Basic set of equations in mesoscale meteorological model associated with local fractional derivative operators involving the cantorian and cantor-type spherical coordinates

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Received 1 March 2014, www.tsi.lv

Abstract

Recently, the local fractional calculus theory was applied to process the non-differentiable phenomena in fractal domain. The main object of this paper is to present the basic set of equations in mesoscale meteorological model on the Cantor sets involving local fractional derivative operators and the corresponding cantor-type spherical coordinate equations. It is shown that these equations are efficient and accurate for describing some of atmospheric motion.

Keywords: Mathematic, basic set of equations, mesoscale meteorological model

1 Introduction

Recently, the local fractional calculus theory was applied to process the non-differentiable phenomena in fractal domain (see [6–11] and the references cited therein). There are some local fractional models, such as the local fractional Fokker-Planck equation [6], the local fractional stress-strain relations [7], the local fractional heat conduction equation [11], wave equations on the Cantor sets [12], and the local fractional Laplace equation [20].

In the different process of atmospheric motion, many problems of dynamics had already been described by a number of classic basic equations, the description of these equations establish on the preferable smoothness. However, in the different process of atmospheric motion, it exists both large scale turbulence and small scale turbulence. Therefore, the problems of dynamics is no longer can be describe by classic basic equation due to the turbulent flows may be of fractal character. Meanwhile, it has relevance between the fractional order calculus and fractal. Thus, it can use the operator with fractional order of gradient, divergence etc. To generalize the atmospheric dynamic equation in the corresponding fractal form.

The main aim of this paper is present in the mathematical structure of the basic set of equations in mesoscale meteorological model in local fractional derivative and to propose their forms in the Cantor-type spherical coordination.

2 Mathematic Tools

2.1 IN THE CARTESIAN COORDINATE SYSTEM

In this part, we will introduce the local fractional derivative. It is need to bring in local fractional derivative, which define as:

$$D_{0+}^{\alpha} f(x_0) = \left.\frac{d^{\alpha} f(x)}{dx^{\alpha}}\right|_{x=x_0} = \lim_{x\to x_0}\frac{\Delta^{\alpha}[f(x)-f(y)]}{(x-x_0)^{\alpha}}, \tag{1}$$

where in,

$$\Delta^{\alpha}\left(f(x)-f(x_0)\right) \cong \Gamma(1+\alpha)\Delta\left(f(x)-f(x_0)\right). \tag{2}$$

For a function of three variables, the vector form can be written in the form Ref. [2]

$$\gamma(x,y,z) = L(x,y,z)e_1^{\alpha} + M(x,y,z)e_2^{\alpha} + N(x,y,z)e_3^{\alpha}. \tag{3}$$

Let $u(x,y,z) = u_1(x,y,z)e_1^{\alpha} + u_2(x,y,z)e_2^{\alpha} + u_3(x,y,z)e_3^{\alpha}$ a local fraction vector field and $\varphi(x,y,z)$ is a local fractional scalar field, the computing rules of Hamilton operator are valid as follows Ref. [2].

(1) The local fractional gradient operator defined as [2]:

$$\nabla^{\alpha}\varphi = \frac{\partial^{\alpha}\varphi}{\partial x_1^{\alpha}}e_1^{\alpha} + \frac{\partial^{\alpha}\varphi}{\partial x_2^{\alpha}}e_2^{\alpha} + \frac{\partial^{\alpha}\varphi}{\partial x_3^{\alpha}}e_3^{\alpha}. \tag{4}$$

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The local fractional divergence and curl of local fractional vector field are written in the form [2]:

$$div^{\alpha}u = \nabla^{\alpha}u = \frac{\partial^{\alpha}u_1}{\partial x_1^{\alpha}} + \frac{\partial^{\alpha}u_2}{\partial x_2^{\alpha}} + \frac{\partial^{\alpha}u_3}{\partial x_3^{\alpha}}, \quad (5)$$

$$curl^{\alpha}u = \nabla^{\alpha} \times u = \left(\frac{\partial^{\alpha}u_3}{\partial x_2^{\alpha}} - \frac{\partial^{\alpha}u_2}{\partial x_3^{\alpha}}\right)e_1^{\alpha} + \left(\frac{\partial^{\alpha}u_1}{\partial x_3^{\alpha}} - \frac{\partial^{\alpha}u_3}{\partial x_1^{\alpha}}\right)e_2^{\alpha} + \left(\frac{\partial^{\alpha}u_2}{\partial x_1^{\alpha}} - \frac{\partial^{\alpha}u_1}{\partial x_2^{\alpha}}\right)e_3^{\alpha}. \quad (6)$$

2.2 IN THE SPHERICAL COORDINATE SYSTEM

The spherical coordinate in the three-dimensional space is the form (λ, φ, r), in the classic differential, the gradient is:

$$\nabla = i\frac{1}{r\cos\varphi}\frac{\partial}{\partial\lambda} + j\frac{1}{r}\frac{\partial}{\partial\varphi} + k\frac{\partial}{\partial r}. \quad (7)$$

It can be generalized to the fractional calculus, the gradient of this definition is:

$$\nabla^{\alpha} = i^{\alpha}\frac{1}{r^{\alpha}\cos^{\alpha}\varphi}\frac{\partial^{\alpha}}{\partial\lambda^{\alpha}} + j^{\alpha}\frac{1}{r^{\alpha}}\frac{\partial^{\alpha}}{\partial\varphi^{\alpha}} + k^{\alpha}\frac{\partial^{\alpha}}{\partial r^{\alpha}}. \quad (8)$$

Similarly, it can get the divergence of this definition, like:

$$div^{\alpha}u = \nabla^{\alpha}u = \frac{1}{r^{\alpha}\cos^{\alpha}\varphi}\frac{\partial^{\alpha}u_2}{\partial\lambda^{\alpha}} + \frac{1}{r^{\alpha}\cos^{\alpha}\varphi}\frac{\partial^{\alpha}\left(u_{\varphi}\cos^{\varphi}\varphi\right)}{\partial\varphi^{\alpha}} + \frac{1}{r^2}\frac{\partial^{\alpha}\left(u, r^2\right)}{\partial r} \cdot \quad (9)$$

For above all, the following equations are valid:

$$div^{\alpha}u = \nabla^{\alpha}u = \frac{\partial^{\alpha}u_1}{\partial x_1^{\alpha}} + \frac{\partial^{\alpha}u_2}{\partial x_2^{\alpha}} + \frac{\partial^{\alpha}u_3}{\partial x_3^{\alpha}}, \quad (10)$$

$$\nabla^{\alpha}(uv) = (\nabla^{\alpha}u)v + u\nabla^{\alpha}v. \quad (11)$$

2.3 THE INTERAL OF FRACTIONAL ORDER

(1) The local fractional line integral of the function $u(x_p, y_p z_p)$ in the local fraction vector form.

$$u(x_p, y_p z_p) = u_1(x_p, y_p, z_p)e_1^{\alpha} + u_2(x_p, y_p, z_p)e_2^{\alpha} + u_3(x_p, y_p, z_p)e_3^{\alpha} \cdot \quad (12)$$

Along a fractal line l^{α} is written as [2]:

$$\int_{l^{\alpha}} u(x_p, y_p z,_p)dI^{\alpha} = \lim_{N\to\infty}\sum_{p=1}^{N} u(x_p, y_p, z_p)\Delta I_p^{\alpha}, \quad (13)$$

where for p = 1,2,…,N and N elements of line ΔI_p^{α}, it is required that all $|\Delta I_p^{\alpha}| \to 0$ as $N \to \infty$.

(2) The local fractional surface integral of the given function across a surface $s^{(\beta)}$ is defined as [2]:

$$\iint s^{(\beta)}u(x_p, y_p, z_p)ds^{(\beta)} = \lim_{N\to\infty}\sum_{p=1}^{N} u(x_p, y_p, z_p)n_p\Delta s_p^{(\beta)}, \quad (14)$$

where for p = 1, 2… N and N elements of area $\Delta s_p^{(\beta)}$ with a unit normal local fractional vector n_p, it is required that all $\Delta s_p^{(\beta)} \to 0$ as $N \to 0$.

(3) The local fractional volume integral of the given function in a fractal region $v^{(r)}$ is given by [2]:

$$\iiint_{v^{(r)}} u(x_p, y_p, z_p)dv^{(r)} = \lim_{N\to\infty}\sum_{p=1}^{N} u(x_p, y_p, z_p)\Delta v_p^{r}, \quad (15)$$

where for p = 1,2,…,N and N elements of volume $\Delta v_p^{(r)}$, it is required that all $\Delta v_p^{(r)} \to 0$ as $N \to 0$.

Let us consider a local fractional vector field $u = u_1e_1^{\alpha} + u_2e_2^{\alpha} + u_3e_3^{\alpha}$, the following result hold [27].

(4) Divergence Theorem of local fractional field states that:

$$\iiint_{r^{(\gamma)}} \nabla^{\alpha}udv^{(\gamma)} = \oiint_{s^{(\beta)}} uds^{(\beta)}. \quad (16)$$

3 Peculiar Properties of Fractional Operator

3.1 THE PROPERTY OF MEMORY

The integral of fractional order has a character of memorability. The role of memory functions for compliance retardation and modules relaxation in viscoelastic materials is examined. The complexity of viscoelastic materials that occurs in the linear domain was explained by the influence of modelling these effects using the fractional calculus, such as Heat equation with memory is established, under some general and reasonable conditions in Ref. [17].

At this point, we can use a brief description to explain the memorability of fractional order, take Newtonian equation as an example, for the particle of unit mass,

$$m\frac{dV}{dt} = F(r, v, t) \cdot \quad (17)$$

In this expression, r as displacement vector, v as speed, t as time, transform the expression (14), it can get a new expression like below:

$$m\frac{dv}{dt}=\int_0^t k(t-\tau)F(r,v,t)dt \cdot \tag{18}$$

In the expression, the $k(t-\tau)$ is given kernel function with memory. The characteristic of non-locality in (17) can be obtained from locality. Where in the retardation effects are taken into account, this kind of equations can open new possibilities of understanding the classical mechanics.

3.2 THE PROPERTY OF GENERALIZED FLUX

The traditional diffusion motions use the second order convection - The classical case of Fick's second law equation - to describe, like below:

$$\frac{\partial C(x,t)}{\partial t}=D\frac{\partial^2 C(x,t)}{\partial x^2} \cdot \tag{19}$$

In the expression, $C(x,t)$ is the particle concentration in the space x at the time t; v is convection velocity; D is diffusion coefficient.

The characteristic of Fick's law is local diffusion, Flux J which in the space of a point is proportional to the concentration gradient in a small area, however, The Fick's law don't consider the affection from particle migration by other particle, also, without considering the impact from history.

However, in the complicated system, the particle movement in different time and the particle movement in different spatial point have influence each other, therefore, when it study the particle movement at one time in one spatial point, it is need to consider the influence of particle movement at other time in other spatial point.

Thus, it is need to consider the relevancy from spatially and temporally respectively, but do not consider the spatially and temporally coupling effect of spatially and temporally, then use the limitation processing method to process the traditional second-order diffusion equation in spatially and temporally, and get the fractional order fluxional anomalous diffusion equation.

In case of one-dimensional, the flux expression of limited particle can be modified to the equality of the relation between particle flux and particle concentration [4].

$$\int_0^t d\tau\int_0^x J(x',t)dx'=\int_0^t d\tau\int_0^x k(x,x';t,\tau)C(x',\tau)dx' \cdot \tag{20}$$

In this expression, $k(x,x';t,\tau)$ is the kernel function with the property of diffusion, due to it is not need to consider the coupling interaction of spatial and temporal, the kernel function with the property of diffusion can be express as:

$$k(x,x';t,\tau)=k_x(x,x')k_t(t,\tau). \tag{21}$$

Without loss of generality, we assume that the process of diffusion in space is uniform on statistics and stationary random process on time, so the spatial diffusion kernel function $k_x(x,x')$ is a function of $(x-x')$, and the diffusion kernel function $k_t(t,\tau)$ of time is a function of $(t-\tau)$.

If we assume $k_x(x,x')$, $k_t(t,\tau)$ with the property of negative power law,

$$k_x(x,x')=\frac{D}{\Gamma(-\alpha)}\frac{1}{(x-x')^{\alpha-1}}, \tag{22}$$

$$k_x(t,\tau)=\frac{1}{\Gamma(\gamma)}\frac{1}{(t-\tau)^{1-\lambda}}. \tag{23}$$

In this expression, D, α, γ are constant, $\Gamma(-\alpha)$, $\Gamma(\gamma)$ are gamma function. Substituting the expression (19) and (20) to expression (18), then derivative the t and x of both side of equality sign in expression (18), it can get the expression like below:

$$\begin{aligned} J=D\frac{1}{\Gamma(\gamma)}\frac{\partial}{\partial t}\int_0^t\frac{1}{(t-\tau)^{1-\gamma}}d\tau \\ \frac{1}{\Gamma(-\alpha)}\frac{\partial}{\partial x}\int_0^x\frac{1}{(x-x')^{\alpha-1}}C(x',\tau)dx' \end{aligned}. \tag{24}$$

Further, bring in the definition of the Riemann-Liouville fractional derivative, the result is like below:

$$J={}_0^{RL}D_t^{1-\gamma}\left(D\frac{\partial^{\alpha-1}}{\partial x^{\alpha-1}}C(x,t)\right). \tag{25}$$

Then, we can get the diffusion equation with fractional order,

$$\frac{\partial C(x,t)}{\partial t}={}_0^{RL}D_t^{1-\gamma}\left(D\frac{\partial^{\alpha}}{\partial x^{\alpha}}C(x,t)\right), 1\le\alpha\le 2. \tag{26}$$

Comparing to the classical diffusion equation, the diffusion equation of fractional order has more implications: the property of generalized flux, it can describe the physical phenomenon more accurate.

In turn, we will study the basic set of equation of atmospheric dynamic with fractional order, although these equations already have no application in practice, however, we can apply one of which when simulation and gradually introduce, we may get some useful results.

4 The Cantor-Type of the Basic Set of Equation in Mesoscale Meteorological Model

4.1 COVERSATION OF MASS WITH FRACTIONAL ORDER

In this section, we start with conversation of mass on Cantor Sets with fractional order. The mass of fractal fluid in $v^{(\gamma)}$ is defined through [2].

$$M=\iiint_{v^{(\gamma)}}\rho dv^{(\gamma)}. \tag{27}$$

And we can get:

$$\frac{\partial^{\alpha}M}{\partial t^{\alpha}}=-\oiint_{s^{(\beta)}}\rho vds^{(\beta)}. \tag{28}$$

Using Divergence Theorem of local fractional field, we can get:

$$\begin{aligned}&\iiint_{v^{(\gamma)}}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\rho dv^{(\gamma)}+\oiint_{s^{(\beta)}}\rho vds^{(\beta)}\\&=\iiint_{v^{\gamma}}\left[\frac{\partial^{\alpha}\rho}{\partial t^{\alpha}}+\nabla^{\alpha}(\rho v)\right]dv^{(\gamma)}=0\end{aligned}. \tag{29}$$

This implies that:

$$\frac{\partial^{\alpha}\rho}{\partial t^{\alpha}}+\nabla^{\alpha}(\rho v)=0. \tag{30}$$

This is called continuity equation with fractional order on fractal materials, which also have the formulation (31):

$$\begin{aligned}&\frac{\partial^{\alpha}\rho}{\partial t^{\alpha}}+\nabla^{\alpha}(\rho v)=\frac{\partial^{\alpha}\rho}{\partial t^{\alpha}}+v(\nabla^{\alpha}\rho)+\rho(\nabla^{\alpha}v)\\&=\frac{D^{\alpha}\rho}{Dt^{\alpha}}+\rho(\nabla^{\alpha}v)=0\end{aligned}. \tag{31}$$

Where in the fractal material derivative of the fluid density ρ is expressed as:

$$\frac{D^{\alpha}\rho}{Dt^{\alpha}}=\frac{\partial^{\alpha}\rho}{\partial t^{\alpha}}+v(\nabla^{\alpha}\rho). \tag{32}$$

If the fractal fluid is incompressible, we also get:

$$\frac{\partial^{\alpha}\rho}{\partial t^{\alpha}}+v(\nabla^{\alpha}\rho)=0\Leftrightarrow\frac{D^{\alpha}\rho}{Dt^{\alpha}}=0\text{, or }\nabla^{\alpha}v=0. \tag{33}$$

The corresponding form of the equation of conservation of mass in spherical coordination as below,

$$\begin{aligned}&\frac{d^{\alpha}\rho}{dt^{\alpha}}+\rho\left(\frac{1}{r^{\alpha}\cos^{\alpha}}\frac{\partial^{\alpha}u}{\partial\lambda^{\alpha}}+\frac{1}{r^{\alpha}}\frac{\partial^{\alpha}v}{\partial\varphi^{\alpha}}+\frac{\partial^{\alpha}w}{\partial r^{\alpha}}\right)\\&+\rho\left(-\frac{v}{r^{\alpha}}\tan^{\alpha}\frac{\partial^{\alpha}w}{r^{\alpha}}\right)=0\end{aligned}. \tag{33a}$$

Can simplify as,

$$\begin{aligned}&\frac{\partial^{\alpha}\rho}{\partial t^{\alpha}}+\frac{1}{r^{\alpha}\cos^{\alpha}\varphi}\frac{\partial^{\alpha}\rho u^{\alpha}}{\partial\lambda}+\frac{1}{r^{\alpha}}\frac{\partial^{\alpha}\rho v^{\alpha}}{\partial\varphi^{\alpha}}\\&+\frac{\partial^{\alpha}\varphi w^{\alpha}}{\partial r^{\alpha}}-\frac{\rho v^{\alpha}}{r^{\alpha}}\tan^{2}\varphi+\frac{2\rho w^{\alpha}}{r^{\alpha}}=0\end{aligned}. \tag{33b}$$

4.1.1 Remark

Similarly, we can get the expression like below:

(1) Conservation of heat with fractional order.

$$\frac{\partial^{\alpha}\rho}{\partial t^{\alpha}}+\nabla^{\alpha}(\rho v)=S_{\theta}. \tag{34}$$

In this expression, S_{θ} represent the source and sink of heat.

(2) Conservation of motion with fractional order.

$$\frac{\partial^{\alpha}V}{\partial t^{\alpha}}=-V\nabla^{\alpha}V-\frac{1}{\rho}\nabla^{\alpha}(\rho v)-gk-2\Omega\times V. \tag{35}$$

(3) Conservation equation of energy with fractional order.

Similarly, fractional order energy conservation equation in fractal media can be express like below:

$$\begin{aligned}&\frac{d^{\alpha}(\rho h)}{dt^{\alpha}}+\frac{\partial^{\alpha}(\rho uh)}{\partial x^{\alpha}}+\frac{\partial^{\alpha}(\rho uh)}{\partial y^{\alpha}}+\frac{\partial^{\alpha}(\rho wh)}{\partial z^{\alpha}}\\&=-Pdiv^{\alpha}u+div^{\alpha}(k\Delta^{\alpha}T)+\Phi+s_{n}\end{aligned}. \tag{36}$$

In the expression (36) above, k is the coefficient of thermal conductivity of fluid. s_n is the thermal source in fluid. Φ called dissipation coefficient is the part of which thermal energy transformed from mechanical energy. $Pdiv^{\alpha}u$ is the force to work on the fluid.

4.2 VORTICITY AND VORTICITY AND EQUATION WITH FRACTIONAL ORDER

Vorticity, that is to describe the feature of tiny clumps in air rotative-field. Due to the Earth's rotation, we can often see that the vortex of motion often occur in the process of movement, such as cyclone, anticyclone, typhoon etc.

In non-uniform three dimensional flow field, air tiny clumps at the same time would rotate around the X, Y, Z axis, that is, there are three vorticity components: ζ,η,ξ, the sum of these three vector known as the speed of vorticity in three-dimensional space, We represent it as rotation.

Based on above, we can get the form of speed vorticity with fractional order:

$$rot^{\alpha}V = \zeta e_1^{\alpha} + \eta e_2^{\alpha} + \xi e_3^{\alpha} = \left(\frac{\partial^{\alpha} w}{\partial y^{\alpha}} - \frac{\partial^{\alpha} u}{\partial z^{\alpha}}\right) e_1^{\alpha} + \left(\frac{\partial^{\alpha} u}{\partial z^{\alpha}} - \frac{\partial^{\alpha} w}{\partial x^{\alpha}}\right) e_2^{\alpha} + \left(\frac{\partial^{\alpha} u}{\partial x^{\alpha}} - \frac{\partial^{\alpha} u}{\partial y^{\alpha}}\right) e_3^{\alpha} . \quad (37)$$

Take a further step; we can get the vortex equation with fractional order by using vector operation, which describe the vortex motion.

The vector form of equations of motion with fractional order like below:

$$\frac{d^{\alpha}\vec{V}}{dt^{\alpha}} = g - \frac{1}{\rho}\nabla^{\alpha} p - 2\Omega \times \vec{V} + F . \quad (38)$$

According to

$$\frac{d^{\alpha}V}{d^{\alpha}t} = \frac{\partial V}{\partial t} + \nabla^{\alpha}\left(\frac{V^2}{2}\right) + 2\Omega \times V . \quad (39)$$

Therefore, equation of motion (39) would rewrite as:

$$\frac{\partial^{\alpha}\vec{V}}{\partial t^{\alpha}} + \omega_0 \times \dot{V} = g - \frac{1}{\rho}\nabla^{\alpha} p - \nabla\left(\frac{\vec{V}^2}{2}\right) + F . \quad (40)$$

Do the rotation operation to (40), we can get:

$$\nabla^{\alpha} \times g = \nabla^{\alpha} \times (\nabla^{\alpha}\phi) = 0, \qquad \nabla^{\alpha} \times \left(-\frac{1}{\rho}\nabla^{\alpha} p\right) = -\nabla^{\alpha}\frac{1}{\rho} \times \nabla^{\alpha} p = B. \quad (41)$$

And using (42), (43),

$$\nabla^{\alpha} \times (A \times B) = (B \times \nabla^{\alpha})A - (A\nabla^{\alpha}B) - B(\nabla^{\alpha}A), \quad (42)$$

$$\nabla^{\alpha} \times (\omega_a \times V) = (V\nabla^{\alpha})\omega_a - (w_a\nabla^{\alpha})V + \omega_a(\nabla^{\alpha}V) - V(\nabla^{\alpha}\omega_a) = (V\nabla^{\alpha})\omega_a - (w_a\nabla^{\alpha})V + \omega_a(\nabla^{\alpha}V) . \quad (43)$$

And (44),

$$\nabla^{\alpha}\omega_a = \nabla(\omega + 2\pi) = \nabla^{\alpha}\omega + 2\nabla^{\alpha}\Omega = \nabla^{\alpha}(\nabla^{\alpha} \times V) = 0 . \quad (44)$$

Therefore, we get the vorticity equation with fractional order as below:

$$\frac{d^{\alpha}\omega_a}{dt^{\alpha}} = -(\omega_a\nabla^{\alpha})V + (\nabla^{\alpha}V)\omega_a = B + \nabla^{\alpha} \times F . \quad (45)$$

5 Conclusions

In the present work, we present some basic set of equations in mesoscale meteorological model on the Cantor sets derived from local fractional vector calculus. These could be applied to describe atmospheric flow. The later used fractional calculus is continuous and differential quantities as classical result; what's more, the latter is local fractional continuous and no differential quantities. The classical result is obtained in case of fractal space-time dimension, which is equal to (1).

Acknowledgements

This research was financially supported by the MWR public sector research and special funds - the most stringent in arid zone water resources management key technologies (201301103) and National Nature Science Foundation of China (NSFC) under Grant No. 41130641, 41201025 and The Ministry of Education Key Laboratory of Eco-Oasis Open Topic-Moisture change in Central Asia and its influence on precipitation in Xinjiang province (XJDX0201-2013-07) and the National Key Technology Research and Development Program of the Ministry of Science and Technology of China (Grant No:2012BAC23B01). This financial support is gratefully acknowledged and appreciated.

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