An improved method of controlling bullwhip effect and the analysis of the bullwhip effect

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Abstract

The bullwhip effect is an important parameter to measure whether the logistics management is good or not. The bullwhip effect affects the production, inventory, transport efficiency in logistics management seriously. In this paper, we establish the structural model of supply chain with multi distribution centre and apply control method to inhibition the bullwhip effect. We analyse the control mechanism of the bullwhip effect and present the control arithmetic to control the bullwhip effect. All the processes are under the circumstance that the demand is worst according to the control theory. At last we processed a stochastic control simulation experiment to control the bullwhip effect. The result shows that the bullwhip effect is inhibited and the bullwhip effect is reduced and stable. The first part of this paper is the related problem description. The second part is basic model and quantitative description of bullwhip effect. The third is control method. The last part is a simulation example.

Keywords: bullwhip effect, multi distribution centre, logistics management

1 Introduction

The bullwhip effect is a phenomenon of demand fluctuations transfer increase in supply chain. This phenomenon means that the orders retailers delivery to manufacture is different to the actually orders in logistics management. This distortion spreads to the upstream in an enlarged form. The consequences of the bullwhip effect are self-evident to the enterprises. So, bullwhip effect is very serious in logistics management. Manufacturers pay excess production cost of raw materials, raw material shortages, manufacturing overtime payment and the high level of inventory due to the poor demand forecasting. These will lead to the extra storage costs, backlog of funds, low efficiency of the transportation process and the extra transportation costs. All of these will cause the enormous economic losses in logistics management.

The first person recognized the bullwhip effect is Forrester. He pointed out that the changes manufacturer perceive is far exceeds the customers perceive through a series of case studies. And he also noticed this effect in the supply chain of each class will be amplified [1]. Many scholars also pointed out the bullwhip effect exist in many industries through the numerical analysis of actual data from the economic angle. SHU Liang you and Yanfeng Ouyang thought that the bullwhip effect can be restrained when the supply chain members share the demand information and the larger the range of sharing demand information is, the bigger the function restraining the bullwhip effect is [2, 3]. V. Gaur and Krane S D believed that the order quantity has a tendency to increase with moving to the upstream in the supply chain [4, 5]. Yanfeng Ouyang researched the stability of bullwhip in system and characterization of the bullwhip effect in linear, time-invariant supply chains. [6, 7]. Senge and Sterman observed management behaviour of beer distribution game in a wider view under the same conditions. They found a small sale volatility of retailer can be amplified to make orders or yield change greatly of each member through each link in the supply chain [8, 9]. Towill confirmed that the inventory management has effect on information distorted in supply chain [10]. Sterman J D first used the (s, S) ordering strategy to prove the existence of bullwhip effect [11]. Sucky, E studied the effect of bullwhip. He thought the effect of bullwhip is overrated [12]. Lee considered the AR(p) model and ARIMA(0,1,1) model. He studied the bullwhip effect caused by fluctuations in prices and supplier out of stock [13]. Xu and Dong considered the retailers and suppliers are in the same AR (1) model and predicted the demand. He got the result that the RMSEP of supplier is high than that of retailer after applying VMI [14]. Chen, Drezner, Ryan and David confirmed the impact of demand forecasting on the bullwhip effect. They not only proved the existence of bullwhip effect theoretically, but also quantized the variability each stage increase in the supply chain [15]. Blackburn, Kahn and Rinks are some of the earliest scholars studied the influence of bullwhip effect on enterprise economic profit. They reviewed the USA manufacturing industry and USA economic history to research the bullwhip effect in the view of system dynamics [16-18]. Metters quantified the bullwhip effect in the supply chain with the heuristic algorithm. He get

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the result: economic profit of enterprises enhance the rate of up to 30%. He also found the promoting effect has a close relationship with scope of business enterprise and the enterprise cost structure [19]. Clark and Hammond studied on the influence of technology for radio frequency identification (RFID) and electronic product code (EPC Network) network technology in the operation of Italy FMCG supply chain system. They found that these advanced technologies can improve the supply chain system by visualization and reduce the safety stock level of the enterprise. These technologies weaken the bullwhip effect to a great extent and promote the profits of fast moving consumer goods supply chain through above principles [20]. Giuliano, Fernanda and Comenges studied a single-product serial supply chain. They considered a control parameter can switch the chain from a series of filters to a series of amplifiers in the bullwhip effect and analyse how the optimal values of the parameters change when discontinuities are in an order policy [21]. Chandra and Grabis quantified the bullwhip effect in the case of serially correlated external demand if autoregressive models are applied to obtain multiple steps demand forecasts. They find the MRP can reduce magnitude of the bullwhip effect while providing the inventory performance comparable to that of a traditional order-up approach [22]. Clark and Hammond discussed the relationship between BPR process of the food industry in USA and channel performance. They thought that VMI has achieved more satisfactory success than pure EDI in the food industry through empirical analysis [23]. Anupindi and Bassok applied the contract model to reduce the bullwhip effect [24, 25]. With the development of economy and the intensification of market competition, the research and control of bullwhip effect have become indispensable part of business management in supply chain.

In this paper, we established a supply chain model with multi distribution centres and introduce a demand disturbance. Then, we will improve the quantization method of the bullwhip effect and apply the robustness to control the bullwhip effect. And, we propose a control theory method of bullwhip effect to make the bullwhip effect minimum in supply chain. At last, we simulate the improved method through numerical analysis and validate the bullwhip is reduced and stable in logistics management.

2 The model of supply chain

Now, we begin to discuss the improved supply chain structure model of distribution centre. At first, we make assumptions in our model:

**Assumption 1**: Demand per period at each retailer location is an independent and equivalently distributed random variable.

**Assumption 2**: A periodic review procedure is used. In each period, the following sequence of events happens at each stocking location: order, delivery and sale.

**Assumption 3**: Orders are delivered after a constant lead time.

**Assumption 4**: Excess demand is backlogged at all levels.

**Assumption 5**: Fixed plus variable ordering costs are happened at the retailer level and the variable costs are charged at the distribution centre only.

**Assumption 6**: Holding costs and shortage costs are charged against expected (end of period) inventory levels.

**Assumption 7**: Pipeline holding costs are paid by the receiving location.

**Assumption 8**: All the costs are stationary.

In assumption 1, the stationary of demand means that the source of demands serial correlation is only the retailers’ ordering decisions. Regular review and backlogging are common practices in many industries. We assume that the ordering cost structure charges fixed replenishment costs to retailers and not to the wholesaler. This condition could occur when wholesalers have adopted effective order filling technology and/or where the manufacturers absorb the cost of filling orders. In the latter case, it could be argued that these costs are passed on to the wholesaler and are reflected in the unit cost (and hence in the holding cost). We have observed these planning when wholesalers have long term contracts with their suppliers.

Next, we consider the nominal system of the supply chain. i.e.:

\[ x_{i,k+1} = x_{i,k} + u_{i,k} - d_{i,k} \]  

\[ x_{i,k+1} = x_{i,k} - Lu_{i,k} + u_{i,k} . \]

Equation (1) is supply chain upstream part inventory dynamic equation for the market customer layers. \( x_{i,k} \) is the order inventory about a customer supply chain upstream portion. It is an \( n \)-dimensional column vector. \( u_{i,k} \) is customer order quantity. It is also an \( n \)-dimensional column vector. \( d_{i} \) is deterministic demand. It is an \( n \)-dimensional vector. Equation (2) is the upstream part inventory dynamic equation for distribution centre. \( x_{i,k} \) is order inventory about distribution centre. It is an \( m \) dimensional vector. \( Lu_{i,k} \) is customer order aggregation amount about distribution centre. Matrix \( L \) translates \( n \)-dimensional vector aggregation about customer order into \( m \)-dimensional demand vector about distribution. Among them, \( L \) is a matrix for \( m \) row \( n \) column. That is:

\[ \lambda_{i,1} \lambda_{i,2} \ldots \lambda_{i,n} \]

\[ \lambda_{j,1} \lambda_{j,2} \ldots \lambda_{j,n} \]

\[ \ldots \ldots \ldots \ldots \]

\[ \lambda_{m,1} \lambda_{m,2} \ldots \lambda_{m,n} . \]

where \( \lambda_{i,j} \geq 0 \) \, \( i = 1,2,\ldots,m \) \, \( j = 1,2,\ldots,n \) \, \( \sum_{i} \lambda_{i,j} = 1 \) \, \( j = 1,2,\ldots,n \). Actually, row vector \((\lambda_{i,1}, \lambda_{i,2}, \ldots, \lambda_{i,n})\) of the
middle aggregation matrix \( L \) is a weighted coefficient vector. It dedicates that the \( i \) intermediate rally point of the supply chain is the order allocation proportion for \( n \) customer.

Equations (1) and (2) can also be written the form of a matrix, that is:

\[
\begin{bmatrix}
    x_{1,k+1} \\
    x_{2,k+1}
\end{bmatrix}
= \begin{bmatrix}
    x_k \\
    -L \
    I
\end{bmatrix}
+ \begin{bmatrix}
    u_{1,k} \\
    -d_{1,k}
\end{bmatrix}
+ \begin{bmatrix}
    -d_{2,k}
\end{bmatrix}.
\]

The nominal system about matrix form of supply chain is as following:

\[
x_{k+1} = x_k + Bu_k + d_k,
\]

where, \( B = \begin{bmatrix} I & 0 \\ -L & I \end{bmatrix} \), \( d_k = \begin{bmatrix} -d_{1,k} \\ -d_{2,k} \end{bmatrix} \).

When the supply chain is subjected of disturbance about the ended uncertain demand, we write the terminal disturbance as \( Fw_k \). When the disturbance transfers to the front supply chain distribution centre and manufacturer (Figure 1), it forms the bullwhip effect:

\[
x_{k+1} = x_k + Bu_k + d_k + Fw_k.
\]

For the quantitative description of the bullwhip effect, we mostly adopt the variance form to descriptive the bullwhip effect quantitatively in supply chain. The description has influence on the quantitative analysis of the bullwhip effect but the description is not convenient for a complex structure, such as the multi-distribution centre in supply chain. It is more difficult to study the dynamic control of the bullwhip effect further. In this thesis, we adopt the deviation description of the bullwhip effect. The concept of the bullwhip effect is that the enhancement effect when the ended demand fluctuation is forward in the supply chain. In this thesis, the object is multiple distribution centre model. The bullwhip effect is the enhanced process that the lower demand fluctuation causes the higher demand fluctuation. We adopt to compare the fluctuation of the front inventory and order with the following demand fluctuation in order to describe the bullwhip effect. That is:

\[
\hat{u}_k = u_k' - u_k.
\]

The deviation system of multiple distribution centres in supply chain is as following:

\[
\dot{x}_{k+1} = x_k + Bu_k + Fw_k.
\]

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\[
r_{ik}^2 = \frac{x_{ik}^T Q_{ik} x_{ik} + \hat{u}_{ik} T_{ik} \hat{u}_{ik}}{w_{ik}^T w_{ik}},
\]

where \( Q \) is positive semi definite matrix \( Q = \text{diag}(Q_1, Q_2), Q_1 \) and \( Q_2 \) are also positive semi definite matrix. \( r_{ik} \) describes the bullwhip effect of the demand fluctuation about customer. \( r_{ik} \) describes the bullwhip effect of the ended demand fluctuation about distribution centre. In this way, the bullwhip effect can be described by more general parameters, such as \( r_{ik} \) and \( r_{ik} \) in supply chain .The bigger the value of \( r_{ik} \) and \( r_{ik} \), the stronger the bullwhip effect. On the contrary, the smaller the value of \( r_{ik} \), the weaker the bullwhip effect.

3 The \( H_\infty \) control of the bullwhip effect

3.1 THE CONTROL MECHANISM OF THE BULLWHIP EFFECT

Bullwhip effect is a high risk exists in the marketing, it is a The result of the game about demand forecast revisions, order quantity decision, price fluctuation and so on between vendors and suppliers which increase the supplier's production, supply, inventory management and marketing instability. The reason of the bullwhip effect is that when supply chain information transfer from the final clients to the original suppliers the information distorted and gradually enlarged, the demand information

\[
\begin{align*}
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\end{align*}
\]
to appear more and more large fluctuations because the share could not be effectively to realize the information.

For the deviation formula in the supply chain, the parameters $r_{1,k}$ and $r_{2,k}$ and Equation (11) in the bullwhip effect describe the process that the ended demand fluctuation causes the front fluctuation about inventory and order in the supply chain. It is a question how to select a $\dot{u}_k$ to weak the bullwhip effect as possible, especially the ended demand fluctuation, that is the worst disturbance conditions. We Select control $\dot{u}_k$ in order to reduce the bullwhip effect to the lowest degree in supply chain. This is a $H_\infty$ control question by using the words of analysis of system control theory. The essence of the question is that when the disturbance $w$ is big, that is the bullwhip effect, we make $J$ had a smallest value by selecting $\dot{u}_k$, or:

$$
\min_{\dot{u}_k} \max_{w} J = \frac{1}{2} \sum_{k=1}^{\infty} (x_k^T Q_1 x_k + \dot{u}_k^T \dot{u}_k - \beta^2 w^T w), \quad (13)
$$

where $Q = Q_1^T Q_1 = \text{diag}(Q_{11}, Q_{12}) = \text{diag}(Q_{21}^T, Q_{22}^T, Q_{31})$ is positive semi definite matrix. $\beta$ is weighted factor which is about the disturbance $w$.

3.2 THE CONTROL ALGORITHM OF $H_\infty$

Its significance is clear for modern logistics that the supply chain Equations (10) and (13) describes the control problem of $H_\infty$ in the bullwhip effect. That is how to consider the worst situation in the condition of uncertain circumstance. The management strategy of supply chain is to make the inventory status and order control deviation minimize, that is make the bullwhip effect minimize.

\[ \dot{x}_i \rightarrow 0, \dot{u}_k \rightarrow 0. \quad (14) \]

For the Equations (10) and (13), we can get the following result easily. That is, if and only if

\[ I - \beta^2 C^T S_k^{-1} F \geq 0, 0 \leq k \leq N. \quad (15) \]

This question has the only saddle point solution:

\[ \dot{u}_k = -B^T S_k^{-1} [I + (BB^T - \beta^2 FF^T)] S_k^{-1}$ \dot{x}_k$, 

\[ w_k = \beta^2 C^T S_k^{-1} [I + (BB^T - \beta^2 FF^T)] S_k^{-1}$ \dot{x}_k$, 

where $S_k$ is fit for the formula of Riccati:

\[ S_k = Q + S_{k+1} [I + (BB^T - \beta^2 FF^T)] S_{k+1}^{-1}, S_n = 0. \quad (18) \]

If $(I, B)$ is positive definite and $(I, Q_k)$ is measured, when $k \rightarrow \infty$, then $S_k \rightarrow S > 0$, it is said that there are feedback control $\dot{u}_k$ in the asymptotically stable system. Obviously, Equations (10) and (13) meet the condition that the $(I, B)$ is positive definite and $(I, Q_k)$ is measured. So the supply chain system has asymptotically stable solution of $H_\infty$. We only need to solve the solution of Equation (18).

In simulation, for Equation (18), when $k \rightarrow \infty$, after sufficiently many iterations calculation, if $\| S_{k+1} - S_k \| \rightarrow 0$, we can regard that we have got the stable solution of Riccati. Then, we can get the stable solution about $u$.

Therefore, the inventory quantity and order quantity in supply chain system are:

\[ x_k' = x_k + x_k, \quad (19) \]

\[ u_k' = \dot{u}_k + u_k, \quad (20) \]

where we can set the inventory status $x_k$ and the quantity $u_k$ by the plan which is formulated by supply chain management.

4 Numerical analysis

We fist get the change of parameters of bullwhip effect in supply chain through $H_\infty$ control. We assume that there is a large-scale supply chain. In addition, in this supply chain, there are ten customer groups and five distribution centres $n = 10, m = 5$. At the same time, we hypothesize that $F = [I_{10}, 0_5]$ and the average price of the products $p = 10$.

The initial condition of the inventory deviation is:

\[ \tilde{x}_1 = [0.45, 0.11, 0.09, 0.32, 0.17, -0.05, 0.02, -0.09, 0.30, 0.35, 0.28, -0.03, 0.11, 0.17, 0.26], \]

(Unit of measurement: thousand)

Customer distribution aggregation layer matrix:

\[ \begin{bmatrix}
0.2 & 0.1 & 0.4 & 0.1 & 0 & 0.3 & 0.2 & 0.1 & 0 & 0.3 \\
0.2 & 0.4 & 0.2 & 0.2 & 0.2 & 0.5 & 0.2 & 0.1 \\
0.3 & 0.1 & 0.2 & 0.1 & 0.4 & 0.1 & 0.2 & 0.1 & 0.4 & 0.2 \\
0 & 0.1 & 0.1 & 0.4 & 0.3 & 0.1 & 0 & 0.2 & 0.2 & 0.1 \\
0.3 & 0.3 & 0.1 & 0.1 & 0.3 & 0.6 & 0.1 & 0.3 & 0.2
\end{bmatrix} \]

The demand and the inventory in supply chain are:

\[ \dot{u}_k = (1.01, 1.07, 1.19, 0.96, 1.03, 1.12, 0.87, 1.22, 1.18, 1.33, 1.12, 0.91, 1.22, 1.21, 1.13), \]

(Unit of measurement: thousand)

\[ \tilde{x} = (1.10, 1.09, 1.21, 0.97, 1.08, 1.17, 0.91, 1.32, 1.21, 1.35, 1.14, 0.95, 1.32, 1.28, 1.16), \]

(Unit of measurement: thousand)

$k$ is limited time and we assume that $k = 10$.

When $k \rightarrow \infty$, here is the gain matrix:
We can get the Figure 2: the change curve of $r_1$ and $r_2$. This picture shows the change of the parameters of the bullwhip effect.

We can see that the bullwhip effect parameter $r_1$ and $r_2$ in supply chain decreased through $H_k$ control. This is means that this method reduced the bullwhip effect when demand disturbance is maximal at the terminal of the supply chain.

The bullwhip effect will lead the financial loss in logistics management. So, we discuss the economic loss. We just show the economic loss of the first and second middle aggregation points to the first five customers demand as the bullwhip effect. $p_{1k}$ means the economic loss of middle aggregation point at $k$ and $p_{2k}$ means the economic loss of distributor at $k$.

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From Figures 3 and 4, we can see clearly that the economic loss of middle aggregation point and distributor reduce obviously and tend to be stable through the $H_s$ control. The numerical analysis shows that the bullwhip effect is well suppressed ($r_{d,k}$ and $r_{k,k}$ diminish) and the economic loss reduces greatly through the $H_s$ control.

5 Conclusions

Bullwhip effect, or demand information distortion, has been a subject of both theoretical and empirical studies in the operations management literature. In this paper, we present a hierarchical model framework for the analysis of the bullwhip effect of inventories in multi-echelon distribution supply chains. The work we have done is as follows:

1) We establish a hierarchical model framework of multi-echelon distributions.

2) We describe the bullwhip effect in a quantitative method.

3) We analyse this quantitative description of bullwhip effect through the $H_s$ control. Through the numerical analysis, we can see that the bullwhip effect is restrained and weakened effectively. This means that the bullwhips effect is controlled in logistics management.

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References

[14] Xu K, Dong Y 1998 Vendor Managed Inventory: Towards a Better Coordination of Information in a Supply Chain ICM98,Shanghai
[22] Chandra C, Grabis J 2005 Application of multi-steps forecasting for restraining the bullwhip effect and improving inventory performance under autoregressive demand European Journal of Operational Research 166(2) 337-50
[23] Clark T, Hammond J 1997 Reengineering channel reordering processes to improve total supply chain performance Production and Operation Management 6(3) 248-65