Quadratic polynomial fitting of total energy of null subcarriers in underwater acoustic OFDM communication

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Abstract

Doppler scale estimation is one key module in underwater acoustic (UWA) orthogonal frequency division multiplexing (OFDM) communication. In this paper, null subcarrier based blind Doppler scale estimation is considered, which is an extremal problem. The cost function is constructed of the total energy of null subcarriers through DFT. The frequencies of null subcarriers are calculated according to non-uniform Doppler shifts at each tentative scaling factor. Then it is proofed that the cost function can be fitted as a quadratic polynomial near the global minimum. So the accurate location of the global minimum can be achieved through polynomial interpolation. This theory is a basement of developing new methods of Doppler scaling estimation and is verified through an experiment conducted in shallow water.

Keywords: orthogonal frequency division multiplexing (OFDM), underwater acoustic (UWA) communication, polynomial interpolation, Doppler scale estimation

1 Introduction

Underwater acoustic (UWA) orthogonal frequency division multiplexing (OFDM) communication has been under extensive investigation in recent years [1-5]. Unlike the radio channel, which has relative short delay spread and slow time variation, underwater acoustic channels suffer from exhibit long delay spread and fast time variation. The latter brings significant Doppler effects to UWA communication systems, so estimation of the Doppler scaling factor is of critical importance [6-9].

As an extension of the blind carrier frequency offset (CFO) estimation method [10], using the energy on the null subcarriers to find the best fit becomes a popular Doppler scaling factor estimation method in UWA OFDM communication [11-15]. The key is an extremal problem on the cost function constructed of energy of the null subcarriers. In [11] a two-step approach was used. First resample the received signal according to the Doppler scaling factor roughly measured by preamble and postamble, followed by resolution of residual Doppler, which is considered to be uniform. The rough measurement leads to a poor real-time performance and residual Doppler could not be considered to be uniform when the carrier frequency is low to achieve long range communication. In [12] the total energy of frequency measurements at null subcarriers of the block resampled with different tentative scaling factors and huge computational complexity is need. As an improvement, in [15] the cost function is sampled sporadically to find the rough place of the global minimum, and then get an accurate estimation by the method of steepest descent.

In this paper, the cost function is constructed of the total energy of null subcarriers through DFT. The frequencies of null subcarriers are calculated according to non-uniform Doppler shifts at each tentative scaling factor. DFT is more convenient and with much less computational complexity than resampling. Then the cost function is investigated and proofed that it can be fitted as a quadratic polynomials near the global minimum. So the accurate location of the global minimum can be achieved through polynomial interpolation, as a new method of Doppler scaling estimation. To verify the theory, an experiment was conducted in shallow water, whose results confirm its validity.

2 Cyclic prefix OFDM with non-uniform Doppler shifts

Let denote the OFDM symbol duration and T_g the cyclic prefix. The total OFDM block duration is $T + T_g$.

The frequency spacing is $\Delta f = 1/T$. The k -th subcarrier is at the frequency

$$f_k = f_c + k\Delta f \, , k = -\frac{K}{2}, \cdots, \frac{K}{2} - 1, \tag{1}$$

where f_c is the carrier frequency and K subcarriers are used so that the bandwidth is $B = K\Delta f$.

Consider one CP-OFDM block. Let $d(k)$ denote the information symbol to be transmitted on the k-th subcarrier. The non-overlapping sets of active subcarriers S_A and null subcarriers S_N satisfy $S_A \cup S_N = \{-K/2, \cdots, K/2 - 1\}$. The transmitted signal in passband is then given by

$$s(t) = \left[\sum_{k \in S_A} d(k) e^{jk2\pi\Delta f t} \right] e^{j2\pi f_c t} \, , t \in \left[-T_g, T \right] . \tag{2}$$

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Consider a multipath underwater channel that has the impulse response

$$c(\tau,t)=\sum_p A_p(t)\delta\left(\tau-\tau_p(t)\right), \tag{3}$$

where $A_p(t)$ is the path amplitude and $\tau_p(t)$ is the time-varying path delay. To develop our receiver algorithms, this paper adopt the following assumptions.

A1) All paths have a similar Doppler scaling factor a such that

$$\tau_p(t)\approx\tau_p-at. \tag{4}$$

In general, different paths could have different Doppler scaling factors. The method proposed in this paper is based on the assumption that all the paths have the same Doppler scaling factor. When this is not the case, part of useful signals is treated as additive noise, which could increase the overall noise variance considerably. However, we find that as long as the dominant Doppler shift is caused by the direct transmitter/receiver motion, as it is the case in our experiments, this assumption seems to be justified.

A2) The path delays $\tau_p(t)$, the gains $A_p(t)$, and the Doppler scaling factor a are constant over the block duration.

The received signal in passband is then

$$\tilde{y}(t)=\sum_p A_p\sum_{k\in S_A}d(k)e^{jk2\pi\Delta f\left(t+at-\tau_p\right)}e^{j2\pi f_c\left(t+at-\tau_p\right)}+\tilde{n}(t), \tag{5}$$

where $\tilde{n}(t)$ is the additive noise.

Base on the expression in (5), each subcarrier experiences a Doppler-induced frequency shifts $(f_c+k\Delta f)a$, which depends on the frequency of the subcarrier. Since the bandwidth of the OFDM signal is comparable to the center frequency, the Doppler-induced frequency shifts on different OFDM subcarriers differ considerably; i.e., the narrow-band assumption does not hold.

3 Quadratic polynomial fitting of total energy of null subcarriers

The total energy of the null subcarriers is used as the cost function. Assume that coarse synchronization is available from the preamble. After truncating each CP-OFDM block from the received signal, CP is removed. The energy of null subcarriers whose frequency is measured according to tentative scaling factors are achieved by DFT as in (6).

$$Y(\gamma_\kappa)=\frac{1}{T}\int_0^T\tilde{y}(t)e^{-j2\pi(f_c+\kappa\Delta f)(1+a+\hat{a})t}dt,\kappa\in S_N\ , \tag{6}$$

where $\gamma_\kappa=(f_c+\kappa\Delta f)(1+a+\hat{a})$ is frequency of null subcarrier, $\tilde{a}=a+\hat{a}$ is a tentative scaling factor, a is the Doppler scale factor, $\hat{a}$ is the deviation of the Doppler scale factor and the tentative scaling factor.

The sum of the energy of null subcarriers is used as the cost function for the Doppler scale estimation.

$$J(\tilde{a})=\sum_{k\in S_N}\left|\frac{1}{T}\int_0^T e^{-j2\pi(f_c+k\Delta f)(1+\tilde{a})t}\tilde{y}(t)dt\right|^2, \tag{7}$$

$$\bar{a}=arg\ min J(\tilde{a})\ , \tag{8}$$

where $\bar{a}$ is the estimation of a.

The Equation (6) can be transformed as follows:

$$\begin{aligned}Y(\gamma_\kappa)&=\frac{1}{T}\int_0^T\sum_p A_p\sum_{k\in S_A}d(k)e^{j2\pi(f_c+k\Delta f)(t+at-\tau_p)}\\ &e^{-j2\pi(f_c+\kappa\Delta f)(1+a+\hat{a})t}dt+\frac{1}{T}\underset{a\ll1,(f_c+\kappa\Delta f)\hat{a}T\ll1}{=}-\\ &\frac{(f_c+\kappa\Delta f)\hat{a}T}{1+a}\sum_p A_p\sum_{k\in S_A,k\neq\kappa\pm1}\\ &d(k)e^{-j2\pi(f_c+k\Delta f)\tau_p}+\frac{1}{T}\int_0^T\tilde{n}(t)e^{-j2\pi\gamma t}dt\\ &=\alpha_\kappa\hat{a}+\beta_\kappa\end{aligned}, \tag{9}$$

where $\beta_\kappa=\frac{1}{T}\int_0^T\tilde{n}(t)e^{-j2\pi\gamma t}dt$,

$\alpha_\kappa=-\frac{(f_c+\kappa\Delta f)T}{1+a}\sum_p A_p\sum_k d(k)e^{-j2\pi(f_c+k\Delta f)\tau_p}$.

Then the cost function is:

$$\begin{aligned}J(\tilde{a})&=\sum_{\kappa\in S_N}Y(\gamma_\kappa)Y^*(\gamma_\kappa)=\\ &\sum_{\kappa\in S_N}(\alpha_\kappa\hat{a}+\beta_\kappa)(\alpha_\kappa\hat{a}+\beta_\kappa)^*=\\ &\hat{a}^2\sum_{\kappa\in S_N}|\alpha_\kappa|^2+\hat{a}\sum_{\kappa\in S_N}\alpha_\kappa\beta_\kappa^*+\beta_\kappa\alpha_\kappa^*+\sum_{\kappa\in S_N}|\beta_\kappa|^2\end{aligned}. \tag{10}$$

Considering the irrelevance of signal and noise, $\sum_{\kappa\in S_N}\alpha_\kappa\beta_\kappa^*+\beta_\kappa\alpha_\kappa^*\to0$ when κ is large enough, thus $J(\tilde{a})$ becomes

$$\begin{aligned}J(\tilde{a})&=\hat{a}^2\sum_{\kappa\in S_N}|\alpha_\kappa|^2+\sum_{\kappa\in S_N}|\beta_\kappa|^2=\\ &(\tilde{a}-a)^2\sum_{\kappa\in S_N}|\alpha_\kappa|^2+\sum_{\kappa\in S_N}|\beta_\kappa|^2\end{aligned}. \tag{11}$$

From Equation (11) it can been seen that the cost function is a quadratic polynomial of the tentative scaling factor $\tilde{a}$, and minimized when $\tilde{a}=a$.

There are two conditions need in the derivation of (9):

$$a\ll1, \tag{12a}$$

$$(f_c+\kappa\Delta f)\hat{a}T\ll1\Rightarrow\hat{a}\ll\frac{1}{\left(\frac{f_c}{\Delta f}+\kappa\right)}\ . \tag{12b}$$

In UWA communication, the Doppler scale factor is normally about the order of 10^{-3}, so (12a) is always meted. The condition (12b) indicates that the cost function can be fitted by a quadratic polynomials only when the tentative scaling factor being limited in a small range around the Doppler scale factor.

Based on (11), a Doppler scale factor estimating method can be proposed. First find the fitting function with several sample of the cost function around its minimum, and then calculate the exact position of the minimum through the fitting quadratic polynomials function as the estimation of the Doppler scale factor.

4 Experimental results

An experiment was conducted in Lianhua lake of Heilongjiang province in September, 2010. The water depth was around 40 m, the transmitter was located at a depth of about 5 m and the receiver submerged at a depth of about 7 m. The receiver boat was anchored and the transmitter boat could move around. The range between the receiver boat and the transmitter boat was 2~3 km. OFDM signals were transmitted while transmitter boat was moving towards the receiver boat in the first voyage and away from in the second voyage.

The bandwidth of the OFDM signal is 4 kHz, and the carrier frequency is 6 kHz. The transmitted signal thus occupies the frequency band between 4 and 8 kHz. CP-OFDM with a CP of 85.3 ms per OFDM block. The number of subcarriers is 341. The subcarrier spacing is 11.72 Hz, and the OFDM block duration is 85.3 ms. QPSK modulation is used. Block-type pilot is adopted and a null subcarrier is inserted in every 4 subcarriers. So the number of active subcarriers is 256 and the number of null subcarriers is 85, as illustrated in Figure 1. Every frame transmitted contains 50 blocks. The transmitter is moving at a speed of up to 5 kn, at which the Doppler shifts of 8 kHz is 13.36 Hz, which is larger than the OFDM subcarrier spacing.

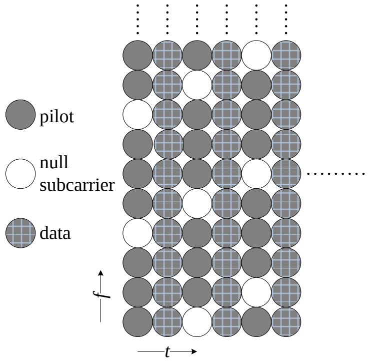

FIGURE 1 Illustration of pilot and null subcarrier pattern

Figure 2 depicts the cost function of one OFDM block. Three samples are made around the minimum, according to which a quadratic polynomials function is fitted. Zoom in Figure 2 to see the details as in Figure 3. The cost function $J(\tilde{a})$ has several minimums but just one unique global minimum, and can been fitted as a quadratic polynomial function around the global minimum. The position of the global minimum is the Doppler scale factor. Figure 4 depicts the errors between the cost functions and the corresponding fitting functions of different OFDM blocks. The error is expressed as $\left(J_p(\tilde{a}) - J_c(\tilde{a})\right)/J_c(\tilde{a})$, where $J_p(\tilde{a})$ is the quadratic polynomial function of (11) and $J_c(\tilde{a})$ is the cost function of (7). In this experiment, the area of accurate fitting is about $[-1.5\times10^{-4},\ 1.5\times10^{-4}]$, which is matched with (12*b*).

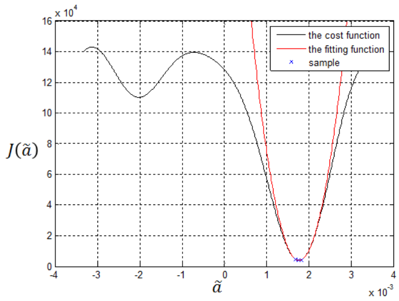

FIGURE 2 The fitting of the cost function of one OFDM block

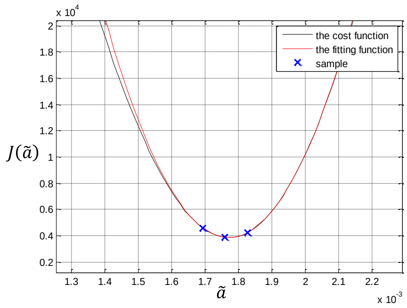

FIGURE 3 Zoom in around the global minimum

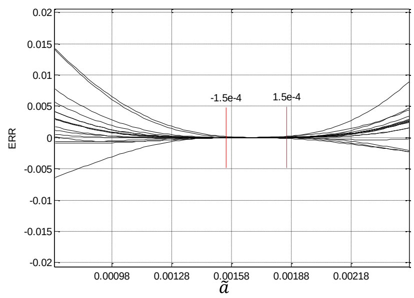

FIGURE 4 The errors between the cost functions and the corresponding fitting functions of different OFDM blocks

5 Conclusions

In this paper, a new cost function of total energy of null subcarriers measurement method is proposed in UWA channels with non-uniform Doppler shifts, which is of less computational complexity. The cost function has several minimums but just one unique global minimum whose position is the Doppler scale factor, and can been fitted as a quadratic polynomial function around the global minimum. To verify the theory, an experiment was conducted in shallow water, whose results confirm its validity.

Based on this theory, new methods of Doppler scaling estimation through polynomial interpolation will be investigated in future research, to develop simple and effective Doppler scaling estimation for UWA OFDM communication systems.

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