Parameter estimation for nonlinear system using intelligent algorithm

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Received 23 November 2014

Abstract

Mathematical models are the basis of all control problems. The movement law of things described by equations is the mathematical model. The traditional paradigm of system identification employs prior information on system structures and environments and input/output observation data to derive system models. Accordingly, system identification becomes one of the current active subjects in engineering problems. It is well known that nonlinear systems widely exist in people's production and life. Consequently, in this paper, a parameter estimation method of nonlinear system is put forward based on an improved artificial fish swarm algorithm. Its basic idea is as follows. Firstly, the parameter estimation problems of nonlinear systems are changed into a nonlinear function optimization problem over parameter space. Then, the estimates of the system parameters are obtained based on an improved artificial fish swarm algorithm. Finally, in simulation, compared with other algorithms, the simulation results indicated that the presented method is rational and effective.

Keywords: Nonlinear system; Model; System identification; Parameter estimation; Intelligent algorithm

1 Introduction

In recent years, aspects of system identification have been discussed in a multitude of papers, at many conferences and in an appreciable number of university courses [1-2]. It is well known that identification theory of linear system has been very mature [1]. To the best of our knowledge, nonlinear systems widely exist in people's production and life [3-8]. In research on the method of nonlinear system identification, because many engineering objects are very complex, the identification methods of nonlinear systems show remarkable advantage in the analysis of the engineering object. However, because of the inherent complexity and diversity of nonlinear systems, the current research on the nonlinear far not reached the degree of maturity; it has not been fully revealed that the "essence problem" of nonlinear is contained behind the complex phenomenon. Moreover, for the identification of nonlinear system, one of the difficulties is short of unified mathematical model to description of various nonlinear system features. Consequently, the complex nonlinear object recognitions are still not well solved via the traditional identification methods; the identification of nonlinear system is the main topics in the current international identification fields. Thereby, this paper proposes a parameter estimation approach of general nonlinear system by using intelligence algorithm.

Artificial fish swarm algorithm, a novel intelligent algorithm, was first proposed in 2002 [9]. It was inspired by the natural social behavior of fish in searching, swarming and following. Each individual fish searches for food based in its own way. Information on searching is passed to others, and the swarm achieves a global optimum. It has been proved in function optimization [9], parameter estimation [10], combinatorial optimization [11], geotechnical engineering problems [12] and radial basis function neural networks [13], among others. Of course, it can be adopted to identify the nonlinear system model.

In this paper, aim at the diversity of nonlinear system models, a parameter estimation method is proposed based on an improved artificial fish swarm algorithm. Finally, three kinds of nonlinear models are taken as simulation examples to show the effectiveness of the proposed approach. This paper is organized as follows. In Section 2 of this paper, we describe the identification problem formulation for the nonlinear system model. In Section 3, we give a brief overview of the artificial fish swarm algorithm and the particle swarm optimization algorithm, moreover, an improved artificial fish swarm algorithm is proposed. Parameter estimation method of general nonlinear system is given in Section 4 using the improved artificial fish swarm algorithm. Section 5 presents the numerical simulations to show the validity of the presented approach. Finally, Section 6 summarizes the contribution of this paper and conclusions.

2 Problem formulation

Here, general nonlinear system model can be described as the following form.

$$y(t) = f(u(t), t, \theta), \tag{1}$$

where, y(t) is the output of the nonlinear system, u(t) is the input of the nonlinear system, $\theta = (\theta_1, \theta_2, \dots, \theta_k)^T$ is the

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unknown parameter vector, the expression form of the function *f* is known, and u(t) is also given. Moreover, the actual values $(y_0(t), t=1, 2, ..., n)$ of the system output y(t) are known. It is necessary that the parameter is estimated by using $y_0(t)$.

To the purpose of estimation, the nonlinear system models, which are represented by Equation (1), must meet the following assumptions. (a) The system output y(t) must be measurable; (b) Each parameter must be related with the output y(t) of the system. That is to say, the parameters can be estimated based on the observed data. (c) As long as the parameters are determined, the value of the system output y(t) can be obtained by the system simulation. (d) The system dose not diverges in finite time. That is to say, the value of y(t) dose not tend to infinity.

3 Improved artificial fish swarm algorithm

3.1 STANDARD PARTICLE SWARM OPTIMIZATION ALGORITHM

Motivated by social behavior of organisms such as fish schooling and bird flocking, Kennedy and Eberhart first introduced particle swarm optimization algorithm in 1995 [14, 15]. Particle swarm optimization algorithm is a population based heuristic searching algorithm guided by individuals' fitness information. In particle swarm optimazation algorithm, candidate solutions of a specific optimazation problem are called particle. Each particle in the Ddimensional search space is characterized by two factors, i. e., its position $X_i = (x_{i1}, x_{i2}, \dots, x_{iD})$ and velocity $V_i = (v_{i1}, v_{i2}, \dots, v_{iD})$..., v_{iD}), where *i* denotes the *i*th particle in the swarm. In the process of search optimal, all particles in particle swarm optimization algorithm fly through the searching space, and adjust its velocity and position to find a better solution (position) according to its own experience and experience of neighboring particles iteratively.

Let $P_i(t)=(p_{i1}, p_{i2}, ..., p_{iD})$ denotes the best position found by particle *i* within *t* iteration steps, $P_g(t)=(p_{g1}, p_{g2}, ..., p_{gD})$ denotes the best position among all particles in the swarm so far. In the standard particle swarm optimization algorithm, particles update their positions and velocities as the following Equations (2) and (3).

$$V_{i}(t+1) = wV_{i}(t) + c_{1}r_{1}(P_{i}(t) - X_{i}(t))$$
(2)

$$+c_{2}r_{2}(P_{g}(t)-X_{i}(t))$$

$$X_{i}(t) = X_{i}(t) + V_{i}(t+1), i = 1, 2, \dots n,$$
(3)

where *n* denotes the number of particles in the swarm, $V_i(t)$ and $X_i(t)$ represent the velocity and position of particle *i* in the solution space at *t*th iteration step, respectively; r_1 and r_2 are two random numbers uniformly distributed in the range [0, 1]; c_1 and c_2 are acceleration constant, usually c_1 = c_2 =2.0; *w* is the inertia weight. Generally, the value of each component in V_i can be clamped to the range [V_{\min} , V_{\max}] to control excessive roaming of particles outside the search space. Each particle flies toward a new position according to Equations (2) and (3). In this way, all particles in the swarm find their new position and apply these new position to update their individual best position $P_i(t)$ and global best position $P_g(t)$ of the swarm. This process is repeated until a user defined stopping criterion, usually maximum iteration number t_{max} , is reached.

3.2 ARTIFICIAL FISH SWARM ALGORITHM

Artificial fish swarm algorithm is a new populationbased optimization technique inspired by the natural feeding behavior of fish. A fish is represented by its *D*dimensional position $X_i = (x_1, x_2, ..., x_k, ..., x_D)$, and food satisfaction for the fish is represented as FS_i . This paper targets *FS* minimization. The relationship between two fish is denoted by their Euclidean distance $d_{ij} = ||X_i - X_j||$. Another parameters include: visual (representing the visual distances of fish), step (maximum step length), and δ (a crowd factor). *n* is used to represent the size of the fish population. All fish try to identify locations able to satisfy their food needs using three distinct behaviors. These include:

(1) Searching behavior: Searching is a basic biological behavior adopted by fish looking for food. It is based on a random search, with a tendency toward food concentration. It is expressed mathematically as

$$x_{i+1}^{k} = x_{i}^{k} + R(S1)(x_{j}^{k} - x_{i}^{k}) / ||X_{j} - X_{i}||,$$

and $FS_{j} < FS_{i}$ (4)

$$x_{i+1}^k = x_i^k + R(S2), (5)$$

where x_i^k represents the *k*th element of fish position X_i . We randomly select for fish X_i a new position X_j within its visual. If the corresponding FS_j is satisfied, Equation (4) is then employed at the next position X_{i+1} . If FS_j is not satisfied after try number trials, a random position within the step range will be directly adopted as Equation (5). In the above equations, R(S1) and R(S2) represent random variables within [0, step] and [-step, step], respectively.

(2) Swarming behavior: Fish assemble in several swarms to minimize danger. Objectives common to all swarms include satisfying food intake needs, entertaining swarm members and attracting new swarm members. Mathematically,

$$x_{i+1}^{k} = x_{i}^{k} + R(S1)(x_{c}^{k} - x_{i}^{k}) / ||X_{c} - X_{i}||$$

$$FS_{i} < FS_{i} \text{ and } (n_{s} / n) < \delta$$
(6)

A fish located at X_i has neighbors within its visual. X_c identifies the center position of those neighbors and is used to describe the attributes of the entire neighboring swarm. If the swarm center has a greater concentration of food than is available at the fish's current position X_i (i.e., $FS_c < FS_i$), and if the swarm (X_c) is not overly crowded ($(n_s/n) < \delta$), the fish will move from X_i to next X_{i+1} , toward X_c . Here, n_s represents number of individuals within the X_c 's visual. Swarming behavior is executed for a fish based on its associated X_c ; otherwise, searching behavior guarantees a next position for the fish. (3) Following behavior: When a fish locates food, neighboring individuals follow. Mathematically,

$$x_{i+1}^{k} = x_{i}^{k} + R(S1)(x_{\min}^{k} - x_{i}^{k}) / || X_{\min} - X_{i} || FS_{\min} < FS_{i} \text{ and } (n_{f} / n) < \delta$$
(7)

Within a fish's visual, certain fish will be perceived as finding a greater amount of food than others, and this fish will naturally try to follow the best one (X_{\min}) in order to increase satisfaction (i.e., gain relatively more food $[FS_{\min} < FS_i]$ and less crowding $((n_f/n) < \delta)$). n_f represents number of fish within the visual of X_{\min} . Searching behavior commences if following behavior is unable to determine a fish's next position.

Besides, artificial fish swarm algorithm should provide a bulletin that records the optimal state and current performance of fish during iterations.

3.3 IMPROVED ARTIFICIAL FISH SWARM ALGORITHM

The artificial fish swarm algorithm visual provides local search attributes. A small visual restricts a fish to interaction with a relatively small number of companions. The artificial fish swarm algorithm step limits maximum step length, with a small step limiting fish to searching a small area and increasing the risk of wasting time. The step values are set based on Euclidean distance calculations and are sensitive to artificial fish swarm algorithm performance (refer to Equations (4), (6), and (7)). As settings are difficult, this paper employs standard particle swarm optimazation algorithm formulation to minimize the impact of the step factor. As a result, artificial fish are able to swim like a particle in standard particle swarm optimization algorithm, subject to the visual factor, but not the step. All the original artificial fish swarm algorithm equations have been modified.

3.3.1 Searching behavior

$$x_{i+1}^{k} = x_{i}^{k} + \phi_{3}^{k} (x_{j}^{k} - x_{i}^{k}) \text{ and } FS_{j} < FS_{i},$$
 (8)

$$x_{i+1}^{k} = x_{i}^{k} + R(V1) , \qquad (9)$$

$$\phi_3^k = c_3 r_3^k \,, \tag{10}$$

where φ_3^{k} is a uniform random number within [0, 2] with a mean value of one. c_3 is 2, and a uniform random number within [0, 1]. X_j is still a new position within X_i 's visual (same as fish swarm algorithm). Therefore, Equation (8) uses the standard particle swarm optimization algorithm formulae to releases step settings in Equation (4). Since Equation (8) is free to step, this paper further modified Equation (5) as Equation (9), providing such with a visual range. R(V1) is a random variable within [-visual, visual]. When step is smaller than visual in artificial fish swarm algorithm, the improved artificial fish swarm algorithm allows fish to swim for greater lengths than permitted by the original artificial fish swarm algorithm. Modified searching behavior is totally free to step. 3.3.2 Swarming behaviour

$$x_{i+1}^{k} = x_{i}^{k} + \phi_{4}^{k} (x_{c}^{k} - x_{i}^{k}) \quad FS_{c} < FS_{i},$$
and $(n / n) < \delta$
(11)

where the definition of φ_4^k is the same as φ_3^k and definition of X_c is same as that in the artificial fish swarm algorithm. Modified swarming behavior is therefore free to step.

3.3.3 Following behavior

$$\begin{aligned} x_{i+1}^{k} &= x_{i}^{k} + \phi_{5}^{k} \left(x_{\min}^{k} - x_{i}^{k} \right) \quad FS_{\min} < FS_{i} \\ \text{and} \quad (n_{f} / n) < \delta \end{aligned}, \tag{12}$$

where the definition of φ_5^k is the same as φ_3^k and the definition of X_{\min} is same as that used in the artificial fish swarm algorithm. Modified following behavior is therefore free to step.

This improved artificial fish swarm algorithm is free of the step parameter of the artificial fish swarm algorithm. All positions of X_j , X_c , X_{min} , and the next position X_j are dependent upon visual only. The major advantage of this is to release a step parameter.

4 Parameter estimation approach by improved artificial fish swarm algorithm

The specific steps of the estimation procedure are as follows.

<u>Step 1.</u> Parameter θ is taken as a fish.

Step 2. Determine the fitness function: On the basis of knowing the values of the parameters, system outputs y(t) can be obtained based on Equation (1) from simulation experiment. The purpose of identification is to make the identification of system output as close as possible obtain the known system output value, which should make this group parameters corresponding to the particles with a smaller fitness value. So we take the following criterion function as the fitness function.

$$FS = \sum_{t} [y(t) - y_0(t)]^2 .$$
(13)

<u>Step 3.</u> Initialize the fish swarm: Let population size is n, initial position and velocity of particles are randomly set in the range allowed, the individual extreme coordinates of each fish is set as the current position, and calculate the corresponding individual extreme (i. e., the individual's fitness value), while the global extreme value (i. e., the global fitness value) is the best among the individuals' extreme, record the fish's serial number of the best value, and set the global extremum as the current best fish position, and many others.

Step 4. Calculate the fitness value of each fish.

<u>Step 5.</u> For each fish, its fitness value is compared with the individual's extreme, if it is excellent, then the current individual extreme is updated.

<u>Step 6.</u> For each fish, its fitness value is compared with the global extreme, if it is excellent, then the current global extreme is updated.

<u>Step 7.</u> Each fish's swarming behavior is carried out by using Equation (11).

<u>Step 8.</u> Each fish's following behavior is carried out based on Equation (12).

<u>Step 9.</u> Each fish's searching behavior is carried out according to Equations (8), (9) and (10).

<u>Step 10.</u> If preset stop criteria (usually it is the maximum number of iterations or minimum error) does not reach, then the procedure returns to Step 4. Otherwise, the procedure end, and the optimal parameter value θ of nonlinear system model is obtained.

5 Numerical simulation

In order to demonstrate the effectiveness of the presented identification method for nonlinear system models, the following illustrative examples are given.

IAFSA represents the improved artificial fish swarm algorithm, AFSA denotes the artificial fish swarm algorithm, SPSOA describes the standard particle swarm optimization algorithm and GA represents the genetic algorithm.

Example 1. Consider the following transfer function model:

$$\frac{\mathbf{y}(t)}{u(t)} = \frac{l\mathrm{e}^{-\tau t}}{kt+1},\tag{14}$$

where the identified parameters are the proportion coefficient *l*, the inertia coefficient *k* and the lag factor τ .

Example 2. Consider the following state space model:

$$x_1(t+1) = k_1 x_1(t) x_2(t), \qquad (15)$$

$$x_{2}(t+1) = k_{2}x_{1}^{2}(t)u(t), \qquad (16)$$

$$y(t) = k_3 x_2(t) - k_4 x_1^2(t), \qquad (17)$$

$$x_1(0) = 1$$
 and $x_2(0) = 1$, (18)

TABLE 3 The estimates of model's parameters of Example 3

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Example 3. Consider the following Hammerstein model:

$$A(z^{-1})y(t) = B(z^{-1})x(t) + C(z^{-1})w(t), \qquad (19)$$

where $A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^2$, $B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2}$, $C(z^{-1}) = 1 + c_1 z^{-1}$, $x(t) = u(t) + d_1 u^2(t) + d_2 u^3(t) + d_3 u^4(t)$ and w(t) is a Gaussian white noise sequence with zero mean, standard variance $\sigma = 0.01$.

Although the above three cases is not the same form, but they can be attributed to the form of Equation (1) through a certain transformation. In the simulation, the parameters of the identification algorithm are set as follows. The number of artificial fish n = 30, the maximum step length step = 150, the visual distances of fish visual = 5, try number is 10, crowd factor $\delta = 0.618$, the trust parameters $c_1 = c_2 = 2$, the inertia weight w = 0.57, the maximum number of iterations $t_{max} = 50$, the parameter $X_{max} = 2$, the maximal rate (boundary value) $V_{max} = 2$. In Example 1, the initialized search ranges of the parameters are [1, 6]. In Example 2, the initialized search ranges of the parameters are [0.2, 1.0]. In Example 3, the initialized search ranges of the parameters are [-0.9, 1.2].

The estimates of parameters of the systems by using the IAFSA are shown in Tables 1, 2 and 3, respectively. In order to show the validity of the presented estimation method, we further adopted the AFSA (or SPSOA, or GA) to identify the parameters of the systems, respectively. And the simulation results are also given in Tables 1, 2 and 3, respectively.

TABLE 1 The estimates of model's parameters of Example 1

Parameters	l	k	τ
True values	5	3	2
IAFSA	4.9989	2.9897	2.0003
AFSA	4.8607	3.0043	1.9130
SPSOA	5.0120	2.8766	2.0111
GA	4.8815	3.0203	1.8770

TABLE 2 The estimates of model's parameters of Example 2

Parameters	k_1	k_2	k_3	k_4
True values	0.4	0.3	0.8	0.9
IAFSA	0.4004	0.3012	0.7980	0.9020
AFSA	0.3769	0.2864	0.8091	0.7453
SPSOA	0.4100	0.3102	0.8093	0.9131
GA	0.3871	0.2799	0.7709	0.8096

Parameters	a_1	a_2	b_1	b_2	<i>C</i> 1	d_1	d_2	d_3
True values	-0.8	0.9	1.1	0.7	0.6	0.2	0.4	0.6
IAFSA	-0.8011	0.8968	1.0980	0.7009	0.6010	0.1988	0.4938	0.6021
AFSA	-0.8109	0.9200	1.1130	0.6097	0.5780	0.2111	0.4205	0.6230
SPSOA	-0.7568	0.9204	1.2007	0.7225	0.6221	0.1809	0.3396	0.5679
GA	-0.8104	0.8395	1.1583	0.6674	0.5833	0.2203	0.4301	0.5833

From the simulation experiment results of Examples 1, 2 and 3, we can see that the parameter estimation values obtained using the IAFSA are almost approximate to the actual values. That is to say, the precision of identification using the IAFSA is remarkably high compared with other methods.

To the best of our knowledge, the above nonlinear models have a certain representation in a variety of the nonlinear system. Consequently, for vast majority nonlinear system models, the proposed parameter estimation method is effective from the above simulation results.

6 Conclusion

A new parameter estimation method for nonlinear system model is put forward by using the improved artificial fish swarm algorithm in this paper, and get satisfactory results. In simulation, the improved artificial fish swarm

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algorithm is shown that it has the advantages of multipoint optimization, simple, easy and so on. In particular, due to it dose not depend on the model form in the search optimization process, it is widely used in various model parameter estimation. Accordingly, it is shown that the presented method is valid and reasonable.

Acknowledgments

This work was supported by the Scientific Research Program Funded by Shaanxi Provincial Education

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Department under Grant No. 14JK1538, the Project Supported by Shaanxi Provincial Natural Science Foundation research of China under Grant No. 2014JM8325, the Doctoral Scientific Research Start-up Funds of Teachers of Xi'an University of Technology of China under Grant No. 108-211006, the National Natural Science Foundation of China under Grant No. 61273127 and the Specialized Research Fund for the Doctoral Program of Higher Education under Grant No. 20116118110008.

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