Strategic decision of competing supply chain network with multi-criteria decision making

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Abstract

In this paper, we propose a rigorous modeling and analytical framework for the design of decentralized supply chain network with a rival chain present, which involved in the production, storage, and distribution of a substitutable product to markets. The different tiers of firms, consisting of manufacturers, distributors, and retailers are assumed to be multi-criteria decision makers who seek to not only maximize the total profits, but also to minimize the emissions quantities with an appropriate weight. Qualitative properties of the equilibrium solution to the infinite-dimensional variation inequality formulation are provided. In particular, the existence and uniqueness of the results are derived. The algorithm is provided and applied to compute solutions to numerical examples in order to illustrate our approach.

Keywords: Chain-to-Chain Competition, Network Design, Nash Game, Variational Inequality, Multi-criteria Decision Making

1 Introduction

As the barriers of new markets become lower, it's easier to enter the market for the entrants. It's important to develop a framework for the modeling and analysis of supply chain network design with a rival chain present. Optimal strategies can be proposed to strengthen their core competitiveness.

The major focus of the supply chain network design literature has been on intra-chain subjects [1], [2], [3]. In recent years, there has been a considerable shift in thinking with supply chain network design on supply chain versus supply chain competition [4], [5], Rezapour [6]. The competition between supply chains in the markets offering the substitutable goods is one of the most key factors when designing supply chain.

Nagurney et al. developed a supply chain network equilibrium model for supply chain decision-making. Such a model is sufficiently general to handle many decision-makers and their independent behaviors[7]. This model has been extended by many researchers in numerous directions [8], [9], [10]. The above academic research present optimal production planning, logistics, warehousing and sales price decision. However, the physical network structure of the supply chain is given in these literatures. Its physical network greatly influences the performance and competitiveness of supply chain.

Reapour and Farahani simultaneously considers inter-chain competition and supply chain network design, developed an equilibrium model of his new supply chain and an already existing chain to design a decentralized SC network in markets with deterministic demands[11].

Indeed, the increase in environmental concerns is significantly influencing supply chains. As noted in Nagurney, firms are being held accountable not only for their own environmental performance, but also for that of their suppliers, distributors, and even, ultimately, for the environmental consequences of the disposal of their products[12].

In this paper, a significant extension of the supply chain network design of Reapour is made by introducing environmental concerns into a supply chain network equilibrium framework. An equilibrium model is developed from a multi-criteria perspective for sustainable supply chain network design in the presence of a rival supply chain .The mathematical model that we propose allows for the simultaneous determination of decision-makers through capital investments, and the product flows, coupled with the emissions generated.

2 Assumptions and notations

We consider a decentralized SC network consisting of one or more manufacturers, distributors, and retailers. These two chains (a pre-existing chain and a new chain) provide heterogeneous products to the consumers, either identical or highly substitutable. We assume all the manufacturers have infinite capacity. All costs such as the production costs of manufacturers, the transaction costs (manufacturer and distributor, distributor and retailer), the inventory costs of distributor and the location costs of DCs are continuous and convex.

In the pre-existing supply chain, we consider I^1 manufacturers produced a substitutable product, which can be purchased by J^1 distributors, who then sell to K^1 retailers, who in turn respond to consumers via demand functions.

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Let $q^1_{i^1j^1}$ denote the amount of products transacted (or shipped) between manufacturer i^1 and distributor j^1, group these variables between all manufacturers and all distributors into the vector Q^1_1.

Then the production output of the product by manufacturer i^1 is $q^1_{i^1} = \sum_{j^1=1}^{J^1} q^1_{i^1j^1}$, the amount of the product purchased by distributor j^1 is $q^1_{j^1} = \sum_{i^1=1}^{I^1} q^1_{i^1j^1}$. Let $p^1_{i^1j^1}$ denote the price charged for the product by manufacturer i^1 to distributor j^1. Let $a^1_{j^1}$ denote the capacity at distributor j^1, group these variables into the vector A^1. Let $s^1_{j^1k}$ denote the amount of the product transacted between distributor j^1 and retailer k, group these variables into the vector Q^1_2. So the amount of the product which bought by retailer k is $s^1_k = \sum_{j^1=1}^{J^1} s^1_{j^1k}$. Let p^1_k denote the demand price of the product associated with market k, group these variables into the vector P^1. Let d^1_k be the demand for the product at the demand price p^1_k at market k, where d^1_k is a random variable with a density function of $f^1_k(x)$. Let $F^1_k(x)$ denote the probability distribution function of d^1_k.

The new supply chain consists of I^2 manufacturers, J^2 distributors, and K retailers. Then we give the notation about the new supply chain. Let $q^2_{i^2j^2}$ denote the amount of the product shipped between manufacturer i^2 and distributor j^2, group these variables into the vector Q^2_1. So the output of the product by manufacturer i^2 is $q^2_{i^2} = \sum_{j^2=1}^{J^2} q^2_{i^2j^2}$, the amount of product purchased by distributor j^2 is $q^2_{j^2} = \sum_{i^2=1}^{I^2} q^2_{i^2j^2}$. Let $p^2_{i^2j^2}$ denote the price charged for the product by manufacturer i^2 to distributor j^2. Let $a^2_{j^2}$ denote the capacity at distributor j^2, group these variables into the vector A^2. Let $s^2_{j^2k}$ denote the amount of the product transacted between distributor j^2 and retailer k, group these variables into vector Q^2_2. Then the amount of the product which bought by retailer k is $s^2_k = \sum_{j^2=1}^{J^2} s^2_{j^2k}$. Let d^2_k be the demand for the product at the demand price p^2_k at market k, where d^2_k is a random variable with a density function of $f^2_k(x)$. Let $F^2_k(x)$ denote the probability distribution function of d^2_k.

3 The competing supply chain network model with multi-criteria decision makers

In this section, the supply chain network model with manufacturers, distributors and retailers is developed. Specifically, we consider two competing supply chains.

We assume that manufacturer i^1 is faced with production cost function $G_{i^1}(Q^1_1)$, which can depend, in general, on the entire vector of production outputs, that is Q^1_1,The transaction costs between manufacturer i^1 and distributor j^1 pair is given by $C_{ij}(q^1_{ij})$.The total costs incurred by manufacturer i^1 are equal to the sum of his production costs plus the total transaction costs. The total profit of manufacturer i^1 is equal to the difference between the revenue and the total costs. So the criterion of profit maximization for manufacturer i^1 can be expressed mathematically as

$$\begin{aligned} &\max \quad \sum_{j^1=1}^{J^1} p^1_{i^1j^1} q^1_{i^1j^1} - G_{i^1}(Q^1_1) - \sum_{j^1=1}^{J^1} C_{i^1j^1}(q^1_{i^1j^1}), \\ &s.t. \quad q^1_{i^1j^1} \geq 0 \qquad \forall i^1, j^1 \end{aligned} \tag{1}$$

In addition to the criterion of profit maximization, each manufacturer also seeks to minimize the total emissions generated both in production of the product as well as in transportation of the product to the various distributors. Letting $e^1_{i^1}$ denote the amount of emissions generated by product produced at manufacturer i^1, and $e^1_{i^1j^1}$ denote the amount of emissions generated in transporting the product from manufacturer i^1 to distributor j^1. So the second criterion of manufacturer i^1 as:

$$\begin{aligned} &\min \quad e^1_{i^1}\left(\sum_{j^1=1}^{J^1} q^1_{i^1j^1}\right) + e^1_{i^1j^1}(q^1_{i^1j^1}), \\ &s.t. \quad q^1_{i^1j^1} \geq 0 \qquad \forall i^1, j^1 \end{aligned} \tag{2}$$

We assume that assign a nonnegative weight of $\varphi^1_{i^1}$ to the emissions-generation criterion (2) and 1 to the profit maximization criterion (1). The multi-criteria decision making problem for manufacturer i^1 is transformed into:

$$\begin{aligned} &\max \quad \sum_{j^1=1}^{J^1} p^1_{i^1j^1} q^1_{i^1j^1} - G_{i^1}(Q^1_1) - \sum_{j^1=1}^{J^1} C_{i^1j^1}(q^1_{i^1j^1}) - \varphi^1_{i^1}\left(e^1_{i^1}\left(\sum_{j^1=1}^{J^1} q^1_{i^1j^1}\right) + e^1_{i^1j^1}(q^1_{i^1j^1})\right). \\ &s.t. \quad q^1_{i^1j^1} \geq 0 \qquad \forall i^1, j^1 \end{aligned} \tag{3}$$

It is assumed that each distributor seeks to maximize its own profits and minimize the total emissions. Hence, the multi-criteria decision making objective function faced by distributor j^1 may be expressed as follows:

$$\begin{aligned}\max &\sum_{k=1}^{K} p_k^1 s_{j^1k}^1 - H_{j^1}^1(q_{j^1}^1) - \sum_{k=1}^{K} T_{j^1k}(s_{j^1k}^1) - \sum_{i^1=1}^{I^1} p_{i^1j^1}^1 q_{i^1j^1}^1 \\ &- \eta_{j^1}^1 (e_{j^1}^1(q_{j^1}^1) + e_{j^1k}^1(s_{j^1k}^1)) \\ s.t. &\sum_{i=1}^{I} q_{ij}^1 \ge \sum_{j=1}^{J} s_{jk}^1 \qquad \forall j \\ &\sum_{i=1}^{I} q_{ij}^1 \le a_j^1 \qquad \forall j \\ &q_{ij}^1, s_{jk}^1 \ge 0 \qquad \forall i,j,k\end{aligned} \qquad (4)$$

The first term in (4) denotes the revenue of distributor j^1. Let $H_{j^1}^1(q_{j^1}^1)$ denote inventory costs, $T_{j^1k}(s_{j^1k}^1)$ denotes the transaction costs between the distributor j^1 and retailer k. The forth term denotes the payments for the product to the various manufacturers. The last term denotes the amount of emissions. Let $e_{j^1}^1(q_{j^1}^1)$ denote the amount of emissions generated by the distributor j^1, and let $e_{j^1k}^1(s_{j^1k}^1)$ denote the amount of emissions of product transacted between distributor j^1 and retailer k. Let $\eta_{j^1}^1$ denote the nonnegative weight to the emissions criterion. The first constraint expresses that the demand of distributor j^1 is no more than its supply. Let $\alpha_{j^1}^1$ denote the Lagrange multiplier of this constraint. The second constraint expresses that the supply of distributor j^1 cannot exceed its capacity. $\beta_{j^1}^1$ is the Lagrange multiplier of this constraint.

We can discuss manufacturer i^2 and distributor j^2 in the new supply chain.The multi-criteria decision making problem for manufacturer i^2 is transformed into:

$$\begin{aligned}\max &\sum_{j^2=1}^{J^2} p_{i^2j^2}^2 q_{i^2j^2}^2 - G_{i^2}(Q_1^2) - \sum_{j^2=1}^{J^2} C_{i^2j^2}(q_{i^2j^2}^2) \\ &- \varphi_{i^2}^2 (e_{i^2}^2(\sum_{j^2=1}^{J^2} q_{i^2j^2}^2) + e_{i^2j^2}^2(q_{i^2j^2}^2)) \\ s.t. &\quad q_{i^2j^2}^2 \ge 0 \qquad \forall i^2, j^2\end{aligned} \qquad (5)$$

The multi-criteria decision making problem for distributor j^2 can be expressed as:

$$\begin{aligned}\max &\sum_{k=1}^{K} p_k^2 s_{j^2k}^2 - H_{j^2}^2(q_{j^2}^2) - \sum_{k=1}^{K} T_{j^2k}(s_{j^2k}^2) - \sum_{i^2=1}^{I^2} p_{i^2j^2}^2 q_{i^2j^2}^2 \\ &- L_{j^2}(a_{j^2}^2) - \eta_{j^2}^2 (e_{j^2}^2(q_{j^2}^2) + e_{j^2k}^2(s_{j^2k}^2)) \\ s.t. &\sum_{i^2=1}^{I^2} q_{i^2j^2}^2 \ge \sum_{j^2=1}^{J^2} s_{j^2k}^2 \qquad \forall j^2 \\ &\sum_{i^2=1}^{I^2} q_{i^2j^2}^2 \le a_{j^2}^2 \qquad \forall j^2 \\ &q_{i^2j^2}^2, s_{j^2k}^2 \ge 0 \qquad \forall i^2, j^2, k\end{aligned} \qquad (6)$$

The difference between the pre-existing supply chain and the new supply chain is the costs of locating a DC. Let $L_{j^2}(a_{j^2}^2)$ denote the costs of locating a DC.

The market equilibrium conditions are now turned to discuss. The transactions between the retailers and the consumers are the stochastic economic equilibrium conditions. The equilibrium conditions mathematically as:

$$d_k^n(p_k^*) \begin{cases} \le \sum_{j^n=1}^{J^n} s_{j^nk}^{n*} \quad a.e., \quad if \quad p_k^{n*} = 0 \\ = \sum_{j^n=1}^{J^n} s_{j^nk}^{n*} \quad a.e., \quad if \quad p_k^{n*} > 0 \end{cases} \qquad \forall k \in K, n = 1,2.$$

We assume that the two supply chains compete in a noncooperative fashion. Given that the governing optimazation/equilibrium concept underlying noncooperative behavior is that of Nash (1951)[13]. The Equilibrium Conditions of the Competing Supply Chain Network is equivalent to the variational inequality problem as follows, determine

$$(Q_1^{1*}, Q_1^{2*}, Q_2^{1*}, Q_2^{2*}, \alpha_1^*, \beta_1^*, A_2^*, \alpha_2^*, \beta_2^*, P^{1*}, P^{2*}) \in K,$$

s.t. $\forall (Q_1^1, Q_1^2, Q_2^1, Q_2^2, \alpha_1, \beta_1, A_2, \alpha_2, \beta_2, P^1, P^2) \in K$ satisfying:

$$\begin{aligned}&\sum_{i^1=1}^{I^1}\sum_{j^1=1}^{J^1}[\frac{\partial G_{i^1}(Q_1^{1*})}{\partial q_{i^1j^1}^1} + \frac{\partial C_{i^1j^1}(q_{i^1j^1}^{1*})}{\partial q_{i^1j^1}^1} + \frac{\partial H_{j^1}^1(q_{j^1}^{1*})}{\partial q_{i^1j^1}^1} - \alpha_{j^1}^{1*} + \beta_{j^1}^{1*} \\ &\quad + \varphi_{i^1}^1 \frac{\partial(e_{i^1}^1(q_{i^1}^{1*}) + e_{i^1j^1}^1(q_{i^1j^1}^{1*}))}{\partial q_{i^1j^1}^1} + \eta_{j^1}^1 \frac{\partial e_{j^1}^1(q_{j^1}^{1*})}{\partial q_{i^1j^1}^1}](q_{i^1j^1}^1 - q_{i^1j^1}^{1*}) \\ &+ \sum_{j^1=1}^{J^1}\sum_{k=1}^{K}[\frac{\partial T_{j^1k}(s_{j^1k}^{1*})}{\partial s_{j^1k}^1} - p_k^{1*} + \eta_{j^1}^1 \frac{\partial e_{j^1k}^1(s_{j^1k}^{1*})}{\partial s_{j^1k}^1} + \gamma_{j^1}^{1*}](s_{j^1k}^1 - s_{j^1k}^{1*}) \\ &+ \sum_{j^1=1}^{J^1}[\sum_{i^1=1}^{I^1} q_{i^1j^1}^{1*} - \sum_{k=1}^{K} s_{j^1k}^{1*}]\cdot[\alpha_{j^1}^1 - \alpha_{j^1}^{1*}] + \sum_{j^1=1}^{J^1}[a_{j^1}^1 - \sum_{i^1=1}^{I^1} q_{i^1j^1}^{1*}]\cdot[\beta_{j^1}^1 - \beta_{j^1}^{1*}] \\ &\sum_{i^2=1}^{I^2}\sum_{j^2=1}^{J^2}[\frac{\partial G_{i^2}(Q_1^{2*})}{\partial q_{i^2j^2}^2} + \frac{\partial C_{i^2j^2}(q_{i^2j^2}^{2*})}{\partial q_{i^2j^2}^2} + \frac{\partial H_{j^2}^2(q_{j^2}^{2*})}{\partial q_{i^2j^2}^2} - \alpha_{j^2}^{2*} + \beta_{j^2}^{2*} \\ &\quad + \varphi_{i^2}^2 \frac{\partial(e_{i^2}^2(q_{i^2}^{2*}) + e_{i^2j^2}^2(q_{i^2j^2}^{2*}))}{\partial q_{i^2j^2}^2} + \eta_{j^2}^2 \frac{\partial e_{j^2}^2(q_{j^2}^{2*})}{\partial q_{i^2j^2}^2}](q_{i^2j^2}^2 - q_{i^2j^2}^{2*}) \\ &+ \sum_{j^2=1}^{J^2}\sum_{k=1}^{K}[\frac{\partial T_{j^2k}(s_{j^2k}^{2*})}{\partial s_{j^2k}^2} - p_k^{2*} + \eta_{j^2}^2 \frac{\partial e_{j^2k}^2(s_{j^2k}^{2*})}{\partial s_{j^2k}^2} + \gamma_{j^2}^{2*}](s_{j^2k}^2 - s_{j^2k}^{2*}) \\ &+ \sum_{j^2=1}^{J^2}[\frac{\partial L_{j^2}(a_{j^2}^2)}{\partial a_{j^2}^2} - \beta_{j^2}^{2*}]\cdot[a_{j^2}^2 - a_{j^2}^{2*}] + \sum_{j^2=1}^{J^2}[\sum_{i^2=1}^{I^2} q_{i^2j^2}^{2*} - \sum_{k=1}^{K} s_{j^2k}^{2*}]\cdot[\alpha_{j^2}^2 - \alpha_{j^2}^{2*}] \\ &+ \sum_{j^2=1}^{J^2}[a_{j^2}^2 - \sum_{i^2=1}^{I^2} q_{i^2j^2}^{2*}]\cdot[\beta_{j^2}^2 - \beta_{j^2}^{2*}] + \sum_{k=1}^{K}(\sum_{j^1=1}^{J^1} s_{j^1k}^{1*} - d_k^1(p_k^{1*})) \times [p_k^1 - p_k^{1*}] \\ &+ \sum_{k=1}^{K}(\sum_{j^2=1}^{J^2} s_{j^2k}^{2*} - d_k^2(p_k^{2*})) \times [p_k^2 - p_k^{2*}] \ge 0,\end{aligned}$$

where $K \subseteq R_+^{I^1J^1 + I^2J^2 + J^1K + J^2K + J^1 + J^1 + J^2 + J^2 + J^2 + K + K}$. (7)

For easy reference in the subsequent sections, variational inequality problem (7) can be rewritten in the following standard variational inequality form: determine X^* satisfying:

$$\langle F(X^*)^T, X - X^* \rangle \geq 0, \quad \forall X \in K, \tag{8}$$

where $F(X) \equiv (F_{i^1 j^1}, F_{i^2 j^2}, F_{j^1 k}, F_{j^2 k}, F^1_{j^1}, F^2_{j^1}, F^1_{j^2}, F^2_{j^2}, F^3_{j^2}, F_k, F_k)_{i^1=1,\cdots,I^1, j^1=1,\cdots,J^1, i^2=1,\cdots,I^2, j^2=1,\cdots,J^2, k=1,\cdots,K}$,

$X \equiv (Q_1^{1*}, Q_1^{2*}, Q_2^{1*}, Q_2^{2*}, \alpha_1^*, \beta_1^*, A_2^*, \alpha_2^*, \beta_2^*, P^{1*}, P^{2*})$, $\langle \cdot,\cdot \rangle$ denotes the inner product in N-dimensional Euclidean space.

4 Qualitative properties

In this Section, some qualitative properties of the solution to variational inequality (7) are discussed. In particular, the existence and uniqueness of the solution are derived. The previous assumptions about the production cost functions, transaction cost functions, the inventory cost functions and the location cost functions that enter into the variational inequality (8) is continuous. However, the feasible set is not compact. Thus, the existence of a solution simply from the assumption of continuity of the functions cannot be derived. Nevertheless, a rather weak condition to guarantee the existence of the solution can be imposed. Let

$$K_b \equiv \{(Q_1^{1b}, Q_1^{2b}, Q_2^{1b}, Q_2^{2b}, \alpha_1^b, \beta_1^b, A_2^b, \alpha_2^b, \beta_2^b, P^{1b}, P^{2b}) \mid$$
$$0 \leq Q_1^{1b} \leq b_1, 0 \leq Q_1^{2b} \leq b_2, 0 \leq Q_2^{1b} \leq b_3, 0 \leq Q_2^{2b} \leq b_4,$$
$$0 \leq \alpha_1^b \leq b_5, 0 \leq \beta_1^b \leq b_6, 0 \leq A_2^b \leq b_7, 0 \leq \alpha_2^b \leq b_8,$$
$$0 \leq \beta_2^b \leq b_9, 0 \leq P^{1b} \leq b_{10}, 0 \leq P^{2b} \leq b_{11}\}$$

Obviously K_b is a bounded closed convex subset of $R_+^{I^1J^1+I^2J^2+J^1K+J^2K+J^1+J^1+J^2+J^2+J^2+K+K}$. Therefore, at least one solution $X^b \in K_b$ can be satisfied the following variational inequality.

$$\langle F(X^b)^T, X - X^b \rangle \geq 0, \quad \forall X \in K_b \tag{9}$$

Theorem 1: (existence)

Variational inequality (8) admits a solution if there exists $b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11} > 0$, such that variational inequality (9) admits a solution in K_b.

Theorem 2: (existence)

Suppose that the production cost functions $G_{i^1}(q^1_{i^1 j^1})$, $G_{i^2}(q^2_{i^2 j^2})$ are additive convex functions, transaction cost functions $C_{i^1 j^1}(q^1_{i^1 j^1})$, $T_{j^1 k}(s^1_{j^1 k})$, $C_{i^2 j^2}(q^2_{i^2 j^2})$, $T_{j^2 k}(s^2_{j^2 k})$ are convex functions, location cost functions $L_{j^2}(a)$ are convex functions, then variational inequality (8) admits a solution.

Theorem 3: (Uniqueness)

Suppose that the production cost functions $G_{i^1}(q^1_{i^1 j^1})$, $G_{i^2}(q^2_{i^2 j^2})$ are additive strictly convex functions, transaction cost functions $C_{i^1 j^1}(q^1_{i^1 j^1})$, $T_{j^1 k}(s^1_{j^1 k})$, $C_{i^2 j^2}(q^2_{i^2 j^2})$, $T_{j^2 k}(s^2_{j^2 k})$ are strictly convex functions, location cost functions $L_{j^2}(a)$ are strictly convex functions. Then the solution to the variational inequality (8) is unique.

Theorem 3 expresses that the equilibrium product shipment pattern between the manufacturers and the distributors, the equilibrium transaction pattern between the distributors and the retailers, and the equilibrium price pattern at the retailers of the pre-existing supply chain, is unique. As well as the equilibrium product shipment pattern between the manufacturers and the distributors, the equilibrium transaction pattern between the distributors and the retailers, the equilibrium price pattern at the retailers and the facility's capacity of the new supply chain, is unique.

5 Algorithm

We utilize the Euler method to obtain the solution to the variational inequality [14]. The statement of the modified projection method is as follows:

(1) Initialization

Set $X^0 \in K$, $t = 0$, and let $\{a_t\}$ be scalar ,s.t. $\sum_{t=1}^{\infty} a_t = \infty, a_t > 0, a_t \to \infty$, let ε be prespecified tolerance.

(2) Computation

Compute $X^{(t+1)} = \max\{0, X^{(t)} - a_t F(X^{(t)})\}$

(3) Convergence Verification

If $| X^{(t+1)} - X^{(t)} | \leq \varepsilon$, then stop; else, set $t = t+1$, and go step 2.

6 Numerical Example

In particular, we consider two competing supply chains. The pre-existing supply chain consisted of two manufacturers, two distributors and two retailers. The new supply chain had two available manufacturers and three candidate distributors and two same retailers. In this Section, the Euler method is applied to numerical example. The algorithm was implemented in VC++. The convergence criterion utilized was that the absolute value of the successive results differed by no more than 0.01.

The production cost functions were given by:

$$G_1^1(q_1^1) = 0.1(q_1^1)^2 + 0.15q_1^1, G_2^1(q_2^1) = 0.2(q_2^1)^2 + 0.1q_2^1,$$
$$G_1^2(q_1^2) = 0.2(q_1^2)^2 + 0.1q_1^2, G_2^2(q_2^2) = 0.1(q_2^2)^2 + 0.1q_2^2$$

The transaction cost functions were given by:

$$G_{11}^1(q_{11}^1) = 0.2(q_{11}^1)^2 + 0.3q_{11}^1, G_{12}^1(q_{12}^1) = 0.25(q_{12}^1)^2 + 0.25q_{12}^1,$$
$$G_{21}^1(q_{21}^1) = 0.3(q_{21}^1)^2 + 0.2q_{21}^1, G_{22}^1(q_{22}^1) = 0.1(q_{22}^1)^2 + 0.3q_{22}^1,$$
$$G_{11}^2(q_{11}^2) = 0.1 \cdot (q_{11}^2)^2 + 0.2q_{11}^2, G_{12}^2(q_{12}^2) = 0.15 \cdot (q_{12}^2)^2 + 0.2q_{12}^2,$$
$$G_{13}^2(q_{13}^2) = 0.7 \cdot (q_{13}^2)^2 + 0.8q_{13}^2, G_{21}^2(q_{21}^2) = 0.2 \cdot (q_{21}^2)^2 + 0.1q_{21}^2,$$

$G_{22}^2(q_{22}^2) = 0.2 \cdot (q_{22}^2)^2 + 0.2q_{22}^2, G_{23}^2(q_{23}^2) = 0.8 \cdot (q_{23}^2)^2 + 0.85q_{23}^2,$
$T_{11}(s_{11}^1) = 0.3 \cdot (s_{11}^1)^2 + 0.35s_{11}^1, T_{12}(s_{12}^1) = 0.35 \cdot (s_{12}^1)^2 + 0.3s_{12}^1,$
$T_{21}(s_{21}^1) = 0.4 \cdot (s_{21}^1)^2 + 0.2s_{21}^1, T_{22}(s_{22}^1) = 0.3 \cdot (s_{22}^1)^2 + 0.2s_{22}^1,$
$T_{11}^2(s_{11}^2) = 0.2 \cdot (s_{11}^2)^2 + 0.2s_{11}^2, T_{12}^2(s_{12}^2) = 0.3 \cdot (s_{12}^2)^2 + 0.1s_{12}^2,$
$T_{21}^2(s_{21}^2) = 0.3 \cdot (s_{21}^2)^2 + 0.1s_{21}^2, T_{22}^2(s_{22}^2) = 0.1 \cdot (s_{22}^2)^2 + 0.1s_{22}^2,$
$T_{31}^2(s_{31}^2) = 0.8 \cdot (q_{31}^2)^2 + 0.9q_{31}^2, T_{23}^2(s_{23}^2) = 0.9 \cdot (s_{23}^2)^2 + 0.9s_{23}^2$

The location cost functions of DCs in the new supply chain were given by:
$L_1^2(a_1^2) = (0.3) \cdot (a_1^2)^2, L_2^2(a_2^2) = (0.3) \cdot (a_2^2)^2, L_3^2(a_3^2) = (1.3) \cdot (a_3^2)^2,$

The inventory coat functions of DCs were given by:
$H_1^1(q_1^1) = 0.15q_1^1, H_2^1(q_2^1) = 0.175q_2^1,$
$H_1^2(q_1^2) = 0.1q_1^2, H_2^2(q_2^2) = 0.15q_2^2, H_3^2(q_3^2) = 0.35q_3^2$.

The emissions functions generated by manufacturer were given by:
$e_1^1(q_1^1) = 0.05(q_1^1)^2 + q_1^1, e_2^1(q_2^1) = 0.1(q_2^1)^2 + 2q_2^1,$
$e_1^2(q_1^2) = 0.1(q_1^2)^2 + q_1^2, e_2^2(q_2^2) = 0.2(q_2^2)^2 + 4q_2^2$

The emissions functions generated by distributor were given by:
$e_1^1(s_1^1) = 1.2(s_1^1)^2 + s_1^1, e_2^1(s_2^1) = 0.8(s_2^1)^2 + 3s_2^1,$
$e_1^2(s_1^2) = 0.5(s_1^2)^2 + 2s_1^2, e_2^2(s_2^2) = 0.65(s_2^2)^2 + s_2^2, e_3^2(s_3^2) =$
$= 1.3(s_3^2)^2 + 4s_3^2$

The emissions functions generated by transactions were given by:
$e_{i^1j^1}^1(q_{i^1j^1}^1) = 0.8q_{i^1j^1}^1, i^1 = 1,2, j^1 = 1,2,$
$e_{i^2j^2}^2(q_{i^2j^2}^2) = 0.8_{i^2j^2}^2, i^2 = 1,2, j^2 = 1,2,3$
$e_{j^1k}^1(q_{j^1k}^1) = 0.8q_{j^1k}^1, j^1 = 1,2, k = 1,2,$
$e_{j^2k}^2(q_{j^2k}^2) = 0.8_{j^2k}^2, j^2 = 1,2,3, k = 1,2$

The demand functions at the demand markets were:
$d_1^1(P_1) = 8000 - p_1^1 + 0.9 \cdot p_1^2,$
$d_2^1(P_2) = 12000 - p_2^1 + 0.9 \cdot p_2^2,$
$d_1^2(P_1) = 6000 - p_1^2 + 0.9 \cdot p_1^1,$
$d_2^2(P_2) = 10000 - p_2^2 + 0.9 \cdot p_2^1$

Example 1

In the first example we assumed that all the weights associated with the different criteria were set equal to 1 by all the decision-makers. Hence, in this example, the decision-makers were assigned the same weight to profit maximization and emissions value minimization.

The equilibrium product shipments between manufacturers and distributors were (Q_1^{1*}, Q_1^{2*}) :
$q_{11}^1 = 4074.63, q_{12}^1 = 3409.58, q_{21}^1 = 1216.64, q_{22}^1 = 4023.84, q_{11}^2 = 5395.89,$
$q_{12}^2 = 2124.04, q_{13}^2 = 1150.18, q_{21}^2 = 4181.11, q_{22}^2 = 3077.16, q_{23}^2 = 417.81,$

The equilibrium product shipments between distributors and retailers were (Q_2^{1*}, Q_2^{2*}) :
$s_{11}^1 = 2076.86, s_{12}^1 = 3214.39, s_{21}^1 = 2468.66, s_{22}^1 = 4964.74, s_{11}^2 = 5040.56,$
$s_{12}^2 = 4541.31, s_{21}^2 = 413.619, s_{22}^2 = 4783.18, s_{31}^2 = 621.746, s_{32}^2 = 946.187,$

The DCs'capacities were A_2^* :
$a_1^2 = 9581.24, a_2^2 = 5196.47, a_3^2 = 1568.01$

We now discuss the behavior of the various decision-makers in the supply chain, as depicted in bold line in Figure 1.

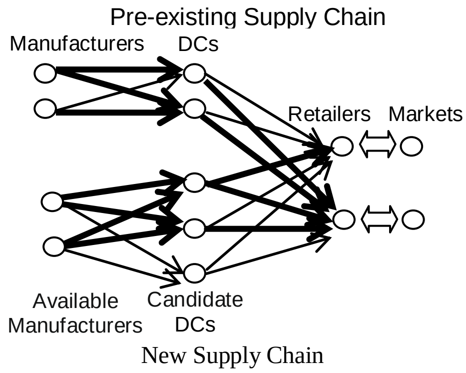

FIGURE 1 The Strategic design of the numerical example 1

Example 2

In this example we assumed that the weights associated with the different criteria were set equal to 1 by all the decision-makers in the pre-existing supply chain, and the weights were set equal to 0.5 in the new supply chain.

The equilibrium product shipments between manufacturers and distributors were (Q_1^{1*}, Q_1^{2*}) :
$q_{11}^1 = 3788.96, q_{12}^1 = 3171.51 q_{21}^1 = 1130.94, q_{22}^1 = 3742.91, q_{11}^2 = 4962.37,$
$q_{12}^2 = 2641.35, q_{13}^2 = 1250.74, q_{21}^2 = 4522.36, q_{22}^2 = 4023.23, q_{23}^2 = 477.016,$

The equilibrium product shipments between distributors and retailers were (Q_2^{1*}, Q_2^{2*}) :
$s_{11}^1 = 1889.72, s_{12}^1 = 3030.17, s_{21}^1 = 2258.15, s_{22}^1 = 4656.27, s_{11}^2 = 5109.82,$
$s_{12}^2 = 4380.69, s_{21}^2 = 934.386, s_{22}^2 = 5725.27, s_{31}^2 = 742.759, s_{32}^2 = 984.79,$

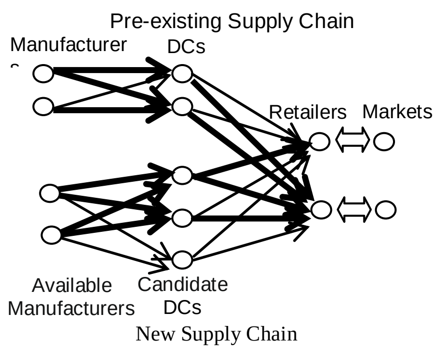

FIGURE 2 The Strategic design of the numerical example 2

The DCs'capacities were A_2^* :
$a_1^2 = 9489.45, a_2^2 = 6659.8, a_3^2 = 1727.62$

We now discuss behaviors of the various decision-makers in the supply chain, as depicted in bold line in Figure 2.

6 Discussion and conclusion

This framework has generalized the recent work of Reapour (2010) with environmental decision making problem. Simultaneously considering competition between supply chains and the physical structure design of supply chain network, this paper has developed an equilibrium model of competitive supply chain network with multi-criteria decision making.

The existence and uniqueness of the equilibrium pattern have been established. If production cost functions of all manufacturers are additive convex functions, transaction cost functions, location cost functions and emissions functions are convex functions, therefore the amount of production, shipment and price pattern are obtained. Furthermore, the conditions in above assumptions are slightly strengthened, the equilibrium solution is unique.

Future research may include other physical structures of the supply chain network, such as reverse and loop supply chains. Moreover, this can be extended to the model of disequilibrium dynamics.

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