Evaluation model of college basketball teaching based on fuzzy AHP theory

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Abstract

Fuzzy analytic hierarchy process (AHP) uses fuzzy matrix on the theoretical basis of original AHPs, so it overcomes difference in judgment among different people well, accords with people’s thinking logics and mental judgment decisions to a larger extent and has a simpler form. Physical education classroom acts as an important approach by which college students do physical exercise and improve physical quality. This thesis takes college students’ basketball teaching for example and establishes an evaluation model about college students’ basketball teaching based on fuzzy AHP theory according to classroom learning status. In accordance with features of fuzzy theory and AHP and in combination with quantitative and qualitative methods, it selects evaluation factors of classroom learning, constructs comprehensive and scientific evaluation system and uses this model to implement instance analysis of students’ classroom learning. Then, it obtains evaluation results, proposes corresponding promotion countermeasures and verifies effectiveness of the model.

Keywords: Fuzzy theory, AHP, Basketball teaching learning, Evaluation

1 Introduction

It was Professor T. L. Saaty that proposed analytic hierarchy process (AHP) [1]. AHP can analyze qualitative problems quantitatively and its application is convenient and flexible, so it is widely applied to research in each field gradually [2]. AHP can classify many influence factors in some complicated puzzles in a stratified way, methodize them orderly, combine some subjective judgments with objective results like researchers’ analyses and experts’ opinions, implement pairwise comparison for influence factors on each hierarchy, obtain primary and secondary importance and then carry out quantitative description. In this process, it is necessary to apply several subject theories, calculate weight values of factors’ importance order on each hierarchy by mathematical method, rank these weighted values of relative importance again and get final ranking results at last [3].

However, AHP still has some shortages in application. On the one hand, when reciprocal scale is used to establish reciprocal judgment matrix in AHP, the judgment matrix fails to satisfy consistency conditions because there is difference between subjective cognizance and objective reality and inaccuracy exists as scale is adapted to measure objects [4]. On the other hand, when normalizing rank aggregation method and root method are used to solve importance ranking of each scheme, impacts of one line of elements are considered in the judgment matrix only, which leads to low computational accuracy and fails to control accuracy according to requirements for accuracy [5].

Although there is difference in impacts of things or elements on system, the difference is transitional with continuous changes and cannot be expressed by accurate numerical values [6]. Besides, the factor which cannot be expressed specifically is an attribute of things. Therefore, concept of fuzziness is put forward. In 1965, Chad a famous American scholar proposed concepts of membership function and membership degree and utilized a mathematical model to describe fuzzy and uncertain factors, i.e., inherent attributed of research objects were expressed by mathematical language. In application of practical problems, many factors have this uncertain and fuzzy character [7]. However, traditional AHPs are qualitative and quantitative analytical approaches which have some problems that cannot be overcome. On the theoretical basis of original AHPs, fuzzy AHP adopts fuzzy matrix so that it overcomes difference in judgment among different people well, accords with people’s thinking logics and mental judgment decisions to a larger extent and has a simpler form [8]. Hence, it is used to measure comparison among elements. For its nature, the possibility that measurement may be inaccurate does not exist. Though the established priority judgment matrix is rough, it can be constructed easily. Since the fuzzy consistency matrix transformed from the priority judgment matrix satisfies consistency conditions, it is no need to carry out consistency test again [9]. Besides, it can satisfy computational accuracy. Physical exercise is an important way to improve physical quality and a survival skill nowadays. As future talent of the country, college students need pay attention to physical education and learning [10]. However, physical education classroom of most colleges cannot draw much attention from students so that the phenomenon that students are absent from school and late for lessons appears and some students even have the mentality that they ‘deal with’ physical education and it will be ok as long as they pass required courses [11]. Evaluation on classroom learning

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mainly focuses on mid-term and final examinations, where teachers’ subjective judgment occupies a large proportion. Thus, students’ classroom learning cannot be evaluated objectively but much one-sidedness exists, which goes against exertion of students’ comprehensive ability. Therefore, it is necessary to implement more effective classroom learning evaluation system.

Fuzzy AHP theory is a comprehensive analytical method for evaluation based on AHP, which combines fuzzy theory with AHP. Holding an important position in multi-objective and multi-factor analysis, the method is one of the analytical methods used by academic world widely [12]. Fuzzy quantization for factors that cannot be determined qualitatively overcomes the shortage that measurement of AHP is inaccurate. This thesis establishes the priority judgment matrix and carries out weight calculation and result evaluation. Besides, it takes college students’ basketball teaching for example, constructs an evaluation model about classroom learning and performs example verification for the model. This provides reference value and theoretical support for colleges to carry out scientific and reasonable course evaluation system and is helpful for such schools to learn students’ classroom learning as well as supervise and manage students’ learning.

2 Construction of the evaluation model on college students’ basketball teaching based on AHP theory

2.1 DETERMINING EVALUATION INDEX SYSTEM

When schools comprehensively evaluate college students’ basketball teaching learning, they should not only use students’ final course scores as a standard but also implement comprehensive evaluation on many aspects of college students in the whole process of basketball teaching learning, such as learning ability, performance and test scores. In detail, schools ought to do field investigation, issue questionnaires, determine factors affecting college student’s basketball course and establish evaluation index system according to practical learning conditions of college students’ basketball lessons. Finally, determine “the number of absence and lateness times for lessons”, ‘enthusiasm for classroom exercise’, ‘situations about participation in classroom activities’, ‘seatwork’, ‘achievement test at ordinary times’ and ‘achievement test at the end of a school term’ as main evaluation indexes and establish a simple evaluation model about basketball teaching learning, as shown in Figure 1.

It is assumed that the first-level set of evaluation factors is U, we will get $U=\{U_1, U_2, \ldots, U_n\}$, where $U_i$ refers to each evaluation factor of U.

Thus, evaluation system of this course involves

$U=\{U_1, U_2\}=\{\text{evaluation on learning process of the course and assessment on achievement test}\}$

$U_1=\{U_{11}, U_{12}, U_{13}, U_{14}\}=\{\text{the number of absence and lateness times for lessons, enthusiasm for classroom exercise, situations about participation in classroom activities and seat work}\}$

$U_2=\{U_{21}, U_{22}\}=\{\text{achievement test at ordinary times and achievement test at the end of a school term}\}$.

2.2 DETERMINING WEIGHTS OF EVALUATION FACTORS

At present, situations about classroom learning of college students’ physical education lessons are basically based on teachers’ subjective judgment. Thus, scientific rationality is lacked. Using AHP can process several influence factors hierarchically and comprehensively and determine weights objectively. Carry out pairwise comparison about each hierarchical structure in Figure 1 and establish comparative judgment matrix. In order to make evaluation more accurate, use the proportional scale of relative importance, which is suggested by Saaty, and then solve relative weight of each factor. Details are shown in Table 1.

In Table 1, the factors $i$ and $j$ represent two indexes or factors compared under the same factor state, respectively. The matrix composed of the scale $a_{ij}$ is a pairwise comparison matrix. If the following relation is satisfied, i.e., $a_{ij}=a_{ik}\times a_{kj}$, it will imply the matrix is completely consistent. At this moment, the feature vector that is corresponding to its maximum characteristic root can express degree of relative importance of each index and then orthogonalize it. In doing so, weight vector can be solved.

<table>
<thead>
<tr>
<th>Index scale</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The two factors i and j are equally important</td>
</tr>
<tr>
<td>3</td>
<td>The factor i is a little more important than j</td>
</tr>
<tr>
<td>5</td>
<td>The factor i is obviously more important than j</td>
</tr>
<tr>
<td>7</td>
<td>The factor i is strongly more important than j</td>
</tr>
<tr>
<td>9</td>
<td>The factor i is extremely more important than j</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>They are scale values that is corresponding to intermediate state of the two kinds of judgment quality in the above</td>
</tr>
</tbody>
</table>

It is assumed that the weight factor vector that is corresponding to the evaluation factor set is A. Then, we will get:

$$\hat{A} = (a_1, a_2, \ldots, a_n)$$

where $a_i$ refers to the role of the evaluation factor $u_i$ and measurement on its position in all evaluation factors and is called weight.
Generally, \( a_i \geq 0 \), and measurement on its position is called weight:

\[
\sum a_i = 1.
\]

### 2.3 Establishing Pairwise Comparison Matrix for Selected Evaluation Factors

In accordance with the evaluation factor system in Figure 1, use fuzzy AHP theory to construct pairwise comparison matrix and formulate weights of mutual impacts among factors according to scale values in Table 1. See details in Table 2, Table 3 and Table 4. This will be a basis on which evaluation on basketball course learning is carried out. In addition, we need calculate maximum characteristic value, priority vector as well as consistency need satisfy the condition that it is less than or equal to 0.1. If not, re-entry is needed until it satisfies the condition.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Comparison matrix about the second hierarchy and the first hierarchy ( U ) and hierarchical ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U )</td>
<td>( U_1 )</td>
</tr>
<tr>
<td>( U_1 )</td>
<td>1</td>
</tr>
<tr>
<td>( U_2 )</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Comparison matrix about the third hierarchy and the first hierarchy ( U_1 ) and hierarchical ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_1 )</td>
<td>( U_{11} )</td>
</tr>
<tr>
<td>( U_{11} )</td>
<td>1</td>
</tr>
<tr>
<td>( U_{12} )</td>
<td>1/3</td>
</tr>
<tr>
<td>( U_{13} )</td>
<td>1/3</td>
</tr>
<tr>
<td>( U_{14} )</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Comparison matrix about the third hierarchy and the second hierarchy ( U_2 ) and hierarchical ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_2 )</td>
<td>( U_{21} )</td>
</tr>
<tr>
<td>( U_{21} )</td>
<td>1</td>
</tr>
<tr>
<td>( U_{22} )</td>
<td>1</td>
</tr>
</tbody>
</table>

### 2.4 Determining Membership Degree of Evaluation Grade

Since fuzzy theory utilizes fuzzy information that cannot be accurate to implement quantitative analog processing for people’s subject ideas, we will use fuzzy data to express evaluation grade applied to students’ basketball course learning in the following. According to practical situations related to course teaching, a five-grade course evaluation standard will be used, i.e.,

\[ V = \{ v_1', v_2', \ldots, v_5' \} = \{ \text{excellent, good, ordinary, qualified and disqualified} \}. \]

Then, set each level’s membership degree which may be a value range. See details in Table 5.

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Evaluation grade of course learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute score range</td>
<td>Corresponding comment</td>
</tr>
<tr>
<td>90 ( \leq x \leq 100 )</td>
<td>Excellent performance</td>
</tr>
<tr>
<td>80 ( \leq x &lt; 90 )</td>
<td>Good performance</td>
</tr>
<tr>
<td>70 ( \leq x &lt; 80 )</td>
<td>Ordinary performance</td>
</tr>
<tr>
<td>60 ( \leq x &lt; 70 )</td>
<td>Working harder</td>
</tr>
<tr>
<td>( x &lt; 60 )</td>
<td>Poor performance</td>
</tr>
</tbody>
</table>

As computational process of fuzzy numbers is quite complicated, it is necessary to choose appropriate fuzzy numbers to provide convenience for following calculation. Besides, select L-R fuzzy function. Computational formula of its five grades’ membership function is expressed as

\[
VL_i(X) = \begin{cases} 
0, x < a, \\
\frac{x-a}{b-a}, a \leq x < b, \\
\frac{x-d}{c-d}, c \leq x < d, \\
0, x > d.
\end{cases}
\]

where values of parameters \( a, b, c \) and \( d \) in the formula are determined according to fuzzy comprehensive evaluation. 

\[ [b - a] = |d - c| = \text{error value of course evaluation}. \]

It is assumed that the course evaluation is 2. Then, we may obtain \( L_1 = (88, 92, 98, 102) \), \( L_2 = (78, 82, 88, 92) \), \( L_3 = (68, 72, 78, 82) \), \( L_4 = (58, 62, 68, 72) \), \( L_2 = (48, 52, 58, 62) \).

### 3 Comprehensive Evaluations in the process of classroom learning

Via evaluation ranking results of each factor’s weight and each factor in the process of classroom learning, we may obtain fuzzy numbers of the foregoing function by using related natures of expansion theory and fuzzy numbers. Computational formulas of the two fuzzy functions are

\[ M = (m_1, m_2, m_3, m_4), N = (n_1, n_2, n_3, n_4); \]

\[ M \oplus N = (m_1+n_1, m_2+n_2, m_3+n_3, m_4+n_4); \]

\[ M \odot N = (m_1 \times n_1, m_2 \times n_2, m_3 \times n_3, m_4 \times n_4). \]

### 3.1 Processing of Bottom Data

As quantitative analysis can be applied to some factors while some others can only accept qualitative analysis in practical evaluation on classroom learning, we need quantify factors that can only accept qualitative analysis by using fuzzy theory. Teachers always need observe student’s course learning in the learning process of basketball courses and keep timely, objective and fair records in order that accuracy and scientific of evaluation can be ensured and then the number of absence or lateness for basketball course learning to evaluate effectiveness of student’s course learning.

### 3.2 Processing of Bottom Data

During learning of different items in basketball teaching, it is set that \( f_{ik} \) stands for the index value of students’ learning degree of the \( kth \) basketball task in basketball course learning, \( t_{ik} \) represents the number of the \( ith \) student’s absence or lateness times in the \( kth \) basketball task learning, \( \text{avg}_{ik} \) refers to average time or times of the \( kth \) index among rating indexes of all learning activities related to basketball, \( t_{\text{min}} \) denotes minimum time or times of the \( kth \) index time in all students and \( t_{\text{max}} \) is maximum time or times of the last index time in all students.
If $t_i = \text{avg}_k$, there will be $f_i = 50$;  
In case $t_i < \text{avg}_k$, we will get  
$$f_i = 50 - \frac{\text{avg}_k - t_i}{\text{avg}_k - t_{\text{min}}} \times 50.$$  
If $t_i > \text{avg}_k$, we will get  
$$f_i = \frac{t_i - \text{avg}_k}{t_{\text{min}} - \text{avg}_k} \times 50 + 50.$$  

Then, carry out fuzzification $f_i$ to get five different grades further. Then,  
$0 \leq f_i < 20$ will mean 'disqualified',  
$20 \leq f_i < 40$ refers to 'qualities',  
$40 \leq f_i < 60$ indicates 'ordinary',  
$60 \leq f_i < 80$ means 'good' and  
$80 \leq f_i < 100$ stands for 'excellent'.

Solve corresponding rating variables $L_1$-$L_5$. According to Weight A, use weighted mean method to obtain the formula:  
$$u_1 = \sum_{i=1}^{n} (A_i \times L_i).$$  
(4)

In this way, numerical value of $U_1$, comprehensive evaluation score in the process of basketball teaching learning and $L_i$ its membership function that is corresponding to five different grades.

### 3.3 DEFUZZIFICATION FOR EVALUATION RESULTS OF CLASSROOM LEARNING PROCESS

Generally, central part of fuzzy numbers of membership functions can express importance degree to the largest degree. Thus, use median method to carry out defuzzification for the fuzzy number $X_i=(a_i, b_i, c_i, d_i)$. The formula about its result is  
When $a_i = b_i = c_i = d_i$, $M(X_i) = a_i$.  
Otherwise,  
$$M(X_i) = \frac{b_i + c_i + [(d_i - c_i) - (b_i - a_i)]}{6} = \frac{2b_i + 2c_i + a_i + b_i}{6}. \quad (5)$$

Then, we may obtain the computational formula about defuzzification value of $U_1$, comprehensive evaluation in the classroom learning process is  
$$M(U_i) = \frac{2b_i + 2c_i + a_i + d_i}{6}. \quad (6)$$

Substitute $a$, $b$, $c$ and $d$ of $L_i$ its membership function that is corresponding to five different grades into the foregoing formula. In doing so, we may get comprehensive evaluation results of the classroom learning process.

### 4 Final evaluation on classroom learning

#### 4.1 COMPREHENSIVE EVALUATION ON CLASSROOM ACHIEVEMENT TEST

Since in achievement test about students’ basketball teaching learning, both achievement test at ordinary times and achievement test at the end of a school term are quantified specifically, i.e., they can be measured by data. Thus, we may obtain comprehensive evaluation on achievement test according to weighted average algorithm directly and its formula is  
$$U_2 = \sum_{i=1}^{n} U_{ik} \cdot A_k, \text{ where } k=1, 2. \quad (7)$$

#### 4.2 FINAL EVALUATION ON CLASSROOM LEARNING

In accordance with comprehensive evaluation results of classroom learning process and achievement test for classroom learning, use the weighted average algorithm to get $U$ the final comprehensive evaluation score of college student’s basketball teaching learning. There is  
$$U = U_1 \times A_1 + U_2 \times A_2.$$  

Substitute the obtained value of $U$ into the membership function formula of corresponding grades and then solve membership degree of each grade to get final values of evaluation grades about classroom learning.

### 4.3 APPLICATION EXAMPLE AND ANALYSIS OF RESULTS

In the following content, this article will carry out investigation and statistics according to basketball teaching learning situations of college students from a certain grade of a college. As a result, it is found that the average number of classroom absence and lateness times of all students in the grade in this semester is 5, the maximum number is 12 and the minimum number is 0; average quantized value of enthusiasm for classroom exercise is 40, its maximum is 80 and minimum is 10; average quantized value of participation in classroom activities is 45, its maximum is 75 and minimum is 12. Now, there is a student and the number of his classroom absence and lateness times in this semester is 3. On the basis of the formula:  
$$f_i = 50 - \frac{\text{avg}_k - t_i}{\text{avg}_k - t_{\text{min}}} \times 50,$$  
we may get his corresponding grade is excellent and quantized value of enthusiasm for classroom exercise is 30; in accordance with the formula, his corresponding grade is ordinary and quantized value of enthusiasm for classroom exercise is 50; according to the formula:  
$$f_i = \frac{t_i - \text{avg}_k}{t_{\text{min}} - \text{avg}_k} \times 50 + 50,$$  
his corresponding grade is good, the student’s score in classroom learning at ordinary times is 86.5 and he scores 78 in achievement test at the end of the semester. Then, we may get the fuzzy judgment matrix $U_1$, as shown in the following:  
$$U_1 = \begin{bmatrix} 0.2 & 0.3 & 0 & 0 \\ 0.3 & 0.1 & 0 & 0 \\ 0.4 & 0.1 & 0.1 & 0 \\ 0.2 & 0.1 & 0 & 0.1 \end{bmatrix}.$$
By carrying out the first-level comprehensive evaluation on U1, we may obtain
U11=0.30 0.12 0.13 0.50,
U12=0.28 0.22 0.16 0.79,
U13=0.30 0.19 0.17 0.48,
U14=0.33 0.20 0.23 0.47.
By carrying out the second-level comprehensive evaluation on U1, we may get:
U1=(0.29 0.10 0.14 0.47).
Thus,
U1=0.29×L1+0.10×L3+0.15×L3+0.47×L2
=0.29×(88,92,98,102)+0.10×(68,72,78,82)+
0.14×(48,52,58,62)+0.47×(78,82,88,92)
=(75.7,79.3,85.5,89.8)
M(U1)=(75.7×2+79.3×2+85.5+89.8)/6=82.5.
Next, we find that the student scores 82.5 in the evaluation on classroom learning process.
According to U2=(0.33 0.67), we know
U2=0.33×85.6+0.67×78=80.805.
Hence, the student’s score in the evaluation on classroom achievement test is 80.805. Based on U=(0.5 0.5), we get:
U=0.5×82.5+0.5×80.805=81.65.
In this way, we find the student’s score in the final evaluation on basketball teaching learning is 81.65.
According to the membership grade formula:
\[ VL_i(x) = \begin{cases} 
0, & x \leq a, \\
\frac{x-a}{b-a}, & a < x < b, \\
\frac{c-x}{d-c}, & c \leq x < d, \\
0, & x > d. 
\end{cases} \] (8)
substitute final score of U into it and solve the membership grade of evaluation, i.e.,

References