Deformation forecasting with a novel high precision grey forecasting model based on genetic algorithm

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Abstract

The precision of prediction of grey forecasting model depends on the conformation of background vale and the selection of the initial condition. Existent literatures optimized grey forecasting model just from one side, respectively. Therefore, a novel model named BIGGM (1,1) is proposed in this paper by integrated optimizing background value and initial condition. In addition, genetic algorithm has also been integrated into the new model to solve the optimal parameter estimation problem. An illustrative example of deformation of Lianzi cliff dangerous rock along the Yangtze River in china is adopted for demonstration. Results show that the BIGGM (1,1) model can increase the prediction accuracy, and it is suitable for use in modelling and forecasting of deformation.

Keywords: GM (1,1) model, background value, initial condition, genetic algorithm, integrated optimization, deformation forecasting

1 Introduction

Deformation monitoring (also referred to as Deformation survey) is the systematic surveying and tracking of the distortion in the shape or dimensions of an object (such as bridges, dams, landslides, high rise buildings, etc.) as a result of stresses induced by applied loads or factors such as changes of ground water level, tectonic phenomena, tidal phenomena, etc. Deformation monitoring is a major component of logging measured values that may be used to for further computation, deformation analysis, predictive maintenance and alarming. Deformation forecasts play an important role in protecting the safety of people's lives and property. Such forecasts results of deformation data is a main basis for decision-making, its quality can directly influence the effect of the whole monitoring work. Therefore, increasing the accuracy of forecasts of deformation is an important issue [1].

Numerous researches have studied the deformation forecasting for a quite long time, they developed various models, including the linear regression model, time series model, artificial neural network (ANNS), etc. Since these methods are easy to use and accurate, they have been used in a wide range of applications. However, these methods have limitations. For instance, the linear regression model assumes that variables are independent and that samples have normal distribution. This model, therefore, requires a larger number of samples. The time series model requires stable trends and patterns of historical data. ANNS demands a great amount of training data and relatively long training period for robust generalization, and the hidden layers in ANNS are difficult to explain [2-5].

Meantime, owing to the complexity of deformation system, only part of the system structure can be fully realized. Therefore, based on the character of deformation system and the advantage of the grey model, this study applies the grey forecasting model to forecast deformation [1]. A deformation monitoring point in Lianzi cliff dangerous rock along the Yangtze River of Hubei province in china is chosen as the research object to explore the applicability of various grey forecasting methods.

The grey system theory was first proposed by Professor Deng [6]. The theory is mainly deals with systems that are characterized by poor information or for which information is lacking. Generally, the grey model is written as GM (m, n), where m is the order and n is the number of variables of the modelling. The first-order one-variable grey model GM (1,1) is most widely employed in various fields and achieved satisfactory results [6, 7].

In order to make the grey forecasting model more precise, a large number of researchers concentrate upon improvements of GM (1,1) model mainly in four aspects [8-13]:

1) improvement of data modelling approach to reduce the variability of data to effectively improve prediction accuracy;
2) application of optimal approach to estimate development coefficient and grey input to improve prediction equation;
3) optimization the initial condition of GM (1,1) model;
4) combination of the GM (1,1) model and the advantages of other forecasting models to increase the accuracy of forecasts.

These types of improvements for GM (1, 1) model can increase prediction precision in some practical applications. However, there still exists some space to improve it. A novel high precision model is proposed to improve its precision by the following three aspects.

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Firstly, the error term resulted from the calculation of background value in the conventional grey forecasting model is discussed, and then an integration term is used to substitute the original calculation of background value to eliminate the error term. Second, based on latest information priority principle, we adopt the *n-th* tem of $\underline{x}(1)$ as the initial condition of the grey differential model to increase prediction precision. Third, genetic algorithm (GA) used for search development coefficient and grey input.

The remaining paper is organized as follows. A brief introduction to the traditional GM (1,1) model is presented and discussed. Based on the concept in Section 2, a novel grey forecasting model named BIGGM (1,1) is proposed in Section 3.Section 4 will illustrate the application of BIGGM (1,1) by a numerical example. Finally, Section 5 concludes this paper.

2 Introduction to the traditional GM (1, 1) model

Grey system theory was first proposed by professor Deng in 1982. From then on the grey model (GM) has been concentrated on by a large number of scholars and adopted in various fields. GM mainly focuses on such systems as partial information known and partial information unknown.GM (1, 1) model is constructed as follows [6]:

Step 1: Establish the one-time AGO series.

Supposing surveying a certain observation point at n different times, thus forms a time observation series $x^{(0)}=\{x^{(0)}(i), i=1,2,3\cdots n\}(n>4)$. $x^{(0)}$ is a non-negative sequence of raw data, where $x^{(0)}(i)$ is the datum at *i-th* time and n is the total number of modelling data. $x^{(1)}(k)$ is the generated datum of $x^{(0)}(i)$ and can be obtained by:

$$x^{(1)}(k)=\sum_{i=1}^{k} x^{(0)}(i),(k=1,2,...,n) . \tag{1}$$

Then, it alters the raw data into data with, which an exponential curve can be produced and it's effective for decreasing noise influence.

Step 2: Define the grey parameters.

$x^{(1)}(k)$ can be modelled by a first order differential equation given as:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=u , \tag{2}$$

where, parameter a is called the developing coefficient and u is called the grey input.

Step 3: Estimate the values of a and u.

In fact, parameters a and u cannot be calculated directly from Equation (2). For practice, the algorithm of GM (1,1) is used first to obtain their approximate values through its difference equation:

$$x^{(0)}(k)+az^{(1)}(k)=u , \tag{3}$$

where $z^{(1)}(k)$ is the background value of the *k-th* datum and defined as:

$$z^{(1)}(k)=\frac{1}{2}(x^{(1)}(k)+x^{(1)}(k-1)) . \tag{4}$$

Then, values of grey parameters a and u can be estimated by using least square (LS) method as:

$$(a,u)^T=(A^T A)^{-1}A^T B , \tag{5}$$

where $A=\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}$, $B=(x^{(0)}_{(2)}, x^{(0)}_{(3)}, \cdots, x^{(0)}_{(n)})^T$.

Step 4: Define the predicting model.

Substituting a and u in Equation (3) with Equation (5), the approximate equation becomes the following:

$$\hat{x}^{(1)}(k+1)=\left(x^{(0)}(1)-\frac{u}{a}\right)e^{-ak}+\frac{u}{a} , \tag{6}$$

where $\hat{x}^{(1)}(k+1)$ is the forecasted value at the time $(k+1)$. After the completion of an inverse accumulate generating operation on Equation (6) the predicted value of the primitive data of $\hat{x}^{(0)}(k+1)$ can be obtained as:

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k) . \tag{7}$$

3 The proposed GM (1, 1) model

3.1 OPTIMIZATION OF THE BACKGROUND VALUE

According to the description of the traditional GM (1,1) model in Section 2 we can know that the prediction accuracy depends on the development coefficient a and grey input u, and the solution a and u depends on the structure of background value $z^{(1)}(k)$. Thus, background value $z^{(1)}(k)$ has direct influences on the precision of GM(1,1), as shown in Equation (4) it is a smoothing formula, Equation (4) is chosen to describe the background value for a very short period of time Δ*t*, however, being a short period of time, Δ*t* is only a relative conception. In this period of time, for a deformable body, it may include mutations or the deformation does not occur at an average rate, under this situation, GM (1,1) model often performs very poor and makes delay errors, so the error term resulted from the original calculation of background value needs to be eliminated [8-10].

Take the integral of both sides of the grey differential Equation (2) in the interval [*k*–1, *k*], we can get:

$$\int_{k-1}^{k}\frac{dx^{(1)}(t)}{dt}dt+a\int_{k-1}^{k}x^{(1)}(t)dt=u . \tag{8}$$

By comparing Equation (3) with Equation (8), it can be found that usage of $\int_{k-1}^{k} x^{(1)}(t)dt$ as the background value is more adaptive than Equation (4). The error term will exist when the value of Equation (4) is not equal to the $\int_{k-1}^{k} x^{(1)}(t)dt$. So how to obtain the accurate value of $\int_{k-1}^{k} x^{(1)}(t)dt$ is the key factor to enhance the performance of GM(1,1).

In order to eliminate the error term resulted from the original calculation of background value, then the $\int_{k-1}^{k} x^{(1)}(t)dt$ is directly taken as the background value:

$$z^{(1)}(k) = \int_{k-1}^{k} x^{(1)}(t)dt \,. \tag{9}$$

Owing to Equation (2) gives the result of the exponential function, so in this paper, set:

$$x^{(1)}(t) = Ae^{Bt} + C \,, \tag{10}$$

where *A*, *B* and *C* are constants. It is substituted in Equation (9), and use *k*, *k*–1, *k*–2 to substitute *t* in Equation (10), we can get:

$$x^{(1)}(k) - x^{(1)}(k-1) = A \cdot e^{Bk} - A \cdot e^{B(k-1)} \,, \tag{11}$$

$$\begin{aligned} &x^{(1)}(k-1) - x^{(1)}(k-2) \\ &= A \cdot e^{B(k-1)} - A \cdot e^{B(k-2)} \,. \end{aligned} \tag{12}$$

Calculations:

$$\begin{aligned} &\frac{x^{(1)}(k) - x^{(1)}(k-1)}{x^{(1)}(k-1) - x^{(1)}(k-2)} = \frac{Ae^{B(k-1)}(e^B - 1)}{Ae^{B(k-2)}(e^B - 1)} \\ &= e^B = \frac{x^{(0)}(k)}{x^{(0)}(k-1)} \end{aligned} ,$$

$$B = \ln x^{(0)}(k) - \ln x^{(0)}(k-1) \,.$$

Substitution of *B* into $x^{(1)}(k), x^{(1)}(k-1)$, we can obtain *A* and *C:*

$$A = \frac{x^{(0)}(k)}{(\frac{x^{(0)}(k)}{x^{(0)}(k-1)})^{k-1} \cdot (\frac{x^{(0)}(k)}{x^{(0)}(k-1)} - 1)} \,,$$

$$C = x^{(1)}(k) - \frac{x^{(0)}(k)^2}{x^{(0)}(k) - x^{(0)}(k-1)} \,.$$

Thus, we can obtain the optimized background value as:

$$\begin{aligned} z^{(1)}(k) = &\frac{x^{(0)}(k)}{\ln x^{(0)}(k) - \ln x^{(0)}(k-1)} + \\ &x^{(1)}(k) - \frac{x^{(0)}(k)^2}{x^{(0)}(k) - x^{(0)}(k-1)} \end{aligned} \,. \tag{13}$$

3.2 OPTIMIZATION OF THE INITIAL CONDITION

In the real world, the developments of a deformation system are always instable. So, under such situation, the author suggests paying more attention on the latest datum rather than on other previous data because the latest datum can offer more useful information about development tendency of deformation.

However, in the above-mentioned algorithm of GM(1,1), the initial condition is set as $x^{(1)}(1)$. In fact, from the description of the original GM (1,1) model we can find that the general solution of the Equation (2) can be expressed as following [11-13]:

$$x^{(1)}(t) = ce^{-at} + \frac{u}{a} \,, \tag{14}$$

where *c* is a constant.

For the general solution of we can let *t* = 1 and *t* = *n*, respectively, then we can get the following equations:

$$x^{(1)}(1) = ce^{-a} + \frac{u}{a} \,, \tag{15}$$

$$x^{(1)}(n) = ce^{-an} + \frac{u}{a} \,. \tag{16}$$

Then we can obtain:

$$c = \left(x^{(1)}(1) - \frac{u}{a} \right) e^{a} \text{ and } c = \left(x^{(1)}(n) - \frac{u}{a} \right) e^{an} \,.$$

When $t = 1$, which is the originally initial condition proposed by professor Deng. This way will pay more importance on the farthest datum of modelling data, so the use of the latest datum may not be enough. Therefore, GM (1,1) model cannot have enough ability to catch the instable variance. Hence, this paper, suggests setting the initial condition as $x^{(1)}(n)$ to fully catch the latest tendency. Thus, the specified solution would be:

$$\hat{x}^{(1)}(k+1) = (x^{(1)}(n) - \frac{u}{a}) e^{-a(k-n+1)} + \frac{u}{a} \,. \tag{17}$$

3.3 GA BASED OPTIMIZED GM (1, 1) MODEL

From above discussion, in this paper we take advantage of the ability of the genetic algorithm (GA) to solve optimization problems. This article builds a GA based GM (1,1) model, termed as BIGGM (1,1) model, by combining GA and optimization integrated background value with initial condition to estimate the parameters a and u, then, enhance prediction ability.

Genetic algorithm is a global random search technique invented by Holland in 1975, which is on the basis of Darwin's theory of nature evolutions and the theory of genetic mutations. It is especially useful for complex optimization problems where the number of parameters is large and the analytical solutions are difficult to obtain. Therefore, GA provides a common framework that is suitable for solving the optimization problems in complex systems, and it does not depend on the problem fields in itself [14-18].

To make the GM (1,1) more adaptive and precise, we use the GA to find the optimal parameters a and u of the BIGGM (1,1) model. The steps of BIGGM (1,1) can be shown as follows.

Steps 1: Same as the traditional GM (1,1) model.

Steps 2: Calculate background values through Equation (13) in Section 3.1.

Steps 3: Search the optimal parameters a and u of BIGGM (1,1) by using GA. The procedure is as follows.

1) Define the fitness function. In GA, each chromosome is decoded as network parameter. Input the training samples and calculate the fitness of each individual. From the previous studies, the MAPE (mean absolute percentage error, MAPE) is used as the fitness function.

2) Roulette selection. The roulette selection method is applied for the selection operation in BIGGM (1, 1). Selection is a process which selects the individuals with higher fitness to the next population.

3) Adaptive crossover. Crossover is carried out by recombining genetic material in two father individuals to produce two child chromosomes that share the characteristics of their parents. Crossover operation can produce fresh individuals, so some new points in the searching space can be checked. The frequency is determined by the probability in crossover operation, the higher is the frequency, and the faster will be the convergence speed. But a high frequency will cause genetic algorithm degenerate to be a stochastic search, and probably result in premature convergence that a local optimum solution is reached. The common ranges of crossover rate P_c are 0.01-0.1. In this paper set crossover rate $P_c = 0.5$.

4) Mutation. Mutation is implemented by the conversion of a child individual with a minor probability, the probability of conversion is inversely proportional to the number of variable, and has nothing to do with the size of population. The mutation operator provides a way of recovery of the loss of genetic diversity. The common ranges of mutation rate P_m are 0.01-0.1. In this paper set mutation rate $P_m = 0.02$.

5) Terminal condition. The terminal condition of the optimal procedure is checked at the end of each generation and the process is terminated when the condition is satisfied. Two termination criteria can be chosen: maximum generation criterion and setting boundary criterion. We select the first way as termination criteria, namely, set the maximum generation $T = 1000$.

By using the GA, the optimal parameters a and u are obtained based on the criterion of minimum MAPE. The development coefficient and grey input becomes $\tilde{a}$ and $\tilde{u}$.

Steps 4: Setting the initial condition of the grey differential model as $x^{(1)}(n)$.

Steps 5: The forecasting equation of the BIGGM (1,1) model is stated as:

$$\tilde{x}^{(1)}(k+1) = (x^{(1)}(n) - \frac{\tilde{u}}{\tilde{a}})e^{-a(k-n+1)} + \frac{\tilde{u}}{\tilde{a}} . \tag{18}$$

Steps 6: Taking the IAGO on series $\tilde{x}^{(1)}(k+1)$ we get:

$$\tilde{x}^{(0)}(k) = \tilde{x}^{(1)}(k+1) - \tilde{x}^{(1)}(k), k = 1,2,...,n . \tag{19}$$

4 Illustrative examples

In this section, in order to demonstrate the effectiveness of the proposed model, we use the deformation of Lianzi cliff dangerous rock along the Yangtze River of Hubei province in china as an illustrating example. Lianzi cliff is one of the most dangerous rocks along Yangtze River, situated in Zigui county of Hubei province in china, and geodetic surveying method used as the most effective ways to monitoring its deformation. In this study, we use the historical deformation of Lianzi cliff dangerous rock from 1978 to 1993 as our research data. There are 10 observations, where 1978 -1987 is used for model fitting and 1988-1993 are reserved for testing. For the purpose of comparison, four forecasting models are used as follows:

1) GM (1,1) forecasting model, termed as GM (1,1),

2) Background value improved GM (1,1) model, termed as BGM (1,1),

3) Initial condition improved GM (1,1) model, termed as IGM (1,1),

4) BIGGM (1,1) model.

In order to examine the precision of grey model, error analysis is necessary to understand the difference between modelled value and actual value. Generally, two criteria are used to validate the GM (1,1) model. The first one is the absolute mean error criterion:

$$AME = \frac{1}{n}\sum_{k=1}^{n} | x^{(0)}(k) - \hat{x}^{(0)}(k) | \tag{20}$$

and the second one is the mean absolute percentage error criterion:

$$MAPE = \frac{1}{n}\sum_{k=1}^{n}\frac{|x^{(0)}(k) - \hat{x}^{(0)}(k)|}{x^{(0)}(k)}. \quad (21)$$

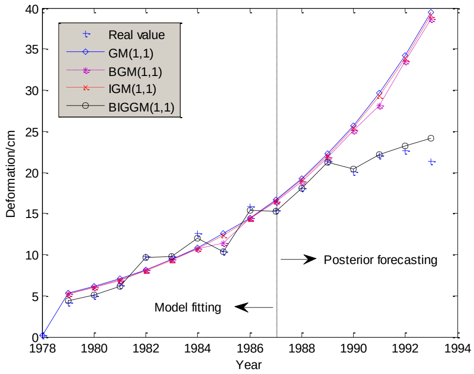

FIGURE 1 Real values and modelling values together with forecasts of deformation by four models

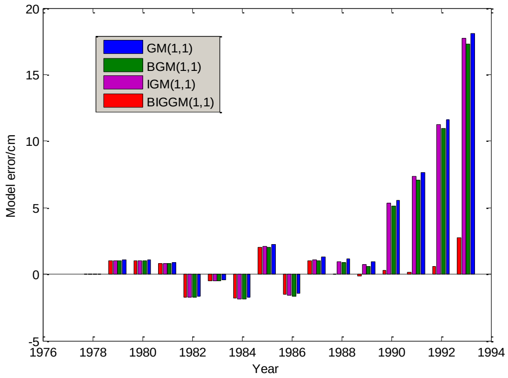

FIGURE 2 Model error distribution from 1978 to 1993

The predicted results obtained by the four models are shown in Figure 1. The model error distribution is also shown in Figure 2. For the purpose of comparison, Table 1 demonstrates that the performance of four models. According to the results shown above, our proposed BIGGM (1,1) model seems to obtain the lowest forecasting errors among these models. So we can say that the BIGGM (1,1) model is an appropriate model for deformation forecasting.

TABLE 1 Performance evaluations of four models

	GM (1,1)	BGM (1,1)	IGM (1,1)	BIGGM (1,1)
AME (1978-1987)	1.313	1.129	1.306	0.217
MAPE (1978-1987)	15.081	13.068	14.811	2.237
AME (1988-1993)	7.504	6.741	7.213	0.613
MAPE (1988-1993)	34.872	31.285	33.494	2.851

5 Conclusions

From the results in this study, the following three conclusions can be drawn,

1) The first main contribution of the paper is to modify the calculation algorithm of background value of GM (1,1). A new integration equation is used to substitute the original calculation of background value to eliminate the error term, such that the precision of prediction can be apparently increased.

2) The second main contribution is based on latest information priority principle, we adopt the *n-th* term of $x(1)$ as the initial condition of the grey differential model to increase prediction precision.

3) The third main contribution is proposed a new model named BIGGM (1,1). The BIGGM (1,1) model has the advantage of grey forecasting model and genetic algorithm; it performs well concerning tendency data. When there are large variations in the data set that will decrease forecasting accuracy. However, no matter what kind of data, the forecasting performance of BIGGM (1,1) model is better than the original GM (1,1) in model forecasting. Therefore, it is a suitable method for forecasting problems with small data sets.

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