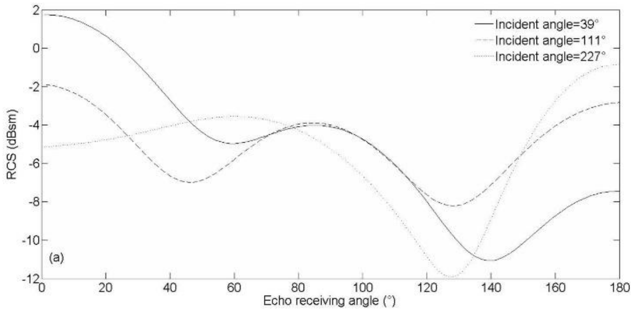
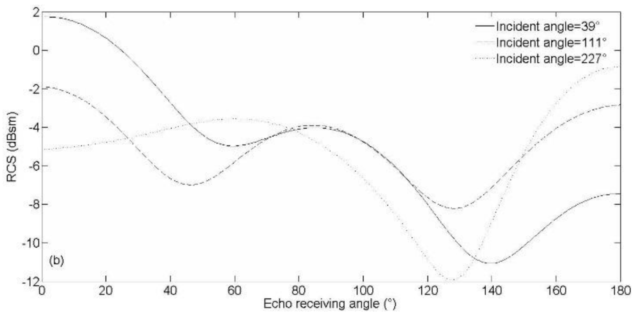


FIGURE 7 The simple missile-like model

Calculate the values of RCS (take *E* surface as example) of this simple missile-like model over some discrete angles among the wide range (take 39°, 111°, 227° as examples) based on both our method and traditional MoM, comparisons of the results of the two solutions are shown in Figure 8.



a)



b)

FIGURE 8 Comparison of the RCS results of the simple missile-like model by a) traditional MoM and b) our method

From Figure 8, one can see that the numerical results calculated by our method are still accurate.

5.4 ERROR STATISTICS

Define the calculation error as:

$$\varDelta=\frac{\left\|\hat{\mathbf{I}}_{\theta}-\mathbf{I}_{\theta}\right\|_2}{\left\|\mathbf{I}_{\theta}\right\|_2}\times 100\%\,, \qquad (17)$$

where $\mathbf{I}_\theta$ stands for the current coefficient matrix over the wide angle calculated by traditional MoM, $\hat{\mathbf{I}}_\theta$ stands for the one calculated by our method. The calculation error statistics for the cuboid, the multi-objective model and the simple missile-like model are shown in Table 1.

TABLE 1 The calculation error statistics

Object	Example 1	Example 2	Example 3
Calculation Error	2.7374×10^{-5}	6.1254×10^{-5}	1.8901×10^{-6}

5.5 COMPARISON OF OPERATION TIME

Table 2 shows the comparison between the computational time of our method and the one of traditional MoM. (The operation environment of the programs is Mathworks Matlab7.0, Pentium(R) Dual-Core CPU at 2.10GHz and an internal memory with capacity of 2GB.)

TABLE 2 Comparison of the computational time (s)

Object	Example 1	Example 2	Example 3
Traditional MoM	402.5571	431.6213	510.1818
Solution of CS	93.6437	99.1613	111.7152

From the tables above, we can see that the calculation results of our method relative to the ones of traditional MoM are still accurate highly but the operation time is reduced a lot.

6 Conclusion

Overall, aiming at fast analyzing wide angle EM scattering problems, this paper proposes a new solution by applying CS into MoM for MFIE, and the feasibility of the method is verified by numerical experiments -- the expected effect of fastening calculating EM scattering problems over a wide angle is achieved.

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