Contract selection between one supplier and two competing retailers

Rongting Sun[1*], Yiqun Guo[2,3]

[1] *School of Economics and Management, Chang'An University, Xi'an 710064, Shaanxi, China.*

[2] *Xi'an Physical Education University, Xi'an 710068, Shaanxi, China.*

[3] *School of Finance and Economics, Xi'an Jiaotong University, Xi'an 710049, Shaanxi, China.*

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Abstract

This paper investigates a contract-selection scheme for coordinating a supply chain consisting of one supplier and two competing retailers. The supplier, as a Stackelberg leader, offers a wholesale-price contract and a quantity-discount contract to two competing retailers; the competing retailers have to choose one of the contracts and decide the quantities they would order. On the basis of anticipated responses and actions of the retailers, the supplier designs the contract combination in order to coordinate the supply chain. Adopting the classic Cournot competition model and using game theory, we show that the contract-selection scheme could coordinate the competing supply chain and provide a relative better performance than the single quantity-discount contract. A numerical study is presented to illustrate the findings.

Keywords: Supply Chain, Coordination, Contract Selection, Competing Retailers

1 Introduction

Multiple decision makers (suppliers and retailers) who look for their own profits in a decentralized supply chain often causes sub-optimal performance, which is referred to as "double-marginalization" in the literature[1,2]. Supply chain coordination aims to resolve this problem by looking for global optima in a supply chain with the development and implementation of the strategies. Coordinating contracts that can achieve the optimal supply chain performance are common used in practice [3].

If there are multiple suppliers/retailers in a supply chain, the horizontal competition will show up because the suppliers/retailers compete for more orders/sales. For example, the CPU manufacturers Intel and AMD compete to supply more products to different PC manufacturers such as HP, Dell, etc; on the other side, PC manufacturers compete with each other to sell more products to consumers.

Lots of contract schemes were studied to coordinate the multiple competitors supply chain especially multi-retailer supply chain, some of them could make the supply chain coordinated [2]. But one critical finding from the existing body of literature is that in most of the coordination models, the supplier affords only a single-type contract to differentiated retailers, i.e. giving same wholesale prices to different retailers; or giving same quantity discount policies to different retailers. However, it is still difficult to coordinate a supply chain with a single-type contract which has a simple form. If the number of competitors increases, the form of the single-type contract which coordinates the supply chain will be more complicated. In practice, suppliers usually provide different contracts to different retailers. It is reasonable that a supplier gives different price schemes to retailers with differentiated "bargaining power" which means abilities in capacity, reputation, finance, etc. But according to Robinson-Patman Act in which price discrimination is forbidden[1,2], it becomes more difficult to coordinate a decentralized supply chain with multiple competitors. Suppliers need a way to serve retailers better and avoid breaking the law at the same time.

In this paper, we have considered a two-stage supply-chain in which two different retailers compete to sell substitutable products provided by a common supplier. Substitution describes the situation where a price increase in a product increases sales of other related products, e.g. if the price of Coca Cola goes higher, the demand for Pepsi increases. There are more examples of substitutable products in reality, PCs from HP, Dell and Lenovo; Smart Phones from Apple, HTC and RIM; Automobiles from Volkswagen, GM and Toyota; etc.

To the best of our knowledge, our work presents the first attempt at studying the determinants of contract selection decisions of competing retailers. The contracts provided by the suppliers follow the Robinson-Patman act. Here, four scenarios are studied in order to coordinate the supply chain without breaking the Robinson-Patman Act. A comparison is made between usual quantity discount contract and our strategy which is partially better. The results are illustrated with numerical examples.

The reminder of the paper is arranged as follows. The related literature is reviewed in Section 2 and then the basic model is presented in Section 3. Section 4 analyzes the effects of this contract-selection scheme which could

* ***Corresponding author's*** e-mail: rongting_sun@live.cn.

coordinate the multi-retailer supply chain and compares our policy with quantity-discount contract. Section 5 presents the results with few numerical examples. Finally, in Section 6 we summarize the results and point out directions for future research.

2 Literature review

Due to the globalization of market, the idea of coordination or integration becomes very popular in the past decades. Supply chain contracts are generally considered as a useful tool to bring supply chain stakeholders in a decentralized setting to operate in coordination. Cachon[3] provided an excellent review about supply chain coordination with contracts. On the other hand, when the number of participants increases sharply in the market, competition among players becomes an important factor which could not be ignored. Considerable research has been devoted to analyzing multi-retailer problems with contract. Ingene and Parry [1] demonstrated that a two-part tariff contract could coordinate a supply chain of a manufacturer and multiple retailers. Cachon and Lariviere [4] concluded that a revenue-sharing contract can perfectly coordinate the supply chain with a supplier and multiple symmetric retailers competing in quantities. Cachon [5] studied competitive behavior in a supply chain with one supplier and N retailers facing stochastic demand. When players have divergent preferences toward consumer backorders, competition can degrade supply chain efficiency enormously, in particular when most of the backorder costs are allocated to the supplier. Bernstein and Federgruen [6] investigated the equilibrium behavior of decentralized supply chains with competing retailers under demand uncertainty, they designed a so-called linear "price discount sharing" scheme to allow the decentralized chain to perform as well as a centralized one. They also compared the situation with which the retailers do not competing. Bernstein and Federgruen [7] investigated a decentralized supply chain, with long-term competition between independent retailers facing random demands. They showed that the coordination mechanisms which lead perfect coordination when retailers compete in price and service. Juan Zhang[8] developed a new mechanism to coordinate the supply chain in which both the manufacturer and the retailer share each other's advertising costs. Xiao[9] examined production and outsourcing decisions for two manufacturers that produce partially substitutable products and play a strategic game with quantity competition. The model in this article is similar with that.

Quantity-discount is one of the most important contract which is widely used because of its simplicity. Jeuland and Shugan [10] showed that the quantity discount contract can coordinate the supply chain. Bernstein and Federgruen [11] showed that a discount scheme based on the sum of three discount components, annual sales volume, order quantity, and order frequency could lead to a perfect coordination of a multi-retailer supply chain when Cournot competition exists between the retailers. Xiao [12] showed that the supply chain with competing retailers is to be coordinated by either a linear quantity discount schedule or an all-unit quantity discount schedule when the demand is under disruption. Bernstein [13] showed that perfect coordination can be achieved with either retailer-specific constant unit wholesale prices or retailer-specific volume discount schemes in a two-echelon supply chain with a single supplier servicing a network of retailers who compete with each other by selecting sales quantities. They also discussed compliance issues with the coordinating schemes in view of the Robinson-Patman act and provide remedies to over come these issues. Under the similar consideration, KarabatI and SayIn [14] also investigated a discount scheme in both the presence and the absence of the non-price discrimination requirement. Lee[15] developed a quality-compensation contract which could fully coordinate the supply chain.

Although wholesale price contract could not coordinate a supply chain, it is still widely used in practice because it is very simple without extra effort or cost to learn [3]. Katok and Wu [16] studied about supply chain coordination with 3 contracts: wholesale price, buy-back, revenue-sharing. They showed that although the buy-back and revenue-sharing contracts improve supply chain efficiency relative to the wholesale price contract, the improvement is smaller than the theory predicts. They also found that although the buy-back and revenue-sharing contracts are mathematically equivalent, they do not generally result in equivalent supply chain performance. Lariviere and Porteus [17] studied a one to one supply chain with the wholesale price contract. Gilbert and Cvsa [18] studied the wholesale price contract with demand uncertainty and costly investment to reduce production costs. They demonstrated that a trade-off exists between the beneficial flexibility of allowing the wholesale price to be adjusted by the market demand and the need to provide incentives to reduce production costs. Bernstein and Federgruen [11] compared the optimal performance of the centralized supply chain with that of various decentralized supply chains operating under given types of wholesale pricing schemes. They have derived an efficiently computable lower and upper bounds, and they have shown that these bounds are tight as long as the gross profit margins of the retailers are not excessively low or the holding cost rate excessively large. El Ouardighi and Kim[19] compared the possible outcomes under a wholesale price contract and a revenue-sharing contract in a supply chain including one supplier and two manufacturers who invest in quality improvement for their products to compete for market demand.

The aim of this study is to build a simple contract-selection scheme and to show that it is partially better compared with the single quantity-discount contract.

3 Basic model

Without loss of generality, we consider a supply chain consisting of one supplier and 2 competing retailers. We assume that the supplier offers two substitutable goods to the two retailers who compete in order quantities, so called Cournot competition [11]. All of the participants are risk-neutral and the information is complete. Retailers compete in order quantity. We suppose that the supplier is more powerful than the retailers in our game, so the sequence they make decision should be: the supplier decides which contract would be offered; the two retailer decide whether accept the contract, if not, the game is end; the two retailers decide how many they would order.

As we know from the literature, several contracts could coordinate the one supplier and one retailer supply chain, but it's hard to do so with a regular contract in a supply chain including multiple retailers. In this research, we try to build a contract-selection scheme which could coordinate the multi-retailer supply chain.

Similar to Ingene and Parry [20] and Xiao [9], we consider the following linear inverse demand function:

$$p_i\left(q_i, q_j\right) = a_i - q_i - dq_j,\ \ 0 < d < 1,\ i, j = 1, 2,\ j \neq i. \qquad (1)$$

where p_i and q_i respectively represent retailer i's retail price and order quantity; a_i means the highest feasible price for retailer i. The retailer with larger a_i has a relative advantage of winning customers due to a better brand, position, reputation, quality, and so on [9]. The retail price of each retailer is an increasing function of his own order quantity, but a decreasing function of his rival's order quantity. d is the substitutability coefficient of the two products, $0 < d < 1$. If $d = 0$, the two products are totally differentiated from each other, and if $d = 1$, they are perfect substitutes.

According to the conclusion above, the total profit of the centralized supply chain is

$$\Pi\left(q_1, q_2\right) = \sum_{i=1, j\neq i}^{2} q_i\left(a_i - q_i - dq_j - c_i - c_0\right). \qquad (2)$$

The optimal order quantity for retailer i which could maximize $\Pi(q_1, q_2)$ are

$$q_i^* = \frac{a_i - da_j + dc_0 + dc_j - c_0 - c_i}{2(1-d^2)},\quad i, j = 1, 2,\ j \neq i. \qquad (3)$$

Similarly, the profit of retailer i is

$$\pi_i = (a_i - q_i - dq_j - c_i - w_i)q_i,\quad i, j = 1, 2,\ j \neq i, \qquad (4)$$

where w_i represents retailer i's unit cost charged by retailer, so called wholesale price; c_0 and c_i are the unit cost of supplier and retailer i, $c_0, c_i > 0$, which could be the cost of transportation, etc.

In this decentralized supply chain, retailers are willing to maximize their own profits, so the optimal order quantity for retailer i is

$$q_i^N = \frac{2a_i - da_j - 2w_i + dw_j - 2c_i + dc_j}{4-d^2},\quad i, j = 1, 2,\ j \neq i. \qquad (5)$$

Proposition 1.
If the supplier only increases wholesale price to one retailer, the profit of this retailer will decrease; to the contrary, his rival's profit increases.

Proof. Suppose the supplier increases retailer 1's wholesale price from w_1 to $w_1'(w_1' > w_1)$, then the profit of retailer 1 becomes

$$\pi_1^{w'} = (a_1 - q_1 - dq_2 - c_1 - w_i')q_1 \qquad (6)$$

and retailer 2's becomes $\pi_2^{w'}$ because the optimal order quantities changed correspondingly.

The optimal order quantity of retailer 1 when wholesale price is w_1 is

$$q_1^N = \frac{1}{2}(a_1 - dq_2^N - c_1 - w_1) \qquad (7)$$

so the optimal order quantity of retailer 1 q_1^N is a decreasing function of wholesale price w_1 and the optimal order quantity of retailer 2 q_2^N.

According to Equation (5), when w_1 increases, q_1^N decreases. So q_2^N increases too. Then $\pi_1^{w'} < \pi_1, \pi_2^{w'} > \pi_2$.

This result is obviously understandable because when the balance of the competition is broken, the one who got a increased wholesale price must lose his competitiveness, which lead to profit loss.

In order to make decentralized profit of the supply chain equal to the centralized situation, let $q_i^N = q_i^*$, then we have

$$w_i = a_i - c_i - 2q_1^* - dq_2^*,\quad i, j = 1, 2,\ j \neq i. \qquad (8)$$

The supplier is not allowed to give different retailers different wholesale price directly due to Robinson-Patman Act [1, 2].

We can find from Equation (3) and (8) that if $a_1 = a_2$ and $c_1 = c_2$, then we have $w_1 < w_2$ while... That indicates the supplier gives the lower wholesale price to the retailer who orders more. Apparently, we could use a quantity-discount contract which was used in the literature above to coordinate the supply chain.

Proposition 2.
If $a_1 - c_1 > a_2 - c_2$, then $q_1^* > q_2^*$.

Proof. According to Equation (3),

$$\begin{aligned} q_1^* - q_2^* &= \frac{a_1 - da_2 + dc_2 - c_1 - a_2 + da_1 - dc_1 + c_2}{2(1-d^2)} \\ &= \frac{(a_1 - a_2) - (c_1 - c_2)}{2(1-d)} \end{aligned} \qquad (9)$$

If $a_1 - a_2 > c_1 - c_2$, $q_1^* > q_2^*$.

From the analysis above we can conclude that a_i and c_i comprise the "power" of retailer i. The retailer who has a higher $a_i - c_i$ has a higher potential to order more.

4 Coordinate supply chain with contract selection

In this section, we design a contract-selection scheme which could coordinate the one supplier and two competing retailers supply chain and compare the results with the single type contract scheme. Let each retailer has 2 options (2 different types of contract) to choose, there will be 4 scenarios once their choices are determined. Obviously, the game leader (the supplier) wishes the retailers choosing the scenario which can maximize the profit of the supply chain, or to make their orders to be (q_1^*, q_2^*).

In this study, wholesale price contract (WP) and quantity discount contract (QD) are the two contracts that the supplier provides to form the combination because these two contracts are most used in practice and they are simple and easy to understand for retailers. Please see Figure 1.

		Retailer 1	
		QD	WP
Retailer 2	WP	(QD,WP)	(WP,WP)
	QD	(QD,QD)	(WP,QD)

FIGURE 1 The 4 scenarios when the retailers choose WP or QD respectively

So, our job is to find out the scenario from Figure 1 and make sure it could be accepted by both participants. And the contracts combination in this scenario should satisfy conditions below:

1. It could maximize the retailers' profits;
2. It is equal to (q_1^*, q_2^*);
3. No one will choose other's choice.

The first two conditions make sure this combination maximize the supply chain profit; the last one is used to fix retailers' choices on this combination because if they deviate this choice combination, the outcome will be out of supplier's control.

We suppose that the supplier will charge the retailer w per unit if he choose WP. And the one chooses QD will have a discount of θ ($\theta > 0$) per unit if his order is larger than the threshold T ($T > 0$). That means if one's order is large enough, QD is a better choice for him than WP.

Without loss of generality, we suppose that $q_1^* > q_2^*$, so that $a_1 - c_1 > a_2 - c_2$, which means retailer 1 has higher potential to order more. Then let's analyze the following 4 scenarios:

4.1 SCENARIO 1: RETAILER 1 CHOOSES QD AND RETAILER 2 CHOOSES WP - (QD, WP)

Let retailer 1 choose QD and retailer 2 choose WP, then the profit functions of two retailers are

$$\pi_1 = [a_1 - q_1 - dq_2 - w - \theta(T - q_1) - c_1]q_1, \tag{10}$$

$$\pi_2 = (a_2 - q_2 - dq_1 - w - c_2)q_2.. \tag{11}$$

Obviously, this choice combination is the supplier's favorite because different contracts are respectively chosen by different retailers and the one with higher order potential chooses QD .

Then solve the first-order condition, we get the optimal order quantities for both retailers to maximize their own profits:

$$q_1^n = \frac{2a_1 - da_2 + dw + dc_2 - 2w - 2\theta T - 2c_1}{4 - 4\theta - d^2}, \tag{12}$$

$$q_2^n = \frac{2a_2 - 2a_2\theta - da_1 + dw + d\theta T + dc_1 - 2w + 2w\theta - 2c_2 + 2c_2\theta}{4 - 4\theta - d^2}, \tag{13}$$

where $q_2^* < T < q_1^*$, which means the retailers only choose QD when their order quantities are larger than T. That are the retailers' best choice in this case if their rival doesn't change his/her mind.

Let $q_i^n = q_i^*$, which makes sure the solution could maximize the profit of the whole supply chain, then we get

$$w = a_2 - 2q_2^* - dq_1^* - c_2 = p_2^* - q_2^* - c_2, \tag{14}$$

$$\theta = \frac{dq_2^* - dq_1^* + a_2 - c_2 + 2q_1^* - 2q_2^* - a_1 + c_1}{2q_1^* - T} = \frac{B}{2q_1^* - T}, \tag{15}$$

where $B = (p_2^* - q_2^* - c_2) - (p_1^* - q_1^* - c_1)$. In this situation, $\theta > 0$ is satisfied.

We can derive from Equation (15) that

$$\theta T = 2\theta q_1^* - B \tag{16}$$

Let $\pi_i^* = \max(\pi_i)$, then $\pi_i^* = \pi_i(q_1^*, q_2^*) \quad i, j = 1, 2, j \neq i$.

Following the derivation above, the combination (QD,WP) satisfied condition 1 and 2. But it still has possibility that retailers would like to choose their rival's choice as described in the other 3 scenarios. The supplier must make sure the retailer are not willing to do like this so that the supply chain could be said to be coordinated.

4.2 SCENARIO 2: BOTH CHOOSE WP - (WP, WP)

If both retailers choose wholesale price contract, the situation is the same as using a single wholesale price contract. The profits of the two retailers are

$$\pi_i' = (a_i - q_i - dq_j - w - c_i)q_i, \quad i = 1, 2, j \neq i, \tag{17}$$

then the optimal order quantities for the two retailers under this scenario are

$$q_i' = \arg\max \pi_i' = \frac{2A_i - dA_j}{4 - d^2}, \quad i = 1, 2, j \neq i, \tag{18}$$

where $A_i = a_i - w - c_i > 0$.

Proposition 3.
If z satisfies

$$z = \frac{A_1}{q_1^*} - \frac{dq_2^*}{q_1^*} + \frac{B}{q_1^*} - \frac{\max(\pi_1')}{q_1^{*2}} - 1, \tag{19}$$

and if

$$0 < \theta < z, \tag{20}$$

then $\max(\pi_1') < \pi_1^*$.

Proof. If $\max(\pi_1') < \pi_1^*$,

$$[a_1 - q_1^* - dq_2^* - w - \theta(T - q_1^*) - c_1]q_1^* > \max(\pi_1'), \tag{21}$$

According to Equation (16),

$$.\theta < \frac{A_1}{q_1^*} - \frac{dq_2^*}{q_1^*} + \frac{B}{q_1^*} - \frac{\max(\pi_1')}{q_1^{*2}} - 1, \tag{22}$$

$$0 < \theta < \frac{A_1}{q_1^*} - \frac{dq_2^*}{q_1^*} + \frac{B}{q_1^*} - \frac{\max(\pi_1')}{q_1^{*2}} - 1 \tag{23}$$

Hence proved.

Proposition 3 demonstrates that if Equations (19) and (20) are satisfied, scenario 1 is better for retailer 1 than scenario 2, so retailer 1 will not choose WP when retailer 2 choose it. But we don't know whether $\max(\pi_2')$ is smaller than π_2^*.

4.3 SCENARIO 3: BOTH CHOOSE QD - (QD, QD)

If both retailers choose quantity discount contract, the situation is the same as using a single quantity discount contract. The profits of the two retailers are

$$\pi_i'' = [a_i - q_i - dq_j - w - \theta(T - q_i) - c_i]q_i, \quad i = 1, 2, j \neq i, \tag{24}$$

The optimal order quantities for the two retailers under this scenario are

$$\begin{aligned} q_i'' &= \arg\max \pi_i'' \\ &= \frac{dA_j + dB - 2d\theta q^*{}_1 - 2A_i - 2B + 4\theta q^*{}_1 + 2\theta A_i + 2B\theta - 4\theta^2 q^*{}_1}{4 - d^2 - 8\theta + 4\theta^2}, \\ & i = 1, 2, j \neq i. \end{aligned} \tag{25}$$

Proposition 4.

$$\text{If } C_5\theta^5 + C_4\theta^4 + C_3\theta^3 + C_2\theta^2 + C_1\theta + C_0 < 0, \tag{26}$$

then we have $\max(\pi_2'') < \pi_2^*$ where

$$C_5 = -16q_1^{*2} \tag{27}$$

$$C_4 = (48 - 16d)q_1^{*2} + 16(A_2 + B)q_1^* - 16\pi_2^*, \tag{28a}$$

$$C_3 = (-48 + 32d - 4d^2)q_1^{*2} + (-48B - 48A_2 + 8A_2d + 16Bd + 8A_1d)q_1^* + 64\pi_2^* - 4B^2 - 8BA_2 - 4A_2^2, \tag{28b}$$

$$\begin{aligned} C_2 &= (4d^2 - 16d + 16)q_1^{*2} + (4d^2B + 48B - 16A_1d + 4d^2A_1 - 16A_2dn + 48A_2 - 32Bd)q_1^* + \left(8d^2 - 96\right)\pi_2^* - 4BA_1d \\ &+ 24BA_2 - 4A_2dB - 4A_2A_1d - 4dB^2 + 12B^2 + 12A_2^2 \end{aligned}, \tag{28c}$$

$$\begin{aligned} C_1 &= \left(8A_1d - 4d^2A_1 - 16B + 16Bd - 4d^2B - 16A_2 + 8A_2d\right)q_1^* + (64n - 16d^2)\pi_2^* - 12A_2^2 - 2d^2BA_1 + 8BA_1d \\ &+ 8A_2dB + 8dB^2 - A_1^2d^2 - d^2B^2 + 8A_2A_1d - 24BA_2 - 12B^2, \end{aligned}, \tag{28d}$$

$$C_0 = \left(-d^4 + 8d^2 - 16\right)\pi_2^* - 4A_2dB + 4B^2 - 4dB^2 + 2d^2BA_1 - 4BA_1d + d^2B^2 + 8BA_2 + 4A_2^2 - 4A_2A_1d + A_1^2d^2. \tag{28e}$$

Proof. If $\max(\pi_2'') < \pi_2^*$,

$$\left[A_2 - q_2'' - dq_1'' - \theta\left(T - q_2''\right)\right]q_2'' < \pi_2^*, \tag{28}$$

$$\left[A_2 - q_2'' - dq_1'' - 2\theta q_1^* + \theta q_2''\right]q_2'' < \pi_2^*, \tag{29}$$

multiplied by $\left(4 - d^2 - 8\theta + 4\theta^2\right)^2$,

$$C_5\theta^5 + C_4\theta^4 + C_3\theta^3 + C_2\theta^2 + C_1\theta^1 + C_0 < 0. \tag{30}$$

Proposition 4 demonstrates that if Equation (27) is satisfied, scenario 1 is better for retailer 2 than scenario 3, so retailer 2 will not choose QD when retailer 1 choose it.

But whether $\max(\pi_1'')$ is smaller than π_1^* is still not known.

4.4 SCENARIO 4: RETAILER 1 CHOOSES WP AND RETAILER 2 CHOOSES QD - (WP, QD)

Let retailer 1 choose WP and retailer 2 choose QD, then the profit functions of two retailers are

$$\pi_1''' = (a_1 - q_1 - dq_2 - w - c_1)q_1. \tag{31}$$

$$\pi_2''' = [a_2 - q_2 - dq_1 - w - \theta(T - q_2) - c_2]q_2, \tag{32}$$

Solve the first-order condition, we get the optimal order quantities to maximize retailers' profits:

$$q_1''' = \frac{2a_1 - 2a_1\theta - da_2 + dw + d\theta T + dc_2 - 2w + 2w\theta - 2c_1 + 2c_1\theta}{4 - 4\theta - d^2}, \tag{33}$$

$$q_2''' = \frac{2a_2 - da_1 + dw + dc_2 - 2w - 2\theta T - 2c_2}{4 - 4\theta - d^2}, \tag{34}$$

In fact, there is no way retailer 1 will choose WP when retailer 2 choose QD.

Proposition 5.
For scenario 4, there exists $q_1''' > q_1^*$ or $q_2''' < q_2^*$.
Proof. According to Equations (12) and (34), we have

$$q_1''' - q_1^n = \frac{-2a_1\theta + dT\theta + 2w\theta + 2c_1\theta + 2T\theta}{4 - 4\theta - d^2} \tag{35}$$

because Equation(16)

$$q_1''' - q_1^n = \frac{[(2 + d + 2\theta) + \dfrac{4\theta - 4 + d^2}{2(1-d)}](a_1 - a_2 - c_1 + c_2)}{4 - 4\theta - d^2} \tag{36}$$

Similarly we have

$$q_2^n - q_2''' = \frac{[(2 + d + 2\theta) + \dfrac{4\theta - 4 + d^2}{2(1-d)}](a_2 - a_1 - c_2 + c_1)}{4 - 4\theta - d^2} \tag{37}$$

so we have $q_1''' - q_1^n = q_2''' - q_2^n$, which means if $q_1''' > q_1^n$, $q_2''' > q_2^n$; else if $q_1''' < q_1^n$, $q_2''' < q_2^n$.

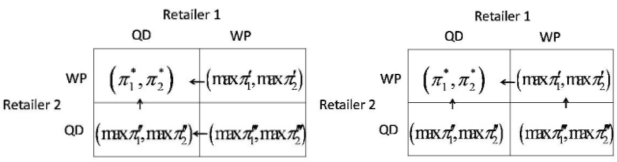
$q_i^n = q_i^*$, so we have $q_1''' > q_1^*$ or $q_2''' < q_2^*$.

Corollary 1. If $q_1''' > q_1^*$, then. $\max(\pi_1''') < \max(\pi_1'')$.; if $q_2''' < q_2^*$, then $\max(\pi_2''') < \max(\pi_2')$.

$q_1''' > q_1^*$ means that retailer 1 has potential to choose lower unit cost (QD). According to Proposition 1, if retailer choose QD, there exists $\pi_1''(q_1'') > \pi_1''(q_1''') > \pi_1''(q_1''')$, so retailer 1 will turn to choose QD. On the other hand, if $q_2''' < q_2^*$, retailer 2 will turn to choose WP. That means scenario 4 could not happen.

Proposition 6.
Retailer 1 will choose quantity-discount contract while retailer 2 will choose wholesale price contract if Equations (20) and (27) are satisfied simultaneously.

Proof. (π_1^*, π_2^*) is the unique Nash Equilibrium for the two retailers (Please see Figure 2).



(a) $q_1''' < q_1^*$ (b) $q_2''' > q_2^*$

FIGURE 2 Decision processes of the retailers

Although (π_1^*, π_2^*) is the unique Nash Equilibrium, it is not Pareto optimal because we only have:

(1). $\pi_1^* > \max(\pi_1')$.;

(2) $\pi_2^* > \max(\pi_2'')$;

(3) $\max(\pi_1''') < \max(\pi_1'')$ or $\max(\pi_2''') < \max(\pi_2')$. There doesn't exist a Pareto optimal solution in this game.

4.5 EFFECT OF θ AND T

θ and T could be varied in their domains when both retailers reach the equilibrium. This section tests the impact of the variation of both parameters on the total supply chain profit.

From Equation (11), retailer 2's maximum profit will not change if θ or T varies. According to Equation (10), we have

$$\pi_1^* = (a_1 - q_1^* - dq_2^* - w - \theta q_1^* + B - c_1)q_1^* \tag{38}$$

We can find from Equation (39) that when θ increases, retailer 1's maximum profit will decrease. And because there is an 1 to 1 map between θ and T, when T decreases retailer 1 will get a better result.

As a result, supplier' s profit will increase when θ or T decreases because the whole supply chain' s optimal profit will not change according to Equation (2).

This result shows that the supplier could adjust the parameter of the scheme to allocate profits to retailers.

4.6 CONTRACT SELECTION VS. SINGLE QUANTITY DISCOUNT CONTRACT

In order to study whether retailers are willing to accept the new policy, we compare the new contracts selection policy with the traditional quantity-discount contract which could also coordinates the two competing retailers.

The single quantity-discount contract could be represented by $(\bar{w}, \bar{\theta})$, where $\bar{w}$ represents the wholesale price and $\bar{\theta}$ represents the all-unit discount. So the profit of the two retailers are

$$\bar{\pi}_i = [a_i - q_i - dq_j - (\bar{w} - \bar{\theta} q_i) - c_i]q_i. \tag{39}$$

Because the total profit of the whole supply chain doesn't change, so the optimal solution is still (q_1^*, q_2^*). We can derive that

$$\bar{w} = \frac{-c_2 q_1^* + a_2 q_1^* + dq_2^{*2} - dq_1^{*2} - a_1 q_2^* + c_1 q_2^*}{q_1^* - q_2^*}, \tag{40}$$

$$\bar{\theta} = \frac{1}{2} \frac{dq_2^* + a_2 - 2q_2^* - c_2 - a_1 + 2q_1^* - dq_1^* + c_1}{q_1^* - q_2^*}. \tag{41}$$

Proposition 7.
Retailer 2's profit increases after applying contract-selection scheme comparing with single quantity discount contract.
Proof.

$$\bar{w} - w = \frac{q_2^*[(p_2^* - q_2^* - c_2) - (p_1^* - q_1^* - c_1)]}{q_1^* - q_2^*} > 0 \tag{42}$$

$$\bar{w} - \bar{\theta} q_2^* - w = \frac{1}{2} \frac{q_2^*[(p_2^* - q_2^* - c_2) - (p_1^* - q_1^* - c_1)]}{q_1^* - q_2^*} > 0 \tag{43}$$

We can find that $\pi_2^* > \max(\bar{\pi}_2)$.

This result implies that at least retailer 2's profit will increase after adopting the new price policy, which would increase their interests to accept it.

5 Numerical Study

In this section, we test how the parameters affect the output of the supply chain members with the simulation method.

The following data is taken for the numerical illustration. The parameters we used to test is the same as that of \cite{xiao2007} (Please see Table 1):

TABLE 1 Parameters we used in this article

Supplier	Retailer 1	Retailer 2	Other
$c_0 = 2$	$a_1 = 10, c_1 = 3$	$a_2 = 8, c_2 = 2$	$d = 0.5$

In Table 1, $a_1 - c_1 = 7 > 6 = a_2 - c_2$ means retailer 1 has higher potential to earn more.

5.1 NUMERICAL EXAMPLE

According to Equation (3), we have $q_1^* = 2$, $q_2^* = 1$, because $q_2^* < T < q_1^*$, we test the value of T from 1 to 2. The result is shown in Table 2.

TABLE 2 Parameter T varies from 1 to 2

No.	T	θ	Z	$Y*$
1	1.0	0.1667	0.2489	0.1696
2	1.1	0.1724	0.2489	0.0443
3	1.2	0.1786	0.2489	-0.0815
4	1.3	0.1852	0.2489	-0.2072
5	1.4	0.1923	0.2489	-0.3321
6	1.5	0.2000	0.2489	-0.4554
7	1.6	0.2083	0.2489	-0.5759
8	1.7	0.2174	0.2489	-0.6924
9	1.8	0.2273	0.2489	-0.8033
10	1.9	0.2381	0.2489	-0.9066
11	2.0	0.2500	0.2489	-1

From Figure 3 and Table 3, we could find that π_1^* and $\max(\pi_1'')$ are larger than the other 2 scenarios, which means retailer 1 will choose QD no matter which contract retailer 2 chooses. And π_2^* is always bigger than $max(\pi_2'')$, so retailer 2 will choose WP in order to get a better result. This observation is the same as what we derived in earlier sections. We also could observe that retailer 1's optimal profit decreases when T and increase while retailer 2's is unchanged.

Also from Figure 3, we observed that those lines are very close even cross near the points $T = 1$ and $T = 2$. Since the decision makers (retailer1 and retailer 2) have more than 1 choice in those areas which will make obstacles to their decision. So the supply chain will not be coordinated in those areas.

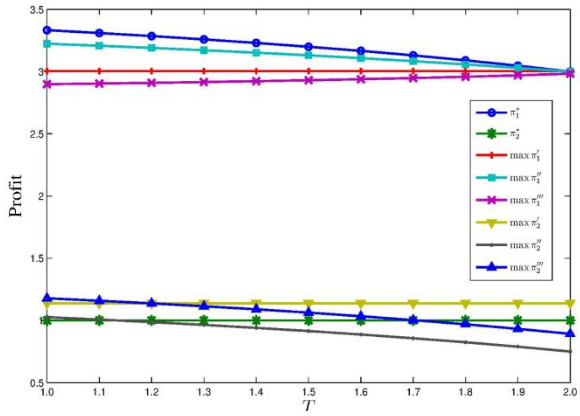


FIGURE 3 Profits of the 4 scenarios when T varies

TABLE 3 Profits of the retailers under 4 scenarios when T varies ($d = 0.5$)

	(QD,WP)	(WP,WP)	(QD,QD)	(WP,QD)
No.	π_1^*, π_2^*	$\max(\pi_1'), \max(\pi_2')$	$\max(\pi_1''), \max(\pi_2'')$	$\max(\pi_1'''), \max(\pi_2''')$
1	3.3333, 1	3.0044, 1.1378	3.2243, 1.0265	2.8992, 1.1785
2	3.3103, 1	3.0044, 1.1378	3.2080, 1.0071	2.9044, 1.1584
3	3.2857, 1	3.0044, 1.1378	3.1906, 0.9864	2.9100, 1.1369
4	3.2593, 1	3.0044, 1.1378	3.1720, 0.9642	2.9162, 1.1139
5	3.2308, 1	3.0044, 1.1378	3.1523, 0.9403	2.9230, 1.0892
6	3.2000, 1	3.0044, 1.1378	3.1311, 0.9174	2.9305, 1.0627
7	3.1667, 1	3.0044, 1.1378	3.1085, 0.8869	2.9388, 1.0340
8	3.1304, 1	3.0044, 1.1378	3.0842, 0.8569	2.9480, 1.0030
9	3.0909, 1	3.0044, 1.1378	3.0581, 0.8243	2.9584, 0.9693
10	3.0476, 1	3.0044, 1.1378	3.0301, 0.7888	2.9701, 0.9326
11	3,1	3.0044, 1.1378	3, 0.7500	2.9835, 0.8926

5.2 COMPARING WITH SINGLE QD

We calculated the optimal profit of the retailers when single QD contract is adopted (see Table 4). The data of the parameters is the same as Table 1.

TABLE 4 Profit of single QD ($d = 0.5$)

retailer	profit	Optimal order	$\overline{w}$	θ
1	3	2	3.5	0.25
2	0.75	1		

Then we will test how parameter d affect the outputs. Let d varies from 0 to 1, the results are in Table 5. T_l, T_h respectively represent the lower and higher value of T which could be nearly seen as the lower bound and upper bound of T.

TABLE 5 Comparing with single QD

d	q_1^*, q_2^*	T_l, T_h	$\pi_{1L}^*, \pi_{1H}^*, \pi_2^*$	$\max(\bar{\pi}_1), \max(\bar{\pi}_2)$
0.1	2.3232, 1.7677	1.82, 2.67	5.2912, 5.2714, 3.1247	4.3066, 2.1973
0.2	2.1875, 1.5625	1.63, 2.13	4.5676, 4.5193, 2.4414	3.7055, 1.6038
0.3	2.0879, 1.3736	1.45, 2.02	4.0173, 3.9268, 1.8868	3.2766, 1.1388
0.4	2.0238, 1.1905	1.36, 1.94	3.5884, 3.4479, 1.4172	3.0000, 0.7500
0.5	2.0000, 1.0000	1.20, 1.90	3.2857, 3.0476, 1.0000	2.8882, 0.4272
0.6	2.0313, 0.7813	1.03, 1.91	3.1051, 2.6909, 0.6104	3.0238, 0.1562
0.7	2.1569, 0.4902	0.66, 1.99	3.1679, 2.3162, 0.2403	3.7500, 0.0000
0.8	2.5000, 0.0000	N/A *	N/A	7.4654, 0.9522

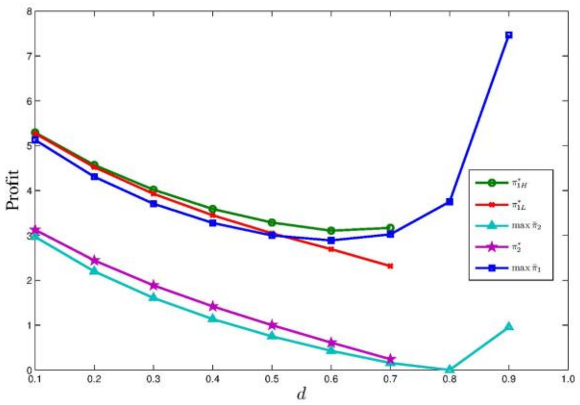


FIGURE 4 Profits comparison with single QD

From Table 5 and Figure 4 we could find that: for the products with relative low substitutability, retailers will be better off under the contract-selection scheme; for the products with relative high substitutability, at least the weaker retailer will be better off.

6 Conclusion

This paper extends the existing related work about the issues of quantity competition and coordination in a two-stage distribution channel where two different retailers compete to sell two substitute products supplied by a common retailer. We first establish the centralized decision model and provide the optimal solution to the model as well. A contract selection scheme, which includes quantity discount (QD) and wholesale price (WP) contract, has been developed to make the supply chain coordinated. Four scenarios are analyzed to make sure the QD+WP contract combination is the best choice of the four for the two competing retailers which could make the profit of the decentralized supply chain equal to the centralized situation. Some important insights are drawn by investtigating the impact of the parameters. The retailers' profits under our scheme are compared with that of the single quantity discount contract are also compared. Through the numerical experiment, one can observe that if substitutability was low, the new scheme is better for both retailers. It is better off for the weaker retailer at least. These results significant expand the application scope of contract types to coordinate the supply chain.

There are many open research issues that remain to be examined within the framework of supply chain coordination with competing retailers. First, while our model focuses on a single supplier and two retailers, exactly the similar approach can be used to analyze the multiple retailers or multiple suppliers situations. Second, more contract types could be analyzed while we only investigate the effects of QD and WP. Third, uncertain demand could be studied to instead the certain demand we used in this article.

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Authors

Rongting Sun, 06. 10. 1983, China

Rongting Sun received the Ph.D. degree in Management science and Engineering from Xi'an Jiaotong University, China in 2011. Currently, he is a researcher at Chang'An University, China. His research interests include supply chain management and logistics management.

Yiqun Guo, 20. 01. 1981, China

Yiqun Guo received her Master degree in Economics from Xi'an Jiaotong University, China in 2008. Currently, she is an associate professor at Xi'an Physic Education University, China. She is working for the Ph.D degree in Xi'an Jiaotong University. Her research interests include industrial economics and supply chain management.