Optimistic and pessimistic decision making based on interval-valued intuitionistic fuzzy soft sets

Xue Wen, Zhi Xiao[1]*

[1] *School of Economics and Business Administration, Chongqing University, Chongqing, China, 400044*

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Abstract

This study explores the application of using the interval-valued intuitionistic fuzzy soft sets (IVIFSS) approach to the multi-criteria group decision making problem. Firstly, we define the concept of the IVIFSS value vector, including the IVIFSS weighted averaging, the optimistic IVIFSS value and the pessimism IVIFSS value. Then, some of their desirable properties are investigated in detail. Furthermore, we prove that the decision makers, as the parameter sets, and their IVIFSS value vector, as mapping rule, may be consider as a new IVIFSS for all decision makers and the vector of this IVIFSS could be considered as the valuation of the alternatives that all the decision makers decide, whose score functions could be rank the alternatives. Finally, an approach to multi-criteria group decision making based on IVIFSS is given and an illustrative example is employed to show the validity of this approach.

Keywords: Fuzzy Soft Sets; Interval-valued Intuitionistic Fuzzy Value; Interval-valued Intuitionistic fuzzy Soft Sets; Multi-criteria Group Decision Making

1 Introduction

To combat the uncertainties and complexities of socioeconomic environments, decision makers converse a manageable model of reality inherently involves a great number of assumptions, which rely on the judgments of the decision makers, including: Laplace, based on the principle of insufficient reason and finding the alternative with the highest average outcome; pessimistic, making the best out of worst possible conditions; Optimistic, the best out of the best alternatives; Savage, the best of the regrets for not taking the right actions; and Hurwicz, giving a range of attitudes from optimistic to most pessimistic ([1, 2]).

On the other hand, in order to deal with the uncertain problems, the theory of probability, the theory of fuzzy sets[3], the theory of intuitionistic fuzzy sets (IFS) and interval-valued intuitionistic fuzzy sets (IVIFS) [4-8], the theory of rough sets[9],interval mathematics and the soft sets [10-18] have been developed and applied in decision making. Especially, the soft set theory, IFS, IVIFS and their combination have received more and more attention since their appearance such as intuitionistic fuzzy soft sets(IFSS) [19-21] and interval-valued intuitionistic fuzzy soft sets (IVIFSS) [8]. Some researches have analyze the methods to relate the judgments of the decision makers in IFS[22, 23], IVFS[24], IFSS decision environment. There is little research relate the judgments of the decision makers in IVIFSS.

In this paper, we incorporate optimistic and pessimistic estimations into IVIFSS for the multi-criteria group decision making (MCGDM) problem.

The remainder of the paper is organized as follows. Section 2 briefly reviews some preliminaries. The concept of the IVIFSS value vector and the properties of it are given in Section 3. Section 4 develops an approach of IVIFSS to MCGDM and gives some illustrative examples. Finally, conclusions are drawn in section 5.

2 Preliminary

In this section, we recall some basic notions related to soft sets, fuzzy soft sets, IFS, IVIFS, IFSS, IVIFSS.

Definition2.1 [25] A pair (F,E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U . For any parameters $\varepsilon \in E, F(\varepsilon) \subseteq U$ may be considered as the set of ε -approximate elements of the soft set (F,E) .

In other words, the soft set (F,E) is a parameterized family of subsets of the set U , where E is the parameterized family and F is the mapping rule. For the decision problem, E is the attributes and F is the decision rules, then (F,E) is the results of the decision problem.

Definition2.2 [11] Let $\mathcal{F}(\mathrm{U})$ denotes the set of all fuzzy sets of U . Let E be a set of parameters and $\mathrm{A} \subseteq \mathrm{E}$. A pair $(\mathcal{F}, \mathrm{A})$ is called fuzzy soft sets over U , where $\mathcal{F}$ is a mapping given by $\mathrm{F}: \mathrm{A} \to \mathcal{F}(\mathrm{U})$.

Definition 2.3 [4] Given a universe of discourse $\mathbf{X} \neq \varnothing$, an IFS A in X is defined as the set of ordered triples $\mathrm{A} = \{< x, \mu_{\mathrm{A}}(\mathrm{x}), \nu_{\mathrm{A}}(\mathrm{x}) > | \mathrm{x} \in \mathrm{X}\}$, where $\mu_{\mathrm{A}}(\mathrm{x}): \mathrm{X} \to [0,1]$ is the degree of membership function of A and $\nu_{\mathrm{A}}(\mathrm{x}): \mathrm{X} \to [0,1]$ is the degree of nonmembership function of A , $0 \le \mu_{\mathrm{A}}(\mathrm{x}) + \nu_{\mathrm{A}}(\mathrm{x}) \le 1$.

Definition 2.4 [19] Consider U and E as an initial universe and a set of parameters respectively, $\iota\mathcal{F}(\mathrm{U})$ denotes the set of all intuitionistic fuzzy sets of U . Let

* ***Corresponding author's*** e-mail: xiaozhicqu@163.com

$A \subseteq E$. A pair $(\iota\mathcal{F}, A)$ is an intuitionistic fuzzy soft set, where $\iota\mathcal{F}$ is a mapping given by $F: A \to \iota\mathcal{F}(U)$. For any parameter $\varepsilon \in A$, $\iota F(\varepsilon)$ is an intuitionistic fuzzy subsets of U and can be written as an intuitionistic fuzzy set $\iota\mathcal{F}(\varepsilon) = \{< x, \mu_{\iota\mathcal{F}(\varepsilon)}(x), \nu_{\iota\mathcal{F}(\varepsilon)}(x) > | x \in U\}$, where $\mu_{\iota\mathcal{F}(\varepsilon)}(x) \in [0,1], \nu_{\iota\mathcal{F}(\varepsilon)} \in [0,1]$ and $0 \le \mu_{\iota\mathcal{F}(\varepsilon)}(x) + \nu_{\iota\mathcal{F}(\varepsilon)}(x) \le 1$.

Definition 2.5 [26] An interval-valued intuitionistic fuzzy sets X on a universe X is a mapping given by: $X: U \to \mathrm{Int}[0,1]$,where $\mathrm{Int}[0,1]$ is the set of all closed subintervals of $[0,1]$ and the inter-valued intuitionistic fuzzy sets can denoted by $\iota\iota\mathcal{F}(U)$. For any parameters $x \in U, \iota\iota\mathcal{F}(U)$ can be defined as $\{< x, [u_X^-(x), u_X^+(x)], [\nu_X^-(x), \nu_X^+(x)] > | x \in X\}$, where $[u_X^-(x), u_X^+(x)] \subseteq [0,1], [\nu_X^-(x), \nu_X^+(x)] \subseteq [0,1]$ and $u_X^+(x) + \nu_X^+(x) \le 1$ .The $u_X^-(x)$ and $u_X^+(x)$ are referred as the degree of membership function of the lower and upper degrees of the membership of x to X . The $\nu_X^-(x)$ and $\nu_X^+(x)$ are referred as the degree of nonmembership function of the lower and upper degrees of the membership of x to X .

Usually we call $([u_X^-(x), u_X^+(x)], [\nu_X^-(x), \nu_X^+(x)])$ is an interval-valued intuitionistic fuzzy number (IVIFN) and the vector component of an IVIFN is interval-valued intuitionistic fuzzy vector (IVIFV).

For comparing IVIFN, Chen and Tan et al.[27] proposed the so-called score function and the so-called accuracy function, where the so-called score function is:

$$S(\alpha) = (1/2)(u_X^-(x) - \nu_X^-(x) + u_X^+(x) - v_X^+(x)) \tag{1}$$

And the so-called accuracy function is:

$$h(\alpha) = \frac{1}{2}(u_X^-(x) + \nu_X^-(x) + u_X^+(x) + \nu_X^+(x)) \tag{2}$$

Functions $S(\alpha)$ and $h(\alpha)$ are similar to the function mean a variance in statistics. Yu et al.in [7] and Ludmila et al. in[28] had extended the functions:

$$S(\alpha) = (1/2)(2 + u_X^-(x) - \nu_X^-(x) + u_X^+(x) - v_X^+(x)) \tag{3}$$

Definition 2.6 [29]Let U and E as an initial universe and a set of parameters respectively, $\iota\iota\mathcal{F}(U)$ denote as the set of all interval-valued intuitionistic fuzzy sets. Let $A \subseteq E$. The pair soft set $(\iota\iota\mathcal{F}(U), A)$ is an interval-valued intuitionistic fuzzy set, where $\iota\iota\mathcal{F}$ is a mapping given by $F: A \to \iota\iota\mathcal{F}(U)$.

An IVIFSS is a special case of soft set because $F(\varepsilon)$ is referred as the interval-valued intuitionistic fuzzy set for any parameters $\varepsilon \in A$. some operations and properties of the interval-valued intuitionistic has been discussed in[29], such as complement, ``and'', ``or'', union, intersection, necessity and possibility operations.

Definition 2.7 [29]Suppose that $(\iota\iota\mathcal{F}(U), A)$ is an IVIFSS over U , $\iota\iota\mathcal{F}(\varepsilon)$ is the interval intuitionistic fuzzy set of parameter ε , then all interval intuitionistic fuzzy set for the soft set $(\iota\iota\mathcal{F}(U), A)$ are referred to as the interval-valued intuitionistic fuzzy value class and is denoted by $C(\iota\iota\mathcal{F}, A)$, where $C(\iota\iota\mathcal{F}, A) = \{\iota\iota\mathcal{F}(\varepsilon) : \varepsilon \in A\}$.

Definition 2.8 [27]Let $\alpha_1 = ([a_1, b_1], [c_1, d_1])$ and $\alpha_2 = ([a_2, b_2], [c_2, d_2])$ be any two IVIFNs, then some operations of α_1 and α_2 can be defined as:

$$\alpha_1 \oplus \alpha_2 = ([a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2], [c_1 c_2, d_1 d_2]) \tag{3}$$

$$\alpha_1 \odot \alpha_2 = ([a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2]) \tag{4}$$

$$\lambda\alpha_1 = ([1-(1-a_1)^\lambda, 1-(1-b_1)^\lambda], [c_1^\lambda, d_1^\lambda]), \lambda > 0 \tag{5}$$

$$\alpha_1^\lambda = ([a_1^\lambda, b_1^\lambda], ([1-(1-c_1)^\lambda, 1-(1-d_1)^\lambda]), \lambda > 0 \tag{6}$$

Definition 2.9[28] Let t $\alpha_1 = ([a_1, b_1], [c_1, d_1])$ and $\alpha_2 = ([a_2, b_2], [c_2, d_2])$ be any two IVIFNs and a is the confidence level, then the interval possibilities $P(\alpha_1 > \alpha_2)$)] as follow:

$$P(\alpha_1 > \alpha_2) = \alpha \frac{[a_1 - d_1, b_1 - c_1] - [a_2 - d_2, b_2 - c_2] + 2}{4} + (1-\alpha)\frac{[a_1 + c_1, b_1 + d_1] - [a_2 + c_2, b_2 + d_2] + 2}{4} \tag{7}$$

3 On interval-valued intuitionistic fuzzy soft sets value

In this section, we shall give the concept of the interval-valued intuitionistic fuzzy soft sets value vector and its properties.

First, let us introduce an example based on the classic in soft sets theory.

Example 1.

Consider there are interval-valued intuitionistic fuzzy soft sets (F, A), (G, B), (H, C) where U is a set of five houses under the consideration of a family to purchase, which is denoted by $U = \{h_1, h_2, h_3, h_4, h_5\}$, A, B and C are parameter sets, where $A = B = C = \{e_1, e_2, e_3, e_4\}$ and $\{e_1\} = \{\text{competitive price}\}$, $\{e_2\} = \{\text{beautiful}\}$, $\{e_3\} = \{\text{in good repair}\}$, $\{e_4\} = \{\text{wooden}\}$

The family consists of the father, mother and daughter. The interval-valued in tuitionistic fuzzy soft sets (F, A), (G, B), (H, C) describes the "attractiveness of the houses" to the father, mother and the daughter. Suppose that

$$(F, A) = \{(F(e_1), e_1), (F(e_2), e_2), (F(e_3), e_3), (F(e_4), e_4)\}$$

$$(G, B) = \{(G(e_1), e_1), (G(e_2), e_2), (G(e_3), e_3), (G(e_4), e_4)\}$$

$$(H, C) = \{(H(e_1), e_1), (H(e_2), e_2), (H(e_3), e_3), (H(e_4), e_4)\}$$

$$F(e_1)=\left\{\begin{array}{l}\langle h_1,[0.6,0.8],[0.1,0.2]\rangle,\langle h_2,[0.4,0.7],[0.0,0.1]\rangle,\\ \langle h_3,[0.3,0.7],[0.2,0.3]\rangle,\langle h_4,[0.7,0.8],[0.1,0.2]\rangle,\\ \langle h_5,[0.5,0.6],[0.3,0.4]\rangle\end{array}\right\}$$

$$F(e_2)=\left\{\begin{array}{l}\langle h_1,[0.2,0.4],[0.4,0.5]\rangle,\langle h_2,[0.5,0.7],[0.1,0.2]\rangle,\\ \langle h_3,[0.2,0.4],[0.4,0.5]\rangle,\langle h_4,[0.2,0.3],[0.4,0.6]\rangle,\\ \langle h_5,[0.7,0.8],[0.0,0.1]\rangle\end{array}\right\}$$

$$F(e_2)=\left\{\begin{array}{l}\langle h_1,[0.2,0.4],[0.4,0.5]\rangle,\langle h_2,[0.5,0.7],[0.1,0.2]\rangle,\\ \langle h_3,[0.2,0.4],[0.4,0.5]\rangle,\langle h_4,[0.2,0.3],[0.4,0.6]\rangle,\\ \langle h_5,[0.7,0.8],[0.0,0.1]\rangle\end{array}\right\}$$

$$F(e_3)=\left\{\begin{array}{l}\langle h_1,[0.6,0.7],[0.2,0.3]\rangle,\langle h_2,[0.7,0.8],[0.1,0.2]\rangle,\\ \langle h_3,[0.1,0.4],[0.4,0.5]\rangle,\langle h_4,[0.6,0.8],[0.0,0.2]\rangle,\\ \langle h_5,[0.2,0.4],[0.4,0.5]\rangle\end{array}\right\}$$

$$F(e_4)=\left\{\begin{array}{l}\langle h_1,[0.4,0.5],[0.2,0.4]\rangle,\langle h_2,[0.7,0.8],[0.1,0.2]\rangle,\\ \langle h_3,[0.3,0.4],[0.4,0.6]\rangle,\langle h_4,[0.6,0.8],[0.0,0.2]\rangle,\\ \langle h_5,[0.1,0.3],[0.4,0.6]\rangle\end{array}\right\}$$

$$G(e_1)=\left\{\begin{array}{l}\langle h_1,[0.2,0.4],[0.4,0.5]\rangle,\langle h_2,[0.6,0.8],[0.0,0.2]\rangle,\\ \langle h_3,[0.1,0.4],[0.4,0.5]\rangle,\langle h_4,[0.6,0.8],[0.0,0.2]\rangle,\\ \langle h_5,[0.2,0.4],[0.5,0.6]\rangle\end{array}\right\}$$

$$G(e_2)=\left\{\begin{array}{l}\langle h_1,[0.6,0.7],[0.1,0.2]\rangle,\langle h_2,[0.2,0.3],[0.4,0.6]\rangle,\\ \langle h_3,[0.8,0.9],[0.0,0.1]\rangle,\langle h_4,[0.3,0.8],[0.0,0.1]\rangle,\\ \langle h_5,[0.6,0.7],[0.2,0.3]\rangle\end{array}\right\}$$

$$G(e_3)=\left\{\begin{array}{l}\langle h_1,[0.5,0.7],[0.1,0.2]\rangle,\langle h_2,[0.2,0.4],[0.4,0.5]\rangle,\\ \langle h_3,[0.1,0.4],[0.2,0.5]\rangle,\langle h_4,[0.2,0.3],[0.4,0.6]\rangle,\\ \langle h_5,[0.6,0.8],[0.0,0.2]\rangle\end{array}\right\}$$

$$G(e_4)=\left\{\begin{array}{l}\langle h_1,[0.2,0.4],[0.4,0.5]\rangle,\langle h_2,[0.2,0.3],[0.4,0.6]\rangle,\\ \langle h_3,[0.7,0.9],[0.0,0.1]\rangle,\langle h_4,[0.3,0.8],[0.1,0.2]\rangle,\\ \langle h_5,[0.7,0.8],[0.0,0.2]\rangle\end{array}\right\}$$

$$H(e_1)=\left\{\begin{array}{l}\langle h_1,[0.2,0.4],[0.4,0.5]\rangle,\langle h_2,[0.2,0.3],[0.4,0.6]\rangle,\\ \langle h_3,[0.7,0.9],[0.0,0.1]\rangle,\langle h_4,[0.3,0.8],[0.1,0.2]\rangle,\\ \langle h_5,[0.7,0.8],[0.0,0.2]\rangle\end{array}\right\}$$

$$H(e_2)=\left\{\begin{array}{l}\langle h_1,[0.2,0.4],[0.4,0.5]\rangle,\langle h_2,[0.2,0.3],[0.4,0.6]\rangle,\\ \langle h_3,[0.3,0.4],[0.4,0.5]\rangle,\langle h_4,[0.1,0.2],[0.4,0.6]\rangle,\\ \langle h_5,[0.3,0.8],[0.0,0.1]\rangle\end{array}\right\}$$

$$H(e_3)=\left\{\begin{array}{l}\langle h_1,[0.4,0.7],[0.0,0.1]\rangle,\langle h_2,[0.6,0.7],[0.2,0.3]\rangle,\\ \langle h_3,[0.1,0.3],[0.3,0.5]\rangle,\langle h_4,[0.2,0.3],[0.4,0.5]\rangle,\\ \langle h_5,[0.4,0.7],[0.2,0.3]\rangle\end{array}\right\}$$

$$H(e_4)=\left\{\begin{array}{l}\langle h_1,[0.7,0.9],[0.0,0.1]\rangle,\langle h_2,[0.5,0.7],[0.1,0.2]\rangle,\\ \langle h_3,[0.2,0.4],[0.4,0.5]\rangle,\langle h_4,[0.3,0.4],[0.4,0.6]\rangle,\\ \langle h_5,[0.6,0.8],[0.0,0.2]\rangle\end{array}\right\}$$

In the above example, $F(e_1)$ means for the father the price of h_1 is at least "competitive" on the membership degree of 0.4 and it is at most "competitive" on the membership degree of 0.5 and the price of house h_1 is not at least "competitive" on the nonmembership degree of 0.2 and it is not at most "competitive" on the nonmembership degree of 0.4.

Based on **Definition 2.7**, we can obtain the value class of the interval intuitionistic fuzzy soft set:

$$C_{(\mathcal{F},\{e_1\})}=\left\{\alpha_{\mathcal{F}(h_1,e_1)},\alpha_{\mathcal{F}(h_2,e_1)},\alpha_{\mathcal{F}(h_3,e_1)},\alpha_{\mathcal{F}(h_4,e_1)},\alpha_{\mathcal{F}(h_5,e_1)}\right\}$$

$$=\left\{\begin{array}{l}([0.2,0.4],[0.4,0.5]),([0.6,0.8],[0.0,0.2]),\\ ([0.1,0.4],[0.4,0.5]),([0.6,0.8],[0.0,0.2]),\\ ([0.2,0.4],[0.5,0.6])\end{array}\right\}$$

Definition 3.1 Suppose $(\mathcal{F},E)$ is an interval-valued intuitionistic fuzzy soft set over U, denote $|U|=n$, $|E|=m$, then the interval-valued intuitionistic fuzzy soft matrix (IVIFSM) could be defined as $\mathcal{F}=(\alpha_{\mathcal{F}_{ij}})_{n\times m}$, where $(\alpha_{\mathcal{F}_{ij}})$ is an interval-valued into-itionistic fuzzy value. We can express it in a tabular form in Table 1.

TABLE 1 Interval-valued intuitionistic fuzzy soft matrix

Objects	e_1	…	e_j	…	e_m
o_1	$\alpha_{\mathcal{F}_{11}}$	…	$\alpha_{\mathcal{F}_{1j}}$	…	$\alpha_{\mathcal{F}_{1m}}$
…		…	…	…	
o_i	$\alpha_{\mathcal{F}_{i1}}$	…	$\alpha_{\mathcal{F}_{ij}}$	…	$\alpha_{\mathcal{F}_{im}}$
…	…	…	…	…	…
o_n	$\alpha_{\mathcal{F}_{n1}}$	…	$\alpha_{\mathcal{F}_{nj}}$	…	$\alpha_{\mathcal{F}_{nm}}$

For the value class of the interval intuitionistic fuzzy soft set, we define its value vector as follows:

Definition 3.2 Let U be an initial universe and E be a set of parameters. Suppose $C_{(\mathcal{F},A)}=\left\{\alpha_{\mathcal{F}_{ij}},i=1,2,...,n;j=1,2,...,m\right\}$ (See table1) be the interval-valued intuitionistic fuzzy value class for soft set $(\mathcal{F},A)$ over U and parameters E, where $A=\left\{e_1,e_2,\ldots,e_j,\ldots e_m\right\}$ and $A\subset E$.

$\omega_j, (j=1,2,\ldots,m)$ is the weights of local criterion for parameter $\omega_j \in [0,1], j=1,2,\ldots,m$, and $\sum_{j=1}^{m}\omega_j = 1$. We say that $\mathrm{IVIFSWA}(\mathcal{F},A)$ is an interval-valued intuitionistic fuzzy soft set weighted averaging vector if:

$$\mathrm{IVIFSWA}(\mathcal{F},A) = [\sum_{j=1}^{m}\oplus\omega_j\alpha_{\mathcal{F}_{1j}},\ldots,\sum_{j=1}^{m}\oplus\omega_j\alpha_{\mathcal{F}_{mj}},\ldots,\sum_{j=1}^{m}\oplus\omega_j\alpha_{\mathcal{F}_{nj}}]' \quad (8)$$

Here:

$$\sum_{j=1}^{m}\oplus\omega_{ij}\alpha_{\mathcal{F}_{ij}} = \omega_1\alpha_{\mathcal{F}_{i1}} \oplus \omega_2\alpha_{\mathcal{F}_{i2}} \oplus \ldots \oplus \omega_m\alpha_{\mathcal{F}_{im}} \quad (9)$$

Especially if $\omega_j = 1/m, (j=1,2,\ldots,m)$ then we called the $\mathrm{IVIFSWA}(\mathcal{F},A)$ as interval-valued intuitionistic fuzzy soft set weighted arithmetic aggregation (IVIFSWAA).

The $\mathrm{IVIFSWA}(\mathcal{F},A)$ is an interval-valued intuition-nistic fuzzy vector and measures or induces to estimate the level of overall evaluation under the interval-valued intuit-tionistic fuzzy environment.

Example 2.

For example 1, the interval-valued intuitionistic fuzzy soft set weighted arithmetic aggregation vector is given as follow:

$$\mathrm{IVIFSWAA}(\mathcal{F},\mathrm{A}) = \begin{array}{l}([0.4736,0.6337],[0.2333,0.3598]) \\ ([0.5946,0.7551],[0.0760,0.1761]) \\ ([0.2293,0.4416],[0.3553,0.4856]) \\ ([0.5573,0.6747],[0.1428,0.3273]) \\ ([0.4267,0.5399],[0.2915,0.4267])\end{array}$$

That is to say, if the five attributes are the same important for the father then house h_1 is at least "attractive" on the membership degree of 0.4736 and it is at most "attractive" on the membership degree of 0.6337 and house h_1 is not at least "attractive" on the nonmem-bership degree of 0.2333 and it is not at most "attractive" on the no membership degree of 0.3598.

Theorem 3.1

Let $\mathrm{IVIFSWA}(\mathcal{F},A)$ be an interval-valued intuitionis-tic fuzzy soft set weighted averaging for interval-valued intuitionistic fuzzy soft set $(\mathcal{F},A)$ over the initial universe U and the set of parameters U, then $\mathrm{IVIFSWA}(\mathcal{F},A)$ is an IVIFV.

Proof. The result follows quickly from definition.

Theorem 3.2

Let $\mathrm{IVIFSWA}(\mathcal{F},A)$, $\mathrm{IVIFSWA}(\mathcal{G},A)$ be two inter-val-valued intuitionistic fuzzy soft sets weighted averaging for interval-valued intuitionistic fuzzy soft set $(\mathcal{F},A)$ and (G,A) over the initial universe U and the set of para-meters E, $\mathrm{A}\subset\mathrm{E}$, then we have the following properties:

$(\mathcal{K},\{\mathcal{F},\mathcal{G}\})$ is an interval-valued intuitionistic fuzzy soft set, where $\mathcal{K}$ is the mapping rule and

$(\mathcal{K},\{\mathcal{F},\mathcal{G}\}) = \{(\mathrm{IVIFSWA}(\mathcal{F},\mathrm{A}),\mathcal{F}),(\mathrm{IVIFSWA}(\mathcal{G},\mathrm{A}),\mathcal{G})\}$

(2) IVIFSWA($\mathcal{K},\{\mathcal{F},\mathcal{G}\}$) is also an IVIFV.

Proof. Accord to Theorem 1, $\mathrm{IVIFSWA}(\mathcal{F},\mathrm{A})$ and $\mathrm{IVIFSWA}(\mathcal{G},A)$ are IVIFV.

For the parameter set $\{\mathcal{F},\mathcal{G}\}$ } and the mapping rule $\mathcal{K}$ as the follows:

$\mathcal{F} \to \mathrm{IVIFSWA}(\mathcal{F},\mathrm{A})$

$\mathcal{G} \to \mathrm{IVIFSWA}(\mathcal{G},\mathrm{A})$

Thus

$(\mathcal{K},\{\mathcal{F},\mathcal{G}\}) = \{(\mathrm{IVIFSWA}(\mathcal{F},\mathrm{A}),\mathcal{F}),(\mathrm{IVIFSWA}(\mathcal{G},\mathrm{A}),\mathcal{G})\}$ is an interval-valued intuitionistic fuzzy soft set and IVIFSWA($\mathcal{K},\{\mathcal{F},\mathcal{G}\}$) is also an IVIFV.

Definition 3.3 Let U be an initial universe and E be a set of parameters. Suppose $(\mathcal{F},A)$ be an interval-valued intuitionistic fuzzy soft set over U, where $\mathrm{A} = \{e_1,e_2,\ldots,e_j,\ldots e_m\}$ and $\mathrm{A}\subset\mathrm{E}$. We say that $\mathrm{OIVIFSV}(\mathcal{F},A)$ is the optimistic interval-valued intuitionistic fuzzy soft set value vector if:

$$\mathrm{OIVIFSV}(\mathcal{F},A) = [\begin{array}{c}([\sup(u^-_{\mathcal{F}_{1.}}),\sup(u^+_{\mathcal{F}_{1.}})],[\inf(\nu^-_{\mathcal{F}_{1.}}),\inf(\nu^+_{\mathcal{F}_{1.}})]) \\ \ldots \\ ([\sup(u^-_{\mathcal{F}_{i.}}),\sup(u^+_{\mathcal{F}_{i.}})],[\inf(\nu^-_{\mathcal{F}_{i.}}),\inf(\nu^+_{\mathcal{F}_{i.}})]) \\ \ldots \\ ([\sup(u^-_{\mathcal{F}_{n.}}),\sup(u^+_{\mathcal{F}_{n.}})],[\inf(\nu^-_{\mathcal{F}_{n.}}),\inf(\nu^+_{\mathcal{F}_{n.}})])\end{array}] \quad (10)$$

And $\mathrm{PIVIFSV}(\mathcal{F},A)$ is the pessimism interval-valued intuitionistic fuzzy soft set value vector for the soft set:

$$\mathrm{IVIFSV}(\mathcal{F},\mathrm{A}) = [\begin{array}{c}([\inf(u^-_{\mathcal{F}_{1.}}),\inf(u^+_{\mathcal{F}_{1.}})],[\sup(\nu^-_{\mathcal{F}_{1.}}),\sup(\nu^+_{\mathcal{F}_{1.}})]) \\ \ldots \\ ([\inf(u^-_{\mathcal{F}_{i.}}),\inf(u^+_{\mathcal{F}_{i.}})],[\sup(\nu^-_{\mathcal{F}_{i.}}),\sup(\nu^+_{\mathcal{F}_{i.}})]) \\ \ldots \\ ([\inf(u^-_{\mathcal{F}_{n.}}),\inf(u^+_{\mathcal{F}_{n.}})],[\sup(\nu^-_{\mathcal{F}_{n.}}),\sup(\nu^+_{\mathcal{F}_{n.}})])\end{array}] \quad (11)$$

Where

$\sup(u^-_{\mathcal{F}_{i.}}) = \max(u^-_{\mathcal{F}_{i1}},u^-_{\mathcal{F}_{i2}},\ldots,u^-_{\mathcal{F}_{im}})$

$\sup(u^+_{\mathcal{F}_{i.}}) = \max(u^+_{\mathcal{F}_{i1}},u^+_{\mathcal{F}_{i2}},\ldots,u^+_{\mathcal{F}_{im}})$

$\inf(\nu^-_{\mathcal{F}_{i.}}) = \min(\nu^-_{\mathcal{F}_{i1}},\nu^-_{\mathcal{F}_{i2}},\ldots,\nu^-_{\mathcal{F}_{im}})$

$\inf(u^+_{\mathcal{F}_{i.}}) = \min(\nu^+_{\mathcal{F}_{i1}},\nu^+_{\mathcal{F}_{i2}},\ldots,\nu^+_{\mathcal{F}_{im}})$

Obviously, $\mathrm{OIVIFSV}(\mathcal{F},A)$, $\mathrm{PIVIFSV}(\mathcal{F},A)$ is the most optimistic (pessimism) interval-valued intuitionistic fuzzy vector and measures or induces to estimate the level of overall evaluation.

Example 3.

For example 1, OIVIFSV and PIVIFSV for the house is given as follows:

TABLE 2 OIVIFS

house	father	Mother	Daughter
h_1	([0.6,0.8],[0.1,0.2])	([0.6,0.7],[0.1,0.2])	([0.7,0.9],[0.0,0.1])
h_2	([0.7,0.8],[0.0,0.1])	([0.7,0.8],[0.0,0.2])	([0.6,0.7],[0.1,0.2])
h_3	([0.3,0.7],[0.2,0.3])	([0.8,0.9],[0.0,0.1])	([0.7,0.9],[0.0,0.1])
h_4	([0.7,0.8],[0.0,0.2])	([0.6,0.8],[0.0,0.1])	([0.3,0.8],[0.1,0.2])
h_4	([0.7,0.8],[0.0,0.1])	([0.6,0.8],[0.0,0.2])	([0.7,0.9],[0.0,0.1])

Theorem 3.3

Let $\mathrm{OIVIFSV}(\mathcal{F},A)$ and $\mathrm{PIVIFSV}(\mathcal{F},A)$ be the optimistic interval-valued intuitionistic fuzzy soft set value and the pessimism interval-valued intuitionistic fuzzy soft set value for interval-valued intuitionistic fuzzy soft set over the initial universe set U and the set of parameters E, then $\mathrm{OIVIFSV}(\mathcal{F},A)$ and $\mathrm{PIVIFSV}(\mathcal{F},A)$ are all IVIFVs.

TABLE 3 PIVIFS

house	father	Mother	Daughter
h_1	([0.2,0.4],[0.4,0.5])	([0.2,0.4],[0.4,0.5])	([0.2,0.4],[0.4,0.5])
h_2	([0.4,0.8],[0.1,0.2])	([0.2,0.3],[0.4,0.6])	([0.2,0.3],[0.4,0.6])
h_3	([0.1,0.4],[0.4,0.6])	([0.1,0.4],[0.4,0.5])	([0.1,0.3],[0.4,0.5])
h_4	([0.2,0.3],[0.4,0.6])	([0.2,0.3],[0.4,0.6])	([0.1,0.2],[0.4,0.6])
h_4	([0.1,0.3],[0.4,0.6])	([0.1,0.4],[0.5,0.6])	([0.3,0.7],[0.2,0.3])

Proof. From the definition we can get the result easily.

Theorem 3.4

Let $\mathrm{OIVIFSV}(\mathcal{F},A)$ and $\mathrm{PIVIFSV}(\mathcal{F},A)$ be two interval-valued intuitionistic fuzzy soft sets weighted averaging for interval-valued intuitionistic fuzzy soft set $(\mathcal{F},A)$ and (G,A) over the initial universe U and the set of parameters E, $\mathrm{A}\subset\mathrm{E}$, then we have the following properties:

$((\mathcal{K}^o,\{\mathcal{F},\mathcal{G}\})$ is an interval-valued intuitionistic fuzzy soft set, where

$(\mathcal{K}^o,\{\mathcal{F},\mathcal{G}\})=\{(\mathrm{IVIFSWA}(\mathcal{F},\mathrm{A}),\mathcal{F}),(\mathrm{IVIFSWA}(\mathcal{G},\mathrm{A}),\mathcal{G})\}$

$((\mathcal{K}^p,\{\mathcal{F},\mathcal{G}\})$ is an interval-valued intuitionistic fuzzy soft set, where

$(\mathcal{K}^p,\{\mathcal{F},\mathcal{G}\})=\{(\mathrm{IVIFSWA}(\mathcal{F},\mathrm{A}),\mathcal{F}),(\mathrm{IVIFSWA}(\mathcal{G},\mathrm{A}),\mathcal{G})\}$

Proof. Similar to the proof of theorem 3.2.

Theorem 3.5

Let $\mathrm{IVIFSV}(\mathcal{F},A)$, $\mathrm{OIVIFSV}(\mathcal{F},A)$ and $\mathrm{PIVIFSV}(\mathcal{F},A)$ then

$\mathrm{PIVIFSV}(\mathcal{F},\mathrm{A})<\mathrm{IVIFSWA}(\mathcal{F},\mathrm{A})<\mathrm{OIVIFSV}(\mathcal{F},\mathrm{A})$

Proof. See the proof theorem 3 in[7]

Definition 3.4 Let U be an initial universe and E be a set of parameters. Suppose $\mathrm{IVIFSWA}(\mathcal{F}_1,\mathrm{A}_1),...,\mathrm{IVIFSWA}(\mathcal{F}_\varkappa,\mathrm{A}_\varkappa)$ be interval-valued intuitionistic fuzzy soft set weighted averaging base on decision maker set $\mathrm{P}=\{\mathrm{p}_1,\mathrm{p}_2\ldots,\mathrm{p}_\varkappa\}$ and the interval-valued intuitionistic fuzzy soft set $(\mathcal{F}_1,\mathrm{A}_1),...,(\mathcal{F}_\varkappa,\mathrm{A}_\varkappa)$ over U and parameters E, where $\mathrm{A}_1=\ldots=\mathrm{A}\varkappa=\{\mathrm{e}_1,\mathrm{e}_2,\ldots,\mathrm{e}_j,\ldots\mathrm{e}_m\}\subset\mathrm{E}$. $\rho_j,(j=1,2,\ldots,\varkappa)$ is the weight of local criterion for decision maker $\mathrm{p}_1,\mathrm{p}_2\ldots,\mathrm{p}_\varkappa$, $\rho_j\in[0,1]$ and $\sum_{j=1}^{\varkappa}\rho_j=1$. We say that $\mathrm{IVIFSDV}(\mathcal{F},\mathrm{P})$ is an interval-valued intuitionistic fuzzy soft set weighted aggregation decision vector if

$$\mathrm{IVIFSWADV}(\mathcal{F},\mathrm{P})=[\sum_{j=1}^{\varkappa}\oplus\rho_j\mathrm{IVIFSWA}(\mathcal{F}_j,\mathrm{A}_j)] \quad (12)$$

Especially if $\rho_j=1/\varkappa,(j=1,2,\ldots,\varkappa)$ then we called the $\mathrm{IVIFSWADV}(\mathcal{F},\mathrm{P})$ as interval-valued intuitionistic fuzzy soft set weighted arithmetic aggregation decision vector (IVIFSWADV).

Definition 3.5 U be an initial universe and E be a set of parameters. Suppose $\mathrm{OIVIFSV}(\mathcal{F}_1,A_1),...,\mathrm{OIVIFSV}(\mathcal{F}_\varkappa,A_\varkappa)$ ($\mathrm{PIVIFSV}(\mathcal{F}_1,A_1),...,P\mathrm{IVIFSV}(\mathcal{F}_\varkappa,A_\varkappa)$) be optimistic (pessimism)interval-valued intuitionistic fuzzy soft set value base on decision maker set $\mathrm{P}=\{\mathrm{p}_1,\mathrm{p}_2\ldots,\mathrm{p}_\varkappa\}$ and the interval-valued intuitionistic fuzzy soft set $(\mathcal{F}_1,\mathrm{A}_1),...,(\mathcal{F}_\varkappa,\mathrm{A}_\varkappa)$ over U and parameters E, where $\mathrm{A}_1=\ldots=\mathrm{A}\varkappa=\{\mathrm{e}_1,\mathrm{e}_2,\ldots,\mathrm{e}_j,\ldots\mathrm{e}_m\}\subset\mathrm{E}$. We say that $\mathrm{IVIFSDV}_{\max-\min}(\mathcal{F},\mathrm{P})$ is an interval-valued intuitionistic fuzzy soft set decision vector with the rule “max-min” if:

$$\mathrm{IVIFSDV}_{\max-\min}(\mathcal{F},\mathrm{P})=$$

$$\left(\left[\inf\left(u^-_{\mathrm{OIVIFSV}_{1.}}\right),\inf\left(u^+_{\mathrm{OIVIFSV}_{1.}}\right)\right],\left[\sup(\nu^-_{\mathrm{OIVIFSV}_{1.}}),\sup(\nu^+_{\mathrm{OIVIFSV}_{1.}})\right]\right)$$

$$\ldots$$

$$[\ (\left[\inf\left(u^-_{\mathrm{OIVIFSV}_{i.}}\right),\inf\left(u^+_{\mathrm{OIVIFSV}_{i.}}\right)\right],\left[\sup(\nu^-_{\mathrm{OIVIFSV}_{i.}}),\sup(\nu^+_{\mathrm{OIVIFSV}_{i.}})\right])]$$

$$\ldots$$

$$\left(\left[\inf\left(u^-_{\mathrm{OIVIFSV}_{\varkappa.}}\right),\inf\left(u^+_{\mathrm{OIVIFSV}_{\varkappa.}}\right)\right],\left[\sup(\nu^-_{\mathrm{OIVIFSV}_{\varkappa.}}),\sup(\nu^+_{\mathrm{OIVIFSV}_{\varkappa.}})\right]\right)$$

And $\mathrm{IVIFSDV}_{\text{min-max}}(\mathcal{F},\mathrm{P})$ is an interval-valued intuitionistic fuzzy soft set decision vector with the rule “min-max” if:

$$\mathrm{IVIFSDV}_{\min-\max}(\mathcal{F},\mathrm{P})=$$

$$(\left[\sup(u^-_{\mathrm{PIVIFSV}_{1.}}),\sup(u^+_{\mathrm{PIVIFSV}_{1.}})\right],\left[\inf(\nu^-_{\mathrm{PIVIFSV}_{1.}}),\inf(\nu^+_{\mathrm{PIVIFSV}_{1.}})\right])$$

$$\ldots$$

$$[(\left[\sup\left(u^-_{\mathrm{PIVIFSV}_{i.}}\right),\sup\left(u^+_{\mathrm{PIVIFSV}_{i.}}\right)\right],\left[\inf(\nu^-_{\mathrm{PIVIFSV}_{i.}}),\inf(\nu^+_{\mathrm{PIVIFSV}_{i.}})\right])]$$

$$\ldots$$

$$(\left[\sup(u^-_{\mathrm{PIVIFSV}_{\varkappa.}}),\sup(u^+_{\mathrm{PIVIFSV}_{\varkappa.}})\right],\left[\inf(\nu^-_{\mathrm{PIVIFSV}_{\varkappa.}}),\inf(\nu^+_{\mathrm{PIVIFSV}_{\varkappa.}})\right])$$

Where

$$\sup\left(u^-_{\mathrm{PIVIFSV}_{i.}}\right)=\max(u^-_{\mathrm{PIVIFSV}_{i1}},u^-_{\mathrm{PIVIFSV}_{i2}},\ldots,u^-_{\mathrm{PIVIFSV}_{i\varkappa}})$$

$$\sup\left(u^+_{u^-_{\mathrm{PIVIFSV}_{i,i.}}}\right)=\max(u^+_{\mathrm{PIVIFSV}_{i1}},u^+_{\mathrm{PIVIFSV}_{i2}},\ldots,u^+_{\mathrm{PIVIFSV}_{i\varkappa}})$$

$$\inf(\nu^-_{u_{\mathrm{PIVIFSV}_{i,i.}}})=\min(\nu^-_{\mathrm{PIVIFSV}_{i1}},\nu^-_{\mathrm{PIVIFSV}_{i2}},\ldots,\nu^-_{\mathrm{PIVIFSV}_{i\varkappa}})$$

$$\inf\left(u^{+}_{u^{-}_{\mathrm{PIVIFSV}_{i,i.}}}\right) = \min(\nu^{+}_{\mathrm{PIVIFSV}_{i1}}, \nu^{+}_{\mathrm{PIVIFSV}_{i2}}, \ldots, \nu^{+}_{\mathrm{PIVIFSV}_{i\varkappa}})$$

Theorem3.6

Let $\mathrm{IVIFSDV}(\mathcal{F},\mathrm{P})$ ($\mathrm{IVIFSDV}_{\text{min-max}}(\mathcal{F},\mathrm{P})$, $\mathrm{IVIFSDV}_{\max-\min}(\mathcal{F},\mathrm{P})$) be an interval-valued intuitionistic fuzzy soft set weighted aggregation decision vector(decision vector with the rule "min-max" , decision vector with the rule "max-min")for decision maker set $\mathrm{P}=\{p_1,p_2\ldots,p_{\varkappa}\}$ and the interval-valued intuitionistic fuzzy soft set $(\mathcal{F}_1,\mathrm{A}_1),...,(\mathcal{F}_{\varkappa},\mathrm{A}_{\varkappa})$ over U and parameters E, where U be an initial universe and E be a set of parameter $\mathrm{A}_1=\ldots=\mathrm{A}\varkappa=\{e_1,e_2,\ldots,e_j,\ldots e_m\}\subset \mathrm{E}$. Then $\mathrm{IVIFSDV}(\mathcal{F},\mathrm{P})$, $\mathrm{IVIFSDV}_{\text{min-max}}(\mathcal{F},\mathrm{P})$, $\mathrm{IVIFSDV}_{\max-\min}(\mathcal{F},\mathrm{P})$ are all IVIFVs and can be denote

$$\begin{pmatrix} [u_1^-,u_1^+],[\nu_1^-,\nu_1^+] \\ \ldots \\ [u_n^-,u_n^+],[\nu_n^-,\nu_n^+] \end{pmatrix}$$

Proof. The result follows quickly from definition.

4 A Multi-criteria group decision making based on inter-valued intuitionistic fuzzy soft sets

In this section, we utilize the proposed operators to group decision base on interval-valued intuitionistic fuzzy soft sets. In a group decision making problem, suppose $\mathrm{X}=\{x_1,x_2,\ldots,x_n\}$ is the set of alternatives, Let $E=\{e_1,e_2,\ldots,e_j,\ldots e_m\}$ be a collection of criteria and that $\mathcal{W}=\{\omega_1,\omega_i,\ldots,\omega_m\}$ c is the weights of local criterion for $E=\{e_1,e_2,\ldots,e_j,\ldots e_m\}$, and $\mathrm{P}=\{p_1,p_2\ldots,p_{\varkappa}\}$ is the set of decision makers and that $\mathcal{P}=\{\rho_1,\rho_2,\ldots,\rho_{\varkappa}\}$ is the weights of decision maker $\mathrm{P}=\{p_1,p_2\ldots,p_{\varkappa}\}$.

Then, we develop an approach to multi-criteria decision making based on interval-valued intuitionistic fuzzy soft sets, the main steps is as follows:

Step 1: All decision makers value the degree range that the alternatives satisfy and do not satisfy the attributes. For decision maker p_k, the degree range that the alternatives x_i satisfy the attribute e_j and the degree range that the alternatives x_i does not satisfy the attribute e_j are all be record. The attribute value $\alpha_{\mathcal{F}_{ijk}}$ denote the alternatives x_i satisfy and does not satisfy the attribute e_j provided by the decision maker p_k .

Step 2: Use formula to construct fuzzy soft sets $(\mathcal{F}_1,\mathrm{E}),\ldots(\mathcal{F}_k,\mathrm{E}),\ldots,(\mathcal{F}_{\varkappa},\mathrm{E})$.

Step 3: compute $\mathrm{IVIFSWA}(\mathcal{F}_1,\mathrm{E}),\ldots,\mathrm{IVIFSWA}(\mathcal{F}_{\varkappa},\mathrm{E})$ and according to the calculation resultscompute $\mathrm{IVIFSWADV}(\mathcal{F},\mathrm{P})$; compute $\mathrm{OIVIFSWA}(\mathcal{F}_1,\mathrm{E}),\ldots,O\mathrm{IVIFSWA}(\mathcal{F}_{\varkappa},\mathrm{E})$ and according to the calculation results compute $\mathrm{IVIFSDV}_{\max-\min}(\mathcal{F},\mathrm{P})$; Compute ($\mathrm{PIVIFSWA}(\mathcal{F}_1,\mathrm{E}),\ldots,P\mathrm{IVIFSWA}(\mathcal{F}_{\varkappa},\mathrm{E})$ and according to the calculation results compute $\mathrm{IVIFSDV}_{\min-\max}(\mathcal{F},\mathrm{P})$.

Step 4: Using the Formula. (3) And (2) calculate the score function and the accuracy function for the element in $\mathrm{VIFSWADV}(\mathcal{F},\mathrm{P})$, $\mathrm{IVIFSDV}_{\max-\min}(\mathcal{F},\mathrm{P})$ and $\mathrm{IVIFSDV}_{\min-\max}(\mathcal{F},\mathrm{P})$.

Step 6: According to the need to choose a decision criterion: "weighted averaging", "min-max" or "max-min".

Example 4.

For example 1,

Step 1, 2 in example above.

Step 3: the decision vector in table 4.

TABLE 4 Decision Matrixes

house	IVIFSWA	IVIFSDV max-min	IVIFSDV min-max
h_1	([0.45,0.65], [0.22,0.33])	([0.6,0.7],[0.1,0.2])	([0.2,0.4],[0.4,0.5])
h_2	([0.50,0.66], [0.21,0.35])	([0.6,0.7],[0.1,0.2])	([0.4,0.8],[0.1,0.2])
h_3	([0.36,0.58], [0.29,0.43])	([0.3,0.7],[0.2,0.3])	([0.1,0.4],[0.4,0.5])
h_4	([0.43,0.63], [0.22,0.39])	([0.3,0.8],[0.1,0.2])	([0.2,0.3],[0.4,0.6])
h_4	([0.46,0.66], [0.21,0.36])	([0.6,0.8],[0.0,0.2])	([0.3,0.7],[0.2,0.3])

Step 4: the decision value in table 5

TABLE 4 Decision Value

house	IVIFSWA		IVIFSDV max-min		IVIFSDV min-max	
	S	D	S	D	S	D
h_1	0.64	0.57	0.5	0.8	0.43	0.75
h_2	0.65	0.59	0.5	0.8	0.83	0.75
h_3	0.55	0.55	0.34	0.75	0.4	0.7
h_4	0.61	0.57	0.45	0.7	0.38	0.75
h_4	0.64	0.58	0.55	0.8	0.38	0.75

Step 5:

According to rule "weighted averaging", the result is:
$h_2 > h_1 > h_5 > h_4 > h_3$

According to rule "max-min", the result is:
$h_2 = h_1 > h_5 > h_4 > h_3$

According to rule "min-max", the result is:
$h_2 > h_1 > h_3 > h_4 = h_5$

Thus the best alternative is the house h2.

5 Concluding and discussions

In this paper, we have explored the applications of using the interval-valued intuitionistic fuzzy soft set approach to the multi-criteria group decision making problem. We present the concept of the interval-valued intuitionistic fuzzy soft sets value vector (including the IVIFSS weighted averaging, the optimistic IVIFSS value and the pessimism IVIFSS value) and prove that the decision makers, as the parameter sets, and their IVIFSS value vector, as the mapping rule, may be consider as a new IVIFSS for all the decision makers. The value vector of this new IVIFSS is calculated and using their scoring function and accuracy function to compare different alternatives. Finally, an example is given to illustrate the given method. Our study is the extension of the interval-valued intuitionistic fuzzy soft set theory and the merits of proposed method are demonstrated using realistic example of multiple criteria decision making problem in interval-valued intuitionistic fuzzy environment...

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Author

< Xue Wen >, <1979.12>,< Chongqing, P.R. China>

Current position, grades: Doctoral student of School of Economics and Business Administration, Chongqing University, China.
University studies: received his B.Sc. in and M.Sc. from Beihang University in China.
Scientific interest: Decision-Making, Information Analysis
Publications: more than 5 papers published in various journals.
Experience: He is a doctoral student of School of Economics and Business Administration, Chongqing University, China.

< Zhi Xiao >, <1961.01>,< Chongqing, P.R. China>

Current position, grades: Professor, Doctor, Postdoctoral, Doctoral supervisor, Professor level 3, Director of Information Management Department, School of Economics and Business Administration, Chongqing University, China.
University studies: received his Ph.D. (Technology Economy and Management), Chongqing University.
Scientific interest: Applied Statistics and Information Analysis, Prediction and Decision-Making, Information Intelligent Analysis and Data Mining, Business Intelligence and Information Management
Publications: more than 30 papers published in various journals.
Experience: He has teaching experience of 28years, has completed 10 scientific research projects.