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EDITORIAL CORRESPONDENCE
Transporta un sakaru institūts (Transport and Telecommunication Institute)
Lomonosova iela 1, LV-1019, Riga, Latvia. Phone: (+371) 67100593. Fax: (+371) 67100535.
E-mail: journal@tsi.lv, http:// www.tsi.lv

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Editors' Remarks

"IN THE SHAPE of a pig?" cried the Mandarin.

"In the shape of a pig," said the messenger, and departed.

"Oh, what an evil day hi an evil year," cried the Mandarin. "The town of Kwan-Si, beyond the hill, was very small in my childhood. Now it has grown so large that at last they are building a wall."

"But why should a wall two-miles away make my good father sad and angry all within the hour?" asked his daughter quietly.

"They build their wall," said die Mandarin, "in the shape of a pig! Do you see? Our own city wall is built in the shape of an orange. That pig will devour us, greedily!"

"Ah."

They both sat thinking. Life was full of symbols and omens. Demons lurked everywhere, Death swam in the wetness of an eye, the turn of a gull's wing meant rain, a fan held so, the tilt of a roof, and, yes, even a city wall was of immense importance. Travelers and tourists, caravans, musicians, artists, coming upon these two towns, equally judging the portents, would say, "The city shaped like an orange? No! I will enter the city shaped like a pig and prosper, eating all, growing fat with good luck and prosperity!"

The Mandarin wept "All is lost! These symbols and signs terrify. Our city will come on evil days."

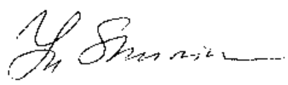
Ray Bradbury,
'The Golden Kite, The Silver Wind'
from 'The Golden Apples of the Sun', 1953

This 13th volume No.2 is devoted to various questions of **computer modelling, logistics and solid state physics**. In particular, we present actual papers from Israel, Ukraine, Byelorussia, Lithuania, USA and Latvia.

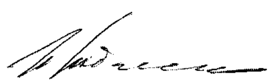
Our journal policy is directed on researches of the fundamental and applied sciences, which are the basement of a full-scale modelling in practice.

This edition is the continuation of our publishing activities. We hope our journal will be interesting for research community, and we are open for collaboration both in research and publishing. This number continues the current 2009 year of our publishing work. We hope that journal's contributors will consider the collaboration with the Editorial Board as useful and constructive.

EDITORS



Yu.N. Shunin



I.V. Kabashkin

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RELIABILITY AND COST ANALYSIS OF A UTILITY COMPANY WEBSITE USING MIDDLEWARE SOLUTION BY MATHEMATICAL MODELLING

S. Sharma[1], S. B. Pandey [1], S. B. Singh[2]

Kumaun University (SSJ Campus), Almora, Uttrakhand, India[1]
G. B. Pant University of Agriculture & Technology, Uttrakhand, India[2]
E-mail: [1]sshweta81@yahoo.com., [2]drsurajbsingh@yahoo.com

This paper studies the reliability measures based on the resilience architecture of the utility company website by mathematical modelling. The architecture of the website has application servers which are used to connect to the middleware boxes and the database servers. The solution assumes to have a load balanced solution for the middleware solution and an active/passive clustering for the database, using RAID 5. With the application of supplementary variable technique, asymptotic behaviour, availability, mean time to failure and cost effectiveness of the system has been obtained. At last some particular cases of the system have also been taken in account.

Keywords: *mathematical modelling, reliability and cost analysis, website architecture*

1. Introduction

A mathematical model represents the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable form. Model updating is a costly and time consuming task, so it is important to rigorously evaluate the quality of a model with respect its end use: the decisions which rely upon the model. Mathematical modelling of complex systems assist the analyst in making decisions for design, control, structural interfacing, reliability and safety analysis and so on. Many authors [1–8] explained different mathematical modelling techniques, did mathematical modelling of different real systems and studied different possibilities.

Here authors have considered a system of which they tried to do the modelling and evaluated various related measures such as mean time to failure, steady state probability, availability and cost analysis. This paper discusses about reliability of a utility company dealing with supplying energy within UK website which uses a middleware solution to integrate with its back end system. The middleware solution is used in software systems to achieve system integration which basically enables seamless information viewing and update.

Keeping all the above facts in view the considered system has been modelled and the failure and repair times for the system follow exponential and general distribution respectively. With the application of supplementary variable technique asymptotic behaviour, availability, mean time to failure and cost effectiveness of the system has been obtained. At last some numerical examples for particular possibilities of the system have also been taken into consideration.

2. System Description

The website in context had five application servers (BEA weblogic) running on Sun v890 fire boxes. The traffic from the website is distributed using load balancer which is responsible to distributing the request to six different application servers. The load distribution is based on the percentage utilization of boxes. All these boxes work in a clustered environment and failure of any of the boxes does not impact the website as the load is always distributed between the active legs and two middleware boxes which are used for connecting to the back end systems which have customer relationship data (CRM) and billing system information. CRM systems are customer relationship models which store the single view of customer data for an enterprise. The details of all the accounts are stored in the billing systems. Website connects to this CRM and the billing system via middleware boxes using Java messaging server request and response messages. The five application server boxes connect to the middleware boxes in a load balanced manner that is any of the six application server boxes can connect to any of the middleware boxes depending upon the load. In the case when one of these middleware boxes is not available the traffic is routed to the other middleware box. The database (oracle) servers are in an active/passive clustering so that at a time there is only one active instance.

3. Assumptions

i. Initially the system is in good state.
ii. Failure of any application server does not impact the system ability to perform the desired task as the load is always distributed between the active application servers.
iii. When one middleware box fails all the weblogic servers automatically start using the other middleware and there is no service disruption.
iv. The session information is replicated across all nodes so the traffic routing change is transparent to the user.
v. It has been assumed that there is no impact on the performance of the website when only one of the middleware boxes is working.
vi. The response time is not impacted when only one middleware solution takes the entire load.
vii. The system is able to do the core view functionality without connecting to the middleware boxes using the information stored in the database.
viii. When the other middleware box also fails then the transactions done on the website are stored in the database store and forward table so that these transactions are processed when the middleware boxes are back up again, from the user point of view the website works but the system is in a partially failed state.
ix. The active/passive failure over of the database is transparent to the user and there is no service disruption.
x. System fails completely when both the middleware boxes and the database instances are down.
xi. The repair of a failed unit starts at once.
xii. The repaired unit works like a new one.
xiii. The failure and repair time for the system follows exponential and general distribution respectively.

4. Block Diagram

Figure 1 below shows the system architecture of the system in context.

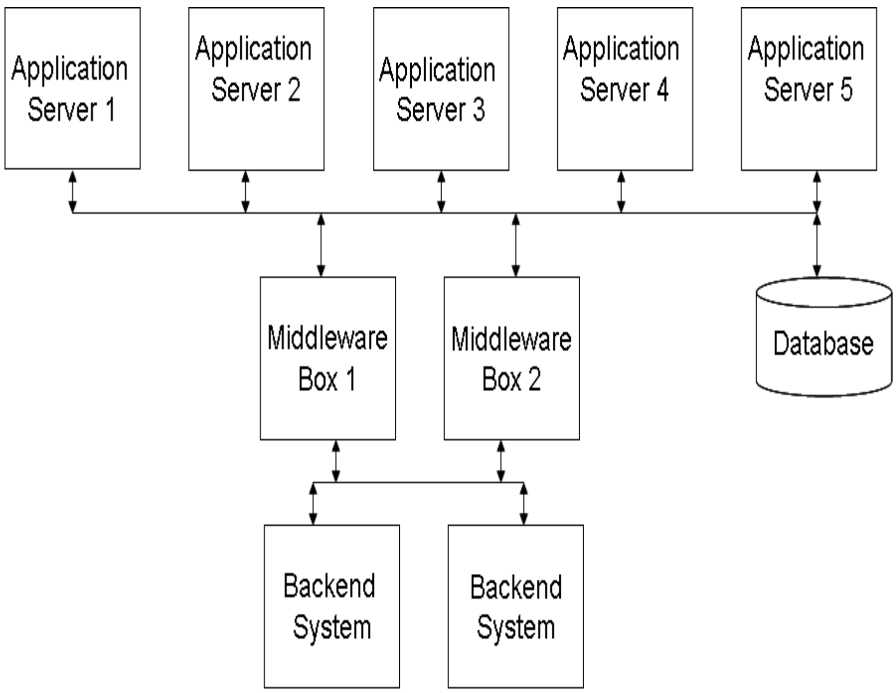

Figure 1. System architecture

5. State Transition Diagram

Figure 2 represents the state transition diagram of the system.

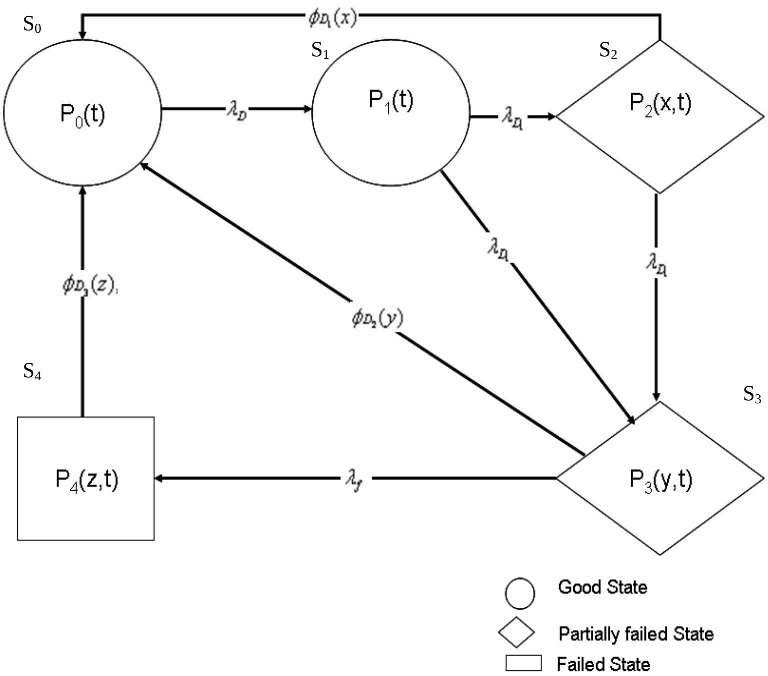

Figure 2. State transition diagram

6. State Descriptions and Notations

- S_0: Denotes the state when all the units (both middleware boxes and active database instance) are in good state.
- S_1: Denotes the state when one middleware box has failed but the website is still functional as the load goes through the other middleware box. There might be degraded response time during this failure but the system works as expected so considered to be good state.
- S_2: Denotes the state when the other middleware box also goes down and the system is in a partially failed state.
- S_3: Denotes the state when the middleware box and the active database instance is down which is now taken over by the standby instance.
- S_4: Denotes the state when the system is in complete failure state.

$\lambda_D, \lambda_{D_1}, \lambda_f$: Constant failure rates of the working/failed units.

$\phi_{D_1}(x), S_{D_1}(x)$: Repair rate and probability density function of the system state S_2 in elapsed repair time x.

$\phi_{D_2}(y), S_{D_2}(y)$: Repair rate and probability density function of the system state S_3 in elapsed repair time y.

$\phi_{D_3}(z), S_{D_3}(z)$: Repair rate and probability density function of the system state S_4 in elapsed repair time z.

$P_i(t)$: Probability that the system is in S_i state at instant 't' for $i = 0$ to 4.

$\overline{P}(s)$: Laplace transformation of P(t).

7. Formulation of Mathematical Model

With the help of continuity arguments and probabilities of consideration the following set of difference-differential equations governing the present mathematical model can be obtained

$$[\frac{\partial}{\partial t}+\lambda_D]P_0(t)=\sum_{i=2}^{i=4}\int_0^\infty P_i(\theta,t)\phi_\alpha(\theta)d\theta, \quad (1)$$

where

$$\begin{bmatrix} i=2,\theta=y,\alpha=D1 \\ i=3,\theta=z,\alpha=D2 \\ i=4,\theta=r,\alpha=D3 \end{bmatrix}$$

$$[\frac{\partial}{\partial t}+2\lambda_{D_1}]P_1(t)=\lambda_D P_0(t) \quad (2)$$

$$[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\lambda_{D_1}+\phi_{D_1}(x)]P_2(x,t)=0 \quad (3)$$

$$[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\lambda_f+\phi_{D_2}(y)]P_3(y,t)=0 \quad (4)$$

$$[\frac{\partial}{\partial r}+\frac{\partial}{\partial t}+\phi_{D_3}(z)]P_4(z,t)=0\ . \quad (5)$$

The boundary conditions are

$$P_2(0,t)=\lambda_{D_1}P_1(t) \quad (6)$$

$$P_3(0,t)=\lambda_{D_1}[P_1(t)+P_2(t)] \quad (7)$$

$$P_4(0,t)=\lambda_f P_3(t) \quad (8)$$

Initial conditions are

$P_0(0) = 1$, and other state probabilities are zero at time $t = 0$. (9)

8. Solution of the Model

Taking Laplace transforms of equations (1)–(8) and using equation (9), one can get equations (10)–(17)

$$[s+\lambda_D]\overline{P_0}(s)=1+\sum_{i=2}^{i=4}\int_0^{\infty}\overline{P_i}(\theta,s)\phi_\alpha(\theta)d\theta \quad (10)$$

$$\begin{bmatrix} i=2,\theta=y,\alpha=D1 \\ i=3,\theta=z,\alpha=D2 \\ i=4,\theta=r,\alpha=D3 \end{bmatrix}$$

$$[s+2\lambda_{D_1}]\overline{P_1}(s)=\lambda_D\overline{P_0}(s) \quad (11)$$

$$[\frac{\partial}{\partial x}+s+\lambda_{D_1}+\phi_{D_1}(x)]\overline{P_2}(x,s)=0 \quad (12)$$

$$[\frac{\partial}{\partial y}+s+\lambda_f+\phi_{D_2}(y)]\overline{P_3}(y,s)=0 \quad (13)$$

$$[\frac{\partial}{\partial z}+s+\phi_{D_3}(z)]\overline{P_4}(z,s)=0 \quad (14)$$

Boundary conditions are

$$\overline{P_2}(0,s)=\lambda_{D_1}\overline{P_1}(s) \quad (15)$$

$$\overline{P_3}(0,s)=\lambda_{D_1}[\overline{P_1}(s)+\overline{P_2}(s)] \quad (16)$$

$$\overline{P_4}(0,s)=\lambda_f\overline{P_3}(s) \quad (17)$$

Solving equations (10) – (14) and using equations (15)–(17), one can get equations (18)–(22)

$$\overline{P_0}(s) = [s+2\lambda_{D_1}][s[s+2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(s+\lambda_{D_1}) + \lambda_D\lambda_{D_1}r_{D_2}(s+\lambda_f)\{1+\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(s)\}]]^{-1} \quad (18)$$

$$\overline{P_1}(s) = \lambda_D[s[s+2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})+\lambda_D\lambda_{D_1}r_{D_2}(s+\lambda_f) \; 1+\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(s)\}]]^{-1} \quad (19)$$

$$\overline{P_2}(s) = \lambda_D\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})[s[s+2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(s+\lambda_{D_1}) + \lambda_D\lambda_{D_1}r_{D_2}(s+\lambda_f)\{1+\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(s)\}]]^{-1} \quad (20)$$

$$\overline{P_3}(s) = \lambda_D\lambda_{D_1}r_{D_2}(s+\lambda_f)[1+\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})][s[s+2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(s+\lambda_{D_1}) + \lambda_D\lambda_{D_1}r_{D_2}(s+\lambda_f)\{1+\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(s)\}]]^{-1} \quad (21)$$

$$\overline{P_4}(s) = \lambda_f\lambda_D\lambda_{D_1}r_{D_2}(s+\lambda_f)r_{D_3}(s)[1+\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})] [s[s+2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})+\lambda_D\lambda_{D_1}r_{D_2}(s+\lambda_f), \{1+\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(s)\}]]^{-1} \quad (22)$$

where

$$r_i(s) = \frac{1-\overline{S_i}(s)}{s}$$

The Laplace transforms of the probabilities that the system is in up (i.e. good) and failed states at time ‘t’, are as follows:

$$\overline{P_{up}}(s) = \sum_{i=0}^{3}\overline{P}_i(s),$$

$$\overline{P_{up}}(s) = [s+2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})+\lambda_D\lambda_{D_1}r_{D_2}(s+\lambda_f) \{1+\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})\}][s[s+2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(s+\lambda_{D_1}) + \lambda_D\lambda_{D_1}r_{D_2}(s+\lambda_f)\{1+\lambda_{D_1}r_{D_1}(s+\lambda_{D_1})\} \{1+\lambda_f r_{D_3}(s)\}]]^{-1} \quad (23)$$

$$\overline{P}_{failed}(s) = \overline{P_4}(s). \quad (24)$$

Also it is interesting to note that

$$\overline{P}_{up}(s) + \overline{P}_{failed}(s) = 1/s.$$

9. Asymptotic Behaviour of the System

Using Abel's lemma $\lim_{s\to 0}[s\overline{F}(s)] = \lim_{t\to\infty} A(t) = F$ (say) in equations (18)-(24), provided the limit on the right hand exists, the following time independent probabilities are obtained

$$P_0 = 2\lambda_{D_1}[2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(\lambda_{D_1})+\lambda_D\lambda_{D_1}r_{D_2}(\lambda_f)\{1+\lambda_{D_1}r_{D_1}(\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(0)\}]^{-1}$$

$$P_1 = \lambda_D[2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(\lambda_{D_1})+\lambda_D\lambda_{D_1}r_{D_2}(\lambda_f)\{1+\lambda_{D_1}r_{D_1}(\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(0)\}]^{-1}$$

$$P_2 = \lambda_D\lambda_{D_1}r_{D_1}(\lambda_{D_1})[2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(\lambda_{D_1})+\lambda_D\lambda_{D_1}r_{D_2}(\lambda_f)\{1+\lambda_{D_1}r_{D_1}(\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(0)\}]^{-1}$$

$$P_3 = \lambda_D\lambda_{D_1}r_{D_2}(\lambda_f)[1+\lambda_{D_1}r_{D_1}(\lambda_{D_1})][2\lambda_{D_1}+\lambda_D+\lambda_D\lambda_{D_1}r_{D_1}(\lambda_{D_1})+\lambda_D\lambda_{D_1}r_{D_2}(\lambda_f) \{1+\lambda_{D_1}r_{D_1}(\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(0)\}]^{-1}$$

$$P_4 = \lambda_f \lambda_D \lambda_{D_1} r_{D_2}(\lambda_f) r_{D_3}(0)[1+\lambda_{D_1} r_{D_1}(\lambda_{D_1})][2\lambda_{D_1} + \lambda_D + \lambda_D \lambda_{D_1} r_{D_1}(\lambda_{D_1}) + \lambda_D \lambda_{D_1} r_{D_2}(\lambda_f)$$
$$\{1+\lambda_{D_1} r_{D_1}(\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(0)\}]^{-1}$$

$$P_{up} = [2\lambda_{D_1} + \lambda_D + \lambda_D \lambda_{D_1} r_{D_1}(\lambda_{D_1}) + \lambda_D \lambda_{D_1} r_{D_2}(\lambda_f)\{1+\lambda_{D_1} r_{D_1}(\lambda_{D_1})\}]$$
$$[[2\lambda_{D_1} + \lambda_D + \lambda_D \lambda_{D_1} r_{D_1}(\lambda_{D_1}) + \lambda_D \lambda_{D_1} r_{D_2}(\lambda_f)$$
$$\{1+\lambda_{D_1} r_{D_1}(\lambda_{D_1})\}\{1+\lambda_f r_{D_3}(0)\}]]^{-1}$$

$$P_{failed} = P_4 \, .$$

10. Particular Case

When repair follows exponential time distribution, setting

$$\overline{S_{D_1}}(s) = \frac{\phi_{D_1}}{s+\phi_{D_1}}, \; \overline{S_{D_2}}(s) = \frac{\phi_{D_2}}{s+\phi_{D_2}} \text{ and } \overline{S_{D_3}}(s) = \frac{\phi_{D_3}}{s+\phi_{D_3}}$$

equations (18)–(24) yields

$$\overline{P_0}(s) = [s+2\lambda_{D_1}][s[s+2\lambda_{D_1} + \lambda_D + \frac{\lambda_D \lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})} + \frac{\lambda_D \lambda_{D_1}}{(s+\lambda_f+\phi_{D_2})}\{1+\frac{\lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})}\}\{1+\frac{\lambda_f}{(s+\phi_{D_3})}\}]]^{-1} \tag{25}$$

$$\overline{P_1}(s) = \lambda_D [s[s+2\lambda_{D_1} + \lambda_D + \frac{\lambda_D \lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})} + \frac{\lambda_D \lambda_{D_1}}{(s+\lambda_f+\phi_{D_2})}\{1+\frac{\lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})}\}\{1+\frac{\lambda_f}{(s+\phi_{D_3})}\}]]^{-1} \tag{26}$$

$$\overline{P_2}(s) = \lambda_D \lambda_{D_1} [s(s+\lambda_{D_1}+\phi_{D_1})[s+2\lambda_{D_1} + \lambda_D + \frac{\lambda_D \lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})} + \frac{\lambda_D \lambda_{D_1}}{(s+\lambda_f+\phi_{D_2})}\{1+\frac{\lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})}\}\{1+\frac{\lambda_f}{(s+\phi_{D_3})}\}]]^{-1} \tag{27}$$

$$\overline{P_3}(s) = \lambda_f \lambda_D \lambda_{D_1} \{1+\frac{\lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})}\}[s(s+\lambda_f+\phi_{D_2})(s+\phi_{D_3})[s+2\lambda_{D_1}+\lambda_D + \frac{\lambda_D \lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})} + \frac{\lambda_D \lambda_{D_1}}{(s+\lambda_f+\phi_{D_2})}\{1+\frac{\lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})}\}\{1+\frac{\lambda_f}{(s+\phi_{D_3})}\}]]^{-1} \tag{28}$$

$$\overline{P_4}(s) = \lambda_f \lambda_D \lambda_{D_1} \{1+\frac{\lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})}\}[s(s+\lambda_f+\phi_{D_2})(s+\phi_{D_3})[s+2\lambda_{D_1}+\lambda_D + \frac{\lambda_D \lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})} + \frac{\lambda_D \lambda_{D_1}}{(s+\lambda_f+\phi_{D_2})}\{1+\frac{\lambda_{D_1}}{(s+\lambda_{D_1}+\phi_{D_1})}\}\{1+\frac{\lambda_f}{(s+\phi_{D_3})}\}]]^{-1} \tag{29}$$

$$\overline{P_{up}}(s) = [s + 2\lambda_{D_1} + \lambda_D + \frac{\lambda_D \lambda_{D_1}}{(s + \lambda_{D_1} + \phi_{D_1})} + \frac{\lambda_D \lambda_{D_1}}{(s + \lambda_f + \phi_{D_2})}\{1 + \frac{\lambda_{D_1}}{(s + \lambda_{D_1} + \phi_{D_1})}\}][s[s + 2\lambda_{D_1} + \lambda_D$$

$$+ \frac{\lambda_D \lambda_{D_1}}{(s + \lambda_{D_1} + \phi_{D_1})} + \frac{\lambda_D \lambda_{D_1}}{(s + \lambda_f + \phi_{D_2})}\{1 + \frac{\lambda_{D_1}}{(s + \lambda_{D_1} + \phi_{D_1})}\}\{1 + \frac{\lambda_f}{(s + \phi_{D_3})}\}]]^{-1} \tag{30}$$

$$\overline{P}_{failed}(s) = \overline{P_4}(s) . \tag{31}$$

11. Numerical Computation

11.1. Availability Analysis

Setting the numerical values, say

$\lambda_D = \lambda_{D_1} = \lambda_f = 0.5$, $\phi_{D_1} = \phi_{D_2} = \phi_{D_3} = 1$

and

$\overline{S_{D_1}}(s) = \frac{\phi_{D_1}}{s + \phi_{D_1}}$, $\overline{S_{D_2}}(s) = \frac{\phi_{D_2}}{s + \phi_{D_2}}$, $\overline{S_{D_3}}(s) = \frac{\phi_{D_3}}{s + \phi_{D_3}}$,

in equation (23) and then taking Inverse Laplace transform, we get

$$P_{up}(t) = \frac{1}{252}[238 + 77e^{-1.5t} + 21te^{-1.5t} + 9e^{-1.25t}\{-7\cos\frac{\sqrt{7}t}{4} + \sqrt{7}\sin\frac{\sqrt{7}t}{4}\}].$$

Also,

$$P_{failed}(t) = \frac{1}{252}[14 - 77e^{-1.5t} - 21te^{-1.5t} + 9e^{-1.25t}\{7\cos\frac{\sqrt{7}t}{4} - \sqrt{7}\sin\frac{\sqrt{7}t}{4}\}].$$

The values of P_{up} (t) and P_{failed} (t) for different values of time ‘t’ is shown in Table 1 and the corresponding graph is shown on Figure 3.

Table 1. System reliability data

S. No.	Time t	Pup(t)	P_{failed} (t)
1	0	1	0
2	1	0.99133	0.00867
3	2	0.97044	0.02956
4	3	0.95501	0.04499
5	4	0.94781	0.05219
6	5	0.94530	0.05470
7	6	0.94460	0.05540
8	7	0.94446	0.05554
9	8	0.94444	0.05556
10	9	0.94444	0.05556
11	10	0.94444	0.05556

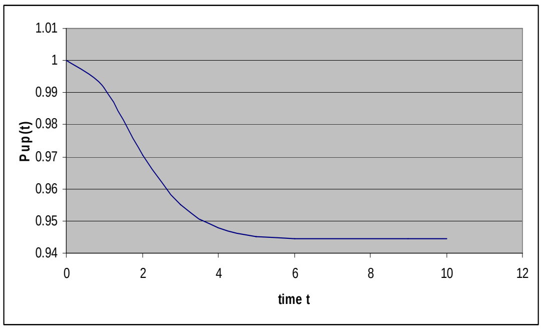

Figure3. Graph of reliability Vs time

11.2. Cost Analysis

Setting the numerical values, say $\lambda_D = \lambda_{D_1} = \lambda_f = 0.5$, $\phi_{D_1} = \phi_{D_2} = \phi_{D_3} = 1$ and

$\overline{S_{D_1}}(s) = \frac{\phi_{D_1}}{s + \phi_{D_1}}$, $\overline{S_{D_2}}(s) = \frac{\phi_{D_2}}{s + \phi_{D_2}}$, $\overline{S_{D_3}}(s) = \frac{\phi_{D_3}}{s + \phi_{D_3}}$,

in equation (20) and then taking Inverse Laplace transform, we get

$$P_2(t) = \frac{1}{12}[1 + 3e^{-2t} - 4e^{-3t/2}].$$

Let 'M' and 'N' be the revenue per unit time and service cost per unit time respectively, then the expected profit E(t) during the interval]0,t] is given by

$$E(t) = M\int_0^t P_{up}(t)dt - N[t - \int_0^t P_2(t)dt] =$$
$$= \frac{M}{252}[238t - 51.33333(e^{-1.5t} - 1) + 14(0.66667 - te^{-1.5t} - 0.66667e^{-1.5t}) + 47.25 - \frac{9\sqrt{7}}{2}e^{-1.25t}(-0.5\sin\frac{\sqrt{7}}{4}t + 1.5\sqrt{7}\cos\frac{\sqrt{7}}{4}t)]$$
$$-N[t - \frac{1}{12}(t - 1.5e^{-2t} + 6e^{-1.5t} - 4.5)]. \quad (32)$$

Setting M = 1 and N = 0.05, 0.1, 0.5 then equation (32) yields Table 2 and the Figure 4.

Table 2. Cost Analysis of the system

S.No.	Time t	Expected profit E(t)		
		N=0.05	N=0.1	N=0.5
1	0	0	0	0
2	1	1.20849	1.14864	0.66983
3	2	2.18843	2.07914	1.20485
4	3	3.10334	2.94735	1.69945
5	4	4.00403	3.80201	2.18582
6	5	4.90262	4.65472	2.67149
7	6	5.80115	5.50740	3.15743
8	7	6.69975	6.36017	3.64351
9	8	7.59837	7.21295	4.12962
10	9	8.49699	8.06574	4.61574
11	10	9.39560	8.91852	5.10185

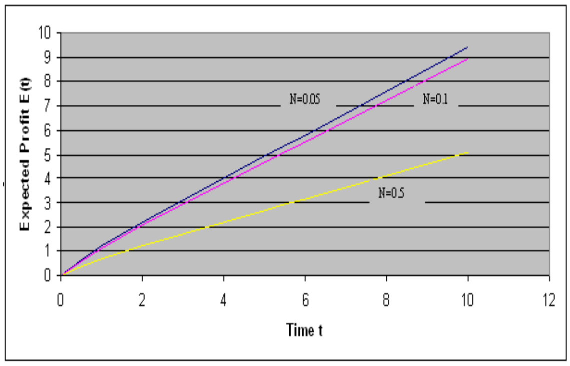

Figure 4. Expected profit Vs time

11.3. Mean Time to Failure (MTTF)

Substituting $\overline{S_i}(s) = 0$ where $r_i(s) = \frac{1 - \overline{S_i}(s)}{s}$ in equation (23), we can obtain

$$M.T.T.F = \lim_{s \to 0} \overline{P}_{up}(s) = \frac{\lambda_{D_1}\lambda_f + \lambda_D\lambda_f + \lambda_D\lambda_{D_1}}{\lambda_D\lambda_{D_1}\lambda_f}$$

Setting the numerical values, say

$\lambda_D = 0.5, \lambda_{D_1} = 0.5$ and $\lambda_f = 0.01, 0.02, 0.03, \ldots$, one can get the Table 3 and the Figure 5.

Table 3. Mean Time to Failure

S. No.	Failure rate	MTTF
1	0.01	104.00000
2	0.02	54.00000
3	0.03	37.33333
4	0.04	29.00000
5	0.05	24.00000
6	0.06	20.66667
7	0.07	18.28571
8	0.08	16.50000
9	0.09	15.11111
10	0.10	14.00000

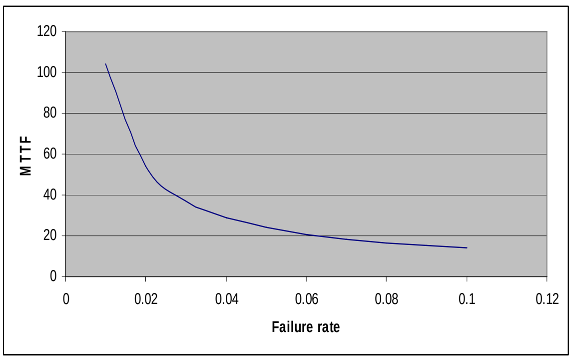

Figure 5. Graph of MTTF Vs Failure rate

Conclusions

Computation of system reliability with respect to time is shown in Table 1 and is depicted in the Figure 3. One can easily conclude by observing these that the reliability of the system decreases with respect to time but attains a constant value in the long run. Further, the graph corresponding to Table 2 reveals that the expected profit increases with the passage of time. The relative decrease in profit is very low, when service cost N is below 0.1 but as the service cost N approaches towards 1 the decrement in profit increases rapidly. Table 3 computes the M.T.T.F. of the system and the graph corresponding to it reveals that the M.T.T.F. decreases rapidly in the beginning, but as the time passes, it decreases approximately at a uniform rate. Thus by mathematical modelling of this system we can interpret actually how reliable the system is and how much cost effective this system can be in the long run.

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Transport and Telecommunication Institute, Lomonosova 1, LV-1019, Riga, Latvia

SIMULATION OF WATER TABLE ELEVATION FLUCTUATION USING FUZZY-LOGIC AND ANFIS

D. C. S. Bisht*, M. Mohan Raju**, M. C. Joshi*

** Department o f Mathematics, Statistics and Computer Science*
College of Basic Sciences and Humanities

***Department of Soil and Water Conservation Engineering, College of Technology*
G.B. Pant University of Agriculture and Technology

Pantnagar-263 145, Uttarakhand, INDIA

E-mail: dcsbisht@gmail.com, mmraju.swce@gmail.com, mcjoshi@69gmail.com

Groundwater is of major importance to civilization, because it is the largest reserve of drinkable water in regions where humans can live. The estimation of the water table elevation is one of the important aspects to understand the mechanism which comprises groundwater resources and to predict what might happen under various possible future conditions. Fuzzy-logic, a soft computing technology of Artificial Intelligence, nowadays have a great concentrated applications importance in water engineering. It is an excellent mathematical tool to handle uncertainty of the system arising due to the fuzziness or vagueness. The soft computing techniques viz. Fuzzy-logic modelling and Adaptive Neuro Fuzzy Inference System were used in present investigation. These systems begin with some basic rules that describe the process. Four models have been developed, two Fuzzy rule based models and two ANFIS models in the prediction of ground water table elevation. On the basis of performance criteria ANFIS yielded the better results out of all the models developed.

Keywords: *Neuro-Fuzzy, ground water modelling, ground water elevation, ANFIS, training, learning*

1. Introduction

Worldwide, groundwater accounts for about one-third of one percent of the earth's water, or about 20 times more than the total of surface waters on continents and islands. The realisation of the concept of natural resources and its conservancy is presently looked upon as one of the main interests of our civilisation. About 97% of the world’s water resources are confined in the sea with no practical value for human consumption. Of the remaining 3%, about 75% is bound in ice sheets, glaciers etc. and only 25% is available as surface water and groundwater. The distribution of this 25% consists of 0.3% in lakes and rivers, and remaining 99.7% is available as groundwater. It is thus clear how important groundwater is, for human consumption, both in present scenario as well as and in the coming future.

2. Groundwater Resource

Groundwater – water found below the surface of the land. Such water exists in pores between sedimentary particles and in the fissures of more solid rocks. Very deep-lying groundwater can remain undisturbed for thousands or millions of years. Most groundwater lies at shallower depths, however, and plays a slow but steady part in the hydrologic cycle.

Groundwater may appear at the surface in the form of springs, or it may be tapped by wells. During dry periods it can also sustain the flow of surface water, and even where the latter is readily available; groundwater is often preferable because it tends to be less contaminated by wastes and organisms. Proper development and management of this large natural resource is a concentrated challenging objective for all types of water supply requirements.

Mathematical groundwater models are obtained through solutions of suitably constrained differential equations that describe groundwater behaviour in a system of interest. These equations have been available since the late nineteenth century. Until fairly recently, their solutions required a level of mathematical sophistication not always available to groundwater hydrologists. The fact, which was more important that the availability of analytical solutions applied only to simplified solutions, which did not simulate the real time flow conditions. There are still many groundwater flow problems for which analytical solutions are difficult, if not impossible to obtain. The reason is that these problems are complex, possessing non-linear features that cannot be included in analytical solutions. Sometimes analytical solutions are yet applied to such problems by over simplifying the complex hydro-geological situation. Since the assumptions underlying the solution are approximations, it is obvious that the results will be inaccurate or even totally erroneous.

Fundamental aspect of the many hydro-geological studies is the problem of forecasting the water table depth at given points. The past research developed on this topic for many years often developing complex models. During the last decade of 19th century, the artificial neural network and thereafter fuzzy logic techniques have become popular in data forecast of time series particularly in the application where the deterministic approach presents serious drawback, due to the noisy or random nature of data. These learning based approaches, which can be considered an alternative to classical methods, exploit the statistical relationships between inputs and outputs, without explicitly considering the physical process relationships, which exist between them. Although fuzzy logic attempts to simulate human “Vagueness” of reasoning, in practice many characteristics of this approach, such as ability to learn and generalize, the ability to cope up with noise, the distribute processing, which maintains robustness can be of great help in many engineering tasks. Moreover, in general, this technique can be included in overall concept of soft computing approach [9].

Again the problem of groundwater modelling does not require a very precise measurement, and moreover a precise model can be very complicated and uneconomical in the development time. Secondly the variables involved in the problem are fuzzy in nature. Therefore, a fuzzy logic can provide better solutions in simple way [1, 6 and 7].

3. Fuzzy Logic

The Fuzzy rule-based approach introduced by Zadeh (1965) [10] is being widely utilized in various fields of science and technology. It is a qualitative modelling scheme in which the system behaviour is described using a natural language. The transparency in formulation of fuzzy rules offers explicit qualitative and quantitative insights into the physical behaviour of the system [4]. The application of fuzzy logic as a modelling tool in the field of water resources is a relatively new concept although some studies have been carried out to some extent in the last decade and these have generated much enthusiasm. Bardossy & Duckstein (1992) [3] applied a fuzzy rule-based modelling approach to a karstic aquifer management problem. Bardossy & Disse (1993) [2] used fuzzy rules for simulating infiltration. Fontane et al. (1997) [5] and Panigrahi & Mujumdar (2000) [8] applied fuzzy logic for reservoir operation and management problems.

4. Application of Fuzzy Logic

Fuzzy set theory, which has been proposed in 1965 by Lofti A. Zadeh (1965) [10], is a generalization of classical theory. Fuzzy logic representations found on Fuzzy set theory try to capture the way humans represent and reason with real world knowledge in the face of uncertainty. Uncertainty could arise due to do generality, vagueness, Ambiguity, chance or incomplete knowledge.

A Fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse, a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the Fuzzy set. In other words, fuzzy sets support a flexible sense of membership of elements to a set.

The range of the model input values, which are judged necessary for the description of the situation, can be portioned into fuzzy sets. The process of formulating the mapping from a given input to an output using fuzzy logic is called the fuzzy inference. The basic structure of any fuzzy inference system is a model that maps characteristics of input data to input membership functions, input membership functions to rules, rules to a set of output characteristics, output characteristics to output membership functions, and output membership function to a single valued output or a decision associated with the output. In rule based fuzzy systems, the relationship between variables are represented by means of fuzzy if-then rules e.g. “if antecedent proposition then consequent proposition”.

5. Study Area

Budaun District, the study area is a part of Ganga-Ramganga inter-basin and it lies between longitudes of 78^0 15’ and 79^0 30’ E; and latitudes of 27^0 30’ and 28^0 30’ N. This district of India is situated in south-west of Rohelkhand region, which has about 5163 sq. km area. The study area is surrounded by Bareilly, Shajahanpur, Rampur, Etah, Aligarh, Bulandshahar and Moradabad districts (Fig. 1).

The south-west boundary of Budaun district is marked by Ganga river and east side by Ramganga river. The district has five *Tehsils* namely: Budaun, Dataganj, Sahaswan, Gunnaur and Bisauli, which is categorized in 18 development blocks in the district.

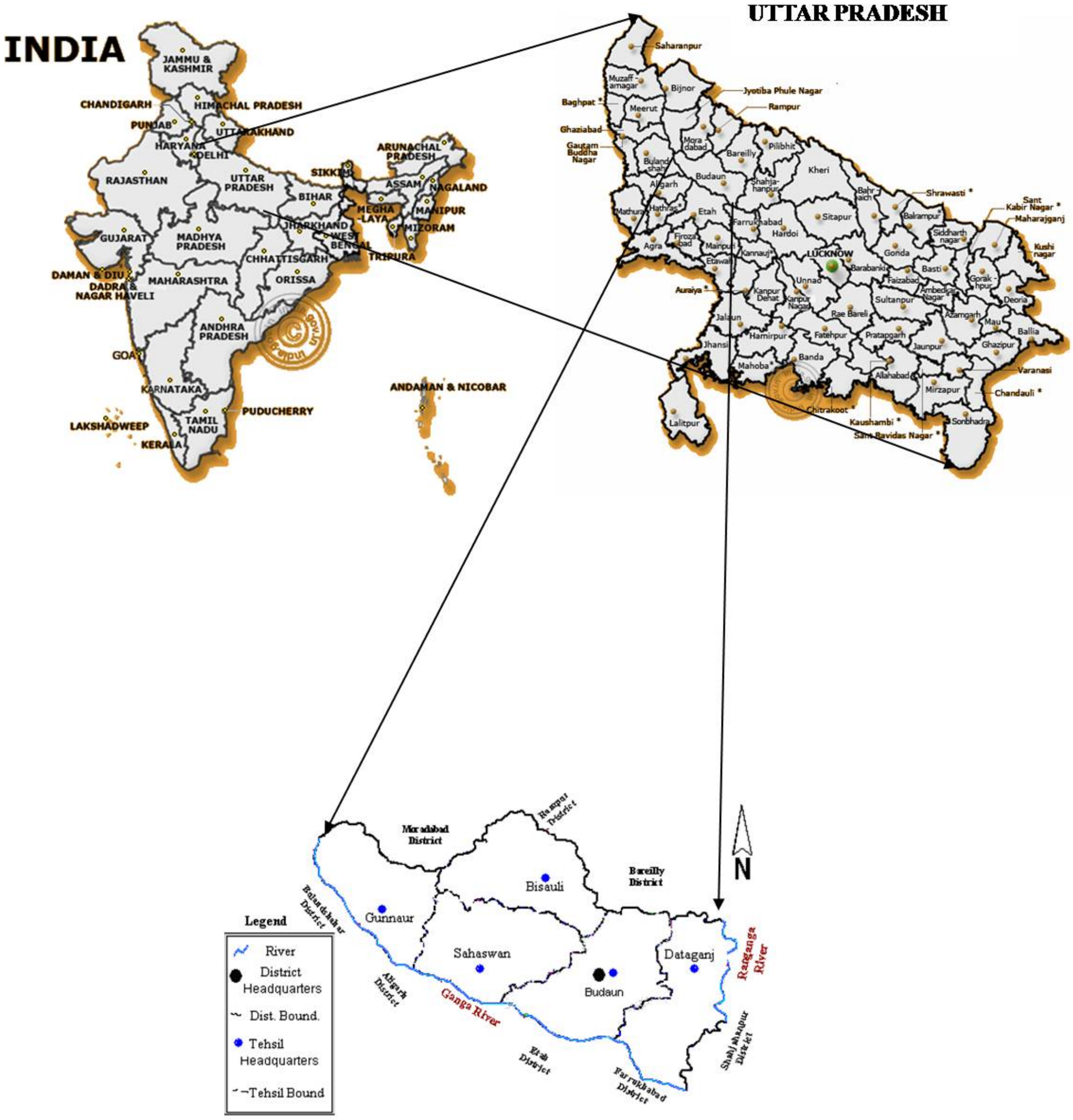

Figure 1. Index map of Buduan district

The Budaun district is a part of Gangetic alluvial plain of quaternary to recent age with flat topography and slopes from north-west to south-east. The height above mean sea level varies from 245 m in the extreme east to 298 m in the extreme north. It is plain of Ganga and Ramganga Rivers (Fig. 1).

6. Methodology

6.1. Fuzzification of Input and Output Data

Since in the present study the data were qualitative form rather to linguistic, hence these need to be fuzzified first. Groundwater recharge, Groundwater discharge, previous groundwater table and present water table elevation above mean sea level were fuzzified in to fuzzy subsets, in order to cover the whole range of changes. The criterion of defining fuzzy subsets is based on subjective perception of specific linguistic level by relevant experts.

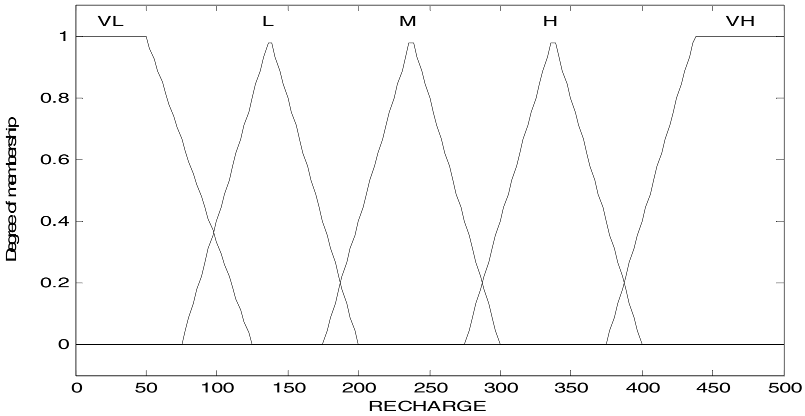

Figure 2. Membership function for recharge

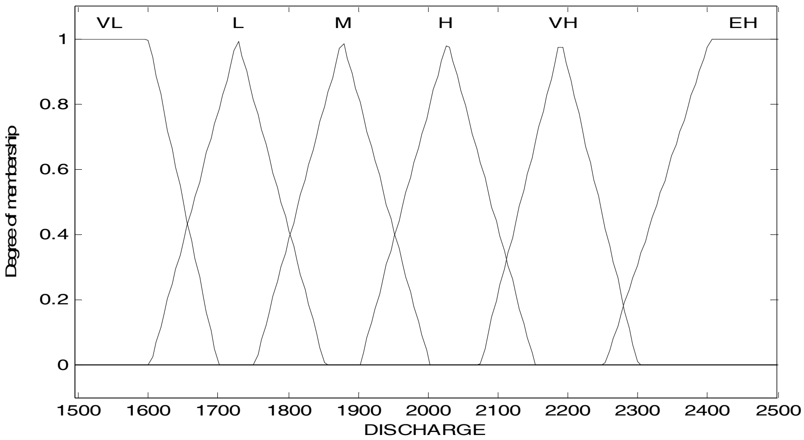

Figure 3. Membership function for recharge

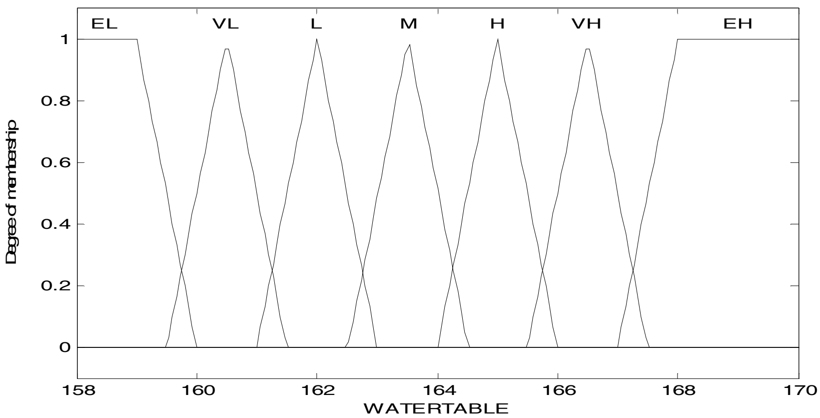

Figure 4. Membership function for water table elevation

Ground water recharge is divided into five subsets, as very low (VL), very low (VL), low (L), medium (M), high (H) and very high (VH) (Fig. 2). Ground water discharge is divided into six fuzzy subsets, as very low (VL), very low (VL), low (L), medium (M), high (H), very high (VH) and extremely high (EH) (Fig. 3). Water table elevation for both input and output is divided into seven fuzzy subsets, as extremely low (EL), very low (VL),very low (VL), low (L), medium (M), high (H), very high (VH) and extremely high (EH) (Fig. 4).

6.2. Fuzzy Rules

On the basis of the available data, we define relation between fuzzy inputs and out puts, these are fuzzy rules for the analysis. To combine input data we used fuzzy intersection rule for fuzzy sets. There are different rules for each model. Fuzzy rules are as in Table 1 and Table 2.

6.3. Defuzzification Method

The result obtained from the implication is in the form of a fuzzy set. This is defuzzified to get a crisp output. In the present study, we used centroid method to defuzzify the data, which is given by algebraic expression: $Z^* = \frac{\int \mu_c(z).zdz}{\int \mu_c(z)dz}$.

6.4. ANFIS Model

The acronym ANFIS derives its name from adaptive neuro-fuzzy inference system. In this model the fuzzy system is configured in a parallel fashion based on a corporative relationship a conceptual ANFIS consists of primarily five components: inputs and output database, a Fuzzy system generator, a FIS, and an adaptive neural network. The Sugeno-type FIS, which is the combination of a FIS and an adaptive neural network, was used in this study for the prediction purposes.

7. Model Development

Model 1:

A simple model was developed by taking two parameter ground water recharge and ground water discharge as input and water table elevation as output using the representation. *W (t) = f{R (t), D (t)}).*

Fuzzy Logic Rule base model and Adaptive Neuro Fuzzy Inference System are used to prediction of this model (FLM 1, ANFIS 1).

*Model 2***:**

The water table elevation at the time t-1 was added as an additional input variable to the model M1. Hence the water table elevation was expressed as a function of ground water recharge at time t, ground water discharge at time t and water table elevation at time t-1 *(i.e. W(t)=f{R(t),D(t),W(t-1)}).*

In this model our inputs are ground water recharge, ground water discharge for the current year and water table elevation for the previous year’s post monsoon season and the output is water table elevation for the current year.

Fuzzy Logic Rule based model and Adaptive Neuro-Fuzzy Inference System are used to prediction of this model (FLM 2, ANFIS 2).

8. Results and Discussion

8.1. Prediction of Ground Water Elevation for Model 1 and Model 2 Using Fuzzy Logic

The first model has been developed with recharge and discharge as input and in second model recharge, discharge and previous year water table as input. In both models the output variable is current year water table elevation above mean sea level. Models were constructed using fuzzy logic toolbox with MATLAB 7.0. The membership functions are constructed using the observed data. Fuzzy Inference System prepared using Fuzzy logic toolbox in MATLAB. Rules which are constructed from observed data are then added in fuzzy logic rule editor. The FIS was then evaluated to obtained output data which is predicted value of the water table elevation for the particular model. Predicted water table elevation values and observed water table elevation values were then plotted on ordinate against years on abscissa, to evaluate the model performance, on qualitative basis. Figures 5–6 show the close agreement between observed and predicted value can be easily observed.

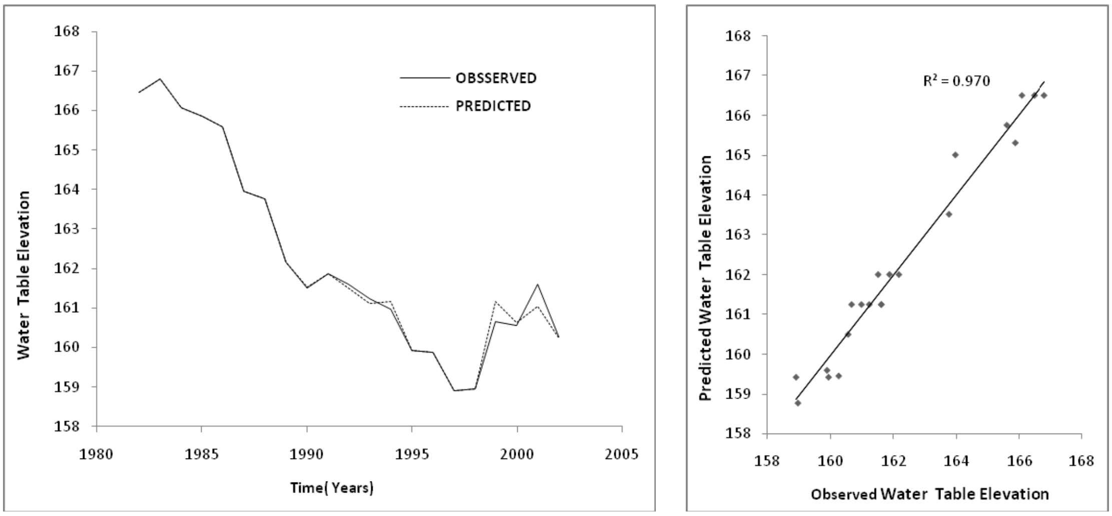

Figure 5. Prediction of ground water elevation for model 1 using fuzzy logic

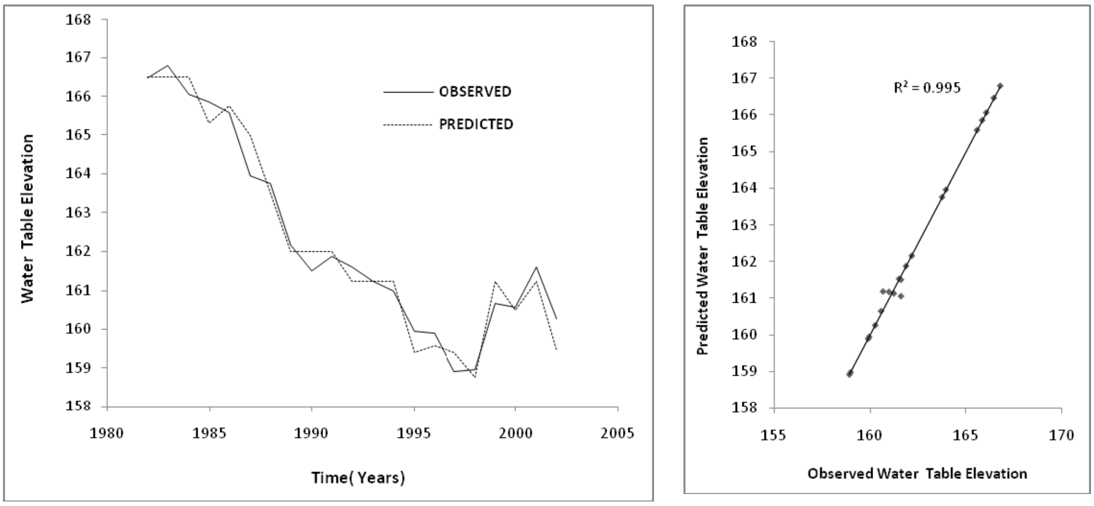

Figure 6. Prediction of ground water elevation for model 2 using fuzzy logic

8.2. Prediction of Ground Water Elevation for Model 1 and Model 2 Using ANFIS

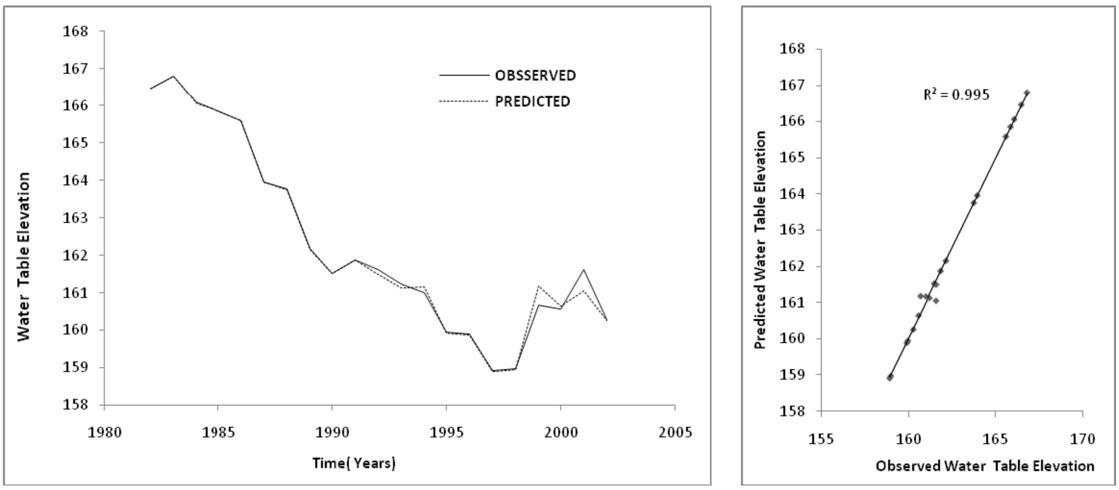

Figure 7. Prediction of ground water elevation for model 1 using ANFIS

The Sugeno type Fuzzy Inference System is used to construct the ANFIS model. The hybrid ANFIS model with 5 subsets of membership functions of trapezoidal shape for input and linear output membership function gives the best results. The 300 epochs were given to train the model. The observed and predicted values of water table elevation for both models were plotted (Figs. 7 and 8). These figures show that there exist close relation between observed and predicted values of water table elevation using ANFIS technique.

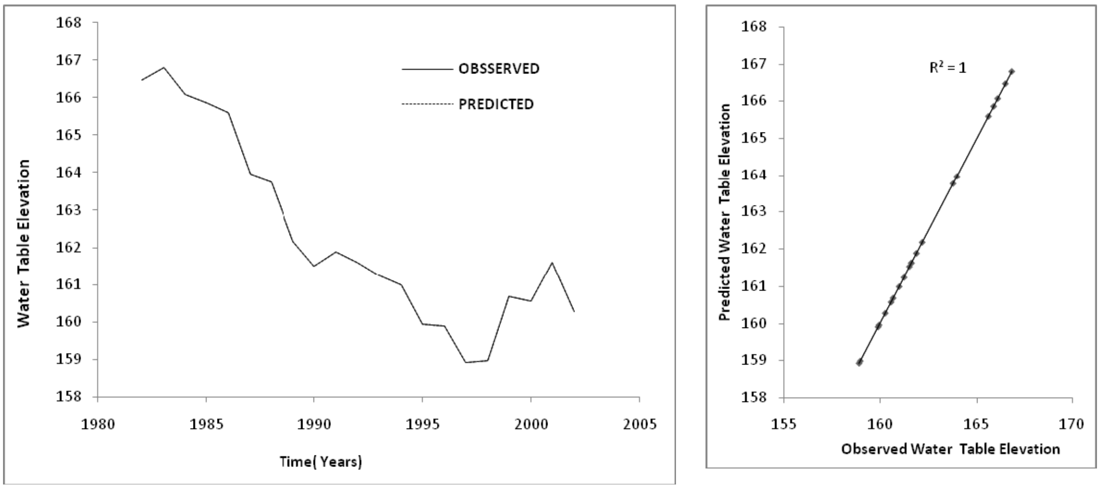

Figure 8. Prediction of ground water elevation for model 2 using ANFIS

8.3. Comparison between Fuzzy Logic Rule Based Models and ANFIS Models

The results shown in Table 3, represents the best possible models of water table elevation prediction and indicates that ANFIS Models performed better than Fuzzy logic rule based models shown graphically on Figs. 5 to 8.

Summary and Conclusions

- Soft computing techniques like Fuzzy Logic rule based models and ANFIS are reliable and more accurate than conventional methods. The performance of Fuzzy Logic rule based models and an ANFIS models was found to be satisfactory on the basis of performance evaluation of models.
- Model 2 (*i.e.* $W(t) = f\{R(t),D(t),W(t-1)\}$). Gives better results than the Model1 (*i.e.* $W(t) = f\{R(t),D(t),\}$).
- On the basis of performance evaluation of models, ANFIS model performed better than Fuzzy Logic rule based models.
- The present study yielded good results and has shown superior performance and the application of modelling ground water table fluctuation.
- ANFIS M 2 gives best results among all developed 4 models.

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Transport and Telecommunication Institute, Lomonosova 1, LV-1019, Riga, Latvia

PRODUCTION FUNCTION OF LATVIA

Yu. Kochetkov

Baltic International Academy
Lomonosova Str. 4, LV-1003, Riga, Latvia

The paper, based on statistical data, presents the mathematical model of the economy of Latvia during the transition period as the production function of Ch. Cobb-P.Douglas. Scientific-technological progress is treated as a function of time. The production function is used in a rate notation form. The evaluation of the parameters of the model is based on the least squares method. During calculations data of the Central Statistical Bureau of Latvia have been used for the period of 1990– 2003. After the production function has been calculated the checking of its adequacy and exactness was carried out by using standard procedures.

Keywords: *production function, least squares method, growth rate, multiple correlation, regression equation, forecast*

When studying economic processes under modern conditions, it is very difficult to obtain the necessary statistical data for construction of closed linear models of economy, e.g., Neumann models, which take internal structure of production into account. At the same time using enlarged economic indices it's easy to get conclusions of great content with the help of production functions mechanism and other econometric methods. The objective of the given article is to construct the mathematical model of economy of Latvia in a form of the production function and to analyse it by using statistical data.

One of the most widespread production functions is the neo-classical Ch.Cobb-P.Douglas function. This function, which is related to the multiplicative class, is remarkable with its structural simplicity. During the working process of specifying the production function, the Ch.Cobb-P.Douglas production function was chosen, in which scientific and technological progress is accounted as time function and is not directly connected either with labour or with capital. According to Hicks, such technological progress is called neutral and at this the marginal rate of substitution is a constant in time function of fund endowment. In a volumetric form this production function looks as follows:

$$Y_t = A \cdot e^{jt} \cdot K_t^{\alpha} \cdot L_t^{\beta}, \tag{1}$$

where Y_t – cumulative production output volume, K_t – cumulative fixed capital volume, L_t – total labour expenses, α and β – capital and labour production elasticity coefficients, t – time, $A \cdot e^{jt}$ – function, which reflects influence of many production factors on effectiveness, including scientific and technological progress (j). This function increases exponentially with increase of argument *t*. Coefficient A – is a progress multiplier.

Such a notation form of a *Ch.Cobb-P.Douglas* production function (1) is a mathematical model that shows connections between volumetric indices of production output and resource expenses. Along with the connections of indicated volumetric indices, connections among growth rates of these indices are often examined. There is a connection of *Ch.Cobb-P.Douglas* production function in a volumetric and a rate form [1]. Equation (1) transformed into linear by taking the logarithm:

$$\ln Y_t = \ln A + jt + \alpha \ln K_t + \beta \ln L_t . \tag{2}$$

We regard variables K_t and L_t as continuous differentiable time functions. After determining, that time periods equal to one year are to be examined, and taking a full differential from both sides of equation (2), when dt=1, we get:

$$\frac{dY_t}{Y_t} = j + \alpha \frac{dK_t}{K_t} + \beta \frac{dL_t}{L_t} . \tag{3}$$

Equation (3) represents a linear dependence of growth rates, corresponding to volumetric indices of the *Ch.Cobb – P.Douglas* production function. This will be a production function in a rate form. In equation (3), production output growth rate $\frac{dY_t}{Y_t}$ consists of 3 parts: j – contribution of scientific and technological forwarding, $\alpha \frac{dK_t}{K_t}$ – contribution from capital growth, $\beta \frac{dL_t}{L_t}$ – contribution from an increase in the number of the employees. Sometimes j is interpreted as a contribution into the development speed of intensive factors that reflect intensification of production on the macro level, and reflect the sum $\alpha \frac{dK_t}{K_t} + \beta \frac{dL_t}{L_t}$ – as a contribution of extensive factors.

We get an approximation formula with discreet growth rates if we substitute differentials in equation (3) for accretions ΔY_t, ΔK_t and ΔL_t. Formulas (1) and (3) will be equivalent only at a continuous time examination. However, statistical data used for evaluation of production functions, is discreet (usually these are annual data of the Central Statistical Bureau). That is why equations (1) and (3) represent different production functions, which have to be evaluated separately. *Ch.Cobb – P.Douglas* production function in the rate notation form (3) was used in the given paper.

It was assumed, that effectiveness level does not depend on the production scale. Therefore a regular limitation is imposed: $\alpha + \beta = 1$, from a priori considerations α and β values have to satisfy the following conditions: $0 < \alpha < 1$ and $0 < \beta < 1$. By using the indicated limitation and presuming that $\beta = 1 - \alpha$, equation (3) was transformed into a single-factor linear model in a form of a polynomial of the first power:

$$\frac{dY_t}{Y_t} - \frac{dL_t}{L_t} = j + \alpha \left(\frac{dK_t}{K_t} - \frac{dL_t}{L_t} \right). \tag{4}$$

Evaluation of the parameters of the given model was carried out with the help of the least squares method by means of solving the system of normal equations:

$$\begin{cases} j \cdot n + \alpha \sum \left(\frac{dK_t}{K_t} - \frac{dL_t}{L_t} \right) = \sum \left(\frac{dY_t}{Y_t} - \frac{dL_t}{L_t} \right) \\ j \sum \left(\frac{dK_t}{K_t} - \frac{dL_t}{L_t} \right) + \alpha \sum \left(\frac{dK_t}{K_t} - \frac{dL_t}{L_t} \right)^2 = \sum \left(\frac{dY_t}{Y_t} - \frac{dL_t}{L_t} \right) \left(\frac{dK_t}{K_t} - \frac{dL_t}{L_t} \right), \end{cases} \tag{5}$$

where n – number of row levels and summation sign applies to all levels of the initial time row.

During construction of the model of the gross domestic product of Latvia (GDP) was shown as variable Y and the number of the employed population was shown as variable L. Since, in fact, reliable fixed capital statistics does not exist; non-financial investments were used instead of this index, as they are closely connected with the volume of accumulated capital. Besides that, new technologies are introduced into economy exactly because of investments, but the model was constructed in growth rates. Data of the Central Statistical Bureau of Latvia were used from 1990 to 2003 incl., GDP and investments were converted into constant prices of the year 2000. Using the data of the Central Statistical Bureau of Latvia discreet growth rates of indices (Table 1) were calculated, which afterwards were used for statistical calculations at parameterisation of the production function.

Table 1. Discreet growth rates of indices

Group number	$\frac{Y_t - Y_{t-1}}{Y_{t-1}}$	$\frac{L_t - L_{t-1}}{L_{t-1}}$	$\frac{K_t - K_{t-1}}{K_{t-1}}$
1	-0.12595	-0.00845	-0.31839
2	-0.32118	-0.07345	-0.54041
3	-0.11412	-0.06892	-0.40793
4	0.02190	-0.10124	0.07072
5	-0.00945	-0.03453	0.07006
6	0.03681	-0.09239	0.51448
7	0.08376	0.04320	0.25564
8	0.04758	-0.00404	0.36246
9	0.02839	-0.01826	-0.05186
10	0.06844	-0.02789	0.22441
11	0.07935	0.02232	0.07938
12	0.06067	0.02807	0.11841
13	0.16288	0.01820	0.13168
Average values	**0.00147**	**-0.02441**	**0.03913**

As a result of system of equations (5) solution by data application from Table 1, model parameters were found: $j = 0{,}0065$; $\alpha = 0{,}3$; $\beta = 0{,}7$. Accomplished form of the production function of Latvia during the transition period can be presented as *Ch.Cobb – P.Douglas* function in the rate form of notation:

$$\frac{dY_t}{Y_t} = 0{,}0065 + 0{,}3 \cdot \frac{dK_t}{K_t} + 0{,}7\frac{dL_t}{L_t} . \qquad (6)$$

Then verification of the obtained production function was carried out. Coefficient of multiple correlation R_{ykl} was calculated that determines the closeness degree of the connection among the change rate of GDP, the non-financial investments change rate and the change rate of the number of the employed population in Latvia:

$$R_{ykl} = \sqrt{\frac{r_{yk}^2 + r_{yl}^2 - 2r_{yk} \cdot r_{yl} \cdot r_{kl}}{1 - r_{kl}^2}} , \qquad (7)$$

where r_{yk} , r_{yl} , r_{kl} – paired correlation coefficients, that determine connection closeness of respective factors. Here the following notations are introduced: $y = \frac{dY_t}{Y_t}$, $k = \frac{dK_t}{K_t}$, $l = \frac{dL_t}{L_t}$.

Paired correlation coefficients calculation was carried out according to the following type of formula: $r_{yl} = \frac{cov(y,l)}{\sigma_y \cdot \sigma_l}$, where $cov(y,l)$ – covariance y and l , σ_y , σ_l – average quadratic drift of factors. Calculation results are presented in Table 2.

Table 2. Correlation and determination coefficients

Paired correlation coefficients	r_{yk}	r_{yl}	r_{kl}
	0.79800	0.26064	0.11234
Multiple correlation coefficient R_{ykl}	0.81634		
Cumulative determination coefficient R^2_{ykl}	0.6664		

Since multiple correlation coefficient $R_{ykl} > 0{,}7$ is sufficiently close to 1, we can say that closeness degree of connection among the change rate of the GDP of Latvia and the change rate of the number of the employed population and the non-financial investments change rate is sufficiently high. This shows good approximation of the model to the actual data. Cumulative determination coefficient $R^2_{ykl} = 0{,}6664$ shows the variation part of the resulting indication $\frac{dY_t}{Y_t}$ under the influence of factorial indications $\frac{dK_t}{K_t}$ and $\frac{dL_t}{L_t}$. We can say, that almost 67% of the rate of the GDP change variation is explained by the presented multiple linear regression equation (6), and 33% of the GDP variation results from the influence of unaccounted factors in the model.

Apart from cumulative determination coefficient R^2_{ykl} , such statistical accuracy index as regression equation average quadratic drift or average quadratic mistake was calculated in order to evaluate the accuracy of the model. For this, theoretical resulting values of the indication (y'_i), excesses ($e_i = y_i - y'_i$)

and their squares were calculated according to model (6). Average quadratic mistake was found using the formula:

$$S_e = \sqrt{\frac{1}{n-h} \cdot \sum_{i=1}^{n} (y_i - y_i')^2} \ , \tag{8}$$

where $h = 3$ – number of parameters in the regression model; n – number of groups.

As a result of the calculation, the average quadratic drift of the reviewed resulting indications (y_i) values were obtained from theoretical values of (y_i') that were calculated according to the model: $S_e = 0{,}06647$. Average quadratic mistake of regression equation (S_e) was found and equalized with the average quadratic drift of the resulting indication (σ_y =0.11936) that was calculated applying empirical data. Since it was ascertained that $S_e < \sigma_y$, then it is expedient to use the regression model.

In order to find out whether the regression equation (6) is suitable for practical use, e.g., for prognoses, the evaluation of significance of this equation as a whole was carried out. At that, the fundamental hypothesis H_o about non-significance of the equation (6) was set up. F – the statistics that has Fisher-Snedekor distribution was used for its check-up:

$$F = \frac{R^2_{ykl}}{1 - R^2_{ykl}} \cdot \frac{n - m - 1}{m} , \tag{9}$$

where m – number of explanatory variables (m = 2).

Calculated value $F = 9{,}988$. By using Fisher-Snedekor distribution tables the F value has been found – criterion subject to significance degree of γ and to the number of degrees of freedom. At the significance level of $\gamma = 5\%$ (confidence probability 95%) $F_{crit} = 4{,}10$, and at $\gamma = 1\%$ $F_{crit} = 7{,}56$. Since in both cases it was ascertained that $F > F_{crit}$, then zero hypothesis H_o is rejected and an alternative hypothesis H_1 about statistical significance of the regression equation as a whole is accepted. Therefore equation (6) is suitable for practical use.

Since determination coefficient value does not prove the high quality of the regression equation yet, autocorrelation of the accidental constituent was checked. Excess (e_i) is the evaluation of the accidental constituent. In order to detect autocorrelation of excesses the Durbin-Watson criterion was used (DW), which comes to examination of the main hypothesis H_o about absence of autocorrelation. The calculated value of this criterion was determined in accordance with the formula:

$$DW = \frac{\sum_{2}^{n} \left(e_i - e_{i-1}\right)^2}{\sum_{1}^{n} e_i^2} \ . \tag{10}$$

Calculations showed that $DW = 1.6201$. According to Durbin-Watson distribution tables, lower and upper critical borders of DW– statistics were found: taking the given n, the number of discretion ranges $k' = 2$ and the significance level of $\gamma = 5\%$ $d_L = 0{,}861$ and $d_U = 1{,}562$. Since it was ascertained that the calculated value of the criteria is more than the upper critical border, but less than 2 ($d_u < DW < 2$), then hypothesis H_o about absence of autocorrelation is accepted. DW statistics confirmed the non-correlativeness of drifts from the regression line, and it can be said, that the obtained formula (6) is satisfying.

The characteristic of normal balance distribution is of great importance for determining confidence intervals of the model. Therefore the hypothesis about normal distribution character of indicated accidental components was checked, basing on the asymmetry and excess indices analysis. The following formulas were used to calculate asymmetry (A) and excess (E) indices [2]:

$$A = \frac{\frac{1}{n}\sum_{i=1}^{n} e_i^3}{\sqrt{\left(\frac{1}{n}\sum_{i=1}^{n} e_i^2\right)^3}} , \qquad E = \frac{\frac{1}{n}\sum_{i=1}^{n} e_i^4}{\left(\frac{1}{n}\sum_{i=1}^{n} e_i^2\right)^2} - 3 \ . \tag{11}$$

Also average quadratic mistakes of sample characteristics of asymmetry (σ_A) and excess (σ_E) were calculated:

$$\sigma_A = \sqrt{\frac{6(n-2)}{(n+1)(n+3)}}; \qquad \sigma_E = \sqrt{\frac{24n(n-2)(n-3)}{(n+1)^2(n+3)(n+5)}}. \tag{12}$$

The following results were obtained: $A = -0{,}128117$; $E = -0{,}662366$; $\sigma_A = 0{,}542810$; $\sigma_E = 0{,}779739$. Since inequalities are simultaneously fulfilled:

$$|A| < 1{,}5\sigma_A; \qquad \left|E + \frac{6}{n+1}\right| < 1{,}5\sigma_E, \tag{13}$$

it is possible to say, that the hypothesis about normal character of excesses distribution is not rejected.

Thus, verification of function (6) that was obtained as a result of parameterisation of the production function showed that the given model, constructed on the basis of temporary rows of analysed indices, is adequate and sufficiently precise. Graphical results of the calculation according to the model, which illustrates its quality, are presented on Figure 1.

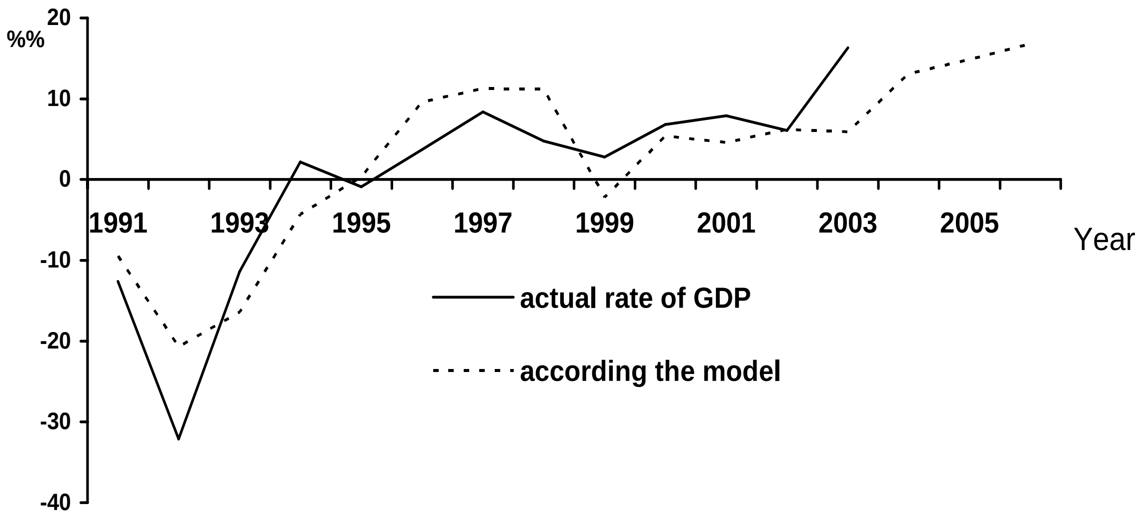

Figure 1. Production function of Latvia

In order to forecast the resulting indication according to model (6), i.e. to predict the rate of the GDP growth several steps ahead, at first it is necessary to determine forecast values of all factors that are included in the model (the rate of growth of the employed population L and of non-financial investments K). With this aim, models of paired regression were used that were calculated according to empirical data, in a form of linear functions:

$$\bar{y}_i = a_0 + a_1 \cdot t, \tag{14}$$

where $\bar{y}_1 = l$, $\bar{y}_2 = k$; a_0, a_1 – model parameters.

Model parameters evaluation was carried out according to the least-squares method resulting from the solution of simultaneous normal equations [1]:

$$\begin{cases} n \cdot a_0 + \left(\sum t\right) \cdot a_1 = \sum y_i \\ \left(\sum t\right) \cdot a_0 + \left(\sum t^2\right) \cdot a_1 = \sum y_i \cdot t. \end{cases} \tag{15}$$

Verification of the obtained models was carried out according to the above method. Calculation results are presented in Table 3.

Table 3. Regression equations $\bar{y}_i = f(t)$ and the results of models verification

No	Index name	Function l	Function k
1	Regression equation	$l = -0{,}077236 + 0{,}007546t$	$k = -0{,}268908 + 0{,}044005t$
2	Correlation coefficient	$r_{lt} = 0{,}520137$	$r_{kt} = 0{,}442904$
3	Determination coefficient	$r_{lt}^2 = 0{,}2704$	$r_{kt}^2 = 0{,}196164$
4	Average quadratic mistake of the regression equation	$S_{lt} = 0{,}038962$	$S_{kt} = 0{,}261033$
5	Average quadratic drift of selection	$S_l = 0{,}045618$	$S_k = 0{,}291146$
6	F – statistics, significance level 5% ($F_{crit} = 4{,}84$)	$4{,}077 < F_{crit}$	$2{,}684 < F_{crit}$
7	Durbin – Watson (DW) criterion, significance level 5%, $d_L = 1{,}01$; $d_U = 1{,}34$	2.212 $2 < DW < 4 - d_U$	1.102 $d_L < DW < d_U$
8	Asymmetry (A) and excess (E)	$A = 0{,}242388$ $E = -0{,}311286$	$A = 0{,}527771$ $E = -0{,}529337$
9	Asymmetry (σ_A) and excess (σ_E) average quadratic mistakes	$\sigma_A = 0{,}54281$ $\sigma_E = 0{,}779739$	$\sigma_A = 0{,}54281$ $\sigma_E = 0{,}779739$

Data analysis in Table 3 shows the following. Correlation coefficient $r_{lt} > 0{,}5$, therefore there is a characteristic moderate connection between l and t. Since coefficient $r_{kt} < 0{,}5$, then the connection between k and t is weak. Average quadratic mistake of the regression equation in both cases is less than the corresponding average quadratic drift of selection, which points out to the expediency of practical use of obtained regression equations. On the other hand, evaluation of the obtained regression equations significance, by calculating F – statistics (significance level 5%), showed, that one cannot reject zero hypothesis about the insignificance of regression equations: calculated values of the criterion are less than F_{crit}. This shows that functional connections l and t, k and t are not strong enough. Relatively small determination coefficient' values also confirm this.

For autocorrelation excesses check-up of Durbin-Watson criteria were calculated. In case of l function DW statistics confirmed H_o hypothesis about absence of autocorrelation. Though, in the second case (k function) it was ascertained that the calculated value of the criterion is situated in the area of ambiguity. Therefore, there is no sufficient reason to make one or the other conclusion about independence of level values of accidental component, i.e. about absence of a sufficient autocorrelation of the first order in a residual sequence at the given quantity of row levels. Also the hypothesis about the normal character of excesses distribution, when calculating according to regression levels was checked. Asymmetry and excess analysis showed that in both cases both inequalities (13) are simultaneity fulfilled. Consequently, the hypothesis about normal character of distribution is not rejected for both functions.

The final stage was tendency extrapolation on the basis of the calculated regression equations. Forestalling period of 1, 2 and 3 years was taken. In the addition to the point forecast, levels of possible borders within the changes of the forecast indices were determined. For this, forecast confidence intervals were calculated, that take into consideration the uncertainty, which is connected with the trend's position, the possibility of drifting from this trend was calculated as well [2]:

$$y_{n+L'} \pm t_\alpha \cdot S_y \sqrt{\frac{n+1}{n} + \frac{(t_1 - \bar{t})^2}{\sum_1^n (t - \bar{t})^2}}, \tag{16}$$

where n – time row length, L' – forestalling period, $y_{n+L'}$ – point forecast at the moment $n + L'$, t_α – t value of Student's statistics, S_y – average quadratic value of factual observations' drift from calculated,

t_1 – forestalling time, for which the extrapolation is made, $\bar{t}$ – ordinal number of level, that is standing in the middle of the row, t – ordinal number of levels of the row. Prediction calculation results are presented in Table 4, confidence probability is 90% and it is also reflected on Figure 1.

Table 4. Point and interval indices of predictions

Factor	Forestalling time	Point forecast	Confidence probability forecast	
			Lower level	Higher level
l	Year 1	0.0284	-0.0528	0.1096
	Year 2	0.0359	-0.0477	0.1196
	Year 3	0.0435	-0.0428	0.1298
k	Year 1	0.3472	-0.1968	0.8911
	Year 2	03912	-01691	0.9514
	Year 3	0.4352	-0.1431	1.0135
y	Year 1	0.1305	-0.0080	0.2690
	Year 2	0.1490	0.0064	0.2917
	Year 3	0.1675	0.0202	0.3148

The forecast for the economy of Latvia is favourable – production output growth rate will be increasing in the nearest future. Taking into consideration that adequacy check-up of models (14) did not give a positive result on all points (Table 3), model (6) forecast was also carried out with application of average arithmetical data l and k for the last 3 years. It is ascertained, that if the formed economic development pattern is maintained, the predicted value of the GDP growth rate will amount to 0.0554. The given value is situated within the limits of the calculated predictions confidence intervals closer to interval of lower borders. Thus, the model that was constructed on the basis of discovering patterns of the dynamics of the country's economic development for a definite period, in a shape of Ch.Cobb-P.Douglas production function allows to make medium-term forecast for future.

Conclusions

The testing of the production function of Latvia showed that the chosen mathematical model was adequate and sufficiently accurate. In order to forecast the rate of the GDP growth in Latvia for some period ahead the forecasted amounts included in the model of factors were determined. With this purpose models of pair regressions were calculated on the basis of empirical data as well as their adequacy checked. The last stage was extrapolation of the tendencies, basing on the obtained models. The accepted period was 1, 2 and 3 years. Single prospects were also calculated as well as their confidence intervals. The obtained result showed that the forecast for the economy of Latvia is favourable – the rate of production growth will be increasing in the nearest future. The forecast was confirmed for a few last years.

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Transport and Telecommunication Institute, Lomonosova 1, LV-1019, Riga, Latvia

EXPERTS SYSTEM FOR STUDENTS' KNOWLEDGE ASSESSMENT IN THE AREA OF HOSPITALITY TECHNOLOGY

I. Kulish, R. Muhamediyev

Information Systems Management Institute
Department of Natural Sciences and Computer Technologies
Ludzas 91, LV-1003, Riga, Latvia
E-mail: irena.kulish@inbox.lv, ravil@inbox.lv

The deficiency of single level additive model of evaluation is considered. The model is frequently used by test systems and other systems of evaluation. It is very simple but the single level additive model of evaluation can't realize linear indivisible function. To do away with deficiency of additive model expert systems has a proposal. An output mechanism would be based on neural network, productions, fuzzy logic or Bayes logical conclusion. The examples of expert systems to evaluate the students' knowledge in hospitality technology is considered.

Keywords*: experts system, neuron nets, knowledge evaluating*

1. Introduction

In order to perform an assessment in practice, the one-level additive models are broadly applied. The most characteristic feature of this kind of assessment – a decision about the result is made based on the total amount of the received points (marks). The broad usage of this method is guaranteed by its simplicity. There is a great amount of systems of this type. One of the most common methods is the knowledge assessing test system, in which the result is formed by the total amount of received points for correct and incorrect answers, also various risk assessment systems, e.g. the examined company risk evaluation system [1]. The many test systems for determining the psychological portrait or condition can be added to this category [2].

In systems of this type the main principle of operation is the calculation of total points (marks). The amount of points for each indicator or the proportion of the indicator is determined by the expert. The acquired assessment (additive convolution) is the reflection of the to-be-tested object's current condition. In the case of one-level assessment's additive system:

$$T = w_1 x_1 + w_2 x_2 + ... + w_n x_n \ , \qquad (1)$$

where T – the total of received points, w_i – specific weight of the indicator (amount of points), x_i – indicator or answer.

The shortcomings of systems like these, which are often perceived on an intuitive level, become apparent in the expressive constraints of one-level assessment's additive model. E.g. assessing students' knowledge, the knowledge in separate essential issues becomes a critical point. This means that in the case of an incorrect answer to a question of this type a correctly formed system will give a negative assessment. It is complicated to complete this in a one-level model, as the negative answer is compensated with other positive answers. In separate cases this shortcoming can be eliminated by defining penalty points, e.g. in the total amount of potential positive points. But there are also more complicated cases, when this approach will not give a result, e.g. when the assessment is performed by a summing up of the answers.

For example, assessing a student's term paper, the method of co-operative assessment is often applied in accordance with criteria developed by experts beforehand. Four groups of experts are involved in the examined assessment: students themselves, their fellow students, employees, and teacher. Before the assessment students define the extent of importance of each expert and calculate the expert's ratio. Then the paper is assessed by all experts, and the results are discussed together. However, with a formal approach this assessment is not objective enough. For a joint assessment of the paper a mutual proportion of the separate parts can carry a vital importance.

Example 1. If the introduction part does not correspond with the conclusions, then without considering that both parts separately can be assessed positively, the paper can not be accepted.

Example 2. It can turn out that practically the indicators of all chapters are rather weak, but that the most important chapter (or several chapters) possess several outstanding features; in this case the importance of other indicators can be raised.

It is observed from the mentioned examples that a conclusion can be made based on the summing up of features (Example 1) or in the assessment process a correction of indicators must be done (Example 2). It is obvious that performing the correction of Formula 1 with weight correction, it is impossible to reach a solution in Example 1 and Example 2 (the change of expression is necessary as a minimum).

Thus in the terms of a simple additive model it is difficult or even impossible for the assessor to find solutions for the tasks of knowledge assessment encountered in practice. (Similar examples in the area of company risk evaluation have been shown earlier [1].

2. Application of Neuron Nets and Productive Systems in Assessment of Knowledge

Below, it can be observed that Examples 1 and 2 can be solved based on the base of neuron net (http://center.fio.ru/vio/VIO_01/Present/ITO/1999/II/5/5121.html).

We will apply neuron net with a basic element pade-neuron with an activation threshold function (http://bukhnin.chat.ru/). Indicators are displayed in Table 1.

Table 1. Risk indicators and their values (Example 1)

Symbol	Indicator	Value
x1	Introduction	1, if the first assessment is positive, in the opposite instance 0
x2	Conclusions	1, if the first assessment is positive, in the opposite instance 0
x3	Correspondence of introduction and conclusions	1, if introduction corresponds with conclusions, in the opposite instance 0

A neuron model is as follows:

$$y = f\left(\frac{\sum_{i=1} a_i x_i + a_0}{\sum_{i=1}^{n} b_i\, x_i + b_0} \right). \qquad (2)$$

The threshold function of activation can be displayed with an expression:

$$f(g) = \begin{cases} "accept", g < 1 \\ "decline", g \geq 1 \end{cases}, \qquad (3)$$

Assuming that $a_0 = b_0 = 0$, $a_1 = a_2 = a_3 = 1$ and $b_1 = b_2 = 1$, $b_3 = 2$ one can make sure, that with existence of quantities $x_3 = 0$, $x_2 = x_1 = 1$, the application result of expression 2 and 3 will be $g > 1$, accordingly *f(g) = ”decline”,* i.e. the paper is not accepted, but with existence of quantities $x_3 = x_2 = x_1 = 1$, $g < 1$, *f(g) = ”accept”,* the paper will be accepted, which corresponds with the conditions of Example 1.

Considering that threshold function (3) is equivalent to the condition:
if g < 1, then “accept” else „decline”, then one can continue the solving of Examples 1 and 2, applying the system of rules (production).

Let us show that the task can be solved by means of productive system, applying the chain of direct reasoning (forward chaining) (http://bukhnin.chat.ru/).

We will mark the quantity of the indicator as *True* or *False,* accordingly 1 and 0.

Then

Rule 1: IF x_1 *AND* x_2 *AND* x_3 *THEN «accept»*

Rule 2: IF NOT x_3 *THEN «decline».*

In order to solve Example 2 another rule must be introduced:

Rule 2: IF x_4 *THEN «to raise the assessment for all parts of the paper»,*

where x_4 = *True*, if one or several chapters display outstanding indicators and *False* in the opposite instance, but *«to raise the assessment for all parts of the paper»* – a function which can change the assessment of paper's parts.

It seems that the application of the single-layer neuron net is equivalent to the application of the productive system with a direct chain of reasoning. This is not true. Let us examine another hypothetical example.

Example 3. A system of training management is defined. The system must solve a question about further education, based on testing. In the simplest case there can be 2 basic questions (Q1 and Q2). Besides one can assume, if the trainee gives positive answers to both questions (1), then training is not compulsory, if both questions are answered negatively (0), then training has not been put into effect. Both cases can serve as evidence for the inefficiency of the training system.

Table 2. Example 2 formalization

Q1 \ Q2	1	0
1	A	B
0	B	A

A – inefficient system, B – efficient system.

Applying pade-neuron weights, it is impossible to separate situations A and B (if the function of activation is not changed), while a corresponding rule, for example,
IF $Q_1 = Q_2$ *THEN A ELSE B* easily solves the situation.

It must be mentioned that changing pade-neuron activation threshold function (expression 3) to $f(g) = \begin{cases} A, g = 0 \\ B, g \neq 0 \end{cases}$ and assuming following quantities of neurons $a_0 = 0$, $b_0 = 1$, $a_1 = 1$, $a_2 = -1$ and $b_1 = b_2 = 1$, it is not difficult to make sure that, with a condition $x_2 = x_1$ the result of the neuron will be 0, less or more than zero (0.5 or - 0.5). But these kinds of changes are a kind of deviation from the ideology of neuron net, which examines nets with equal pair functions, most often sigmoidal. In the opposite instance net training is a problem difficult to solve.

On the whole, the situation viewed in Example 3 can not be solved on principle, applying in assessment one-level additive system or single-layer neuron net in its classical understanding.

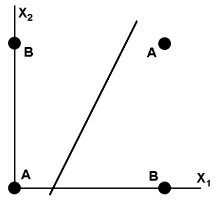

Figure 1. XOR problem

Thus in the system of two indicators the feature vector is two-dimensioned. In this case the equation:

$$T = w_1 x_1 + w_2 x_2, \tag{4}$$

will be a straight line (one-dimension hyper-plane) equation. Obviously there are situations when a straight line (more generally hyper-plane) can not separate the categories of situations (images). If we view two situations A and B, placed in system of rectangular coordinates x_1 and x_2, as shown in Fig. 1, then with no conditions the straight line will be able to divide the plane in such a way, that points belonging to various situations (categories). Division in categories as shown on Figure 1, implements the logical function of XOR excluding for input signals. This problem is entitled the excluding XOR problem, i.e. impossibility to model excluding XOR by means of a linear function. First time it was examined in connection with the research of single-layer perceptron (2, 3). Functions unable to realize within a single-layer network, were entitled linearly inseparable. Thus, the mathematic model of one-level additive assessment (1) is equivalent to perceptron with corresponding restrictions for the category of recognizable images. Thus, the additive model of one-level assessment can not realize a linearly inseparable function.

In testing an empiric exit from such a condition is multistage (adaptive) systems, in which the rating of objects is displayed as convolution of indicators in sections and each indicator is determined by the convolution of criteria, which, in their turn, can be defined as elementary indicators of functional dependence (in output data) [4, 5]. In this case the mathematical model, for example, can be

$$T = f_2\left(\sum_{i=1}^{N} w_{2i} f_1\left(\sum_{k=1}^{K} w_{1k} x_k\right)\right), \tag{5}$$

where f_1, f_2 functional dependencies in elementary indicators and accordingly in sections, w – specific weight, x – quantity of elementary indicators, N – number of sections, K – number of elementary indicators in a section.

It is obvious, that expression 5 corresponds to the mathematical model of two-layer neuron network with direct links [3] with a precision to the signal of initial excitation and designations. Two-layer model can solve XOR problem [6]. Thus, solutions of linearly inseparable tasks are possible applying two-layer and multi-layer networks, networks with nonlinear synapses, productive systems and systems of Bayes' decision-making, which can be viewed below.

3. Bayes' Decision-Making Method

Bayes' decision-making method is mathematically well-grounded. According to it an implementation of conclusion mechanism is possible, which provides a possibility to solve tasks of diagnostics, testing and planning. A characteristic of these tasks is a necessity to consider uncertainty peculiar to the answers of trainees. Bayes' method provides a possibility to consider this uncertainty. Bayes' method is based on probability of an event as an assessment, given by a person and it can change receiving any additional information. The mathematical foundation of this method is formed by Bayes' Theorem. The theorem views pairs of certain amount of incompatible events $H_1, H_2, \ldots, H_n$, considering that one of the events will definitely come true, event S with probability P(S) > 0. Then, according to theorem, the probability of the event H_i, with a condition that event S has come true, can be calculated with following formula:

$$P(H_i / S) = \frac{P(S / H_i)P(H_i)}{\sum_{j=1}^{n} P(S / H_i)P(H_i)}, \qquad (6)$$

Following terminology has been accepted: events H_i are hypotheses, $P(H_i)$ – a priori probabilities of hypothesis, $P(H_i|S)$ – a posteriori probabilities of hypothesis, event S – symptoms, $P(S|H_i) = P_i^+$ – prove probability of hypothesis H_i proved with symptom S, $P(S|{\sim}H_i) = P_i^-$ – disprove probability of hypothesis H_i disproved with symptom S.

In order to assess the influence of symptom S on hypothesis H_i, it is necessary to mark the symptom, for instance, by asking the user if the event S has realized. If it has realized (answer “Yes”), then for calculations of the new probability (a posteriori) the following formula is used:

$$P(H_i|S) = P_i^+ P(H_i)/[P_i^+ P(H_i) + P_i^- (1 - P(H_i))], \qquad (7)$$

If the event has not realized (answer “No”), formula:

$$P(H_i|{\sim}S) = (1-P_i^+) P(H_i)/[(1 - P_i^+ P(H_i) - P_i^- (1 - P(H_i))], \qquad (8)$$

If an answer “Don't know” is given, then a posteriori probability of hypothesis H_i remains invariable. In order to list the uncertainty of the knowledge, user can expand the list of possible answers, offering to describe the symptom according to scale from –n to +n, where n > 1, –n answer “No”, 0 – “Don't know”, but +n – “Yes”. For example, if n = 5, then answer 4 corresponds to extent of certainty “Very probable a yes”, whereas an answer -3 corresponds to extent of certainty “More probable a no”. Below a process of decision-making is viewed for a hypothesis. Assuming, that H – a hypothesis with an a priori probability P(H) and it depends on the symptoms $S_1, S_2, \ldots, S_k$, where $k \gtrsim 1$. Each of the symptoms S_i is connected with two probabilities $P_i^+ = P(S_i|H)$ and $P_i^- = P(S_i|{\sim}H)$. First probability characterizes symptom's extent of participation in this hypothesis (e.g. in the system of medical experts a probability of high temperature S, catching the flu H), but the other probability characterizes symptom's extent of participation in other hypotheses (carrying on the previous illustration, the probability of high temperature in case if the patient does not have the flu). It must be noted that P_i^+ and P_i^- are not mutually dependent, each of them can be assessed from 0 to 1. If P_i^+ can be assessed using the stored data (a book with students' grades e.a.), then calculations of P_i^- practically impossible, P_i^- assessment is nearly always dependent on the expert.

Applying Bayes' method in decision-making, Example 3 can be solved in a fragment of the following primitive experts' system:

Questions:

1. Is only one task completed correctly?
2. Are both tasks completed correctly?

Conclusions:

1. Both tasks are solved or no tasks are solved (A), 0.5, 1,0,1, 2,1,0
2. One of the tasks is not solved (B), 0,5 1,1,0, 2,0,1

In conclusions the a posteriori probability is recorded and probabilities of proving or disproving hypotheses. Viewing, for instance, a conclusion “None of the tasks were solved (B), 0,5 1,1,0, 2,0,1”, then 0,5 is the probability of an a priori conclusion (result). Then it is followed by three numeral combinations, in which the first number corresponds to the number of questions (symptom), but second and third – to probability, respectively P_i^+ – for approval of hypothesis and P_i^- disproval of hypothesis. For example, if the answer to the first question is “Yes”, then $P_i^+ = 1$, $P_i^- = 0$. The calculation of conclusion's a posteriori probability is performed according to formulas 7 and 8. Asking questions and receiving user's answers, the system with certainty models a situation, when tasks are solved completely or completely unsolved (A), and situations when only one task is solved (B).

4. Experts' System for Assessing Knowledge in Hospitality Culture

Based on the performed analysis, the multi-layer neuron systems and Bayes' decision-making systems can be recommended for application in systems of knowledge assessment. The latter of the listed methods is chosen for realization of experts' system, Evaluating Knowledge of Hospitality Service Culture (further EKHSC)). EKHSC structure is displayed on Figure 2. System is formed by 28 local systems of experts, in diagram shown as circles and are divided into four groups. Each of the groups assesses an aspect of hospitality culture: ethical, aesthetic, organizational, organizing and psychological. Colour circles refer to Technological culture), uncoloured – to Humanitarian culture.

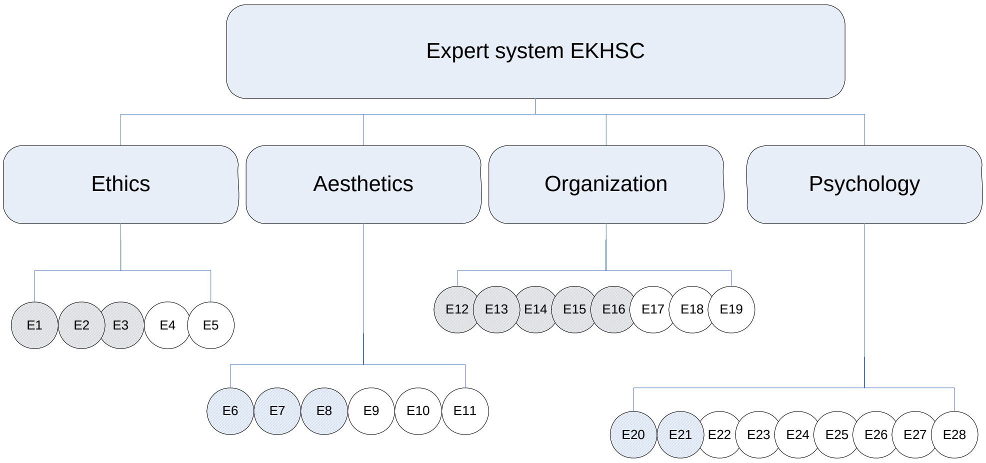

Figure 2. Structural schema of experts' system for assessing knowledge in hospitality service culture

A complete list of local experts' systems is given in Table 3.

Table 3. EKHSC of local experts systems

1.	To have a good knowledge of international ethical standards in serving guests
2.	To know and obey the organizational and professional ethical norms
3.	To improve professional proficiency
4.	Observance and application of professional ethic and norms of behaviour in interaction with the guest
5.	Knowledge and application of international norms of politeness
6.	Knowledge and application of aesthetic values in serving the guests
7.	Knowledge and application of production norms and norms of professional etiquette
8.	Knowledge and application of protocol and etiquette
9.	Observation of personnel's professional aesthetic norms.
10.	Skills and application of speech and culture in various languages
11.	Knowledge of one's own and foreign norms of etiquette
12.	Knowledge of the company as an organization activity
13.	Application of hospitality standards
14.	Knowledge of company's technological documentation
15.	Organization of personnel's working space
16.	Professional application of service technologies
17.	Skills and abilities in selling and advertising the services and goods
18.	To know and to foresee the development of services
19.	To apply knowledge and skills while interacting with guests
20.	Knowledge of organization's resources
21.	Planning of organization's activities
22.	Knowing one-self
23.	Knowledge of psychological communication principles
24.	Ability to apply verbal and non-verbal means characteristic to a certain culture
25.	Knowledge and skills in service of foreign guests
26.	Development of personality's psychological stability
27.	Solving of problem situations
28.	Promotion of guests' motivation

One of the local expert systems will be examined in the following.

5. An Example of Local Experts System That Evaluates Ethical Knowledge of Culture of Service

Ethical aspect is one of the most important aspects in hospitality culture, determining the level in which the hospitality employees apply and develop ethical values in the process of serving a client. Ethical aspect reflects the knowledge of ethics, moral principles and behaviour, expressed through the actions of a specialist. The ethical aspects of service culture are formed from the views about world, moral values, determining hospitality specialist’s professional behaviour and regulate a relationship with guests. Ethical principles urge a hospitality specialist to form such relationships with guests, which in society are considered desirable and supportable, are stimulated in practice of modern service and at the same time relieve the serving process making it efficient and pleasant for both parties. Ethical aspect is a component of hospitality specialist's professional culture.

The aspect is formed by following criteria: knowledge of international standards in guest serving, knowledge of the organization and professional norms of ethics and its observation, observation of professional, ethical and behavioural norms, application in communication with the guests and observation of international norms of politeness.

Questions of expert system No. 5: “Knowledge and observation of international norms of politeness”.

1. *Do you respect generally accepted human values?*
2. *Is your attitude towards your own culture respectful and understanding?*
3. *Is your attitude towards foreign cultures respectful and understanding?*
4. *Do you wish to get to know behavioural rules of foreign culture representatives and to acquire them?*
5. *How do you assess your knowledge of foreign cultures' etiquette?*
6. *Do you hold a view that norms of etiquette promote mutual understanding and making of contacts?*
7. *Does your attitude towards a person depend on their position and social status?*
8. *Do you treat others as you would like them to treat you?*
9. *When greeting people, do you always smile?*
10. *Do you always try to leave a good impression?*
11. *Do you know that politics and religion are the most delicate conversation topics?*
12. *Do you hold a view that sports, arts and music are neutral conversation topics?*
13. *Do you always express gratitude for the attention and help given to you?*
14. *Does your behaviour and appearance make others feel uncomfortable?*

Conclusions:

High level: 0.5, 1, 0.75, 0.2, 2, 0.8, 0.25, 3, 0.85, 0.2, 4, 0.75, 0.2, 5, 0.85, 0.25, 6, 0.7, 0.3, 7, 0.15, 0.9, 8, 0.7, 0.25, 9, 0.9, 0.15, 10, 0.7, 0.3, 11, 0.8, 0.2, 12, 0.8, 0.2, 13, 0.9, 0.1, 14, 0.2, 0.8
Average level: 0.3, 1, 0.45, 0.2, 2, 0.55, 0.35, 3, 0.65, 0.25, 4, 0.6, 0.4, 5, 0.5, 0.35, 6, 0.6, 0.45, 7, 0.25, 0.55, 8, 0.55, 0.35, 9, 0.65, 0.2, 10, 0.52, 0.3, 11, 0.5, 0.25, 12, 0.5, 0.25, 13, 0.6, 0.25, 14, 0.35, 0.65
Low level: 0.2, 1, 0.2, 0.7, 2, 0.2, 0.8, 3, 0.2, 0.78, 4, 0.25, 0.75, 5, 0.3, 0.8, 6, 0.15, 0.85, 7, 0.8, 0.15, 8, 0.15, 0.7, 9, 0.2, 0.85, 10, 0.25, 0.7, 11, 0.25,0.8, 12, 0.25, 0.8, 13, 0.25, 0.85, 14, 0.75, 0.2

In this expert system probabilities P_i^+ and P_i^- are expert's assessment, provided by one of the authors of the article. The other 27 local experts' systems have an analogue scheme. The expert systems are united by the “Little system of experts” [7, 8].

6. Testing Results of Expert Systems' EKHSC

Applying the developed local expert systems, a survey of becoming hospitality specialists was conducted. An observation expressed by students of 1st year (assessment of low, average and high level respectively 0.776, 0.650, 0.159) and 2nd year students (assessment of high, average and low level respectively 0.912, 0.077, 0.069), demonstrated improvement of results depending on the training time.

The authors plan to introduce with a detailed analysis of results in the next paper.

Conclusions

One-level additive systems possess principal restrictions related to the fact that systems can not realize linearly inseparable functions.

The improvement of assessment systems' quality is expressed in formation of such assessment models, which could realize linearly inseparable functions. For this purpose a multi-layer neuron network can be used, a productive system or Bayes' decision-making method.

An assessment system can be chosen applying additional conditions, which are not related to the essence of the set tasks. For example, a neuron network on principle is able to provide conclusion drawing with incomplete output data, but it requires a complicated training process. A productive experts' system does not require training, but it is essentially dependent on experts' assessment.

In this work the system of Bayes' decision-making has been applied. Experts' system for assessing hospitality culture knowledge has been developed – formed by 28 local subsystems, which assess the knowledge of becoming specialists in 4 areas. A testing of system has been performed, which approves a self-evident improvement of results depending on the training time.

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Transport and Telecommunication Institute, Lomonosova 1, LV-1019, Riga, Latvia

A TANDEM QUEUE WITH TWO MARKOVIAN INPUTS AND RETRIAL CUSTOMERS

O. Taramin

Department of Applied Mathematics and Computer Science, Belarusian State University
Nezavisimosti Ave. 4, 220030, Minsk, Belarus
Phone: +375 172095486. E-mail: taramin@mail.ru

A tandem retrial queue consisting of two stations is studied. The first station has a single server. An input flow at the first station is described by the Markovian Arrival Process (MAP). If a customer from this flow meets the busy server, it goes to the orbit of infinite size and tries its luck later on in exponentially distributed random time. The service time distribution at the first station is assumed to be general. After service at this station the customer proceeds to the multi-server second station. If this customer meets a free server at the second station, it starts service immediately; else the customer leaves the system forever. Besides the customers proceeding from the first station, an additional MAP flow of customers arrives at the second station directly, not entering the first station. A customer from this flow is lost if there is no available server at the second station. The service time by a second station server is exponentially distributed. We derive the stationary distribution of the system states at embedded epochs and at an arbitrary time, calculate the main performance measures. Numerical results are presented.

Keywords*: tandem retrial queue, Markovian Arrival Process, multi-server second station, asymptotically quasi-Toeplitz Markov chain*

1. Introduction

Queuing networks are widely used in capacity planning and performance evaluation of computer and communication systems, service centres, manufacturing lines and other systems where customers, jobs, packets, etc. are subjected to a successive processing. Some examples of their application to real systems can be found in [1]. Tandem queues are good mathematical models of telecommunication systems and networks. So, their investigation is important for applications.

The theory of tandem queues is well developed, see, e.g., [2–4]. However, most of papers are devoted to the exponential queueing models. Over the last two decades efforts of many investigators in tandem queues were directed to weakening the distribution assumptions on the service times and arrival pattern. In particular, the arrival process should be able to capture correlation and burstiness since real traffic in modern communication networks exhibits these features. One of the most suitable models of such an arrival process is the Markovian Arrival Process (MAP). The MAP has the advantage of being almost as computationally tractable as a stationary Poisson process, as well as it is well suited when correlation in the input flow cannot be ignored.

As far as we know the tandem queues where the effect of retrials is taken into account were investigated only in [5] and [6]. Note that retrial queues allow for the phenomenon that a customer who can not get service immediately upon arrival returns to the system after a random time. The $MAP/PH/1 \to \cdot/PH/1/K+1$ tandem retrial queue was considered in [5]. The system with a stationary Poisson arrival process, general service time distribution at both stations and the constant retrial rate was analysed in [6].

The model considered in the present paper is more general comparing to the ones in [5], [6]. We deal with a tandem retrial queue under the assumption that customers arrive according to the MAP at both stations of the tandem. The operation of a second station of the tandem is described by a multi-server queue. The service time distribution at the first station is general. We assume that both stations have no buffer space.

The rest of the paper is organized as follows. In section 2, the mathematical model is described. In section 3, the results concerning the stationary distribution of the embedded Markov chain at service completion epochs at the first station are presented. The stationary state distribution at an arbitrary time is calculated in section 4. The system performance measures are derived in Section 5. Section 6 contains numerical results illustrated the behaviour of the performance measures depending on system parameters. Finally, section 7 concludes the paper.

2. The Mathematical Model

We consider a tandem queue consisting of two stations in series. The first station has a single server and a general service time distribution function $B(t)$ with the finite first moment $b_1 = \int_0^\infty t dB(t).$

The input flow of customers entering the first station is described by the MAP. This process is coded as $MAP^{(1)}$ and is defined by means of the underlying process $\nu_t, t \geq 0$, which is an irreducible continuous time Markovian chain with the state space $\{0,...,W\}$ where W is some finite integer. Arrivals occur only at the epochs of the process $\nu_t, t \geq 0$, transitions. The intensities of transitions accompanied by an arrival of k customers are combined into the matrices $D_k, k = 0,1,$ of size $(W+1)\times(W+1)$.

The matrix generating function of these matrices is $D(z) = D_0 + D_1 z, |z| \leq 1$. The matrix $D(1)$ is the infinitesimal generator of the process $\nu_t, t \geq 0$.

The stationary distribution vector $\boldsymbol{\theta}$ of this process satisfies the equations $\boldsymbol{\theta} D(1) = \mathbf{0}, \boldsymbol{\theta}\mathbf{e} = 1$. Here and in the sequel $\mathbf{0}$ is a zero row vector and $\mathbf{e}$ is a column vector consisting of 1's.

The average intensity λ (mean rate) in the $MAP^{(1)}$ is defined by $\lambda = \boldsymbol{\theta} D_1 \mathbf{e}$.

The coefficient of variation c_{var} of intervals between customer arrivals is defined by $c_{var} = 2\lambda\boldsymbol{\theta}(-D_0)^{-1}\mathbf{e} - 1$.

The coefficient of correlation c_{cor} of the successive intervals between customer arrivals is given by $c_{cor} = (\lambda\boldsymbol{\theta}(-D_0)^{-1} D_1 (-D_0)^{-1}\mathbf{e} - 1)/c_{var}^2$.

For more information about MAP and related research see, e.g., [7].

If a customer meets a free first station server upon arrival, it automatically starts service. Else this customer goes to so called orbit and from the orbit tries its luck later on after a random amount of time. The times between retrials made by all customers are exponentially distributed with some parameter α_i when the number of customers in the orbit is equal to $i, i > 0, \alpha_0 = 0$. We do not fix the explicit dependence of the intensities α_i on i assuming only that $\lim_{i\to\infty} \alpha_i = \infty$. Note that such dependence describes the classic retrial strategy ($\alpha_i = i\alpha, \alpha > 0$) and the linear strategy ($\alpha_i = i\alpha + \gamma, \alpha > 0, \gamma > 0$) as special cases.

After service at the first station a customer proceeds to the second station which is represented by N independent identical servers. If this customer meets a free server at the second station, it starts service immediately; else the customer leaves the system forever. The service time by a second station server is exponentially distributed with the parameter $\mu > 0$.

Besides the customers proceeding from the first station an additional MAP flow of customers arrives at the second station directly, not entering the first station. This MAP is coded as $MAP^{(2)}$ and is defined by the underlying process $\eta_t, t \geq 0,$ with the state space $\{0,1,\ldots,V\}$ and the matrix generating function $A(z) = A_0 + A_1 z, |z| \leq 1$.

The mean rate of the $MAP^{(2)}$ is $h = \boldsymbol{\vartheta} A_1 \mathbf{e}$ where $\boldsymbol{\vartheta}$ is the unique solution to the system $\boldsymbol{\vartheta} A(1) = \mathbf{0}, \boldsymbol{\vartheta}\mathbf{e} = 1$.

We assume that a customer from the $MAP^{(2)}$ is lost if there is no available server at the second station.

Let us introduce some notation:

- $I (O)$ is an identity (zero) matrix of appropriate dimension. When needed the dimension of the matrix will be identified with a suffix;
- $\otimes$ and $\oplus$ are symbols of the Kronecker product and sum of matrices, see, e.g., [8];
- $\bar{V} = V + 1; \bar{W} = W + 1;$

- $P(n,t),\, n \ge 0,$ are coefficients of the matrix expansion $e^{D(z)t} = \sum_{n=0}^{\infty} P(n,t)z^n,\, |z| \le 1.$ The (ν,ν')th entry of the matrix $P(n,t)$ defines the probability that n customers arrive in the $MAP^{(1)}$ during the interval $(0,t]$ and the state of the underlying process ν_n at the epoch t is ν' given $\nu_0 = \nu,\ \nu,\nu' = \overline{0,W};$
- Q and $\tilde{Q}$ are square matrices of dimension $(N+1)\bar{V}\bar{W}$ and $(N+1)\bar{V}$ respectively:

$$Q = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix} \otimes I_{\bar{V}\bar{W}},\quad \tilde{Q} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix} \otimes I_{\bar{V}}.$$

3. The Stationary Distribution of the Embedded Markov Chain

Let t_n be the n-th service completion epoch at the first station, $n \ge 1$.

Consider the process $\xi_n = \{i_n, r_n, \eta_n, \nu_n\},\ n \ge 1,$ where i_n is the number of customers in the orbit at the epoch $t_n - 0,\ i_n \ge 0;$ r_n is the number of busy servers at the second station at the epoch $t_n - 0, r_n = \overline{0,N};$ η_n and ν_n are the states of the $MAP^{(1)}$ and $MAP^{(2)}$ arrival processes respectively at the epoch $t_n,\ \eta_n = \overline{0,V},\ \nu_n = \overline{0,W}.$

The process $\xi_n,\ n \ge 1,$ is a four-dimensional Markov chain with one countable and three finite state space components.

Enumerate the states of the chain $\xi_n, n \ge 1,$ in the lexicographic order and form the square matrices $P_{i,l},\ i,\, l \ge 0,$ of size $(N+1)\bar{W}\bar{V}$ of transition probabilities from the states having the value i of the first component to the states having the value l of this component.

Lemma 1. The non-zero transition probability matrices $P_{i,l}$ are defined as follows:

$$P_{i,l} = Q\{C_i \alpha_i \Omega_{l-i+1} + C_i (I_{(N+1)\bar{V}} \otimes D_1)\Omega_{i-l}\},\ l \ge \max\{i-1, 0\},\ i \ge 0, \tag{1}$$

where $C_i = \int_0^\infty e^{-\alpha_i t} e^{(C \oplus D_0)t} dt = (\alpha_i I - C \oplus D_0)^{-1},\ i \ge 0,\ \Omega_n = \int_0^\infty e^{Ct} \otimes P(n,t) dB(t),\ n \ge 0,\ \Omega_{-1} = O,$

$$C = \begin{pmatrix} A_0 & A_1 & O & \dots & O & O \\ \mu I_{\bar{V}} & A_0 - \mu I_{\bar{V}} & A_1 & \dots & O & O \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ O & O & O & \dots & A_0 - (N-1)\mu I_{\bar{V}} & A_1 \\ O & O & O & \dots & N\mu I_{\bar{V}} & A(1) - N\mu I_{\bar{V}} \end{pmatrix}. \tag{2}$$

Proof. Formula (1) becomes clear if we take into account the meaning of the matrices which occur in the right-hand side of (1). The matrix Q is used to install the number of busy servers at the second station just after the service completion at the first station. Given the number of busy servers at the second station has installed after the service completion at the first station, the process $\{r_t, \eta_t\},$ which describes the evolution of the number of busy servers and the arrival process at the second station, behaves like Quasi-Birth-and-Death process until the next service completion epoch. The generator C of this process has form (2).

The $(r,\eta,\nu; r',\eta',\nu')$th entry of the matrix $C_i \alpha_i$ defines the probability that, given i customers stay in the orbit and the state of the process $\{r_t, \eta_t, \nu_t\}$ is (r,η,ν) after the service completion epoch at the first station, the next service at this station will be initiated by a customer from the orbit and the state of the process $\{r_t,\eta_t,\nu_t\}$ will be (r',η',ν') at the beginning of the service.

The matrix $C_i(I_{(N+1)\bar{V}}\otimes D_1)$ has the analogous probabilistic sense with the only difference that the next service at the first station is initiated by a customer arriving in the $MAP^{(1)}$.

The $(r,\eta,\nu;r',\eta',\nu')$ th entry of the matrix Ω_n defines the probability that n customers arrive in the $MAP^{(1)}$ and the process $\{r_t,\eta_t,\nu_t\}$ transits from the state (r,η,ν) to the state (r',η',ν') during the service time at the first station.

Taking into account the above explanations and using the total probability formula we readily obtain expression (1) for transition probability matrices. □

Corollary 1. The Markov chain $\xi_n,\ n\geq 1,$ belongs to the class of asymptotically quasi-Toeplitz Markov chain.

Proof. It is seen from (1) that transition probability matrices $P_{i,l}$ depend on i and l and this dependence can not be reduced to the dependence on the difference $l-i$ only. It means that the Markov chain $\xi_n,\ n\geq 1,$ is a level dependent one. At the same time, the dependence of i vanishes as $i\to\infty$ and the matrices $P_{i,l}$ approach to matrices that depend on the value i and l only via the difference $l-i$. It implies that the chain under consideration belongs to the class of asymptotically quasi-Toeplitz Markov chains, see [9]. □

So, the further investigation of the process $\xi_n,\ n\geq 1,$ will be based on the results given in [9].

Let us denote

$$\tilde{Y}_k=\lim_{i\to\infty}P_{i,i+k-1},\ k\geq 0. \tag{3}$$

The matrices $\tilde{Y}_k, k\geq 0,$ can be considered as transition probability matrices of some Markov chain, say, $\zeta_n,\ n\geq 1,$ with the same state space as the chain $\xi_n,\ n\geq 1.$ The chain $\zeta_n,\ n\geq 1,$ is called as limiting chain relative to the chain $\xi_n,\ n\geq 1.$ The limiting chain $\zeta_n,\ n\geq 1,$ is a level independent one, moreover it belongs to the class of quasi-Toeplitz Markov chains, see [9].

Denote by $\tilde{Y}(z)$ the generating function of the matrices $\tilde{Y}_k, k\geq 0.$

Corollary 2. The generating function of the limiting chain $\zeta_n,\ n\geq 1,$ transition probability matrices has the following form

$$\tilde{Y}(z)=Q\int_0^\infty e^{Ct}\otimes e^{D(z)t}dB(t). \tag{4}$$

Proof. Taking into account Lemma 1 and using (3) we get expression (4) for generating function $\tilde{Y}(z)$. □

Theorem 1. The sufficient condition for ergodicity of the Markov chain $\xi_n,\ n\geq 1,$ is the fulfilment of the inequality

$$\rho=\lambda b_1<1. \tag{5}$$

Proof. The matrix $\tilde{Y}(1)$ is an irreducible one. So, it follows from [9] that the sufficient condition for ergodicity of the chain $\xi_n,\ n\geq 1,$ is the fulfilment of the inequality

$$\mathbf{x}\tilde{Y}'(1)\mathbf{e}<1, \tag{6}$$

where $\mathbf{x}$ is the unique solution to the system

$$\mathbf{x}\tilde{Y}(1)=\mathbf{x},\ \ \mathbf{xe}=1. \tag{7}$$

Let the vector $\mathbf{x}$ be of the form $\mathbf{x}=\boldsymbol{\delta}\otimes\boldsymbol{\theta}$, where $\boldsymbol{\delta}$ is a solution to the system

$$\boldsymbol{\delta}Q\int_0^\infty e^{Ct}dB(t)=\boldsymbol{\delta},\ \ \boldsymbol{\delta}\mathbf{e}=1. \tag{8}$$

Substituting this vector $\mathbf{x}$ and $\tilde{Y}(1)$ given by (4) into (7) we get the following relations: $\mathbf{x}\tilde{Y}(1)=(\boldsymbol{\delta}\otimes\boldsymbol{\theta})Q\int\limits_0^\infty e^{Ct}\otimes e^{D(1)t}dB(t)=\int\limits_0^\infty \boldsymbol{\delta}Qe^{Ct}\otimes\boldsymbol{\theta}e^{D(1)t}dB(t)=\boldsymbol{\delta}Q\int\limits_0^\infty e^{Ct}dB(t)\otimes\boldsymbol{\theta}=\boldsymbol{\delta}\otimes\boldsymbol{\theta}=\mathbf{x}.$ So, the vector $\mathbf{x}$ is a solution of (7).

The vector $\boldsymbol{\delta}$ is the unique solution to system (8) since the matrix $Q\int\limits_0^\infty e^{Ct}dB(t)$ is irreducible stochastic. This implies that the vector $\mathbf{x}=\boldsymbol{\delta}\otimes\boldsymbol{\theta}$ is the unique solution to (7). Substituting this vector and $\tilde{Y}'(1),$ calculated by means (4), into inequality (6) we reduce this inequality to form (5) using obvious transformations. □

The value ρ in (5) is the system load. Inequality (5) becomes intuitively clear if we take into account that the value λb_1 is the mean number of customers entering the first station during the service of a customer by the first server.

In what follows we suppose that inequality (5) is fulfilled.

Denote the stationary state probabilities of the Markov chain $\xi_n,\ n\geq 1,$ by $\pi(i,r,\eta,\nu),\ i\geq 0,\ r=\overline{0,N},\ \eta=\overline{0,V},\ \nu=\overline{0,W}.$

Let also $\boldsymbol{\pi}_i$ be the row vector of probabilities $\pi(i,r,\eta,\nu)$ listed in the lexicographic order of arguments $(r,\eta,\nu),\ i\geq 0.$

Denote by $\Pi(z)=\sum\limits_{i=0}^{\infty}\boldsymbol{\pi}_i z^i,\ |z|\leq 1,$ the generating function of these vectors.

To compute the vectors $\boldsymbol{\pi}_i,\ i\geq 0,$ we use the numerically stable algorithm elaborated for asymptotically quasi-Toeplitz Markov chains, see [9]. It is based on censoring technique and asymptotic properties of the chain under consideration.

The algorithm consists of the following steps:

1. Calculate the matrix G as the minimal nonnegative solution of the matrix equation $G=\tilde{Y}(G)$. The matrix G is calculated by the iterative method.

2. For pre-assigned sufficiently large integer i_0 calculate the matrices $G_{i_0-1},G_{i_0-2},\ldots,G_0$ using the equation of the backward recursion $G_i=P_{i+1,i}+\sum\limits_{n=i+1}^{\infty}P_{i+1,n}G_{n-1}G_{n-2}\ldots G_i,\ i=i_0-1,i_0-2,\ldots,0$, with the boundary condition $G_i=G,\ i\geq i_0$.

3. Calculate the matrices $\bar{P}_{i,l}=P_{i,l}+\sum\limits_{n=l+1}^{\infty}P_{i,n}G_{n-1}G_{n-2}\ldots G_l,\ l\geq i,\ i\geq 0,$ where $G_i=G,\ i\geq i_0$.

4. Calculate the matrices F_l using the recurrent formulas $F_l=(\bar{P}_{0,l}+\sum\limits_{i=1}^{l-1}F_i\bar{P}_{i,l})(I-\bar{P}_{l,l})^{-1},\ l\geq 1.$

5. Calculate the vector $\boldsymbol{\pi}_0$ as the unique solution to the system $\boldsymbol{\pi}_0(I-\bar{P}_{0,0})=0,\ \boldsymbol{\pi}_0\sum\limits_{l=0}^{\infty}F_l\mathbf{e}=1.$

6. Calculate the vectors $\boldsymbol{\pi}_l$ as follows: $\boldsymbol{\pi}_l=\boldsymbol{\pi}_0F_l,\ l\geq 1.$

4. The Stationary Distribution at an Arbitrary Time

Define the process of the system states at an arbitrary time as $\xi_t=\{i_t,r_t,\eta_t,\nu_t\},\ t\geq 0,$ where i_t is the number of customers at the first station (in the orbit and in service), r_t is the number of busy servers at the second station, η_t and ν_t are the states of the $MAP^{(1)}$ and $MAP^{(2)}$ arrival processes respectively at time $t,\ t\geq 0.$

The process $\xi_t,\ t \ge 0,$ is non-Markovian. But the stationary distribution of this process can be related to the stationary distribution of the embedded Markov chain $\xi_n,\ n \ge 1,$ using the results for Markov renewal and semi-regenerative processes (see [10]).

Let

$$p(i,r,\eta,\nu) = \lim_{t\to\infty} P\{i_t = i, r_t = r, \eta_t = \eta, \nu_t = \nu\},\ i \ge 0,\ r = \overline{0,N},\ \eta = \overline{0,V},\ \nu = \overline{0,W}, \tag{9}$$

be the steady-state probabilities of the process $\xi_t,\ t \ge 0$. Let also $\mathbf{p}_i$ be the vector of probabilities $p(i,r,\eta,\nu)$ listed in the lexicographic order of components $(r,\eta,\nu),\ i \ge 0.$

Theorem 2. The steady-state probability vectors $\mathbf{p}_i,\ i \ge 0,$ of the process $\xi_t,\ t \ge 0,$ are related to the stationary probability vectors $\boldsymbol{\pi}_i,\ i \ge 0,$ of the embedded Markov chain $\xi_n,\ n \ge 1,$ as follows:

$$\mathbf{p}_0 = \tau^{-1}\boldsymbol{\pi}_0 QC_0, \tag{10}$$

$$\mathbf{p}_i = \tau^{-1}\{\boldsymbol{\pi}_i QC_i + \sum_{l=0}^{i} \boldsymbol{\pi}_l QC_l \alpha_l \tilde{\Omega}_{i-l} + \sum_{l=0}^{i-1} \boldsymbol{\pi}_l QC_l (I_{(N+1)\bar{V}} \otimes D_1)\tilde{\Omega}_{i-l-1}\},\ i \ge 1, \tag{11}$$

where $\tilde{\Omega}_n = \int_0^\infty e^{Ct} \otimes P(n,t)(1-B(t))dt,\ n \ge 0,$ and τ is the mean value of inter-departure time at the first station, $\tau = b_1 + \sum_{i=0}^{\infty} \boldsymbol{\pi}_i QC_i \mathbf{e}.$

Proof. The process $\xi_t,\ t \ge 0,$ is a semi-regenerative one with the embedded Markov renewal process $\{\xi_n, t_n\}, n \ge 1$. By [10], limits (9) exist if the process $\{\xi_n, t_n\}$ is an irreducible aperiodic process and the value τ of the mean inter-departure time at the first station is finite. It is easily verified that ergodicity of the Markov chain $\xi_n,\ n \ge 1,$ implies that all these conditions hold true. So, limits (9) exist if inequality (5) is satisfied.

Formulas (10), (11) for the steady-state probability vectors $\mathbf{p}_i,\ i \ge 0,$ are derived using the limiting theorem for semi-regenerative processes given in [10]. □

Corollary 3. The generating function $\mathbf{P}(z) = \sum_{i=0}^{\infty} \mathbf{p}_i z^i, |z| \le 1,$ of the stationary distribution of the process $\xi_t,\ t \ge 0,$ is related to the generating function $\Pi(z)$ of the stationary distribution of the embedded Markov chain $\xi_n,\ n \ge 1,$ as follows: $\mathbf{P}(z)[C \oplus D(z)] = \tau^{-1}\Pi(z)[zI - Q].$

5. Performance Measures

Having the stationary distributions $\boldsymbol{\pi}_i,\ i \ge 0,$ and $\mathbf{p}_i,\ i \ge 0,$ been calculated, we can find different stationary performance measures of the system under consideration:

- Mean number of customers at the first station at the service completion epoch at this station $L = \Pi'(1)\mathbf{e}.$
- Mean number of customers at the first station at an arbitrary time $\tilde{L} = \mathbf{P}'(1)\mathbf{e}.$
- Mean number of busy servers at the second station at the service completion epoch at the first station $N_{busy} = \Pi(1)(I_{N+1} \otimes \mathbf{e}_{\bar{V}\bar{W}})diag\{r, r = \overline{0,N}\}\mathbf{e}.$
- Mean number of busy servers at the second station at an arbitrary time $\tilde{N}_{busy} = \mathbf{P}(1)(I_{N+1} \otimes \mathbf{e}_{\bar{V}\bar{W}})diag\{r, r = \overline{0,N}\}\mathbf{e}.$
- Probability that an arbitrary customer from the $MAP^{(2)}$ will be lost $P_{loss}^{(2)} = h^{-1}\mathbf{P}(1)(\hat{\mathbf{e}}_{N+1} \otimes A_1\mathbf{e} \otimes \mathbf{e}_{\bar{W}}),$ where $\hat{\mathbf{e}}_{N+1}$ is a column vector of size $N+1$ having 1 as the last entry and zeroes as the rest entries.

- Probability that an arbitrary customer from the $MAP^{(1)}$ will be lost $P_{loss}^{(1)} = 1 - \lambda^{-1}[\mu \tilde{N}_{busy} - h(1 - P_{loss}^{(2)})]$.
- Probability of immediate access to the first station server $P_{imm} = (\lambda\tau)^{-1} \sum_{i=0}^{\infty} \boldsymbol{\pi}_i Q C_i (\mathbf{e}_{(N+1)\bar{V}} \otimes D_1 \mathbf{e})$.
- Probability that a customer from the $MAP^{(1)}$ will be successfully served at the both stations without visiting the orbit $P_{success}^{(1)} = (\lambda\tau)^{-1} \sum_{i=0}^{\infty} \boldsymbol{\pi}_i Q C_i (I_{(N+1)\bar{V}} \otimes D_1 \mathbf{e}) \int_0^{\infty} e^{Ct} dB(t) \tilde{Q} \mathbf{e}$.

6. Numerical Examples

The aim of the numerical examples is to demonstrate an impact of correlation in the input flows arriving at both stations on the performance measures of the queue under consideration.

Experiment 1. In this experiment we analyse the influence of the correlation in the $MAP^{(1)}$ entering the first station on the key performance measures. The effect of correlation is investigated for different values of system load ρ.

For this purpose we consider three $MAP^{(1)}$ processes defined by the matrices D_0 and D_1. All these MAP s have the same average intensity $\lambda = 1$ and different coefficients of correlation.

The $MAP_1^{(1)}$ has the coefficient of correlation $c_{cor} = 0.2$ and is characterized by the matrices

$$D_0 = \begin{pmatrix} -1.349076 & 1.09082 \times 10^{-6} \\ 1.09082 \times 10^{-6} & -0.043891 \end{pmatrix}, D_1 = \begin{pmatrix} 1.340137 & 0.008939 \\ 0.0244854 & 0.0194046 \end{pmatrix}.$$

The $MAP_2^{(1)}$ has the coefficient of correlation $c_{cor} = 0.1$ and is defined by the matrices

$$D_0 = \begin{pmatrix} -1.17494 & 0.34832 \times 10^{-6} \\ 0.34832 \times 10^{-6} & -0.025736 \end{pmatrix}, D_1 = \begin{pmatrix} 1.171346 & 0.0036006 \\ 0.0200534 & 0.0056824 \end{pmatrix}.$$

These processes have the same coefficient of variation $c_{var} = 3.5$.

The $MAP_3^{(1)}$ is a Poisson process with $D_0 = -1,\ D_1 = 1$. It has the coefficient of correlation $c_{cor} = 0$ and the coefficient of variation $c_{var} = 1$.

The $MAP^{(2)}$ arrived at the second station is identical to the $MAP_2^{(1)}$. It has the coefficient of correlation $c_{cor} = 0.1$ and the mean rate $h = 1$.

The service time distribution at the first station is assumed to be Erlangian of order 3 with the intensity 15. The mean service time $b_1 = 0.2$ and the squared coefficient of variation $c_{var}^2 = 1/3$.

We consider the classical retrial strategy $\alpha_i = i\alpha,\ \alpha = 5,\ i \geq 0$. The number of servers at the second station $N = 10$, the mean service rate $\mu = 0.5$.

Let us vary the mean rate λ for all arrival processes in the interval $[0.5, 4.5]$ by multiplying the matrices D_0, D_1 by some positive constant. Any desired value of λ can be obtained while c_{cor} does not change. Note also that in this experiment the system load ρ takes values from 0.1 to 0.9.

Figures 1, 2 illustrate the dependence of the mean number of customers L and $\tilde{L}$ at the first station, the mean number of busy servers N_{busy} and $\tilde{N}_{busy}$ at the second station on the mean rate λ.

Figures 3, 4 show the dependence of the loss probabilities $P_{loss}^{(1)}$ and $P_{loss}^{(2)}$, the probabilities $P_{success}^{(1)}$ and P_{imm} on the mean rate λ.

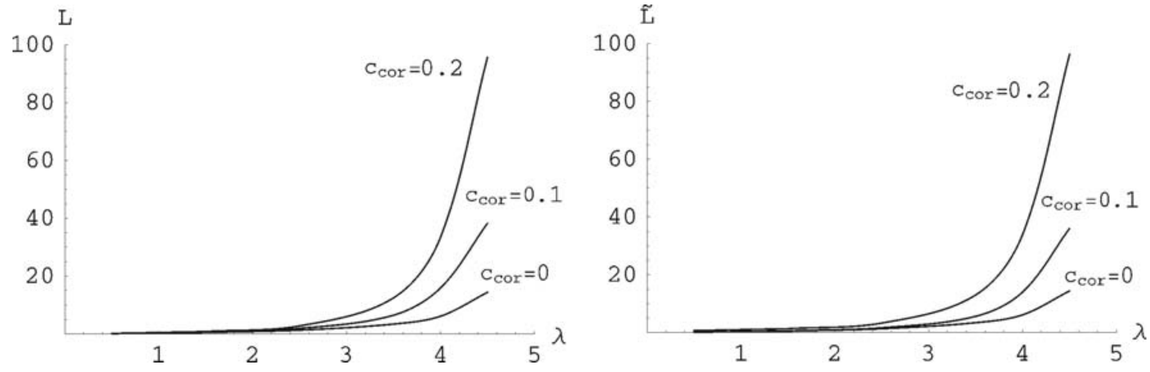

Figure 1. L and $\tilde{L}$ as functions of the mean rate in the arrival process $MAP^{(1)}$

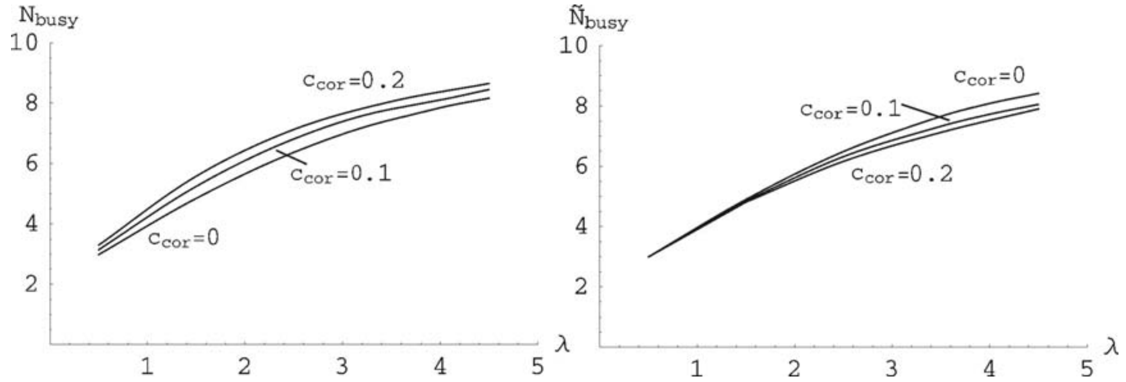

Figure 2. N_{busy} and $\tilde{N}_{busy}$ as functions of the mean rate in the arrival process $MAP^{(1)}$

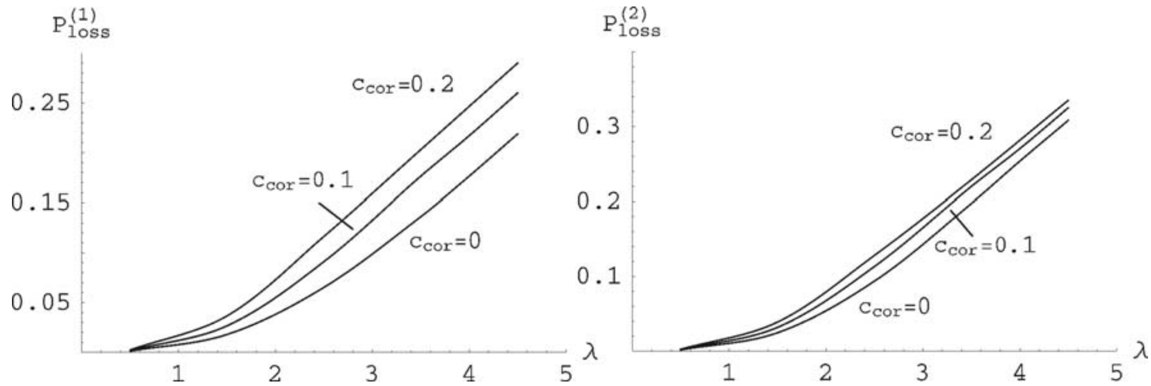

Figure 3. $P_{loss}^{(1)}$ and $P_{loss}^{(2)}$ as functions of the mean rate in the arrival process $MAP^{(1)}$

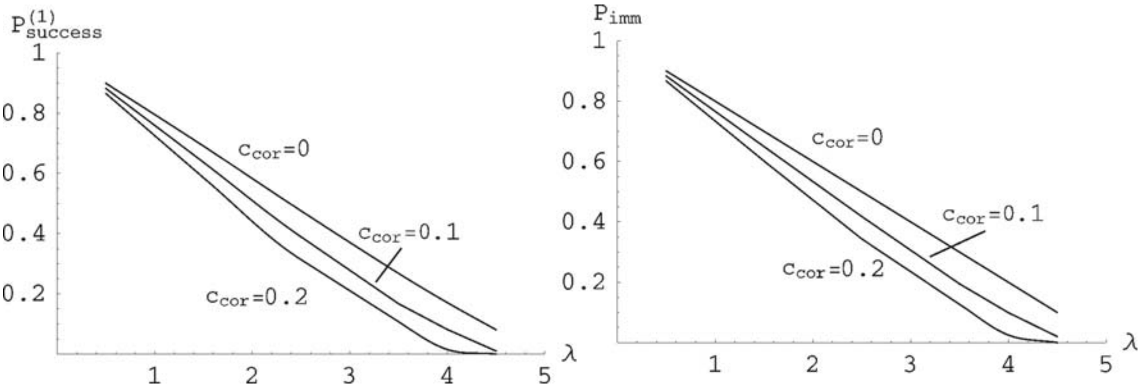

Figure 4. $P_{success}^{(1)}$ and P_{imm} as functions of the mean rate in the arrival process $MAP^{(1)}$

Basing on the figures one can conclude that the mean number of customers at the first station, the number of busy servers at the second station, the loss probability of customers from both arrival flows increase, while the probability of immediate access to the first station server and the probability of successful service at both stations without visiting the orbit decrease when the input intensity λ (and the system load ρ) grows. It confirms the evident fact that increase of the arrival intensity makes worse the quality of service in the system.

More important conclusion is that, under the same value of the arrival rate λ, the increase of correlation in the arrival process essentially affects the value of the system performance measures, including measures relating to the second station of the system. Thus, assumption that the input flow is not correlated and can be approximated by means of stationary Poisson process with the same intensity while actually it is correlated can imply huge errors in prediction of system operation.

Experiment 2. In this experiment we are interesting in how the coefficient of correlation in the arrival process $MAP^{(2)}$ impacts on the system performance measures.

We assume that the arrival process at the first station is identical to $MAP_2^{(1)}$ with the average intensity $\lambda = 1$.

Here we consider the $MAP_1^{(1)}$, $MAP_2^{(1)}$, and $MAP_3^{(1)}$ presented in the first experiment as the $MAP_1^{(2)}$, $MAP_2^{(2)}$, and $MAP_3^{(2)}$ input flows at the second station. These processes have the mean rate $h = 1$ and different coefficients of correlation: $c_{cor} = 0.2$, $c_{cor} = 0.1$, and $c_{cor} = 0$.

The rest system parameters are the same as in the first experiment.

Let us vary the mean rate h for all arrival processes in the interval $[0.5, 8]$. The system load ρ does not change in this experiment.

Figures 5, 6 show the dependence of the loss probabilities $P_{loss}^{(1)}$ and $P_{loss}^{(2)}$, the mean number of busy servers $\tilde{N}_{busy}$ at the second station and the probability $P_{success}^{(1)}$ on the mean rate h in the arrival process $MAP^{(2)}$.

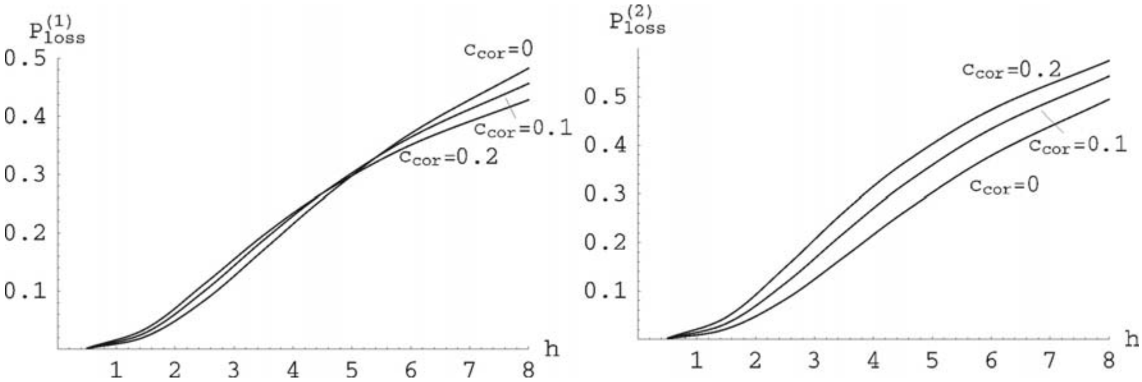

Figure 5. $P_{loss}^{(1)}$ and $P_{loss}^{(2)}$ as functions of the mean rate in the arrival process $MAP^{(2)}$

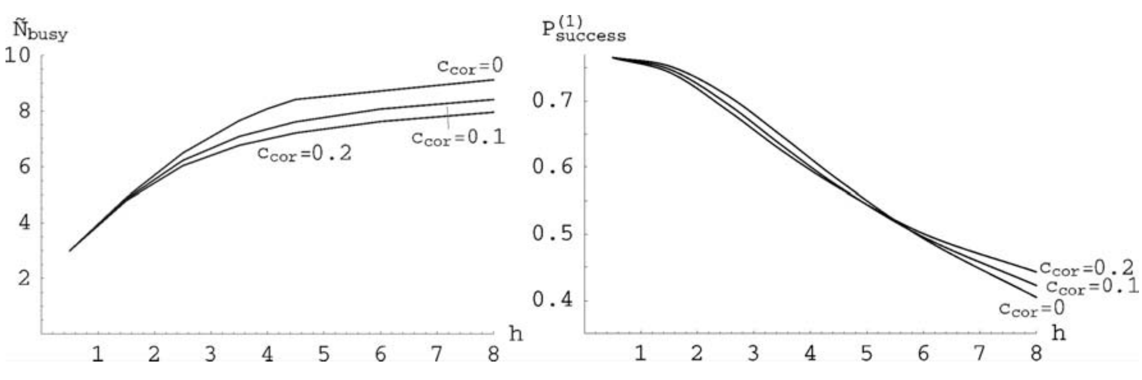

Figure 6. $\tilde{N}_{busy}$ and $P_{success}^{(1)}$ as functions of the mean rate in the arrival process $MAP^{(2)}$

It is clear from these figures that the loss probabilities $P_{loss}^{(1)}$ and $P_{loss}^{(2)}$, the mean number of busy servers $\tilde{N}_{busy}$ at the second station increase, while the probability $P_{success}^{(1)}$ of successful service at both stations without visiting the orbit decreases when the intensity of the additional arrival process at the second station grows. Note that numerical calculations confirm that system characteristics relating to the first station, such as the mean number of customers at the first station and the probability of immediate access to the first station, do not depend on the intensity h.

As well as in the first experiment, based on the presented figures, we have to conclude that the system performance measures are sensitive with respect to the correlation in arrival process. One more interesting fact is that, under $h < 5$, the probabilistic measures $P_{loss}^{(1)}$ and $P_{success}^{(1)}$ become worse when the coefficient of correlation increases, but, when h becomes greater than 5, the behaviour of these probabilities is reversed, i.e. the increase of the correlation has positive influence on $P_{loss}^{(1)}$ and $P_{success}^{(1)}$.

Conclusions

In this paper, the tandem queue with two Markovian inputs and retrial customers is investigated. The processes of system states at embedded epochs and an arbitrary time are studied. The condition for stationary distribution existence of these processes is derived and the algorithms for calculating the steady state probabilities are presented. Expressions for the probability of immediate access, the loss probabilities, the probability of successful service and other important performance characteristics of the system are obtained. The dependence of the system performance measures on the correlation in the input flows at both stations is numerically illustrated.

The results can be exploited for capacity planning, performance evaluations and optimisation of real-life tandem queues and two-node networks with the random multiple access to the first station as well as for validation of general networks decomposition algorithms in case of correlated bursty traffic.

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Transport and Telecommunication Institute, Lomonosova 1, LV-1019, Riga, Latvia

SHIPS SAFETY IN OPEN PORTS

V. Paulauskas [1] ***, D. Paulauskas*** [2] ***, J. Wijffels*** [3]

[1]*Klaipeda University, Manto 84, LT-92294 Klaipeda, Lithuania*
E-mail: donataspakas.lt

[2]*JSC "Arijus"*
Jurininku ave 27, LT-91225, Klaipeda, Lithuania
E-mail: d.paulauskas@arijus.lt

[3]*Lievense N.V.Bijenstraat 26, B-9051, Gent, Belgium*
E-mail: jwijffels@lievense.be

Open ports or terminals, which are not protected from wind at all, are problematic for ships, which have high boards and big influence by aerodynamic forces is arising. Mooring systems in such cases must take in account static and dynamic (harmonic) forces. Evaluation methods in case of open sea ports are presented in this article. Possibilities to evaluate aerodynamic forces for mooring ships and possibilities to prepare right ships mooring schemes in concrete conditions can positively increase ships safety in ports preventing navigational and environmental accidents. In this paper are presented analysis of situations, theoretical basis for study and practical calculations.

Keywords: *aerodynamic forces, ships mooring, open sea port, mooring ropes*

1. Introduction

Some ports are built in open sea parts, which are named as avant-ports. Such ports as Ventspils (oil and gas terminals), Zeebriugge, Gdansk Polnotzny port, planned Klaipeda deep sea port, usually are protected from short and long wave's penetration, minimizing currents acting, but can not be protected by wind action at all. Open port places without any resistance against the wind some time make difficulties for moored ships with big wind surface (area), such as Ro-Ro vessels, Container ships, bulk ships and tankers in ballast.

In some ports, like Europort (Rotterdam) have been constructed wind protection walls, but it is necessary to have more deep studies for the wind protection or decreasing possibilities with minimum investments.

2. Open Sea Ports Analysis

Open sea ports built in sea part and safe land, which is very expensive in countries, which has short sea shore, like Belgium, Lithuania and others, decrease dredging works, especially in hard soil and some time decrease investments for the port infrastructure [Baublys, A. 2003; Paulauskas, V., 2004]. In such cases there are built breakwater systems, which protect quay walls from short and long waves penetration (Fig. 1) and minimize currents influence in case if a port is constructed on river's delta (Fig. 2, Fig. 3).

Wind acting in open sea port, without any or minimum wind resistance, exists in big number of such type ports and terminals [Baublys, A., 2007], because there are open sea areas for the wind. In some ports, which are built on a very flat sea shore, has very low resistance for the wind, like Europort, which has the same problems as open sea ports (Fig. 4).

In open sea ports wind creates very high aerodynamic forces on moored ships, which have big influence on ships mooring systems and quay walls fenders and mooring bollards and finally on quay walls [EAU, 2004, 2006].

3. Aerodynamic Forces Which Create Wind Acting on the Ships

In the open sea ports wind acting on the ships is the same as in open sea, that means wind velocity change from 0 near ground (water) up to average or maximum on level above water level 5–7 m. Aerodynamic forces, which are acting on ship, spread on constant and periodical (harmonic) wind force components [Vensel, 1969]. Constant wind velocity component creates forces, which can be calculated as follows [Paulauskas, V., 1998]:

$$F_C = C_a \frac{\rho_1}{2}(S_x \cdot \cos q_a + S_y \cdot \sin q_a) v_{aC}^2 , \qquad (1)$$

where: C_a – aerodynamic coefficient, can be taken from concrete ship data, which model has been tested in aerodynamic tube; ρ_1 – air density, for the calculations can be taken as 1,25 kg/m³; S_x – wind surface area on ship's diametric direction; S_y – wind surface area on ship's middle direction; q_a – wind course angle; v_{aC} – average wind velocity.

Figure 1. Zeebrugge port in Belgium

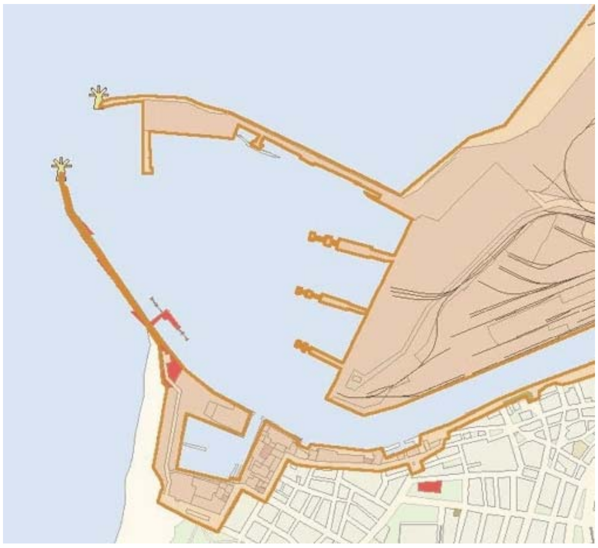

Figure 2. Ventspils open sea port

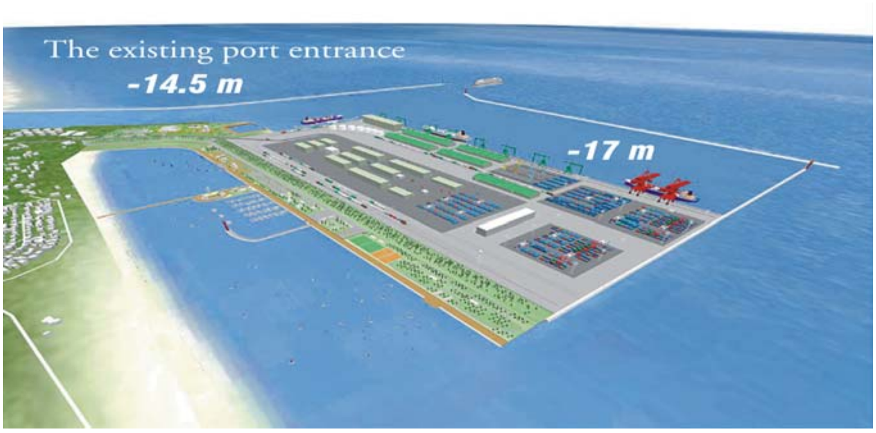

Figure 3. Klaipeda deep sea port (planned)

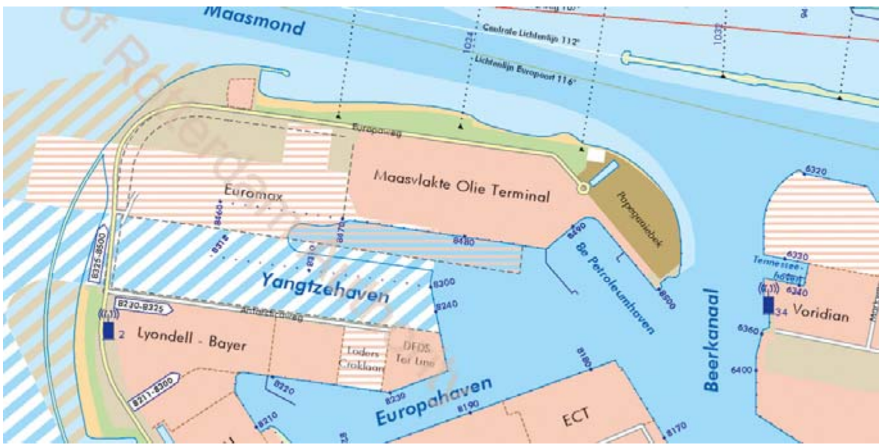

Figure 4. Europort (Rotterdam)

Periodical forces can be calculated via acceleration as follows:

$$F_P^{''} = \frac{4\pi^2 t}{\tau^2} a \cdot \sin\frac{2\pi t}{\tau}, \quad (2)$$

and finally periodical force can be expressed as:

$$F_P = F_P^{''} \cdot m, \quad (3)$$

where: m – ship's mass; τ – period of wind guess; a – integration constant, which can be found as:

$$a = C_a \frac{\rho_1}{4} \Delta v_a^2 (S_x \cdot \cos q_a + S_y \cdot \sin q_a). \quad (4)$$

Maximum forces, which can create periodical component of the wind, will be in case $\sin\frac{2\pi t}{\tau} = 1$, and maximum periodical forces in such case will be:

$$F_{P\max} = \frac{4\pi^2 t}{\tau^2} a \cdot m. \quad (5)$$

Finally, maximum aerodynamic forces, which could act on the ship, will be as follows:

$$F_a = F_C + F_{P\max}. \quad (6)$$

In the same time it is necessary to take in account these directions, which are dangerous for the ships and quay walls mooring systems and is possible to take one-criterion method [Catmale, T. et al., 2007]. In case wind direction to quay wall, ship will be pressed to quay wall and just fenders must absorb ships energy, which is created by wind acting. In case of wind direction from quay wall, mooring ropes mainly must compensate forces and energies, which are created by wind acting on ship. In the same time periodical wind forces component has not big influence, because ship has not big movements and does not create big forces, which can be calculated by formula (5). More dangerous situation for the ship and quay wall mooring systems could be in case when wind is acting with some angle to quay wall, when constant and periodical wind forces will be maximum, that means the ship is pushed from quay wall by constant aerodynamic force and moved along quay wall by harmonic wind force and creates big inertia forces (Fig. 5) [Paulauskas, V., 2006].

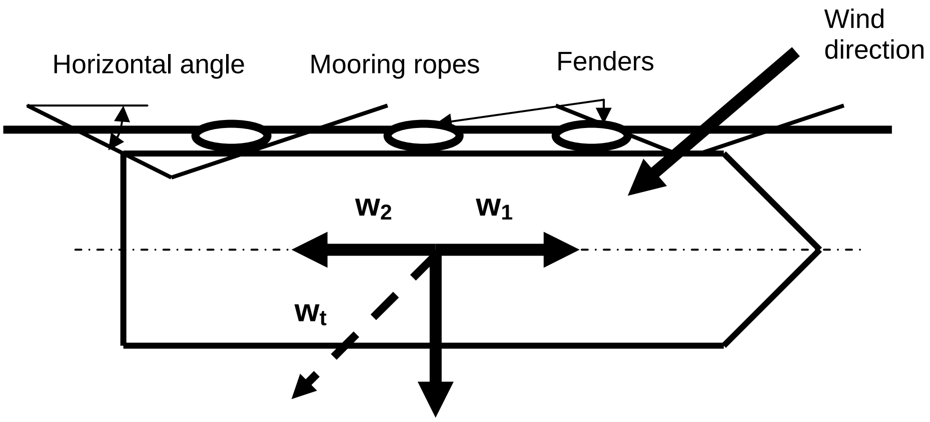

Figure 5. Dangerous wind direction on the ship and quay wall mooring system

In case of course angle of the wind to moored ship, total aerodynamic force can be calculated as follows:

$$F_{a\alpha} = F_C \cdot \sin q_a^{'} + F_{P\max} \cdot \cos q_a^{'}, \quad (7)$$

where: $q_a^{'}$ – wind angle to the quay wall. Presented mathematical model can be used in concrete conditions for the aerodynamic forces calculations in case if no big resistance against the wind before moored ship.

4. Wind Protection Walls in Open Ports and Its Theoretical Basis

In some places in the open ports' parts for the decreasing wind acting on moored to quay walls ships, there are constructed special wind protection walls (Fig. 6).

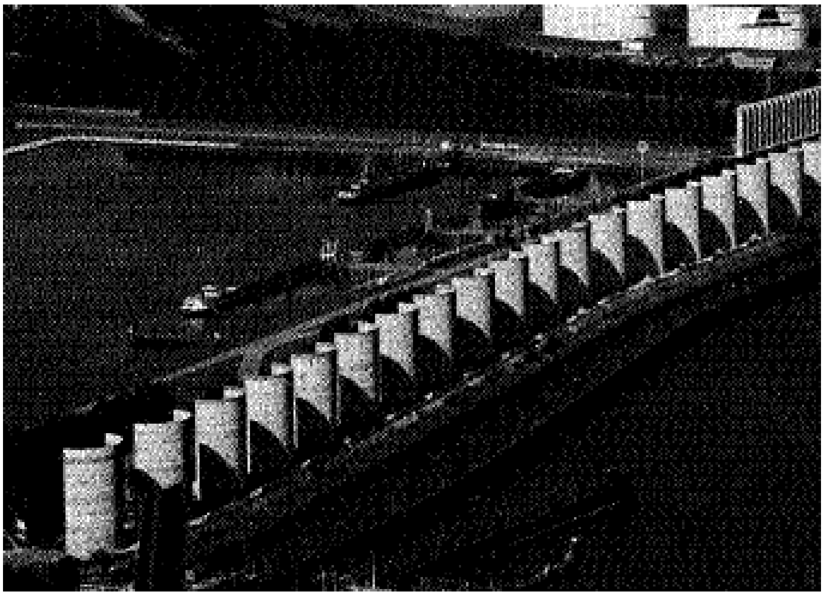

Figure 6. Wind protection wall in Europort

Wind protection wall's effectiveness is very important, because constructions are very big and cost a lot and can be evaluated as wind velocity or wind forces decreasing on the ship. Theoretical and experimental studies of the wind protection walls effectiveness have been made on model shown on Fig. 7,

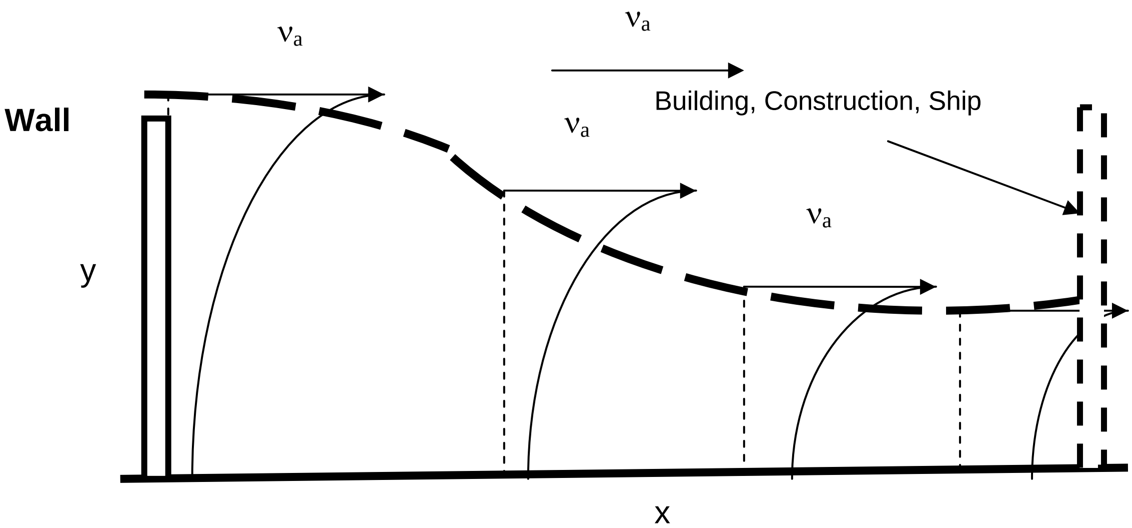

Figure 7. Wind velocity distribution after clause wall (between protection wall and ship)

and on basis of tests results, have been received mathematical dependence as follows:

$$y = \exp(-ax)^2 , \tag{8}$$

where y – the altitude on which wind has initial velocity (v_a); a – the coefficient; x – the distance from protection wall.

In case of non close wind protection wall, it very much depends on the wall configuration and distances between wall elements. In general distance between elements should be short, that energy which absorbs protection wall elements would be bigger then energy, which goes between protection wall elements.

Wind energy, which is absorbed (protect) by protection wall elements, can be calculated as follows:

$$E_a = \frac{m_a \cdot v_a^2}{2}, \tag{9}$$

where: m_a – air mass, which is acting on protection wall element, can be calculated as add mass:

$$m_a = k_a \cdot V_a^{'} \cdot \rho_1, \tag{10}$$

where $V_a^{'}$ – volume (circumference) of the protection wall element, k_a – air add mass coefficient, could be taken as 1,7–1,8. Horizontal distribution is the same as in close wall, which is shown on Fig. 7. Experiments, which have been provided near close and non close walls, results are the same and presented on Fig. 8.

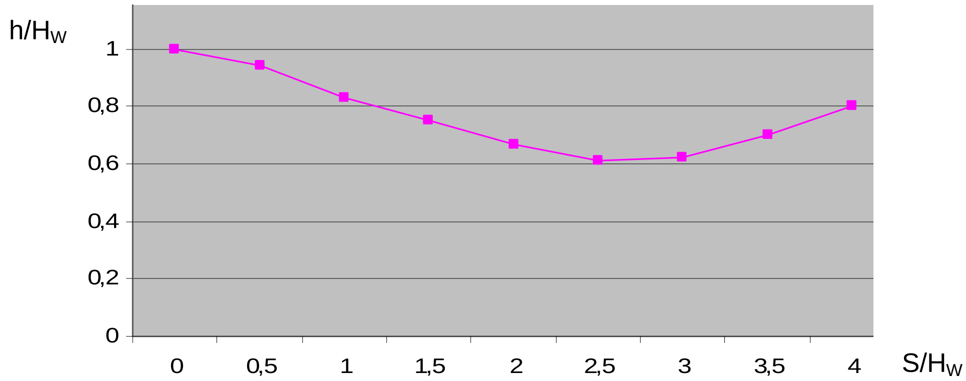

Figure 8. Wind distribution on height (*h*) depends on relatives distance from protection wall (*S/Hw.*) (protection wall and building, construction, ship etc.)

Mathematical dependence can be expressed by regression formula as follows:

$$h / H_w = 0{,}0149\left(\frac{S}{H_w}\right)^2 - 0{,}1836\frac{S}{H_w} + 1. \tag{11}$$

Results, which have been received during testing and study, can be used for the practical tasks in open sea ports or other places.

5. Practical Wind Protection Wall Influence on Moored Ship Evaluation

For the practical evaluation one North Sea port with tidal altitude 5,5 m and in case of moored Ro-Ro ship is taken, which has high board about 25 m above the water level. Evaluation scheme is presented on Fig 9.

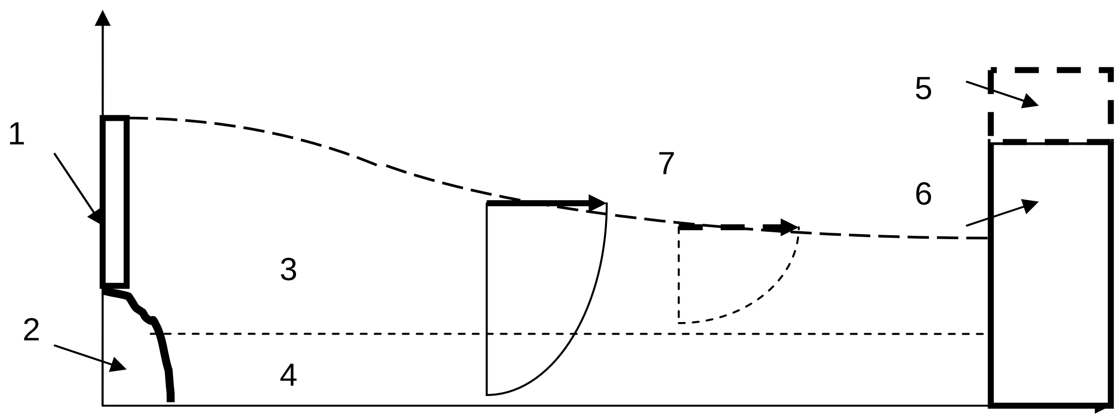

Figure 9. Calculation model for the buildings, constructions, ships: 1 – wind protection wall, high about 15,5 m; 2 – breakwater, high + 9,50; 3 – high water level (TAW + 5,0 m); 4 – low water level (TAW – 0,5 m); 5 – ship position in high water level; 6 – ship position in low water level; 7 – maximum wind distribution line

Wind velocity distribution calculation is made by methodology, presented in section 4 of this article. Wind protection walls can be constructed by containers with different positioning, that means constructing containers in vertical position or using horizontal position walls. In bough cases final result that means reduction of the wind velocity will be the same (for the same height of the wall and distances between wind protection wall elements).

Aerodynamic coefficient of such type of the walls will be also independent of the containers positioning. According to testing in aerodynamic tubes, aerodynamic coefficient will be close to 2.

Maximum wind velocity distribution line for the protection constructions or ships in case distance between wind protection wall and protected construction is about 80 m, will be above water level in low water level case on altitude 1/3 of total protection wall (breakwater and protection wall) height. It means, wind velocity in case of initial wind velocity 30 m/s in low water level and in case tidal amplitude is about 7,5 m and total wind protection wall height is 25 m (breakwater and protection wall), will be about 21,8 m/s (decreasing about 27%) and in high water level mean wind velocity will be about 22,8 m/s (decreasing about 24%).

Received calculation results for the presented case in this article have good correlation with results, received in Europort. Differences between Europort experimental results and calculation results for the study case in this article mainly are linked with difference in aerodynamics coefficients of the wind protection wall construction elements.

Possibilities of reducing forces on vessels mooring ropes in open sea ports can be done in few ways, but the best solution – combination of the options:

- decrease ship's inertia forces;
- reduction of the real acting wind on vessel;
- preparing new mooring schemes, which take in account maximum tensions on mooring ropes and decrease changing length and angles of mooring ropes.

Decreasing of ship's inertia forces are linked with mooring schemes, that means mooring schemes must prevent ship movement along quay wall in case acting of periodical forces. As mentioned above, ship's inertia forces could be very high in case of short mooring ropes.

Decrease of the ship's inertia forces could be made by harder mooring to quay wall. In such case is very important to use longer mooring ropes and uses combination long mooring ropes in different places of the ship. The main task for decreasing ship's inertia forces as result of the ship moving along quay wall to minimize ships free steps and in the same time to avoid very short mooring ropes that would be possible to increase period τ (formula (5)). Increased period τ decreases acceleration and in the same time ship's inertia forces.

Reduction of the real acting wind velocity on constructions or vessels could be made by wind protection wall and as were shown before, such decreasing can be up to 20– 30%, but in the same time in case of very strong storm, when wind whiffs reach up to 50 m/s, just wind protection wall can not fully improve situation and combination of the wind velocity reduction and ship's inertia forces decreasing is necessary.

Ship's inertia forces decreasing can be resulted on basis preparation of good mooring scheme in case to avoid very short mooring ropes and increasing long mooring ropes and springs. In this case as an example for the Ro-Ro ship mooring scheme for the storm conditions can be as an example shown on Fig. 10.

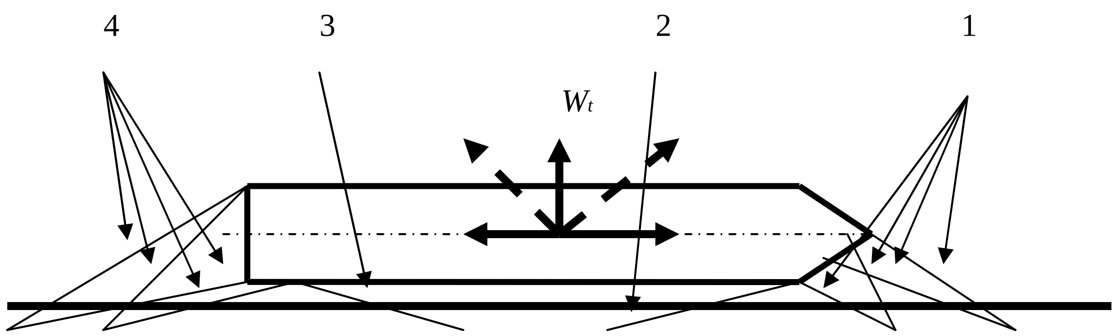

Figure 10. Ro-Ro ship mooring scheme in strong storm conditions: 1 – fore long mooring ropes (from 4 up to 6); 2 – fore springs (at least 2); 3 – astern springs (at least 2); 4 – astern long mooring ropes (from 4 up to 6)

On Fig. 10 in the mooring scheme no short mooring ropes are used and in case of ship's moving along the quay wall period τ increases up to the shortest mooring rope working period, which can be calculated on the basis of moving distance and ship moving speed, that means:

$$\tau = \frac{\Delta S}{v'}, \tag{11}$$

where: ΔS – movement distance, equal to the shortest mooring rope extension in horizontal direction, m; v' – average ship movement speed near the quay wall, m/s.

Mooring ropes position very much influences on the forces, which are created in the mooring system for the protection of outer forces, such as wind loads, ship's inertia forces and other.

Ro-Ro, bulk vessels and tankers in ballast, usually has high hull and mooring ropes have big vertical and horizontal angles as stated by Paulauskas V. et al., 2008. For such type of ships mooring rope tension, which against wind loads perpendicular to ship (quay wall) direction W_t, can be calculated as follows:

$$F_w = F \cdot \cos\alpha \cdot \sin\beta, \tag{12}$$

where: F – real mooring rope tension; α – mooring ropes vertical angles; β – mooring ropes horizontal angles.

As an example, the mooring rope with real tension F = 50 T, vertical angle α = 60º, and horizontal angle β = 30º is taken. In this case force by mooring rope against wind loads will be: F_w = 50 x 0,5 x 0,5 = 12,5 T.

It shows, that in case of maximum mooring rope tension (50 T), just 12,5 T is against wind loads.

As mentioned before, ship's inertia forces try moving ship along the quay wall and inertia forces must be compensated by mooring ropes and fenders. Investigations in many terminals have shown that about 50% of inertia forces are compensated by mooring ropes and about 50% are compensated by fenders (Fig. 11).

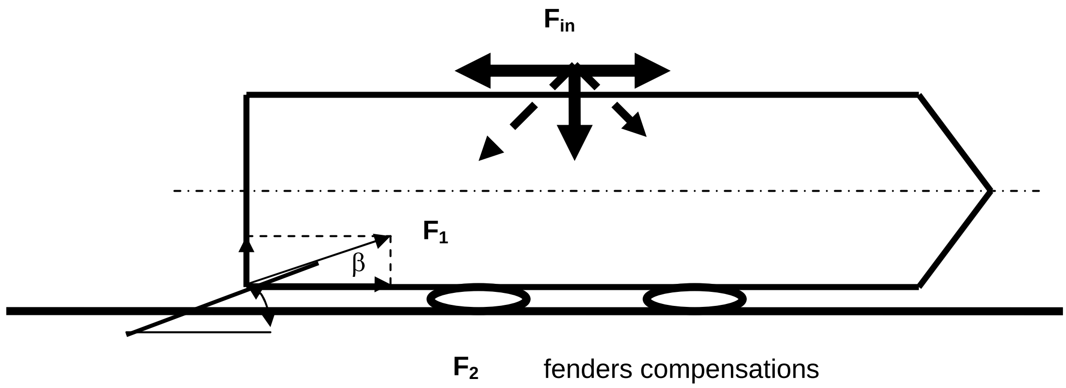

Figure 11. Ship's inertia forces, mooring ropes and fenders compensations

In case of moored ship is moving along the quay wall, mooring ropes together with fenders must compensate ship's inertia force that means vertical angle of the moored rope works the same as in case of compensating the wind loads, and horizontal angle of the mooring rope creates tension, which is calculated as follows:

$$F_{v1} = F_{in} \cdot \cos\alpha, \tag{13}$$

$$F_{h1} = F_{in} \cdot \cos\beta. \tag{14}$$

Finally, mooring ropes take ship's inertia forces, which can be calculated as follows:

$$F_1 = F \cdot \cos\alpha \cdot \cos\beta. \tag{15}$$

As an example, the mooring rope with real tension F = 50 T, vertical angle α = 60º, and horizontal angle β = 30º is taken, in this case force by mooring rope against ship's inertia force will be: F_1 = 50 x 0,5 x 0,866 = 21,7 T.

The presented example shows clearly, that mooring scheme with proper positioning of mooring ropes, plays very important role and can prevent the breaking of mooring ropes during storm conditions, that means compensation wind loads and ship's inertia forces.

Conclusions

1. During the storm conditions ship is influenced by wind loads and inertia forces, which must be compensated by mooring ropes and fenders.
2. Wind loads can be decreased partly by wind protection wall and partly by mooring ropes.
3. Wind protection wall for big ships such as Ro-Ro vessels can not be very complicated, that means high, and it creates limit of the wind protection walls capability.
4. Wind protection wall, constructed by containers (vertical or horizontal positioning) with height up to 13–15 m can decrease wind velocity for the Ro-Ro vessels up to 25% and wind loads up to 30%.
5. Correct or proper mooring schemes are very important part for the Ro-Ro ship safety guaranty and together with wind protection wall, can be very useful in open sea port conditions.
6. Before wind protection wall implementation, simulation in aerodynamic tube is recommended for the wind protection wall in concrete conditions' optimisation.

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Transport and Telecommunication Institute, Lomonosova 1, LV-1019, Riga, Latvia

PRINCIPLES FOR MODELLING OF TECHNOLOGICAL PROCESSES IN TRANSPORT TERMINAL

A. Baublys

Vilnius Gediminas Technical University, Transport Research Institute
Plytinės 27, LT-10105, Vilnius, Lithuania
E-mail: Adolfas.Baublys@ti.vgtu.lt

Technological process is evaluated as a random process, it is also assessed in respective models. Methodology for formalization of technological processes in terminal is suggested and criteria for optimal control and quality of technological process are suggested. Models and algorithms for optimal control of freight clearing technological process are also proposed.

Keywords: transport terminal, technological process, optimal control, models and algorithms

1. Introduction

The following technological operations are carried out with freight in transport terminal: unloading from vehicles, storage, grouping, distribution according to transportation routes, packing in larger transport units, direct reloading from one vehicle to another, loading of goods to vehicles. In case if containers are cleared in the terminal, technological freight clearing processes have higher nomenclature of executed operations. According to the world experience on transportation in containers, 10–20 percent of freight is delivered to container terminals in small consignments. Afterwards these consignments are grouped according to dispatch routes and loaded to containers. There are also cases, when forwarders who haven't their own containers, deliver all goods to terminal in light packages (after having agreed that with terminal authorities) and later the staff of terminal loads the goods to containers (upon the agreement).

Relevant technological equipment is needed for execution of technological processes in the terminal, namely: loading-unloading and sorting equipment, as well as equipment for transportation of goods in the territory of terminal, etc.

Besides, it shouldn‘t be forgotten that several transport modes interact in the terminal. For instance, in the sea terminal there are road transport, railway transport and maritime transport.

Therefore a key task of the terminal staff is to optimally coordinate various technological freight clearing processes, distribute available technological equipment and other relevant resources according to separate transport modes.

The article analyses how technological processes of a terminal could be optimally controlled via mathematical methods and computer hardware.

2. Formalization of Technological Processes in Terminal

Let $J = \{j, j = \overline{1, m}\}$ a set of stages of technological process in the freight clearing terminal. For each stage $j \in J$ the amount and time of equipment K_{jp} is needed and their efficiency is m_{jp}, where p – indicates the index of type of equipment, realising operations of the given stage; m_j – amount of equipment during the stage j.

Technological stage of the process can also be characterised by the amount of available resources R_j^t, $t \in [0, T]$.

Functioning of terminal is analysed by time interval $[0, T]$, described by the set of quantum time $T = \{\Delta t, \forall \Delta t \in N\}$, characterising the fund of working time of terminal. There is a set of goods I, which have to be cleared, $I = D \bigcup S = \{i \in I, i = di, di \in D_{vi} = S_i, S_i \in S\}$, where D – a set of planned goods; S – a set of unplanned goods.

It is assumed that planned goods have to be fully cleared, whereas a certain part of unplanned goods might be un-cleared in case of lack of necessary resources.

Each consignment $i \in I$ is expressed (described) in suite:

$$i =< \alpha_i, \beta_i, \gamma_i, \delta_i, \pi_i, \qquad (1)$$

$$\alpha_i = \begin{cases} t_i^1, i = di - planned \quad end \quad time \quad of \quad clearing \\ t_i^1, i = S_i - time \quad for \quad which \quad part \quad of \quad resource \quad is \quad allocated \quad for \quad consignment\, i. \end{cases}$$

$$\beta_i = \begin{cases} t_i^2, i = di, \\ t_i^2, i = S_i, \end{cases}$$

Time when consignment is delivered for clearing; Time, from which part of resource W_i is allocated for consignment S_i.

Where $\gamma_i = \begin{cases} V_{ij}, i = di, \\ W_{ij}, i = S_i, \end{cases}$

Work volumes according to consignment i in stage j; Part of resource to which consignment i is subject for clearing in stage j;

$$\delta_i = \begin{cases} \delta_i - \\ 0, \\ \delta_i - \\ 0, \end{cases}$$

Fine for incompliance with clearing term; if $t_i^a \le t_i^2$, $i = di$; fine for refucal to clear; if consignment is cleared.

Thus value δ_i is function t_i^a on $i \in D$ and function of allocated resource on $i \in S$.

Clearing time t_i^a of consignment i will be analysed as a linear function of value reverse to the amount of conditional units resource stage j, allocated for consignment i.

In the expression (1) π_i – a set of technological routes, possible for i consignment. $\pi_i = \{p_{ik}\}$ – where p_{ik} – technological route k, for consignment i $k = \overline{1\theta_i}$. More exactly π_{ik} can be defined as a set $\bar{I} \subset I$, a set of technological stages, arranged for consignment i according to binary preference connection, which is distinguished by reflexivity, anti-symmetry and transitivity according to the rule $\forall a$, $\beta \in I$, $a\{B \leftrightarrow |a| < |B|\}$.

In general terms we will call a set of coordinated operations distributed between technological equipment of terminal in clearing freight a technological route. The summation of technological routes for consignment $i \in I$ is expressed as a set $\pi = \pi_a \cup \pi_s$, anticipating all possible options for clearing the goods delivered to terminal.

Subset $\pi_a = \left\{ \bigcup_i \pi_i, i = d_i, d_i \in D \right\}$ demonstrates technological routes in clearing planned freight. In clearing planned freight $i = a_i$, $di \in D$, having stable characteristics, fixed technological routes can be established; they guarantee planned and efficient clearing of goods.

Subset $\pi_s = \left\{ \bigcup_i \pi_i, i = S_i, S_i \in S \right\}$ indicates technological routes in clearing unplanned freight. For unplanned freight $i = S_i$, $S_i \in S$, a set π_i is formed depending on nature of consignment, its dependency on various consumers, state of technological processes of terminal in clearing other consignments. Formation of a set π_i very much depends on the system of priorities and selection of clearing procedure.

Intersection of a set π_i indicates possible loading of equipment (resources) of technological stage, $V_j = \left\{ \bigcap_i \pi_i,\ i = d_i,\ d_i \in D \right\}$, $W_{i(j)} = \left\{ \bigcap_i \pi_i,\ \ i = S_i,\ S_i \in S \right\}$. Here the following situations are possible:

1) Resources of available equipment meet the emerging demand:

$$V_j \le \sum_p K_{jp} M_{jp}\ ;\ \ W_{i(j)} \le 1\ ; \tag{2}$$

2) Resources of existing equipment for insufficiently emerging demand:

$$V_j > \sum_p K_{jp} M_{jp}\ ;\ \ W_{i(j)} > 1\ ; \tag{3}$$

3) Loading of equipment is much lower than its capacity:

$$V_j \ll \sum_p K_{jp} M_{jp}\ ;\ \ W_{i(j)} \ll 1\ . \tag{4}$$

It is clear that the second situation corresponds with the peak loadings and cannot be considered as satisfactory. This can be solved by better organisation of technological process or introduction of additional equipment. With regard to third situation we can speak about insufficient organisation of the process, since expression (4), as a rule, comply with the expression (3) of the second situation.

iT-echnological route sometimes has to be adjusted due to breakdowns of equipment and other disturbances in separate stages of freight clearing technological process as well as due to occurrence of single unplanned consignments.

Technological process as a controlled system P can be formalized as follows. For technological process P the incoming flow of planned consignments is attributed $D = \{d_i\}$. Flow of unplanned freight $S = \{S_i\}$ is analysed as a flow of disturbing impacts and is also attributed to technological process P. Each consignment $i \in I = D \bigcup S$ is described by a set of parameters $i = \langle \alpha_i,\ \beta_i,\ \gamma_i,\ \pi_i \rangle$. Goods are cleared by the existing resources of terminal, meanwhile one technological route $\pi_{ik} \in \pi_i$ is realised for each consignment.

Passage of consignment i by route π_{ik} is regulated by introducing impacts $U = U(t)$. Function $U = U(t)$ is discrete.

Control of technological process of freight clearing is operative impact on technological routes in changing sequence and connection between technological operations or stages.

Passage of consignment by technological route is characterised by parameters:

$$\delta_i = \delta_i(\alpha_i, \beta_i, \gamma_i, \Delta t, \pi_i, u), \Delta t \in [0, T]. \tag{5}$$

Meanwhile generally movement of freight by technological route is restricted by parameters:

$$\delta_i = (\Delta t, \pi_i, u) \in [\alpha_i, \beta_i], \Delta t \in [0, T], \tag{6}$$

and system resources, described in expressions (2)–(4) or general restrictions:

$$\delta_i(\pi_i, u, r_{ij}^t) \in R_j^t,\ \Delta t \in [0, T], \tag{7}$$

where r_{ij}^t – amount of resource j, allocated for consignment i by time quantum t.

At the end of technological process P flow of un-cleared consignment $\tilde{\tilde{I}} = D \bigcup \tilde{S}$ is released. Subset $\overline{S} = S / \tilde{S}$ consists of un-cleared unplanned consignments.

Formalisation of technological process as a controlled system P allows creating a task for control of technological process on the basis of mathematical theory of optimal control according to various quality criteria of its realisation.

3. Optimal Control of Technological Process and Its Quality Criteria

Technological process of freight clearing is a complex control system. The following control aspects could be distinguished: 1) control in clearing planned freight; 2) control in clearing unplanned freight; 3) control in clearing planned and unplanned freight at the same time.

Control in clearing planned freight is distribution of total resources of terminal according to types of operations, stages of technological process and operations. Each consignment $i = d_i$, $d_i \in D$ is characterised by a set of technological routes $\pi_i = \{\pi_{ik}\}$. A respective set of criteria $P = \{P_{k1}, ..., P_{kq}\}$ can be envisaged for a technological route, characterising technological route π_{ik}. The above criteria might indicate duration, price, reliability, stability etc.

Duration of technological route t_{ik} is defined as follows

$$t_{ik} = \sum_{j \in \pi_{ik}} \sum_{P} V_{ij} / \mu_{jp} K_{jp} ,$$

where $\sum_{P} V_{ij} / \mu_{jp} K_{jp}$ – duration of stage j for i consignment.

Reliability in executing j stage can be expressed by the preparedness coefficient $0 < \eta_j < 1$. Usually three states of technological process are possible: 1 – adequate, non-operational; 2 – adequate, operational; 3 – inadequate. Let's say that probabilities of indicated states during moment t are known, i.e. $Q_j^1(t)$, $Q_j^2(t)$, $Q_j^3(t)$, whereas $\sum_{n=1}^{3} Q_j^n(t) = 1$ – coefficient of preparedness of j stage, defined as summation of probabilities, that it will be in states 1 and 2, i.e. $n_j = Q_j^1(t) + Q_j^2(t)$.

Intensity of shifts from one state to another α, β, γ, ξ are values, reverse to certain time parameters and are specified via expressions:

$\alpha = 1/t_n$, where t_n – duration of interval of allocation for operations; $\beta = 1/t_j$, where t_j – duration of operation during stage j; $\gamma = 1/t_{\theta T}$, where $t_{\theta T}$ – average run-in time for breakdown; $\xi = 1/t_b$, where t_b – average restoration time.

These values can be defined by analysing operations of terminal according to statistical data for a respective time interval.

Having assessed the coefficient of preparedness η_j duration of technological process t_{ik} is specified as

$$t_{ik} = \sum_{j \in \pi_{ik}} 1/\eta_j \sum_{P} V_{ij} / \mu_{jp} K_{jp} . \qquad (8)$$

Price criteria are used in the tasks of technological process of terminal. Clearing price of consignment i according to technological route C_{ik} can be estimated depending on clearing time:

$$C_{ik} = t_{ik} C_t , \qquad (9)$$

where C_t – consignment clearing price per time unit.

Clearing price C_{jk} can be estimated as an additive function of prices of technological stages:

$$C_{jk} = F(C_j) = \sum_{j \in \pi_{ik}} V_{ij} C_j / \eta_j , \qquad (10)$$

where C_j – consignment clearing price during j stage.

The task of optimal control of clearing of planned freight on the basis of the listed criteria can be formed as follows. We have to define a set $\pi_d = \left\{ \bigcup_i \pi_i, i = d_i, d_i \in D \right\}$, which anticipates possible options of clearing of the planned freight. In the set π_d we can define a subset of permitted technological routes $\pi_d = \left\{ \pi_{ik}, \forall \pi_{ik} \in \pi_d \middle| t_{ik} \in \left[t_j^1, t_j^2\right];\ t_j^1, t_j^2 \in [0, T] \right\}$. In the subset pd we can find a subset of optimal technological routes.

Thus the above task assesses restrictions of duration of technological routes and minimises the price of clearing of planned freight.

Clearing of unplanned freight is executed at the same time by delivering it from a set of customers $G = \{q\}$; each of them is characterised by a set of technological equipment $K_g = \{K_{qp}\}$. Control of clearing of unplanned freight is related to determination of clearing technology, determination of priorities and queues.

Meanwhile unplanned freight can be cleared with relative, absolute and dynamic priorities. Relative and absolute priorities are fixed, whereas stable priorities are allocated for a respective period. When $t_i \to t_j^2$ we deal with dynamic priority, which is given by priority function $V_i(t) = \omega_i(t - t_i)$, where ω_i – a coefficient defining the change of priority of consignment during the time of its storage in the terminal, where t_i – time of storage of consignment in the terminal, $i \subset S$.

Optimal determination of priorities can be done by assessing economic criteria. Let's say that flows P of goods of various consumers are delivered to terminal with parameter λ_p, general loading of terminal equals to $\sum_{K_p \in K} \lambda_p t_p > 1$; waiting in the queue is detrimental to a consumer α_{ip}, and terminal C; we consider that goods of consumers are distributed by declining order α_p / t_p

$$\max_P(\alpha_{ip}/t_p), \min_P(\alpha_{ip}/t_p)] \tag{11}$$

in the entire range of options. We have to select the number of priority groups $\eta = \overline{1, N}$, $N \le r$, $\bigcup_\eta K_\eta = K$ and the distribution of consignment according to these groups, so as to minimise the functional:

$$F(\eta) = F_1(y_\eta) + F_2(y_\eta) \to \min , \tag{12}$$

where $F(\eta)$ – total loss due to waiting in the queue during the time unit of terminal operations.
The functional consists of two constituents. The first constituent $F_1(\eta)$ can be expressed as a function:

$$F_1(y_\eta) = \sum_{K_p \in K} \sum_{i \in S} \alpha_{ip} \lambda_p \bar{t}_p^I - -\sum_{\eta \in N} \sum_{K_p \in K_\eta} \sum_{i \in S} \alpha_{ip} n_\eta t_p^I ,$$

where $F_1(y_\eta)$ – difference between the loss due to waiting of goods in the queue before and after their linkage into y_η priority group; $\bar{t}^I$ – average waiting time in the queue; n_η – amount of goods combined to y_η priority group. The second constituent $F_2(y_\eta)$ indicates expenditure of terminal in grouping and clearing consignments and we will refer to these as technological expenditure (costs):

$$F_2(y_\eta) = \sum_{\eta \in N} \sum_{K_p \in K_\eta} C_p \lambda_p \bar{t}_p^I .$$

Function $F_1(y_\eta)$ is a declining function and it reaches the optimal value when $\eta = 1$ and zero, when $\eta = N$. Function $F_2(y_\eta)$ is evenly increasing with the increasing $\eta \to N$, and when $\eta = 1$ has minimal value, then F_η is unimodal cost function.

General control of freight clearing set $I = D \bigcup S$ in principle is control of technological process of freight clearing and is displayed by distribution of terminal resources between the goods cleared during the period [0, *T*]. Control should be executed so as to clear as many consignments as possible and gradually use terminal resources. Then the following optimal control criteria could be specified.

Criteria $F(\delta_i)$ for minimising fines for un-cleared (untreated) $i = S_i$ consignment i or inobservance of finalisation directive terms:

$$F(\delta_i) = \sum_{i \in I} \delta_i (R_j^t, \pi_i) \to \min . \tag{13}$$

In applying criteria $F(\delta_i)$ restrictions should be observed according to freight clearing terminals (6) and restrictions according to terminal resources (7).

Terminal loading minimisation criteria $F(\Delta R)$,

$$F(\Delta R) = \sum_{t\in[0,T]}\sum_{j\in J}\left(R_j^t - \sum_{i\in J} r_{ij}^t\right) \to \min \tag{14}$$

i.e. the criteria $F(\Delta R)$ for elimination of peak loading situations, when terms (3) and (4) are executed at the same time in various time quanta $t \in [0,T]$.

In realising criteria $F(\Delta R)$ freight clearing terms (6) and terminal resources (7) should be observed (7).

Thus we analyse various aspects and criteria of control of technological process of freight clearing as well as general control task. Further we will specify separate tasks and algorithms of their solving.

4. A Model for Optimal Control of Clearing of Planned Freight

Model is created by applying the graph theory:

1. Structure of freight clearing system is modelled in the form of graph $G = \{I, V\}$, where I – graph peak, indicating a set of stages, meanwhile each stage $j \in I$ can be presented either via one element or a set of units of technological devices $K_j = \{K_{jp}\}$; V – graph link, indicating possible relations between graph extremes.

In order to facilitate analysis and formalisation, graph G will be presented in the form of floors. We‘ll arrange the graph G, assuming that peak of j floor – technological equipment K_{jp}, j stage, p type. Peak of the upper floor have no incoming link and peak of the lower floor have no outgoing link.

2. Subgraph $G_i \subset G$ is found on graph $G = \{I, V\}$; where each subgraph G_i corresponds to subset π_t, $i \in D$ and has peaks and links corresponding to a set of technological routes to i consignment $G_i = \pi_i = \{j, v \in \pi_{ik}, i = di, di \in D\}$, $\bigcap_i G_i \neq \varnothing$; $\bigcup_i G_i \neq G$.

A set $G_0 = \bigcup_{i\in D} G_i$ indicates a general structure of options of possible technological routes.

According to task formulation, presented in the first part, control of planned freight is expressed as follows:

1. To define rules for minimisation of a set G_0 down to a subset $\tilde{G}$, corresponding to subset π_d. Here the subset $\tilde{G} \subset G_0$ connects technological routes for which

$$\tilde{G} = \left\{\pi_{ik} \in \tilde{\pi}_d \middle| t_{ik} \in \left[t_i^1, t_i^2\right], t_i^1, t_i^2 \in [0,T], i \in D\right\}. \tag{15}$$

2. In subset $\tilde{G}$ to define subset $G^* \in \tilde{G}$, for which conditions are fulfilled:

$$F(G^*) = \min_{i\in D,\, G_{ik}\in\tilde{G}} F(c_{ik}, G_{ik}); \tag{16}$$

$$\sum_{i\in D} V_{ij} / \eta_j \mu_{jp} \leq K_{jp},\ j \in I; \tag{17}$$

$$1/\eta_j \sum_{K_{jp}\in K_j} V_{ij} / \mu_{jp} K_{jp} \in \left[t_i^1, t_i^2\right],\ i \in D,\ j \in I. \tag{18}$$

In other words, subset G^* creates subgraphs $G_i \subset \tilde{G}$, formed by technological routes $\pi_{\delta k} \in \pi_d^*$, meeting in j stage resources restrictions $j \in I$ (2.17) and duration of clearing of separate consignments (18) and minimises the total price of technological process $F(C_{ik}, G_{ik})$ (16).

Subset $\tilde{G} \subset G$ can be found by different means. Subset $\tilde{G} \subset G$ can be formed by choosing between the permitted $\pi_{ik} \in \tilde{G}_i$ critical technological routes $\pi_{i\varphi}$, defined as

$$\pi_{i\varphi} = \max_{e.r.\mu}\left\{\sum_{j=1}^{K} t_l a_l\right\}, \tag{19}$$

where t_l – the time loading extremes of graph G_i; j – the number of graph floor (Lt. – *aukštas*); a_l^j – the matrix of ratios of subgraph G_i extremes j and $j-1$ floors; l – the index of edge of subgraph G_i; $j=\overline{1,m}$, $K \le m$; r, μ – the subset of indexes, highlighting links and peaks of road π_{ik}.

Matrix is equivalent to subgraph of floors j and j–1 of subgrapg G_i. In the case, when link in subgraph crosses $j-1$ floor, the zero extreme is introduced in floor $j-1$. In forming a critical technological route $\pi_{i\varphi}$ values t_l are taken according to expressed binary connections of matrix $\{a_l^j\}$.

Formation of subset $\widetilde{G}$ of critical technological routes $\pi_{i\varphi}$ comprises a respective structure of technological process, increasing a probability that its option optimal according to (16) does exist.

Critical technological route is defined via the method of dynamic programming the key principle of which is recurrent ratio, expressing the principal of optimality of R. Belman, i.e. with each step a decision is made and guarantees optimal continuation of the process with respect to the achieved state at the given moment. Thus by step K, G_i of subgraph corresponds the ratio $\pi_{i\varphi}^k(l^{k-1}) = \max\{t_l a_l^k\}$, where $1 \le l \le m$, l – graph G_i number of edge in the critical road in floor K. If maximum exists in this road, then in search for it function $l^k = l^k(l^{k-1})$ will be found.

G_i of graph for floor $K-1$ will be $\pi_{i\varphi}^{k-1}(l^{k-2}) = \max\limits_{l^k, l^{k-1}}\left\{t_l, a_l^{k-1} + \max\limits_{l^l}(l^{k-1})\right\}$, where $1 \le l \le m$, $l^l << l^{k-1} << m$.

Having distributed the maximum by floors we get: $\pi_{i\varphi}^{k-1}(l^{k-2}) = \max\limits_{l^k, l^{k-1}}\left\{t_l, a_l^{k-1} + \max\limits_{l_k}(l^{k-1})\right\}$.

Having assessed the obtained expressions: $\pi_{i\varphi}^{k-1}(l^{k-2}) = \max\limits_{l^{k-1}}\{t_l a_l^{k-1} + \pi_{i\varphi}^k(l^{k-1})\}$.

Recurrent ratio can be obtained to any floor of subgraph G_i.

$$\left.\begin{aligned} &\pi_{i\varphi}^{k-r}\left(l^{(k-r-1)}\right) = \max_{l^{k-2}}\{t_l a_l^{k-r} + \pi_{i\varphi}^{k-r}(l^{k-r})\} \\ &l^{k-r} = l^{k-r}(l^{k-r-1}); 1 \le e^{k-r} \le m, r \in j, j = \overline{1,m} \end{aligned}\right\}. \tag{20}$$

Ratio (20) is the main functional equation of R. Belman for this task. In subgraph G_i of critical technologies the process of finding is repeated until the subset of technological routes is found $\widetilde{G} = \bigcup\limits_i \breve{G}_i$, meeting restrictions (17), (18).

However, in the above case we have to define a full set G_i for each consignment or flow by applying a special procedure in order to reduce it to subset $\widetilde{G}_i$. Calculations can be shortened if for formation of subset $\widetilde{G} \subset G_0$ we‘ll use a formal procedure, omitting the stage of formation of a full set $G_0 = \bigcup\limits_{i \in D} G_i$. Meanwhile criteria (10) of the applied price $F(c_j)$.

Subset $\breve{G}$ is formed under the following assumptions.

When analysing the operating actual terminals, and referring to theoretic assumptions, it becomes clear that if we compare technological routes with various indicators c_{ik} and t_{ik}, we can formulate the following propositions, providing for the establishment of subset $\widetilde{G}_i$.

Proposition 1. If $\pi_{ik} \in G_i$, and $F(c_j)$ price increases when $c_j = \text{const}$, then

$$\breve{G}_i = \left\{\pi_{in} \in G_i, n \in \theta_i / c_n \le c_{t_{\min}};\ t_n \le t_{c_{\min}}\right\}.$$

Proposition 2. If $\pi_{ik} \in G_i$, and $F(c_j)$ increases with increasing t_j and $c_j = \text{const}$, and exist π_{in} and π_{in-1} such as $t_n \ge t_{n-1}$; $c_n \le c_{n-1}$, it means that $c_{nj} / t_n < c_{(n-1)j} / t_{n-1}$, therefore $\pi_{in} \in \widetilde{G}_i$.

On the basis of these propositions subset $\widetilde{G}$ is formed as follows.

Let Ω_k – space of states in step k ;

$\omega_{kj} \in \Omega_k$ – element of set Ω_k ;

D_k – space of solutions in step k ;

$b_{kj} \in D_k$ – element of set D ;

m – total number of steps;

$c_{kj} t_{kj}$ – projection of function in k step.

Consequently we assume that this task belongs to the type of recursive tasks, and subset is formed as follows:

$$c_j = \sum_{k=1}^{m} c_{kj}\left(\omega_{k-1}, j; b_{kj}\right) \le c_{t\min} ;$$

$$t_j = \sum_{k=1}^{m} t_{kj}\left(\omega_{k-1}, j; b_{kj}\right) \le t_{c\min} .$$

Subset $\widetilde{G}$ is established by the following algorithm:

1. In the subset D_m for step m a set of arranged pairs has to be formed $A_{mj} = \left(t_{mj}, C_{mj}\right)$, where $t_{mj} = t_{mj}\left(\omega_{m-1}, b_{mj}\right)$, $C_{mj} = C_{mj}\left(\omega_{m-1}, j, b_{mj}\right)$.

2. To define a set $A_m^1 \subset A$, where $A_m^1 = \left\{A_{mj} \middle| 0 \le t_{mj} \le t_{c\min} ;\ 0 \le C_{mj} \le C_{t\min}\right\}$.

3. A_m^1 finding corresponds to definition of p solutions in step m $b_m^* = \left\{b_{mj}^*\right\}$, tai yra $a_{mj}^* \sim b_{mj}^*$, $a_{mj}^* \in A_m^1$, $b_{mj}^* \in \left\{b_{mj}^*\right\}$. $t_{(m-1,\,j)m}^{t} C_{(m-1,\,j)m}^{t}$ – a full winning in the last two steps at any solutions in step $m-1$ and in step of perspective solutions $m\left\{b_{mj}\right\}$.

4. In the set D_{m-1} for step $m-1$, a set of arranged pairs has to be defined $A_{m-1,\,j} = \left\{t_{(m-1,\,j)m}^{+}; C_{(m-1,\,j)m}^{+}\right\}$,

where

$$t_{(m-1,\,j)m}^{+}\left(\omega_{m-2}, b_{m-1}\right) = t_{m-1,\,j}\left(\omega_{m-2}, b_{m-1}\right) + t^{*}\left[\omega_{m-1}\left(\omega_{m-2}, b_{m-1}\right)\right];$$

$$C_{(m-1,\,j)m}^{+}\left(\omega_{m-2}, b_{m-1}\right) = C_{m-1,\,j}\left(\omega_{m-2}, b_{m-1}\right) + C^{*}\left[\omega_{m-1}\left(\omega_{m-2}, b_{m-1}\right)\right]$$

5. To specify subset $A_{m-1}^1 \in A_{m-1}$,

where

$$A_{m-1}^1 = \left\{A_{m-1} \middle| 0 \le t_{(m-1,\,j)m}^{+} < t_{c\min} ;\ 0 \le C_{(m-1,\,j)m}^{+} < C_{t\min}\right\}.$$

6. Finding of A_{m-1}^1 corresponds to specification of a set in the step of perspective solutions $(m-1)b_{m-1}^* = \left\{b_{m-1,\,j}^*\right\}$, i.e.

$$a_{m-1,\,j} \sim b_{m-1,\,j}^* ;\ a_{m-1,\,j} \in A_{m-1}^1 ;\ b_{m-1,\,j}^* \left\{b_{m-1,\,j}^*\right\}.$$

When continuing, we receive, for the first stage, the set $b_1^* = \left\{b_{1j}^*\right\}$, each element of which, together with its continuation (extension) specifies technological route depending to set $\widetilde{G}_i$.

In order to extend the set $\widetilde{G}_i$, by including not one, but several technological routes for i consignment, the selective procedure changes and is of the following sequence:

1. In the set D_m , the set $A_m = \left\{C_{mj}\right\}$, where $C_{mj} = C_{mj}\left(\omega_{m-1}, b_m\right)$ is formed – this is a criteria according to which we execute optimisation.

2. The subset $A_m^1 \subset A_m, A_m^1 = \left\{C_m^*(\omega_{m-1}),\ C_m^{**}(\omega_{m-1})\right\} \subset\ A_m$ is formed,

where $C_m^*(\omega_{m-1}) = \max\{C_{mj}(\omega_{m-1}, b_m)\}$;

$C_m^{**}(\omega_{m-1}) \sim b_m^{**}(\omega_{m-1})$; $C_m^{**}(\omega_{m-1}) = \max\{m_j(\omega_{m-1}, b_m)\}$; $C_m^{**}(\omega_{m-1}) \sim b_m^{**}(\omega_{m-1})$.

Consequently, in step m we get a set of perspective answers $\{b_m^*, b_m^{**}\}$. Here $C_m^{**}(\omega_{m-1}) = = C_m^*[\omega_{m-1}(\omega_{m-2}, b_{m-1})]$. Whereas $C_{(m-1)m}^+$ – a full winning in two last steps at any step of answer $(m-1)$ and rational answers $b_m^*(\omega_{m-1})$, $b_m^{**}(\omega_{m-1})$ in step m. In the set D_{m-1} (step $m-1$) the set $A_{m-1}^* = \{C_{m-1,j}^+\}$ is formed, where $C_{m-1,j}^+ \mu_j(\omega_{m-2}, b_{m-1}) = C_{m-1,j}(\omega_{m-2}, b_{m-1}) + C_m^+[\omega_{m-1}(\omega_{m-2}, b_{m-1})]$.

3. The set $A_{m-1}^1 = \{C_{m-1}^*(\omega_{m-2}), \{C_{m-1}^{**}(\omega_{m-2}) \subset A_{m-1}^*$ is formed.

Here $C_{m-1}^*(\omega_{m-2}) = \max\{C_{m-1,j}(\omega_{m-2}, b_{m-1}) + + C_{mj}^+[\omega_{m-1}(\omega_{m-2}, b_{m-1})]\}$;

$C_{m-1}^{**}(\omega_{m-2}) = \max\{C_{m-1,j}(\omega_{m-2}, b_{m-2}) + C_{m-1,j}(\omega_{m-2}, b_{m-2}) + C_{mj}^+[\omega_{m-1}(\omega_{m-2}, b_{m-1})]\}$

proceeding this procedure until the end, we get $C_{1,2,\ldots,m}^* = \max_{\omega_0 \in \Omega_0}\{C(\omega_0, b_1) + C_2^*[\omega_1(\omega_0 b_1)]\}$, $C_{1,2,\ldots,m}^* \sim b_1^*$ – rational technological route included into $\tilde{G}_i$.

The above pressure is executed by applying general methods of successive analysis, and its peculiarity is expressed only by the rules of selection of options and ways for usage of results. Procedures of compression of options allow to indirectly establish a subset $\breve{G}$, corresponding to respective restrictions and criteria, by omitting creation of a full set G_0 and reviewing all its elements. Since usually the set $\tilde{G}$ has few options, the subset G^* can be established through direct review.

5. Models and Algorithms for Optimal Control of Technological Process of Freight Clearing

As mentioned above, clearing of goods in various stages of technological route is affected by various (random) obstacles; due to this the technological process in the terminal also is random. Therefore the moment of finalisation of freight clearing can be defined only with a certain probability. Besides, peak loadings and situations when needs of consumers are not in compliance with terminal capacities, also occur. In the above case a distribution task is established and criteria (10) have to be optimised.

Capacity of terminal (throughput) during period $[0, T]$ is N and it can clear a set of consignments $I = D \bigcup S = \{i = d_i,\ d_i \in D \forall_i = S_i,\ S_i \in S\}$, $i = \overline{1, m}$, each consignment is described by $i = \langle \alpha_i, \beta_i, \gamma_i \rangle$, where α_i – beginning of freight clearing; β_i – planned end of freight clearing; $\gamma_i = t_i$ – time of clearing of consignment i.

Let's say that period $[0, T]$ is divided into a set by equal interval $T = \{\Delta t_z\}$, $z = \overline{1, m}$, $\bigcup_z \Delta t_z = T$, meanwhile $\Delta t_z >> \max \gamma_i$.

We'll define the set $\{N_z^z\}$, $z = \overline{1, m}$, where terminal capacity (budget of main operational time in the interval Δt_z). Each consignment i has defined set of intervals $\{\Delta t_\varphi, t_{\varphi+1}, \ldots, \Delta t \psi\} \in T$; $\varphi_1 \psi \in z$, $\varphi < \psi$; which can be given as a section $\lfloor \Delta t_j, \Delta t \psi \rfloor$, where $\Delta t_{i\varphi} = \alpha_i$ term of beginning of goods clearing $\Delta t_{i\psi} = \beta_i$ – obligatory term for finalisation of goods clearing. If $\Delta t_{i\varphi} = \Delta t_{i\psi}$, then consignment can be cleared only during interval $\Delta t_{i\varphi} = z$. If $\Delta t_{i\varphi} \neq \Delta t_{i\psi}$, then consignment is cleared in any interval $\{\Delta t_\varphi, t_{\varphi+1}, \ldots, \Delta t \psi\} \in [0, T]$.

It is necessary to distribute freight clearing so as to gradually load terminal during the entire T. For that in the set of consignments $I = D \bigcup S$ we have to find subsets $I_z \in I$, $z \in Z$, which guarantee optimal loading of technological equipment of terminal and clearing according to terms envisaged for all consignments.

$$x_{iz} = \begin{cases} 1, & \text{If consignment } i \text{ is cleared in interval } z; \\ 0 & \text{– otherwise.} \end{cases}$$

In general the task for optimisation of loading technological of terminal can be formulated as follows. To find vector

$$\bar{x} = \{x_{iz}\},\ i = \overline{1,m};\ i \in I;\ Z = \overline{1,m}. \tag{21}$$

Minimising target function

$$F = \sum_{r=1}^{m} \left(N_z - \sum_{i=1}^{n} t_i x_{ir} \right) \to \min \tag{22}$$

under restrictions

$$x_{iz} \in \left[\Delta t_{i\varphi}, \Delta t_{i\psi}\right],\ \forall z,\ \varphi,\ \psi \in [0,T],\ i \in I; \tag{23}$$

$$\sum_{i=1}^{n} t_i x_{iz} \le N_z,\ Z = \overline{1,m}; \tag{24}$$

$$\sum_{z=1}^{m} x_{iz} = 1,\ i \in I; \tag{25}$$

$$x_{iz} \in \{0,1\},\ Z = \overline{1,m},\ i \in I. \tag{26}$$

Peculiarity of the given task is that it is moderate and condition (24) restricts margins of $\bar{x}$ existing. For task solving, algorithm created on the basis of margins and branches is used.

Vector $\bar{x}$, meeting (24)–(26) restrictions, will be referred to as answer, vector $\bar{x}$, meeting (23)–(26) restrictions – a permitted answer, whereas a permitted answer optimising (22) function – optimal answer. The main idea of the suggested algorithm is to find the base vector $\bar{x}^0$, which is the answer of task (23)–(26) and to execute later its gradual optimisation.

The base of vector $\bar{x}^0 = \{x_{iz}^0\}$ is found as follows. For each i consignment according to a given term $t_{i\psi}$ is defined Δt_z, for which $x_{iz}^0 = 1$, if $z = \psi$, z, $\psi \in [0,T]$, and $x_{iz}^0 = 0$ for all $z \ne \psi$. A received vector $\bar{x}^0 = \{x_{iz}^0\}$ a permitted answer, since term (23)–(26) is fulfilled. However, vector $\bar{x}^0$ is not within the margins of the optimal formulated task as in case of other criteria of schedule theory.

We'll analyse possibilities for optimising vector $\bar{x}^0$. Having distributed consignments $\{x_{iz}^0\}$ – it is considered that loading of terminal equipment is uneven, therefore in the set of intervals $\{\Delta t_z\} \in [0,T]$, $z = \overline{1,m}$ the set of several intervals $\{\Delta t_z\}$ can be defined, $r \in Z$, for which the condition (23) is a strict inequality. All other intervals $\Delta t_z / \Delta t_r \in [0,T]$, r, $z = \overline{1,m}$ will be referred to as full. In order to get the permitted answer, it is necessary to fill in the pursued intervals Δt_r, and in order to get optimal answer, it is necessary to highlight answer in the set of permissible answers $\{\bar{x}^S\} = \bar{x}$, $S = \overline{1,s}$, minimising target function (22).

Vector $\bar{x}^0$ is optimised by iterative procedure in the freely chosen full interval $\Delta t_x \in \Delta t_z / \Delta t_r \in [0,T]$, α, z, $r \in m$ in the set of consignments $I_\alpha^1 = \{i \in I \mid x_{i\alpha} = 1, \alpha \in z\}$ the subset $\tilde{\tilde{I}}_\alpha^1 = \{i \in I_\alpha^1$, $t_{i\varphi} \ne t_{i\alpha}$, $\varphi < \alpha$; φ, $\alpha \in z_j$ is found. The subset $\tilde{\tilde{I}}_\alpha^1$ is redistributed according to intervals $\{\Delta t_r\}$, where $r = \alpha - 1, \alpha - 2, ..., \varphi$, by optimising target function (22).

Permissible answers $\{\bar{x}^S\}$, $S = \overline{1,s}$ are found via oriented movement according to extremes of tree of options of freight clearing distribution. Ramification strategy is as follows. At tree level p option k of distribution of consignment p is formed, meanwhile $p_i \in I_\alpha^1$, $t_k \in t_r \subset [1,T]$, $\varphi_{pi} \le 2 \le \alpha$. At each tree level p the received distribution options are assessed according to condition (23). The set of received options $\{\bar{t}_k\}$ is defined according to assessment $t_{pi} \le N_k - \sum_{i \in n} t_i x_{ik}^0$, $\forall k \in m$. Later in the set of permissible options $\{\Delta t_k\}$ lower evaluations of distribution are introduced which, based on the optimum (22), can be estimated according to formula

$$F_{pk} = \min\left\{\sum_{i\check{z}1}^{n} t_i x_{ik}^0 + t_p\right\}. \quad (27)$$

The extreme, complying with the option with lowest assessment (evaluation) (27), is chosen as active for further split. The remaining extremes of the given level are final.

If (27) complies with several indices k, then we select the smallest index $\bar{k} = \min\{k\}$. Further we read $x_{pk} = 1$ and $x_{iz} = 0$ for all $z \in m$, $z \neq m$. The process continues until further split becomes impossible. The answer is optimal if tree of options has no final extremes with evaluations.

$$F(\bar{x}^S) < F^*, \ \forall i \notin I_k^1, \quad (28)$$

where $F^* = \min_s F(\bar{x}^S)$ – value of target function of the received answer. Otherwise the answer is verified, and split from extremes, corresponding to (28), is specified. Verification should start from lower levels, since then it is possible to find the answer quite quickly; besides the number of options of upper levels to be verified would decrease. The split from the verified extreme is terminated, if assessment of the lower margin in some of the levels reaches or exceeds F^*. When new answer is received, a respective value of a target function is used for verification. Optimisation procedure of vector $\bar{x}^0$ iteratively is repeated for intervals $(\Delta t_{\alpha+1}, \Delta t_{\alpha+2}, ..., \Delta t_z) \setminus \Delta t_r \in [0, T]$, where $\alpha+1$, $\alpha+2$, ..., z, $k \in m$ until $\{\Delta t_r\} \neq \varnothing$. If the above condition is not fulfilled, it is considered that further optimisation is not possible and calculation is finished.

For further vector $\bar{x}^0$ optimisation iterations $\alpha+1$, $\alpha+2$, ..., γ, ..., z. In estimating assessments (27) we assume that $\bar{x}_{1r}^0 = x_{ir}^{\gamma-1}$, where $x^{\gamma-1} = \{x_{iz}^{\gamma-1}\}$ – distribution vector, formed $\gamma-1$ iteration.

Algorithm of calculations:

1. Base vector $\bar{x}^0$, meeting (23)–(26), is formed.
2. Condition (24) is verified for the received distribution and in the set $\{\Delta t_z\}$ subset $\{\Delta t_r\}$ is found.
3. In the subset $\{\Delta t_r\}$ interval Δt_α is selected. The subset $I_\alpha^1 = \varnothing$ is defined, the procedure is repeated three times due to $\Delta t_{\alpha+1} \in \{\Delta t_r\}$.
4. Extremes of level p are formed according to (23) and (24), margins of options of answers are assessed. Options of level p are gradually re-selected until Δt_k is defined.
5. Extreme $\Delta t_{\bar{k}} = \min\{\Delta t_k\}$ is defined, and read $x_{p\bar{k}} = 1$, $x_{iz} = 0$, $z \in m$, $z \neq k$.
6. Answer $\{x_s^*\}$ is fixed, for which (28) is met, for verification procedure No 3 is repeated.
7. In case if during verification it turns out that $\{\bar{x}_s\}$ exists, $F\{\bar{x}_s\} < F^*$ then value of answer is renewed and we return to 3, otherwise a shift to 8.
8. End of calculations.

Optimal control of technological process of freight clearing can be executed by applying criteria (13).

Let's introduce additional markings. Let $T_i \in [0, T]$ time interval, during which consignments can be cleared $i \in I$, $T_j = [t_i^1, t_i^2]$, x_{ij}^t – a pursued variable, resource *j* stage, given for clearing of consignment during *t*, x_{ij}^t quantum has discrete values and equals to

$$x_{ij}^t = \begin{cases} 0 \leq r_{ij}^t \leq R, \ t \in T_i; \\ 0, \ t \notin T_i. \end{cases} \quad (29)$$

From the set I we will specify the subset of cleared consignments $\theta_1 = \{i \in I \mid t - t_i^2 \geq 0,\ x_{ij}^t \neq 0,\ t \in [1, T]\}$ and subset of un-cleared consignments $\theta_2 = \{i \in I \mid t - t_i^2 < 0,\ x_{ij}^t = 0,\ t \in [1, T]\}$, meanwhile $I = \theta_1 \cup \theta_2$, $\theta_1 \cap \theta_2 = \varnothing$, $\theta_2 = I / \theta_1$.

Formally the task would look as follows:

$$F(\delta_i) = \sum_{i \in I} \delta_i - \sum_{i \in \theta_1} \delta_i \sum_{j \in \pi} \sum_{t \in T_i} x_{ij}^t \Big/ \sum_{j \in \pi_i} V_{ij} \to \min \tag{30}$$

$$\sum_{i \in \theta_1} X_{ij}^t \le R_j^t,\ j = \overline{1, J};\ t \in [0, T],\ 0 \le X_{ij}^t \le R_j^t\ , \tag{31}$$

where x_{ij}^t – discrete values

$$\forall j \in \pi_i, \forall i \in I, t \in [0, T]\ , \tag{32}$$

where the upper t indicates time $t \in [0, T]$ quantum; π_i – technological route, run during clearing i by consignment $i \in \theta_i$. Target function (30) minimises the sum of fines for un-cleared consignments.

Task (30)–(32) is a task of dynamic distribution of the vector resource in the set; its meaning – to re-distribute transformation of the arranged phases of resources between competing processes according to the fine minimum for unfulfilled planned terms of freight clearing.

Conclusions

1. The suggested methodology for formalisation of technological processes in transport terminal provides for management of these processes by means of a dialogue between a worker and computer and solve tasks for optimal control of freight clearing.
2. Technological process of freight clearing is a complex control system. The following aspects of its control could be specified: 1) control in clearing planned freight; 2) control in clearing unplanned freight; 3) control in clearing planned and unplanned freight at the same time.
3. The above specified compression (Lt. – *suspaudimas*) of technological routes is executed through general methods of the successive analysis, and its peculiarity is expressed only by option selection rules and ways of usage of results.
4. Clearing of goods in various stages of technological route is influenced by various (random) obstacles; therefore the technological process itself is also random. Thus the moment of the end of goods clearing can be defined only with a certain probability. Besides, peak loadings are also possible, as well as situations when needs of consumers do not comply with terminal capacities. In such case distribution task is formed and it has to be optimised.
5. The task for dynamic distribution of the vector resource in the set is formed; its meaning – to re-distribute transformation of the arranged phases of resources between competing processes (consignments) according to the fine minimum for unfulfilled planned terms of freight clearing.
6. Generally, technical-economic indicators of terminal operation should be analysed in terms of random factors which are random with respect to any argument value as well. The arguments include time or other parameters of terminal operation (technological process). Thus, the criteria of optimality should also be considered as being random rather than determined.
7. Most criteria used for the assessment of terminal operation and individual technological processes are interlinked and this should be taken into account when using them as optimality criteria. In determining the numerical characteristics of technical-economic indicators according to statistical data obtained during the process of terminal operation, mathematical expectation as well as correlation and variance functions should be calculated.

7\. Random emergency situations cause failures at the transport terminal. In considering them, the theory of probability value functions should be used.

8\. A great number of various factors influence the operation of the terminal and may cause its malfunction; however, their influence may differ to a great extent. Therefore, the simulation data should be optimised for usage during further decision-making process.

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MICROFABRICATED ONE-ELECTRODE In_2O_3 AND Fe_2O_3-In_2O_3 COMPOSITE SENSORS

V. Golovanov, C. C. Liu, A. Kiv[1], D. Fuks[1] , M. Ivanovskaya[2]

Electronic Design Center, Case Western Reserve University, Cleveland, OH 44106, USA
[1]Ben-Gurion University of the Negev, P.O.Box 653, Beer-Sheva 84105, Israel
[2]Belarus State University, Leningradskaya St. 14, 220050 Minsk, Belarus

This paper reports the development of planar sensor based on one-electrode concept. In a typical one electrode sensor construction, a thin platinum coil is coated by oxide semiconductor material. The platinum wire spiral is replaced by a microfabricated palladium or gold resistors **in our prototype sensors**, and the sols of oxide semiconductor are spin-coated over the electrode. The different types of indium oxide–based composites have been tested as a sensing layer. The constructed sensors demonstrate a great sensitivity and stability in detection of carbon monoxide (CO) as compared with commercial infrared analyser. The multi-phase Fe_2O_3–In_2O_3 sensors are characterized by the highest sensitivity to methanol (CH_3OH).

Keywords: multi-phase Fe_2O_3–In_2O_3 composite sensors, microfabrication technology, gas sensing elements

1. Introduction

On-line monitoring of emission gases from different combustion processes has become an important issue when developing techniques for pollution control. In this respect, one electrode sensors may offer advantages in the form of simple construction, resistance to severe conditions, and capability being in the actual source of the emissions. This sensor type may also have a short response time so as to enable the possibility of real time control.

Indium oxide material is widely used for different applications in gas sensing elements [1–3]. The indium oxide based sensors are featured by a low threshold of sensitivity to reducing gases and can operate well in the ppm range [4]. Earlier we have reported the gas sensing characteristics of one electrode sensor [5], where thin platinum wire (diameter 20μm) spiral is embedded inside a sintered In_2O_3 button. The one-electrode spiral acts both as the heating resistor and measuring electrode and the sensor works under a stabilized current feed. The operation principle of the one-electrode sensor is based on the shunting of the electrode by semiconductor oxide, coating the metal meander. At the gas exposure, the sensor response is obtained as a voltage (U) change during the interaction of oxide with surrounding atmosphere. The planar design of one electrode sensor based on different thick-film printing techniques has been reported by us elsewhere [6]. Such approach permitted us to construct the integrated hybrid sensor structures.

However, the inherent shortcomings of the previously employed thick-film technologies were practical limits for the minimum line width and distance between the elements of the meandering heater (~50–100 μm). Indeed, the optimal matching between metal oxide and heater resistances may be achieved through the control of the metal electrode thickness and width. The distance between the strips also became a crucial factor that affects the sensor sensitivity. The most optimal sensor design must be oriented to a sufficient decrease of the distance between the electrode strips. The considerable progress could be obtained as a result of development of high precision and resolution patterning processes, like microfabrication technology. The lithographic pattern reduction techniques permit extensive diversity in the size consideration of the electrode elements with a line width below 0.1 μm.

2. Experimental

We have employed CMOS-compatible processing facilities in order to deposit palladium and gold electrodes on alumina substrates. Their initial resistances (140 Ohm and 80 Ohm, respectively) match well with resistance of the metal oxide composite material. The use of microfabricated electrode made possible low power consumption at the operation temperature about 500°C. The electrodes were coated by nano-sized In_2O_3, Fe_2O_3 and Fe_2O_3–In_2O_3 mixed oxides using sol-gel technology.

a)

b)

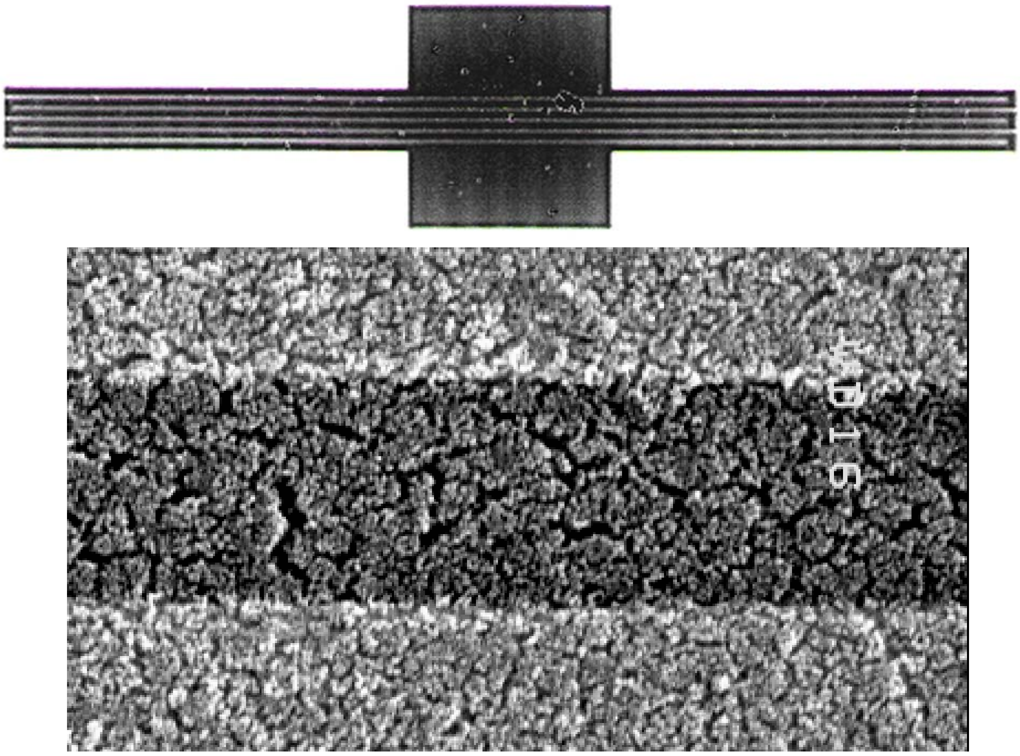


Figure 1. SEM images of microfabricated palladium electrode (a) coated by In_2O_3 film (b)

The sensing layers were synthesized as stabilized sols of the corresponding metal hydroxides. The synthesis procedures consisted of the following steps: (i) forced hydrolysis of an inorganic metal salt solution (0.5 mol l^{-1}) with a base agent (water solution of NH_3, > 99.99% purity, 0.5 mol l^{-1}); (ii) precipitation of a metal hydroxide and its separation from the liquid phase; (iii) formation of a sol by peptisation of the deposit with peptising agent (HNO_3) or as a result of self-peptisation. Table 1 summarizes the synthesis conditions under which the Fe_2O_3–In_2O_3 along with individual indium and iron oxides were obtained.

Table 1. Synthesis conditions of the samples

Sample designation	Synthesis conditions
In_2O_3	Introduction of NH_3 solution into $In(NO_3)_3$ solution
α-Fe_2O_3	Introduction of NH_3 solution into $Fe(NO_3)_3$ solution
γ-Fe_2O_3	Introduction of $Fe(NO_3)_3$–$FeSO_4$ (2:1 mol) solution into NH_3 solution and oxidation of suspension with air (5 h, 100°C)
γ-Fe_2O_3–In_2O_3 (9:1)	Mixing of γ-Fe_2O_3 and In_2O_3 (9:1 mol) sols

In order to obtain thin-film sensors, the sols were deposited by spin-coating onto microfabricated one electrode structure. The sensor elements were successively annealed at 300–400°C for 96 h in air. A further thermal treatment up to 500–800°C was performed in order to characterize the crystallization process of the Fe_2O_3–In_2O_3 composites.

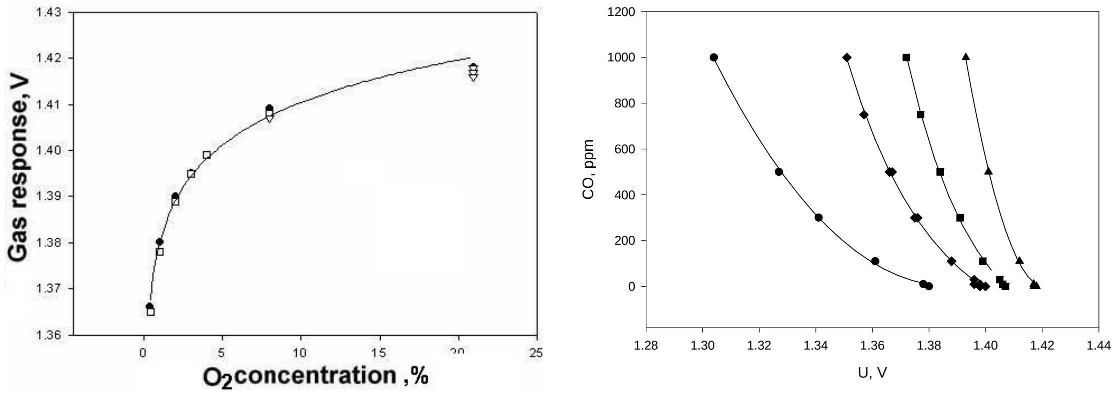
The sensor response values to O_2 (0–20%), CO (1–1000 ppm), NO_2 (10–100 ppm) and CH_3OH (1–20 ppm), were recorded. The sensor was operated at the constant current feed of 10 mA and the measurements were performed in a flow chamber at 0.3 min^{-1} flow rate, 20°C temperature and 5–50% relative humidity. The commercial infrared device IR-880A manufactured by Fisher-Rosemount Inc. was used as a reference analyser for monitoring of carbon monoxide in the gas mixtures.

3. Results and Discussion

3.1. In_2O_3-Based Sensors

The luck of selectivity, being one of the main troubles with semiconductor gas sensors, has been at focus in this study. Motivated by the practical need in real time monitoring of the carbon monoxide in the actual combustion emissions we aimed at developing a calibration procedure for quantitative evaluation of the CO concentration in the gas mixture with oxygen, where both concentrations are subjected to variation. Indeed, the combustion process appeared to show some instability when high concentrations of CO were caused by reducing of the feed of air.

The response of pure In_2O_3 sensors to oxygen is presented on Figure 2a and the calibration curves to carbon monoxide at different concentrations of oxygen in gas mixture are presented on Figure 2b.

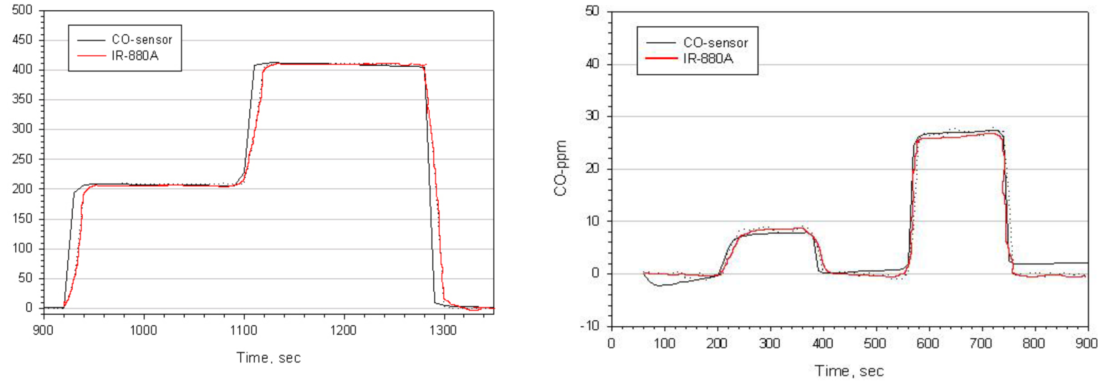


a) b)

Figure 2. Influence of the gas concentrations on the response of In_2O_3-based one electrode sensors to (a) – O_2 and (b) – CO in 20,95% O_2 (▲); 8% O_2 (■); 4% O_2 (◆) and 1% O_2 (●)

Next, the calibration surface has been constructed, which allowed us direct determination of the CO concentration during on-line monitoring of the emission gases. The quantitative assessment of the carbon monoxide presence in the gas mixture with oxygen is illustrated on Figure 3 in comparison with commercial infrared analyser. All variations of the concentration of CO were, faithfully echoed by the developed sensors. The response and recovery times of the sensors were only few seconds. One can see that constructed sensor demonstrate a very good performance in comparison with expensive optical device and able to recognize a very low concentrations of CO with the same level of accuracy. The forms of response curves from reference analyser are very similar.

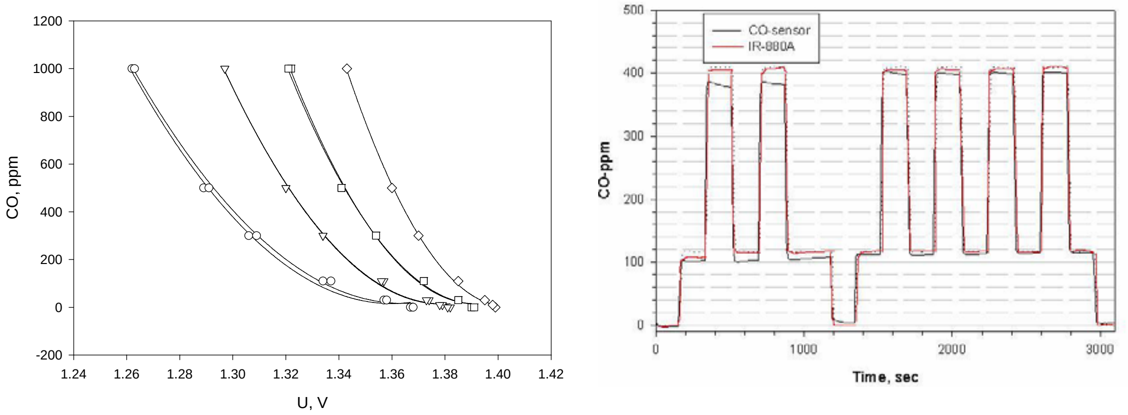
Water vapour, usually present at high concentrations in a combustion gas was not found to impair the performance of microfabricated one electrode sensors as indicators of CO over a wide range. This correlates with our previous findings [5] revealing that main changes caused by the water vapour take place in the range of 1–20% relative humidity. In the exhaust gases air humidity varies in the range of 20–75% RH. Thus, the obtained results demonstrate that designed In_2O_3-based sensors have a good stability of operating parameters with respect to air humidity in the conditions of real exploitation.



a) b)

Figure 3. The quantitative monitoring of the (a) high and (b) low concentrations of the carbon monoxide in the gas mixture with oxygen and water vapour in comparison with infrared device IR-880A. The black line is response of the one electrode sensor and the red line is response of the reference analyser

The long-term stability of sensors was another central aspect in the study. The calibration curves measured within the 24 hours test (shown on Figure 4a) demonstrate a very good reproducibility.



a) b)

Figure 4. (a) – The response of microfabricated one electrode sensors to CO in 20,95% O_2 (◇); 8% O_2(□); 4% O_2 (▽) and 1% O_2(○) measured within the 24 hours after the first measurement.
(b) – Repeatability tests of the sensors recorded together with the reference analyser

A stability test of the sensors was continued up to twelve months in the actual source of the emissions from combustion of natural gas. After a few deviations within a few weeks the sensitivity of the sensors was found to remain stable as it is shown on Figure 5. Such a small deviations can be explained by the fact that the initial resistance of sensor is determined by the resistance of the metal heater, which is not subjected to the time drift. This ensures a very small deviation of the initial sensor parameters during the long-term measurements of one year and secures a good stability and repeatability characteristics.

3.2. In_2O_3-Fe_2O_3 Composite Sensors

It was found that multi-phase γ-Fe_2O_3–In_2O_3 (9:1 mol) composites containing metastable γ-Fe_2O_3 structure are characterized by the greatest sensitivity to both reducing (CH_3OH) and oxidizing (NO_2) gases, but these layers exhibit much lower sensitivity to CO in the same temperature range. γ-Fe_2O_3–In_2O_3 thin films excel substantially in sensitivity the sensors based on individual α-Fe_2O_3 and γ-Fe_2O_3 oxides, see Figure 6. It is worth noting that in contrast to the individual γ-Fe_2O_3 or α-Fe_2O_3 layers, the γ-Fe_2O_3–In_2O_3 thin-film sensors appear to be more stable and selective to methanol vapours in the presence of CO and NO_2.

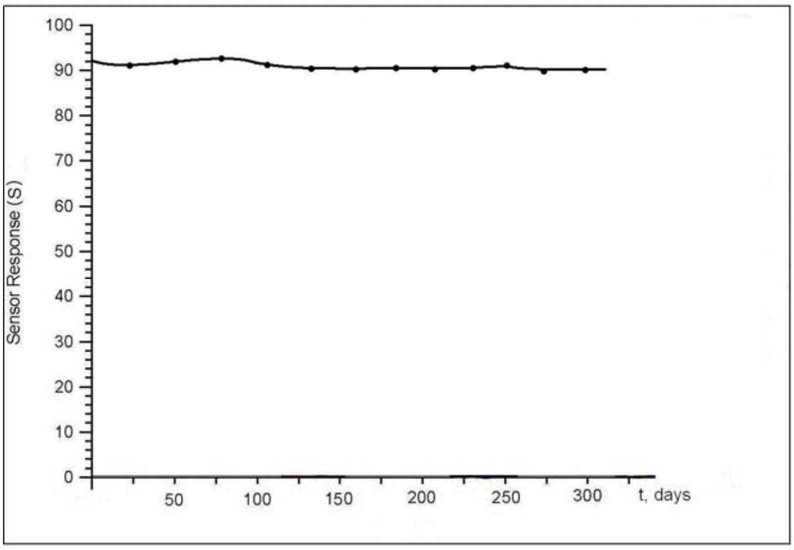


Figure 5. The sensitivity of the sensors to carbon monoxide recorded in the actual source of the emissions from combustion of natural gas during the one year. The response (S) of the sensors was calculated as U_{air}/U_{gas}

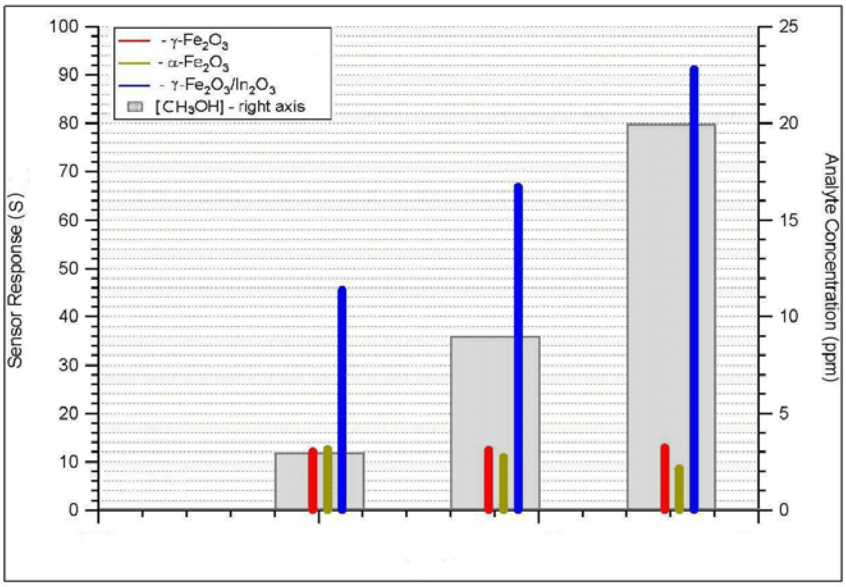


Figure 6. Response of α-Fe_2O_3-, γ-Fe_2O_3- and composite γ-Fe_2O_3–In_2O_3-based sensors to CH_3OH at 300ºC. The response (S) of the sensors was calculated as U_{air}/U_{gas}. Increased response of γ-Fe_2O_3–In_2O_3 samples as compared with individual γ-Fe_2O_3 and α-Fe_2O_3 films can be explained by the presence of a higher contact interface between In_2O_3 and γ-Fe_2O_3 phases within γ-Fe_2O_3–In_2O_3 composite. The heterojunctions between these phases appears to be very active in course of both adsorption and oxidation of methanol

Conclusions

In summary, microfabrication technologies provide a unique and practical means of manufacturing gas sensors. It is clear that developed sensors are sensitive enough for detection of CO in the combustion emissions. The simplicity of measurement scheme (needs one power supply only), better energy efficiency and fast stabilization time are the main advantages of one-electrode sensor design as compared with traditional semiconductor sensors. An advantage of this sensor construction for practical application is the small deviation of initial parameters. The initial resistance of sensor is being determined by the resistance of the metal heater, which is not subjected to the time drift. The small heat capacity of the microfabricated platform and its associated small thermal time constant will allow the implementation of time-depended temperature methods, suitable for rapid identification of hazardous chemicals and air pollutants using gas sensors arrays operating in temperature modulation mode. It was demonstrated that the scientific and commercial rewards for successful development of microfabricated sensors are immense.

Acknowledgment

This work has been funded by the Binational Science Foundation (No. 2006056).

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Transport and Telecommunication Institute, Lomonosova Str.1, Riga, LV-1019, Latvia

Authors' index

Yuri N. Shunin (born in Riga, March 6, 1951)

- Vice-Rector on Academic Issues (Information Systems Management Institute), professor, Dr.Sc. Habil., Member of International Academy of Refrigeration.
- Director of Professional Study Programme Information Systems (Information Systems Management Institute).
- Director of Master Study Programme Computer systems (Information Systems Management Institute).
- **University study**: Moscow physical and technical institute (1968–1974).
- Ph.D. (physics & mathematics) on solid state physics (1982, Physics Institute of Latvian Academy of Sciences), Dr. Sc. Habil (physics & mathematics) on solid state physics (1992, Ioffe Physical Institute of Russian Academy of Sciences).
- **Publications:** 370 publications, 1 patent.
- **Scientific activities**: solid state physics, physics of disordered condensed media, amorphous semiconductors and glassy metals, semiconductor technologies, heavy ion induced excitations in solids, mathematical and computer modelling, system analysis

Igor V. Kabashkin (born in Riga, August 6, 1954)

- Vice-rector for Research and Development Affairs of Transport and Telecommunication Institute, Professor, Director of Telematics and Logistics Institute.
- PhD in Aviation (1981, Moscow Institute of Civil Aviation Engineering), Dr.Sc.Habil. in Aviation (1992, Riga Aviation University), Member of the International Telecommunication Academy, Member of IEEE, Corresponding Member of Latvian Academy of Sciences (1998).
- **Publications:** 390 scientific papers and 67 patents.
- **Research activities:** information technology applications, operations research, electronics and telecommunication, analysis and modelling of complex systems, transport telematics and logistics.

Arnold E. Kiv, Professor, Dr.Sc.Habil. (phys&math)

- Chief of the Department of Mathematical and Computer Modelling.
- **Education:** South Ukrainian Pedagogical University, Professor of the Department of Materials Engineering, Ben-Gurion University of the Negev P.O.B. 653, Beer-Sheva, 84105, Israel.
- **Research activities:** sub-threshold radiation effects in solids.
  Present scientific activities:
  computer modelling of processes in physics, psychology and social sciences.

Ravil Muhamediyev (born in Irkutsk, Russia, June 23.1959)

- The Head of the Department of Natural Sciences and Computer Technologies, Associate Professor, Dr.Sc. (Information Systems Management Institute).
- **Publications:** about 90 scientific papers.
- **Research activities:** programming, artificial intellect and intellectual systems, computer training systems and developing games, simulation of anisochronous systems, multimedia.

Irena Kulish (born in Riga , July 7, 1966)

- Information System Management Institute.
- Management and Marketing Department. Assist. Professor.
- MBA, PhD students of Latvia University of Agriculture.
- **Publications:** 14 publications.
- **Research activities**: restaurant business, specialist's professional competencies, service culture, expert systems of the knowledge assessment.

Dinesh C. S. Bisht

- Lecturer in the Dept. Of Mathematics, Quantum School of Technology, Roorkee, India. Prior to Joining the institute he worked as a visiting faculty at ICFAI Tech. University, Dehradun, India.
- **Education:** he got his doctoral degree from the G. B. Pant University of Agriculture and Technology, Pantnagar, India.
- **Research activities**: Fuzzy Logic, Artificial Neural Networks.

S. B. Singh

- Dr. S. B. Singh is an Associate Professor in the Department of Mathematics, Statistics and Computer Science, G. B. Pant University of Agriculture and Technology, Pantnagar-263145, INDIA. He has more than fourteen and half years of teaching experience to Undergraduate and Post Graduate students at different Engineering Colleges and University.
- **Publications:** He has authored and co-authored eight books in Applied Mathematics. He is an editor of the journal 'Journal of Reliability and Statistical Studies.
- **Research activities**: his area of research is Reliability Theory. He has been conferred with four national awards in the field of education.

Shweta Sharma

- Research scholar in Kumaun University, Almora. The present work is a part of her thesis entitled "Reliability Assessment of Complex Systems by Mathematical Modelling".
- **Publications:** Three of her research papers on different aspects in reliability theory are currently being published in reputed national and international journals.
- **Research activities**: her main area of expertise includes reliability assessment of complex systems using mathematical modelling techniques.

Olga Taramin

- **University study:** graduated from the Faculty of Applied Mathematics and Computer Science of the Belarusian State University in 2007.
- Now she is a PhD student of the department of the Probability Theory and Mathematical Statistics.
- **Research activities**: main field of interests is tandem queueing systems with correlated arrival flows.

Adolfas Baublys (born in Laukininkai, Varena district, Lithuania, September 5, 1941)

- Professor, Dr. Sc.Habil.
- **University study:** Kaunas Politechnical Institute. (1967)
- **Short description of work experience.** Worked as an engineer-technologist in the Vilnius Drill Plant in 1959–1962. During 1962–1980 he worked in the branch office of the Experimental Scientific Research Institute of metal cutting machines as the Head of the Sector and Head of the laboratory. In 1969 he defended a thesis of Doctor of technical sciences; in 1978 – Habil. Dr. thesis. In 1980–1997 he was the Head-Professor of the Transport Management Department at the Vilnius Gediminas Technical University. Since 4 May 1997 – Director of the Transport Science Institute.
- **Field of Scientific Research.** Transport policy; Strategy of Transport System Development, Modelling and Forecasting. *Monographs:* "Cargo Transportation by Railway, Water and Air Transport" (1995); "Introduction to Transport System Theory" (1997); "Cargo Transportation" (1998). *Textbooks:* "Passenger and Cargo Transportation by Road Transport" (1994, 1995); "Transport System" (1995, 1996); "Transport Policy" (1996); "International Transportation by Road Transport" (1996). He is the author of 210 scientific articles. Twenty doctor's theses were defended under his guidance.
- **Social-organisational activities.** In 1991 he was elected a member-expert of the Lithuanian Science Academy. He is also the Chairman of the Commission on Transport Technology for science at the Vilnius Gediminas Technical University; Chief and Scientific Editor of "Transport", the prestige journal of Lithuania.
- **Office address:** Vilnius Gediminas Technical University, Transport Research Institute, Plytines 27, LT-2040 Vilnius, Lithuania, ph: 370 2 31 56 13; fax: 370-2 31 56 13, e-mail: abaublys@takas.lt

CUMULATIVE INDEX

COMPUTER MODELLING and NEW TECHNOLOGIES, volume 13, No. 2, 2009

(Abstracts)

S. Sharma, S. B. Pandey, S. B. Singh. Reliability and Cost Analysis of a Utility Company Website Using Middleware Solution by Mathematical Modelling, *Computer Modelling and New Technologies*, vol. 13, No 2, 2009, pp. 7–15.

This paper studies the reliability measures based on the resilience architecture of the utility company website by mathematical modelling. The architecture of the website has application servers which are used to connect to the middleware boxes and the database servers. The solution assumes to have a load balanced solution for the middleware solution and an active/passive clustering for the database, using RAID 5. With the application of supplementary variable technique, asymptotic behaviour, availability, mean time to failure and cost effectiveness of the system has been obtained. At last some particular cases of the system have also been taken in account.

Keywords: *mathematical modelling, reliability and cost analysis, website architecture*

D. C. S. Bisht, M. Mohan Raju, M. C. Joshi. Simulation of Water Table Elevation Fluctuation Using Fuzzy-Logic and ANFIS, *Computer Modelling and New Technologies*, vol. 13, No 2, 2009, pp. 16–23.

Groundwater is of major importance to civilization, because it is the largest reserve of drinkable water in regions where humans can live. The estimation of the water table elevation is one of the important aspects to understand the mechanism which comprises groundwater resources and to predict what might happen under various possible future conditions. Fuzzy-logic, a soft computing technology of Artificial Intelligence, nowadays have a great concentrated applications importance in water engineering. It is an excellent mathematical tool to handle uncertainty of the system arising due to the fuzziness or vagueness. The soft computing techniques viz. Fuzzy-logic modelling and Adaptive Neuro Fuzzy Inference System were used in present investigation. These systems begin with some basic rules that describe the process. Four models have been developed, two Fuzzy rule based models and two ANFIS models in the prediction of ground water table elevation. On the basis of performance criteria ANFIS yielded the better results out of all the models developed.

Keywords: *Neuro-Fuzzy, ground water modelling, ground water elevation, ANFIS, training, learning*

Yu. Kochetkov. Production Function of Latvia, *Computer Modelling and New Technologies*, vol. 13, No 2, 2009, pp. 24–30.

The paper, based on statistical data, presents the mathematical model of the economy of Latvia during the transition period as the production function of Ch. Cobb-P.Douglas. Scientific-technological progress is treated as a function of time. The production function is used in a rate notation form. The evaluation of the parameters of the model is based on the least squares method. During calculations data of the Central Statistical Bureau of Latvia have been used for the period of 1990–2003. After the production function has been calculated the checking of its adequacy and exactness was carried out by using standard procedures.

Keywords: *production function, least squares method, growth rate, multiple correlation, regression equation, forecast*

I. Kulish, R. Muhamediyev. Experts System for Students' Knowledge Assessment in the Area of Hospitality Technology, *Computer Modelling and New Technologies*, vol. 13, No 2, 2009, pp. 31–37.

The deficiency of single level additive model of evaluation is considered. The model is frequently used by test systems and other systems of evaluation. It is very simple but the single level additive model of evaluation can't realize linear indivisible function. To do away with deficiency of additive model expert systems has a proposal. An output mechanism would be based on neural network, productions, fuzzy logic or Bayes logical conclusion. The examples of expert systems to evaluate the students' knowledge in hospitality technology is considered.

Keywords: *experts system, neuron nets, knowledge evaluating*

COMPUTER MODELLING and NEW TECHNOLOGIES, 13.sējums, Nr. 2, 2009 (Anotācijas)

S. Sharma, S. B. Pandey, S. B. Singh. Drošums un komunālā uzņēmuma izmaksu analīze saits, lietojot starpprogrammatūras risinājumu ar matemātiskās modelēšanas palīdzību, *Computer Modelling and New Technologies,* 13.sēj., Nr.2, 2009, 7.–15. lpp.

Dotajā rakstā tiek izskatīti drošuma pasākumi, kas pamatojas uz elastības arhitektūru komunālā uzņēmuma saitam ar matemātisko modelēšanu. Saita arhitektūrai ir pielikuma serveri, kas tiek pielietoti, lai savienotu starpprogrammatūras kastes ar datubāzes serveriem.

Ar papildu mainīgā lieluma pielietošanu tika iegūti tehnika, asimptotiskā uzvedība, pieejamība, vidējais laiks zudumiem un sistēmas izmaksu efektivitāte. Visbeidzot, daži īpaši sistēmas darbības gadījumi arī tika ņemti vērā.

Atslēgvārdi: *matemātiskā modelēšana, drošums un izmaksu analīze, saita arhitektūra*

D. C. S. Bišts, M. Mohan Raju, M. C. Joši. Gruntsūdeņu augstuma svārstību modelēšana, lietojot faziloģiku un adaptīvo neiro fazi izveduma sistēmu (ANFIS), *Computer Modelling and New Technologies,* 13.sēj., Nr.2, 2009, 16.–23. lpp.

Gruntsūdeņi tie ir civilizācijas pastāvēšanas svarīgākais faktors tāpēc, ka tas ir dzeramā ūdens vislielākā rezerve tajos reģionos, kur dzīvo cilvēce. Gruntsūdeņu atrašanās vietu noteikšana ir viens no svarīgākajiem aspektiem, lai saprastu mehānismu, kurš aptver gruntsūdeņu rezerves, un paredzētu, kas varētu notikt dažādos iespējamos nākotnes apstākļos. Mūsdienās faziloģikai, mākslīgā intelekta skaitļošanas tehnoloģijām ir milzīgi koncentrēta lietošana hidroenerģētikā. Tas ir lielisks matemātisks līdzeklis sistēmas nenoteiktības apstrādē, kas rodas saskaņā ar netiešumu un neskaidrību. Autori savā pētījumā pielieto faziloģikas modelēšanu un adaptīvo neiro-fazi izveduma sistēmu *(ANFIS)*. Šīs sistēmas sākas ar dažiem pamata principiem, kas apraksta procesu. Četri modeļi tika izstrādāti, divi fazi noteikumu bāzēti modeļi un divi ANFIS modeļi ir izveidoti gruntsūdeņu augstuma paredzēšanai.

Atslēgvārdi: *neiro-fazi, gruntsūdeņu modelēšana, gruntsūdeņu augstums, ANFIS*

J. Kočetkovs. Latvijas ražošanas funkcija, *Computer Modelling and New Technologies,* 13.sēj., Nr.2, 2009, 24.–30. lpp.

Rakstā, kas pamatojas uz statistiskiem datiem, tiek parādīts Latvijas pārejas perioda ekonomikas matemātiskais modelis kā Koba-Duglasa ražošanas funkcija. Zinātniski tehnoloģiskais progress tiek uzskatīts kā laika funkcija. Ražošanas funkcija tiek lietota likmes notācijas veidā. Modeļa parametru novērtēšana tiek bāzēta uz mazāko kvadrātu metodi. Aprēķinu laikā tika izmantoti Latvijas Statistiskās pārvaldes dati par laika periodu no 1990. līdz 2003. gadam. Pēc tam, kad bija aprēķināta ražošanas funkcija, ar standarta procedūru palīdzību tika veikta tās adekvātuma un precizitātes pārbaude.

Atslēgvārdi: *ražošanas funkcija, mazāko kvadrātu metode, pieauguma likme, regresijas vienādojums, paredzējums*

I. Kulišs, R. Muhamedijevs. Ekspertsistēma studentu zināšanu vērtējumam viesmīlības tehnoloģiju jomā, *Computer Modelling and New Technologies,* 13.sēj., Nr.2, 2009, 31.–37. lpp.

Rakstā tiek izskatīti novērtēšanas vienlīmeņa savienošanās modeļa trūkumi. Modelis tiek bieži lietots testa sistēmās, kā arī citās novērtēšanas sistēmās. Tas ir ļoti vienkārši, savukārt novērtēšanas vienlīmeņa savienošanās modelis nespēj īstenot lineāro nedalāmo funkciju. Lai novērstu savienošanās modeļa trūkumus ekspertsistēmām ir priekšlikums. Izejošam mehānismam ir jābūt bāzētam uz neironu tīklu, ražošanu, faziloģiku vai *Bayes* loģisko slēdzienu. Rakstā tiek sniegti ekspertsistēmu piemēri studentu zināšanu novērtēšanai viesmīlības tehnoloģiju jomā.

Atslēgvārdi: *ekspertsistēmas, neironu tīkli, zināšanu novērtējums*

O. Taramins. Tandēma rinda ar divām Markova ievadēm un jauna klientu izskatīšana, *Computer Modelling and New Technologies,* 13.sēj., Nr.2, 2009, 38.–47. lpp.

Rakstā tiek pētīta tandēma atkārtotas izskatīšanas rinda sastāvoša no divām stacijām. Pirmajai stacijai ir viens vienīgs serveris. Datu ievades plūsma pirmajā stacijā ir aprakstīta ar Markova ierašanās procesu – *Markovian Arrival Process (MAP).* Ja klients no šīs plūsmas sastopas ar aizņemtu serveri, tas aiziet uz bezgalīga izmēra orbītu un izmēģina savu veiksmi vēlāk eksponenciāli sadalītā nejaušā laikā. Apkalpes laika sadale pirmajā stacijā tiek pieņemta kā vispārēja. Pēc apkalpošanas šajā stacijā klients dodas uz multi-serveru otro staciju.. Ja šis klients sastop brīvu serveri otrajā stacijā, tas uzsāk apkalpošanu uzreiz; citādi klients atstāj sistēmu uz visiem laikiem. Bez klienta rīcības no pirmās stacijas, papildus klientu MAP plūsma ierodas otrajā stacijā tieši, neieejot pirmajā stacijā. Klients no šīs plūsmas ir zaudēts, ja otrajā stacijā nav pieejams serveris. Otrās stacijas servera apkalpošanas laiks ir eksponenciāli sadalīts. Rakstā tiek sniegti skaitliskie piemēri.

Atslēgvārdi: *tandēma atkārtotas izskatīšanas rinda, Markova ierašanās procesu – Markovian Arrival Process (MAP), multi-serveru otrā stacija*

V. Paulauskas, D. Paulauskas, J. Vijfels. Kuģu drošība atklātās ostās, *Computer Modelling and New Technologies,* 13.sēj., Nr.2, 2009, 48.–55. lpp.

Atklātās ostas vai termināļi, kuri pilnībā nav aizsargāti no vēja, ir problemātiski kuģiem, kuriem ir augsti borti, un līdz ar to palielinās ietekme no aeronautiskiem spēkiem. Tauvas sistēmās šādos gadījumos ir jābūt paredzētiem statiskiem un dinamiskiem (harmoniskiem) spēkiem. Novērtēšanas metodes atklāto ostu gadījumos tiek izskatītas dotajā rakstā. Aerodinamisko spēku novērtēšanas iespējas, lai kuģi pietauvotos, un iespējas sagatavot pareizas kuģu pietauvošanās shēmas konkrētajos apstākļos var palielināt kuģu drošību ostās, novēršot navigācijas un vides negadījumus. Dotajā rakstā autori sniedz situāciju analīzi, teorētisko pamatu pētījumam un praktiskus aprēķinus.

Atslēgvārdi: *aerodinamiskie spēki, kuģu pietauvošanās, atklātās jūras ostas, tauvas*

A. Baublis. Tehnoloģisko procesu modelēšanas principi transporta terminālos, *Computer Modelling and New Technologies,* 13.sēj., Nr.2, 2009, 56.–67. lpp.

Tehnoloģiskais process tiek vērtēts kā nejaušs process, tas arī ir novērtēts atbilstošajos modeļos. Metodoloģija tehnoloģisko procesu veidošanai terminālos, kā arī tehnoloģisko procesu optimālā kontrole un kvalitāte tiek izskatīta dotajā rakstā. Bez tam rakstā tiek izstrādāti modeļi un algoritmi optimālai kontrolei, un arī tiek izskatīts kravas klīringa tehnoloģiskais process.

Atslēgvārdi: transporta termināls, tehnoloģiskais process, optimālā kontrole, modeļi un algoritmi

V. Golovanovs, C. C. Liu, A. Kivs, D. Fuks, M. Ivanovskaja. Mikroveidoti vienelektrodu In_2O_3 un Fe_2O_3-In_2O_3 kompozītu sensori, *Computer Modelling and New Technologies,* 13.sēj., Nr.2, 2009, 68.–73. lpp.

Autori rakstā izskata plaknes sensora attīstību, kas pamatojas uz vienelektroda koncepciju.

Tipiskajā vienelektroda sensora uzbūvē plāns platīna tinums ir pārklāts ar oksīda pusvadītāja materiālu. Platīna stieples spirāle tiek aizvietota ar mikroveidotiem pallādija vai zelta rezistoriem mūsu prototipa sensoriem, un oksīda pusvadītāja koloīda šķīdums tiek riņķī pārklāts pāri elektrodam. Dažādi indija oksīda pamata kompozīta veidi ir testēti zondēšanas slānī. Izveidotais sensors parāda lielu jutīgumu un stabilitāti karbona monoksīda (CO) atrašanā, salīdzinot ar komerciālo infrastaru analizatoru. Daudzfāzu Fe_2O_3–In_2O_3 sensori tiek raksturoti ar visaugstāko jutīgumu pret metanolu (CH_3OH).

Atslēgvārdi: multifāze Fe_2O_3–In_2O_3 kompozīta sensori, mikroveidojumu tehnoloģija, gāzes jutīgie elementi

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Transporta un sakaru institūts (Transport and Telecommunication Institute)
Lomonosova iela 1, LV-1019, Riga, Latvia. Phone: (+371) 67100593. Fax: (+371) 67100535
E-mail: journal@tsi.lv, http:// www.tsi.lv

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