Finite element simulation of stray currents on subway shield tunnel

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Abstract

Three-dimensional Finite element model and a circuit element model were simulated respectively in homogeneous soil media and stratified soil media by loading different currents of railway. Simulation results show that the potential attenuation is nonlinear from the subway tunnel to the surrounding underground environment and along far away rail loading current of direction; With the loading current increasing, the potential of surrounding media advanced, to increase the resistance of the ragion near the subway railway can reduce the effect scope of the stray current. Compared to the traditional model of circuit elements, the three-dimensional finite element model can calculate the potential value of each position on the running rails and surrounding, resulting in stray current sphere of influence.

Keywords: subway shield tunnel, finite element model, circuit element model, homogeneous soil media, stratified soil media

1 Introduction

In subway DC traction power systems, the running rails are used as the return path of the train's current back to the supply source. Due to full insulation of the rails to the ground is impossible, a part of current leaves rails to the ground. These leakage currents are called stray currents. Stray current causes a series of serious problems that electrical corrosion for buried metal structures [1, 2]. Seriously affects the normal operation of urban rail transit. Therefore, all over the world attach great importance to the distribution rules and influence ranges of stray current.

Presently, W.V.Baeckamnn, Mou Longhua and Liu Yan had established a stray current model, respectively for variable with traction power current and the resistance of rail and the resistance of track-to-earth [3-5], Potential and spillage of stray current of rail can be solved effectively by the circuit element model. In this paper, on this basis, the actual physical model of the subway system is equivalent to the resistor network model under ideal conditions, and as before, traction current, running track resistance and rail transition resistance is used for variable calculation. A two-dimensional finite element model of stray current was built by Hu Yunjin, it can calculate the potentials of different locations at the ground caused by stray current [6]; A two-dimensional finite element model of subway shield tunnel in Milano was performed by M.Brenna and A.Dolara, simulation results were also given, the results indicate that potential of the location where current flows out tunnel was significantly higher in comparison with the location where current flows into tunnel [7, 8]. A three-dimensional finite element model analysis has been performed considering actual geometrical structures of tunnel and geological conditions.

But there is a difference between the results of these models and actual values. The major reason is that these models were established and simulated under the ideal conditions, it can only macroscopically and qualitatively analyze the effects of different factors on stray current; and the two dimensional finite element models cannot know the condition of stray current caused by an electric field in the earth generated by running rails current.

Thereby, In this paper, to verify the three-dimensional finite element model is better than the traditional model of circuit elements on solving and analysis the distribution and effects of stray current range, Finite element model and a circuit element model were simulated respectively in homogeneous soil media and stratified soil media by FEM method [9-12] and differential equation.

2 Stray current field model in subway

Based on actual size of subway shield tunnel, a three-dimensional model of stray current field computational domain in subway shield tunnel was built. The sectional view of computational domain of subway stray current field, as shown in fig.1. Fig.1 illustrates that the ground was modeled with a length of 100m, a height of 60m and a width of 1000m cuboid. Fig.2 shows the structure of subway shield tunnel and railway.

3 Comparison analysis of two stray current field models in homogeneous soil media

3.1 SIMULATION OF CIRCUIT ELEMENT MODEL

Differential equation was basis to simulate for circuit element model, made assumptions as follows [1]: A homogeneous rail-to-earth resistance; A homogeneous rail resistance; The value of soil resistance is zero; Feeder line impedance r is ignored.

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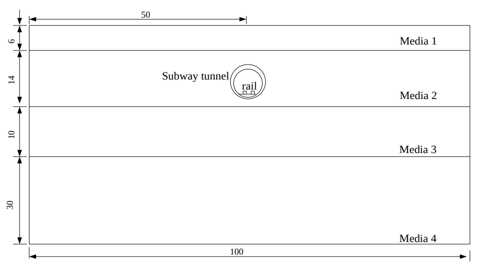

FIGURE 1 Sectional view of computional domain of subway stray current field

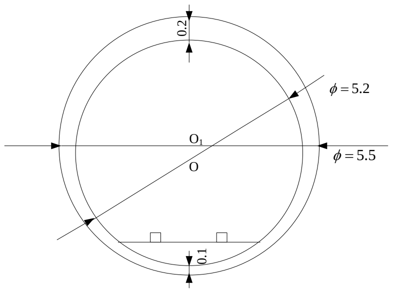

FIGURE 2 Structure of subway shield tunnel and railway

According to the fig.1, subway system model is equivalent to a resistor network of rail-tunnel-ground diagram in homogeneous soil media, as shown fig.3.

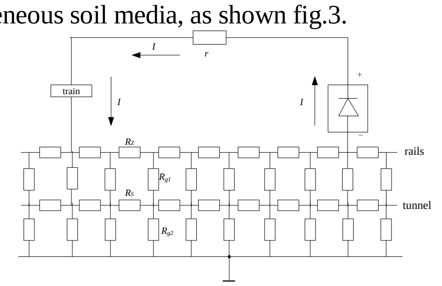

FIGURE 3 Resistor network diagram in homogeneous soil media

Where RZ represents rail resistance Ω / km ; RS represents tunnel resistance Ω / km ;Rg1 represents rail-tunnel resistance Ωkm ; Rg2 represents tunnel-ground resistance Ωkm .

3.1.1 Derivations of formula

According to the fig.3, mathematical formulae was derivated [1], the rail potential equation is gived as formula (1):

$$u(x) = -IZ \frac{R_Z}{R_Z + R_S} \text{th} \frac{\alpha L}{2} \cdot \text{ch}\alpha x + IZ \frac{R_Z}{R_Z + R_S} \cdot \text{sh}\alpha x \text{ , } \quad (1)$$

where $Z = \sqrt{R_{g1} \cdot (R_S + R_Z)}$; $\partial = \sqrt{\frac{R_S + R_Z}{R_{g1}}}$; L represents the distance between train and traction substation; x is a distance between the point of rail and traction substation.

Rail current equation is gived as formula (2):

$$i(x) = -I \frac{R_Z}{R_Z + R_S} \text{th} \frac{\alpha L}{2} \cdot \text{sh}\alpha x + I \frac{R_Z}{R_Z + R_S} \cdot \text{ch}\alpha x + I \frac{R_S}{R_Z + R_S} \quad (2)$$

Rail leakage stray current equation is gived as formula (3):

$$i_S(x) = I \frac{R_Z}{R_Z + R_S} \text{th} \frac{\alpha L}{2} \cdot \text{sh}\alpha x - I \frac{R_Z}{R_Z + R_S} \cdot \text{ch}\alpha x + I \frac{R_Z}{R_Z + R_S} \quad (3)$$

3.1.2 Computer simulation

Based on formula (1)- formula (3), computer simulation was conducted by MTLAB, and made assumptions as follows: $R_z = 0.026\Omega / km$; $R_{g1} = 15\Omega\, km$; $R_z = 0.01\Omega / km$; $L = 1.0km$, when the value of train traction current was 1000A, 2000A, 3000A, 4000A, Simulation diagrams of rail-to-ground potential, rail current, stray current for railway to ground were shown as fig.4-fig.6.

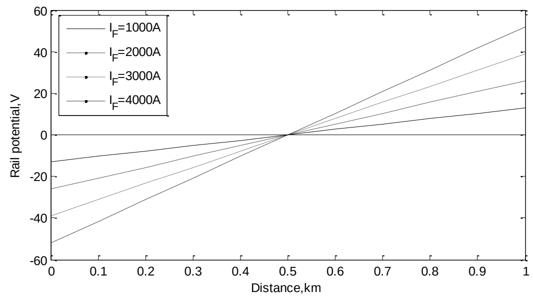

FIGURE 4 Simulation diagrams of rail-to-ground potential

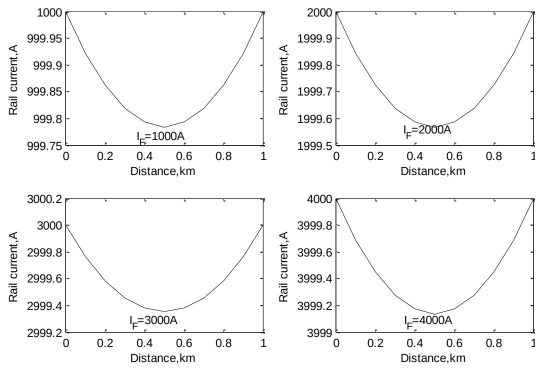

FIGURE 5 Simulation diagrams of rail current

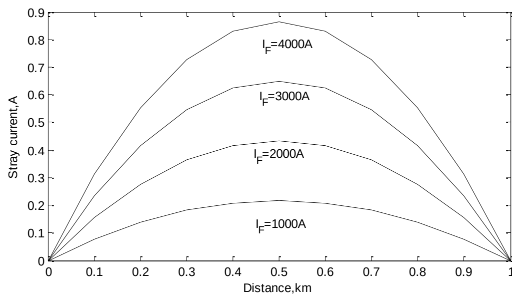

FIGURE 6 Simulation diagrams of stray current for railway to ground

3.2 SIMULATION OF THREE-DIMENSIONAL FINITE ELEMENT MODEL

Since the actual boundary shape of the stray current field computational domain in subway is complex and changeable, and the geological condition is diverse, the finite element method can handle all types of boundary conditions. Also, it can simulate structure of varieties material [9].

Due to two-dimensional finite element model had been verified its correctness in apparatus [6], so, it is reasonable and reliable to build a three-dimensional finite element model. Running rail current generates electric field, and stray current caused by electric potential gradient as shown apparatus [13], and running rail potential was calculated theoretically. Therefore, to actually reflect the distribution of stray current field near the tunnel, FEM method was applied by ANSYS.

3.2.1 Basic equation and boundary condition

According to Ohm's law, the current density components of three-dimensional steady current field are formula (4):

$$\begin{cases} j_x = -\gamma_x \dfrac{\partial \phi}{\partial x} \\ j_y = -\gamma_y \dfrac{\partial \phi}{\partial y}, \\ j_z = -\gamma_z \dfrac{\partial \phi}{\partial z} \end{cases} \quad (4)$$

where jx, jy and jZ represent current density component in the direction of x,y and z and γ_x, γ_y and γ_z represent conductivity in the direction of x,y and z; φ represents potential(V).

The current continuity equation in the direction of x, y and z will be as follow (5):

$$\frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z} = 0. \quad (5)$$

According to equations (4) and (5), we can get the potential basic equations of the stray current of the three-dimensional subway shield tunnel, as shown formula (6):

$$\frac{\partial}{\partial x}(\gamma_x \frac{\partial \varphi}{\partial x}) + \frac{\partial}{\partial y}(\gamma_y \frac{\partial \varphi}{\partial y}) + \frac{\partial}{\partial z}(\gamma_z \frac{\partial \varphi}{\partial z}) = 0. \quad (6)$$

For a constant current field, only the boundary conditions are needed to list. Equation (7) gives the definite conditions of equation (6).

$$\begin{cases} \dfrac{\partial \varphi}{\partial n} = 0 \\ \varphi = 0 \end{cases}. \quad (7)$$

3.2.2 Computer simulation

Figure 1 shows that media1-4 are the same media in homogeneous soil media. The parameters of calculation are shown as table.1.

TABLE 1 Simulation parameters of homogeneous soil media

Material name	Ralative permittivity	Resistivity (Ω·m)	Element
Tunnel	6.4	150	SOLID231 PLANE230
Rail	1×107	2.1×10-7	
Soil	30	100	

In order to analyze the effects of different values of traction current on distribution of stray current field in homogeneous soil media, and the values of current[7] of 10A, 20A, 50A, 100A were loaded separately on one end of each rail. Fig.7 displays that the electric potential contours in homogeneous soil media when the value of carry-current is 10A, and fig.8 shows the neighboring tunnel in the same situation.

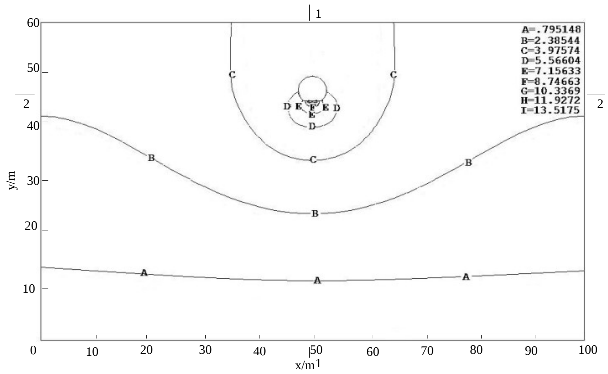

FIGURE 7 Electric potential contours in homogeneous soil media (10A)

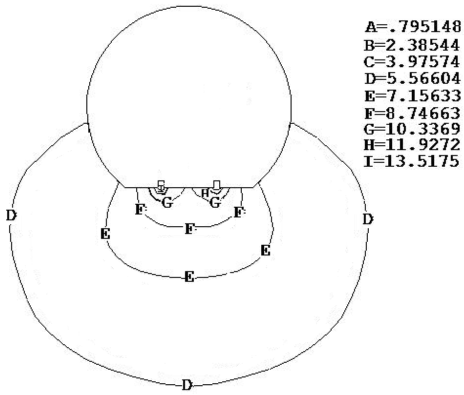

FIGURE 8 Electric potential contours of neighboring tunnel in homogeneous soil media (10A)

Different loading currents have the same contours. The only differences are potential value of contours.

In order to analyze the potential from the rail and rail to the surrounding, the longitudinal section was installed through the centre of rail on the left, as shown in profile 1-1 of fig.3, the path of z=0 was selected, Fig.9 shows that the potential curve of different depths in the longitudinal section. The cross section was installed through the bottom of rail, as shown in profile 2-2 of fig.3, the path of z=0 was selected, Fig.10 shows that the potential curve of different locations in the cross section; Fig.11shows that the Potential curve of railway in homogeneous soil media. On the diagram, 0 is the position of current-carrying.

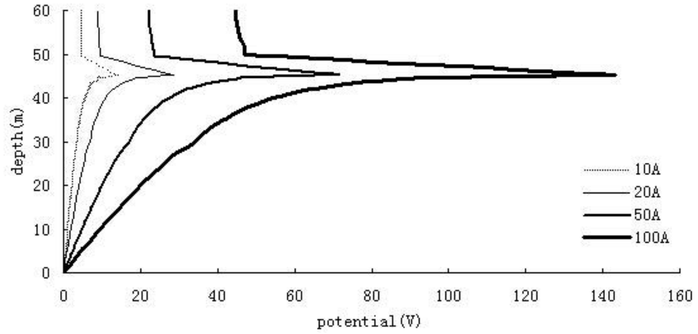

FIGURE 9 Potential curve of 1-1 in homogeneous soil media

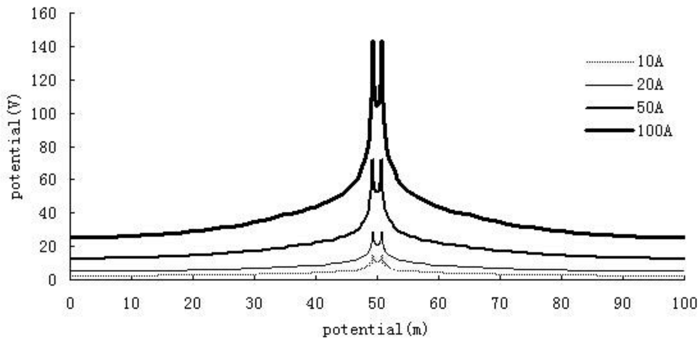

FIGURE 10 Potential curve of 2-2 in homogeneous soil media

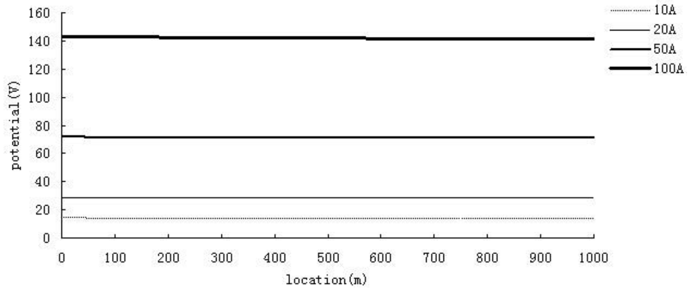

FIGURE 11 Potential curve of railway in homogeneous soil media

Figures 9-11 show that the distribution of stray current from the rail to the surrounding with the different values of traction current, as shown in table.2.

TABLE 2 Potential distribution of the rail and rail to the surrounding underground environment

Current(A)	Profile 1-1potential(V)		Profile 2-2 potential(V)		Rail potential(V)	
	Maximum value	*Minimum value*	*Maximum value*	*Minimum value*	*Maximum value*	*Minimum value*
10	14.3127	0	14.3127	2.5272	14.313	14.154
20	28.6254	0	28.6254	5.0544	28.625	28.308
50	71.5633	0	71.5633	12.636	71.5633	70.77
100	143.127	0	143.127	25.272	143.13	141.54

Figures 7-11 and table.2 show that the potential attenuation is nonlinear from the rail to the surrounding underground. The farther from the rail and the more gently the curve is, the smaller the stray current strength it has.

The path of z=0 was selected in figs.9-11, and we can also select other paths, then each points where rail to the surrounding underground can be known clearly. Baesd on Geramy standard VDE0150: the maximum allowable voltage drop is 0.1V between corroded metal construction and surrounding media, then we can see the in this electric field of buried metal, whether or not it is subjected to corrosion, so that confirm protection range of stray current and take actions accordingly.

3.3 COMPARISON ANALYSIS OF TWO STRAY CURRENT FIELD MODELS

As we know from simulatin of the three-dimensional finite element model and the circuit element model:

(1) Traditional circuit element model can solve running rail voltage, running current, stray current for railway to ground.

(2) Three-dimensional finite element model not only can solve rail voltage, but also can calculate potential anywhere in the underground.

(3) Compared to the traditional model of circuit elements, the three-dimensional finite element model can calculate the potential value of each position on the running rails and surrounding, resulting in stray current sphere of influence.

4 Comparison analysis of two stray current field models in stratified soil media

4.1 SIMULATION OF CIRCUIT ELEMENT MODEL

In actual subway, rail-to-earth resistance andrail resistance is stratified because of geological condition is diverse, as shown fig.1, media1-4 is the different medias, in this paper, a circuit element model was performed considering the different soil medias. Likewise, made assumptions as follows [1]: A homogeneous rail-to-earth resistance; A homogeneous rail resistance; The value of soil resistance is zero; Feeder line impedance r is ignored.

RD2 is soil media2 resistance, Ω/km ; RD3is soil media3 resistance, Ω/km ; RD4is soil media4 resistance, Ω/km ; Rg1is rail-tunnel resistance, Ω/km ; Rg2 is tunnel-soil media2 resistance, Ω/km ; Rg3 is soil media2- soil media3 resistance, Ω/km ; Rg4 is soil media3- soil media4 resistance, Ω/km ; I is tr- action current, A. Resistor network diagram in stratified soil media (see, fig.12).

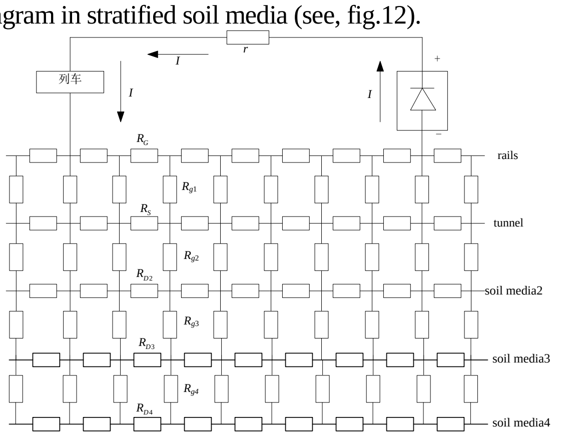

FIGURE 12 Resistor network diagram in stratified soil media

Figure 12 shows a very complicated circuit element model, the more complicated the model is, the more consideration it has, with increasing assumption, the calculative reliability of differential equation becomes lower. So, derivations of formula and simulation were not shown.

4.2 SIMULATION OF THREE-DIMENSIONAL FINITE ELEMENT MODEL

In stratified soil media, fig.1 shows that media1-4 is the different medias. The type of element and parameters of tunnel and rail chosen are not changing. The Rsistivity Parameters of each Soil, as shown in table.3.

TABLE 3 Resistivity parameters of each soil

Number	Name	Ralative permittivity	Resistivity (Ω·m)
Media 1	Clay(wet)	8	10
Media 2	Soft soil	30	100
Media 3	Clay layer	40	500
Media 4	gravel	6	1000

In stratified soil media, similarly, the values of traction current of 10A, 20A, 50A, 100A were loaded separately on one end of each rail. Fig.13 displays that the electric potential contours in stratified soil media when the value of carry-current is 10A. Fig.14 shows the neighboring tunnel in the same situation.

Different loading currents have the same contours. The only differences are potential value of contours.

Figures 15-17 show that the distribution of stray current from the rail to the surrounding with the different values of traction current, as shown in table.4.

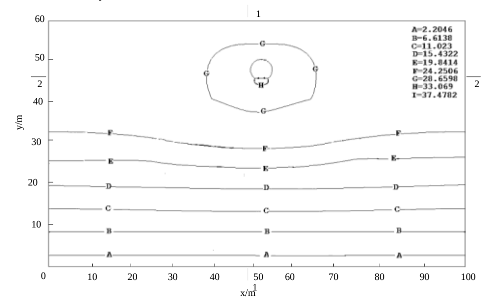

FIGURE 13 Electric potential contours in stratified soil media (10A)

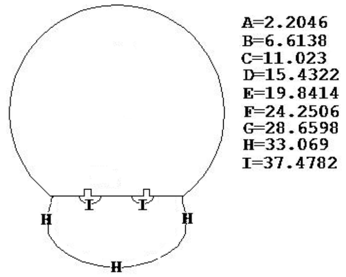

FIGURE 14 Electric potential contours of neighboring tunnel in stratified soil media (10A)

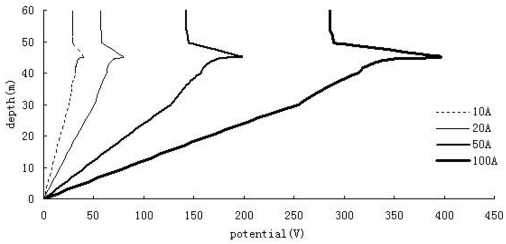

FIGURE 15 Potential curve of 1-1 in stratified soil media

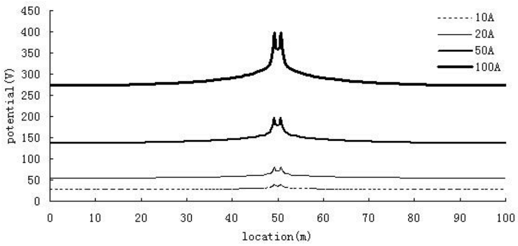

FIGURE 16 Potential curve of 2-2 in stratified soil media

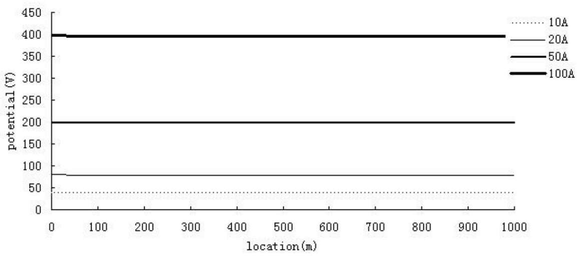

FIGURE 17 Potential curve of railway in stratified soil media

Figures 13-17 and Table 4 show that the potential attenuation is nonlinear from the rail to the surrounding underground. The farther from the rail and the more gently the curve is, the smaller the stray current strength it has.

TABLE 4 Poential distribution of the and rail to the surrounding underground environment

Current(A)	**Profile 1-1potential(V)**		**Profile 2-2potential(V)**		**Rail potential(V)**	
	Maximum value	*Minimum value*	*Maximum value*	*Minimum value*	*Maximum value*	*Minimum value*
10	39.6828	0	39.6828	27.299	39.682	39.524
20	70.3656	0	70.3656	54.598	70.3656	79.048
50	198.414	0	198.414	136.49	198.414	197.62
100	396.828	0	396.828	272.99	396.82	395.24

Different loading currents have the same distribution rule, in stratified soil media.

Compared to the traditional model of circuit elements, the three-dimensional finite element model can calculate the potential value of each position on the running rails and surrounding, this advantage is consistent with in homogeneous soil media.

Whether it is homogeneous or stratified soil media, the potential attenuation is nonlinear from the rail to the surrounding under environment. In the same value of carry-current, 10A, the Comparison of homogeneous soil media and stratified soil media, as shown in table 5 and in table 6.

TABLE 5 Comparison of and stratified soil media

Profile 1-1	**y=54m potentia (V)**	**y=60m potentia (V)**	**Voltage drop(V)**	**y=30m potentia (V)**	**y=40m potentia (V)**	**Voltage drop (V)**
Homogeneous soil media	4.57	4.47	0.1	3.42	5.63	2.21
Stratified soil media	28.58	28.55	0.03	25.42	31.42	6

TABLE 6 Comparison of homogeneous soil media and stratified soil media

Profile 2-2	**Minimum potential (V)**	**Maximum potential (V)**	**Voltage drop (V)**
Stratified soil media	27.30	39.68	12.38
Homogeneous soil media	2.53	14.31	11.78

As can clearly be seen in table 5, the voltage drop in homogeneous soil media is obviously larger than the voltage drop in stratified soil media in 50-60m area of the profile 1-1. On the contrary, the voltage drop in homogeneous soil media is obviously smaller than the voltage drop in stratified soil media in 30-40m area of the profile 1-1. The main reason is that the voltage drop is mainly concentrated in the areas of high resistivity ($\rho_1 < \rho_2 < \rho_3 < \rho_4$).

Table 6 shows that the voltage drop in homogeneous soil media is close to the voltage drop in stratified soil media in the profile 2-2. This reason is consistent with the above-mentioned. Similarly, table 2 and table 4 show the minimum potential maximum potential of rails and the voltage drop is no difference.

The greater the Resistivity is, the less stray current leaks surrounding soil can get.

5 Conclusions

Through the above analysis, the paper comes to the conclusion as follows:

(1) Three-dimensional finite element model of subway shield tunnel can calculate potential anywhere in the underground, which the traditional circuit element model cannot get. And not only that, electric field caused by running rail current also can be simulated, then we can see the in this electric field of buried metal, whether or not it is subjected to corrosion, so that confirm protection range of stray current and take actions accordingly.

(2) For the complex boundary shape and diverse geological condition of subway, it is difficult for derivations of formula, but finite element model will, and the greater the resistivity of the surrounding media is, the less stray current it has.

(3) With the increasingly value of current-carrying, the rail potential of the surrounding media and leakage current raise constantly. The potential attenuation is nonlinear from the subway tunnel to the surrounding underground and along far away rail.

So, for studying stray current sphere of protection, finite element model is more effective than the traditional model.

Acknowledgements

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