Vibration-modal analysis model for multi-span pipeline with different support conditions

Jun Liu[1, 2], Xing He[1], Qingyou Liu[2*], Jiang Naibin[3], Huang Chen[1]

[1]*School of Civil Engineering and Architecture, Southwest Petroleum University, Chengdu 610500, China*

[2]*Modern Design and Simulation Lab for Oil&Gas Equipment's, Southwest Petroleum University, Chengdu 610500, China*

[3]*Science and Technology on Reactor System Design Technology Laboratory, Nuclear Power Institute of China, Chengdu 610041, China*

Received 21 May 2014, www.tsi.lv

Abstract

Vibration characteristics analysis is important in the design of multi-span pipeline with different support conditions. In order to analyze the natural frequency and the vibration modal of the multi-span pipeline, a matrix transfer method is proposed in this paper. With the multi-span pipeline divided into single-span pipes, the transmission formulas for the deflection, angle, bending and shear between two adjacent spans are deduced, in combination with the Krylov function solution of the free vibration equation for the single-span pipe, and the constraint condition between the two adjacent spans of the multi-span pipeline. According to the boundary conditions on the starting and ending spans, the natural frequency equation and the vibration modal function between two adjacent spans of the multi-span pipeline are presented. The FORTRAN program based on the above principle is written, and the natural vibration frequencies and the vibration modals of two typical multi-span pipelines are investigated and compared with the results from ABAQUS. It is shown that the model presented in this paper is efficient in the analysis of multi-span pipeline and has the advantages of high computational efficiency and convenience for engineering practice application.

Keywords: Multi-span pipeline, Vibration equation, Natural frequency, Vibration modal

1 Introduction

With widely used in petroleum engineering, chemical engineering and nuclear power, Multi-span flow pipeline is one of the most common industrial equipment's. During the design of the Multi-span flow pipeline, in order to prevent the production of "instability" and "resonance" phenomenon, much attention should be paid to its dynamic characteristics such as natural frequency and vibration modal. In order to do it, various approaches have been used ranging from numerical methods such as finite element method [1, 2] to analytical models [3]. The former, although has high calculation accuracy, is inconvenience to be used. The latter with appropriate accuracy and relatively low computational cost is convenient to be used in engineering.

For the analytical models, traditionally, three-bending-moment model [4], which is based on the continuous condition of the angle and bending moment on the support linked to two adjacent spans, is usually used to establish frequency equation and solved it by numerical analysis method. This model shows good adaptability to low-order frequencies but for the calculation of high-order frequency, the computational precise will decrease due to the increase of the iteration number. Zhang et al and others [5-10] successively presented the frequency equations of multi-span pipeline with flexible and rigid supports. The influence of support to vibration modal had been also investigated by them. These models also appear capability in the determination of low-order frequencies of multi-span pipeline. However, the derivation processes of vibration mode function and frequency function in them are not available and high-order frequency solution with higher precise still remains unsatisfied. Therefore, it is clearly desirable to develop an effective analytical model with better precise for high-order vibration and more convenient for practical application.

The aim of this paper is to provide an efficient method of vibration characteristics analysis to the design of multi-span pipeline with different spans and different supports. The continuity condition of multi-span pipeline is used to derive the vibration modal functions of the multi-span flow pipeline and by solving them with iteration technique, high-order natural frequency and vibration modal with higher precise can be obtained.

2. Free vibration equation for multi-span flow pipeline

2.1 SINGLE-SPAN FLOW PIPELINE

Based on vibration analysis theory for single-span beam, vibration equation for multi-span flow pipeline can be derived [11-13]. Therefore, according to the d'Alembert principle, the differential equation of free vibration for

[*] ***Corresponding author*** e-mail: Liuqy66@aliyun.com

single-span beam with material homogeneous and cross section uniform (see figure 1) can be written as follows

$$EI\frac{\partial^4 y(\mathrm{x},\mathrm{t})}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}=0\,, \tag{1}$$

where EI is bending stiffness, ρ is material density, A is the area of cross section.

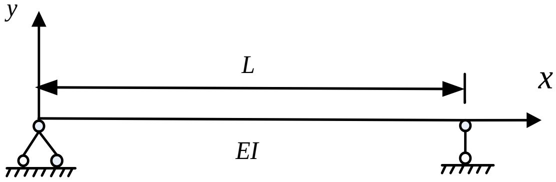


FIGURE 1 Single-span continuous beam with two ends simply supported

Formula (1) is a fourth-order differential equation and its general solution is written as follows, $y(x,t)=[AS(kx)+BT(kx)+CU(kx)+DV(kx)]\sin pt$, where A, B, C, D are constants, $\sin pt$ is the variable that considers the change of displacement with time and has no effect on the natural frequency as well as the vibration mode of the multi-span flow pipeline, $S(kx)$, $T(kx)$, $U(kx)$ and $V(kx)$ are the functions of hyperbolic functions and trigonometric functions, and the expansion of them can be written as follows: $S(kx)=\frac{\mathrm{ch}(kx)+\cos(kx)}{2}$, $T(kx)=\frac{\mathrm{sh}(kx)+\sin(kx)}{2}$, $U(kx)=\frac{\mathrm{ch}(kx)-\cos(kx)}{2}$, $V(kx)=\frac{\mathrm{sh}(kx)-\sin(kx)}{2}$, where k is the function of bending stiffness, material density, area of cross section and natural circular frequency and: $k^4=\frac{EI}{\rho A}\omega^2$, where ω is the natural circular frequency ($\mathrm{rad/s}$).

2.2 MULTI-SPAN FLOW PIPELINE

Multi-span continuous flow pipeline is regarded as the structure that is composed of several single-span be-ams. Each of them has the same vibration mode as a sin-gle-span beam. Therefore, for the multi-span pipeline as shown in Figure 2, the general solution for the free vibration equation of the i$^{\text{th}}$ span can be written as:

$$y_i(x,t)=[A_iS(kx)+B_iT(kx)+C_iU(kx)+D_iV(kx)]\sin pt\,. \tag{2}$$

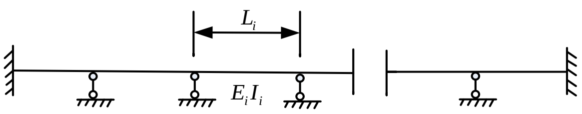


FIGURE 2 Multi-span continuous beams with fixed ends and rigid supports

Based on formula (2), the following expression can be used as the vibration modal function of the span:

$$\varphi_i(x)=y_i(x)=A_iS(kx)+B_iT(kx)+C_iU(kx)+D_iV(kx)\,. \tag{3}$$

It appears that the vibration modal function is composed of the displacements of each cross section at some time. Thus, the displacement, rotation, bending moment and shear force of each cross section of the span can be expressed as formulas (4)-(7):

$$y_i(x)=A_iS(kx)+B_iT(kx)+C_iU(kx)+D_iV(kx)\,, \tag{4}$$

$$\theta_i(x)=\frac{\partial y_i(x)}{\partial x}=k\left[A_iV(kx)+B_iS(kx)+C_iT(kx)+D_iU(kx)\right], \tag{5}$$

$$M_i(x)=E_iI\frac{\partial^2 y_i(x)}{\partial x^2}=E_iIk^2\left[A_iU(kx)+B_iV(kx)+C_iS(kx)+D_iT(kx)\right], \tag{6}$$

$$Q_i(x)=E_iI\frac{\partial^3 y_i(x)}{\partial x^3}=E_iIk^3\left[A_iT(kx)+B_iU(kx)+C_iV(kx)+D_iS(kx)\right]. \tag{7}$$

The force analysis diagram for the i^{th} span is shown as Figure 3, in which y_i, θ_i, M_i and Q_i respectively denote displacement, rotation, bending moment and shear force, superscript L, R respectively denote left end and right end.

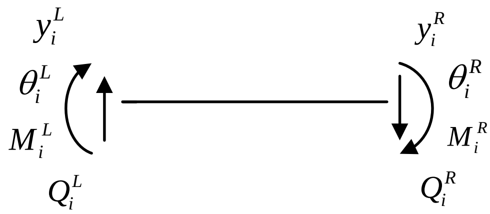


FIGURE 3 Force analysis diagram for i^{th} span

At the left end ($x=0$), formulas (4)-(7) can be simplified as follows:

$$\begin{cases} y_{iL}=A_i \\ \theta_{iL}=kB_i \\ M_{iL}=k^2E_iIC_i \\ Q_{iL}=k^3EID_i \end{cases}. \tag{8}$$

According to equation (8), the four constants of i^{th} span A_i, B_i, C_i, D_i can be determined as follows:

$$A_i=y_{iL}\,,\ B_i=\frac{\theta_{iL}}{k}\,,\ C_i=\frac{M_{iL}}{k^2E_iI}\,,\ D_i=\frac{Q_{iL}}{k^3E_iI}\,.$$

With the four constants substituted into formula (4)-(7), the displacement, rotation, bending moment and shear force equation of the right end ($x=L_i$) of the i^{th} span can be given by:

$$\begin{cases} y_{iR} = S_i y_{iL} + \dfrac{T_i}{k}\theta_{iL} + \dfrac{U_i}{k^2 E_i I} M_{iL} + \dfrac{V_i}{k^3 EI} Q_{iL} \\ \theta_{iR} = kV_i y_{iL} + S_i \theta_{iL} + \dfrac{T_i}{kE_i I} M_{iL} + \dfrac{U_i}{k^2 EI} Q_{iL} \\ M_{iR} = E_i I k^2 U_i y_{iL} + E_i IkV_i \theta_{iL} + S_i M_{iL} + \dfrac{T_i}{k} Q_{iL} \\ Q_{iR} = E_i I k^3 T_i y_{iL} + E_i I k^2 U_i \theta_{iL} + kV_i M_{iL} + S_i Q_{iL} \end{cases} . \tag{9}$$

The equation (9) can be written as matrix form

$$\begin{bmatrix} y_{iR} \\ \theta_{iR} \\ M_{iR} \\ Q_{iR} \end{bmatrix} = [C]_{4\times4} \begin{bmatrix} y_{iL} \\ \theta_{iL} \\ M_{iL} \\ Q_{iL} \end{bmatrix} . \tag{10}$$

The items in the matrix $[C]_{4\times4}$ are: $C_{11} = S_i$, $C_{12} = \dfrac{T_i}{k}$, $C_{13} = \dfrac{U_i}{k^2 E_i I}$, $C_{14} = \dfrac{V_i}{k^3 E_i I}$, $C_{21} = kV_i$, $C_{22} = S_i$, $C_{23} = \dfrac{T_i}{kE_i I}$, $C_{24} = \dfrac{U_i}{k^2 E_i I}$, $C_{31} = E_i I k^2 \mathrm{U}_i$, $C_{32} = E_i IkV_i$, $C_{33} = S_i$, $C_{34} = \dfrac{T_i}{k}$, $C_{41} = E_i I k^3 \mathrm{T}_i$, $C_{42} = E_i I k^2 U_i$, $C_{43} = kV_i$, $C_{44} = S_i$.

The equation (10) also set up the force transitive relation between right end and left end of the span. This means that, as long as the internal force in the left end is determined, the internal force in the right end can also be deduced by equation (10). For example, in the first span shown in figure 2, since the displacement and bending moment of the left end with a fixed bearing are zero, the equation (10) can be simplified as:

$$\begin{bmatrix} y_{1R} \\ \theta_{1R} \\ M_{1R} \\ Q_{1R} \end{bmatrix} = [C]_{4\times4} \begin{bmatrix} y_{1L} \\ \theta_{1L} \\ M_{1L} \\ Q_{1L} \end{bmatrix} = \begin{bmatrix} C_{13}^1 & C_{14}^1 \\ C_{23}^1 & C_{24}^1 \\ C_{33}^1 & C_{34}^1 \\ C_{43}^1 & C_{44}^1 \end{bmatrix} \begin{bmatrix} M_{1L} \\ Q_{1L} \end{bmatrix} . \tag{11}$$

For the second span, considering the supporting condition on the left end of the pipeline, we have: $\begin{bmatrix} y_{2L} \\ \theta_{2L} \\ M_{2L} \\ Q_{2L} \end{bmatrix} = \begin{bmatrix} y_{1R} \\ \theta_{1R} \\ M_{1R} \\ Q_{1R} + F_1 \end{bmatrix}$, where F_1 is the bearing shear in the left end of the 1th span.

Using the above relation and formula (10), the force transitive relation between the right end of 2th span and the left end of the 1th span can be derived as follows:

$$\begin{bmatrix} y_{2R} \\ \theta_{2R} \\ M_{2R} \\ Q_{2R} \end{bmatrix} = \begin{bmatrix} C_{13}^2 & C_{14}^2 \\ C_{23}^2 & C_{24}^2 \\ C_{33}^2 & C_{34}^2 \\ C_{43}^2 & C_{44}^2 \end{bmatrix} \begin{bmatrix} \theta_{2L} \\ Q_{2L} \end{bmatrix} = [C]_{4\times4}^2 \begin{bmatrix} 0 & 0 \\ -C_{23}^1 \dfrac{C_{13}^1}{C_{14}^1} + C_{24}^1 & 0 \\ -C_{33}^1 \dfrac{C_{13}^1}{C_{14}^1} + C_{34}^1 & 0 \\ -C_{43}^1 \dfrac{C_{13}^1}{C_{14}^1} + C_{44}^1 & 1 \end{bmatrix} \begin{bmatrix} Q_{1L} \\ F_1 \end{bmatrix} . \tag{12}$$

Using the similar derivation process as formula (12), the displacement, rotation, bending moment and shear force in the right end of the i^{th} span can be obtained as:

$$\begin{bmatrix} y_{iR} \\ \theta_{iR} \\ M_{iR} \\ Q_{iR} \end{bmatrix} == \begin{bmatrix} C_{13}^i & C_{14}^i \\ C_{23}^i & C_{24}^i \\ C_{33}^i & C_{34}^i \\ C_{43}^i & C_{44}^i \end{bmatrix} \begin{bmatrix} \theta_{2L} \\ Q_{2L} \end{bmatrix} = [C]_{4\times4}^i \begin{bmatrix} 0 & 0 \\ -C_{23}^{i-1} \dfrac{C_{13}^{i-1}}{C_{14}^{i-1}} + C_{24}^{i-1} & 0 \\ -C_{33}^{i-1} \dfrac{C_{13}^{i-1}}{C_{14}^{i-1}} + C_{34}^{i-1} & 0 \\ -C_{43}^{i-1} \dfrac{C_{13}^{i-1}}{C_{14}^{i-1}} + C_{44}^{i-1} & 1 \end{bmatrix} \begin{bmatrix} Q_{1L} \\ F_i \end{bmatrix} . \tag{13}$$

If the right end of the pipeline is supported by hinge support, then $y_{iR} = M_{iR} = 0$. According to formula (13), we have $\begin{vmatrix} C_{13}^i & C_{14}^i \\ C_{33}^i & C_{34}^i \end{vmatrix} = 0$. If the right end is supported by fixed bearing, then $y_{iR} = \theta_{iR} = 0$, we have $\begin{vmatrix} C_{13}^i & C_{14}^i \\ C_{23}^i & C_{24}^i \end{vmatrix} = 0$.

Based on the condition that the determinant value of the matrix C is zero, a test-value method can be used to determine the natural circular frequency of the multi-span continuous pipeline. In the method, different k values are substituted into the determinant of matrix C. if the determinant value corresponding to a certain k is zero, the natural circular frequency is obtained using the formula $k^4 = \frac{EI}{\rho A}\omega^2$. Sorting these obtained k values from small to large order, the natural circular frequencies from low order to high order can be obtained.

If the left end of the first span is supported by hinge support and elastic supports are used on the other span ends (see Figure 4), formula (11) is changed to formula (14).

$$\begin{bmatrix} y_{1R} \\ \theta_{1R} \\ M_{1R} \\ Q_{1R} \end{bmatrix} = [C]_{4\times4} \begin{bmatrix} y_{1L} \\ \theta_{1L} \\ M_{1L} \\ Q_{1L} \end{bmatrix} = \begin{bmatrix} C_{12}^1 & C_{14}^1 \\ C_{22}^1 & C_{24}^1 \\ C_{32}^1 & C_{34}^1 \\ C_{42}^1 & C_{44}^1 \end{bmatrix} \begin{bmatrix} \theta_{1L} \\ Q_{1L} \end{bmatrix}. \quad (14)$$

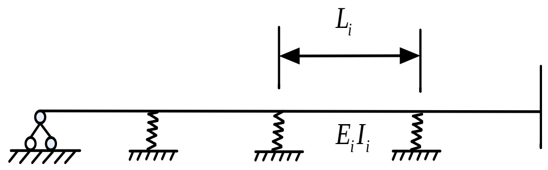


FIGURE 4 Multi-span pipeline with the first span supported by hinge and other spans elastically supported

Since the middle bearings are supported by elastic sup ports, therefore $Q_{1R} = Q_{1L} + K_1 y_1$, where K_1 is the stiffn ess coefficient of the first elastic support (KN/m). Thus, i n this case, the formula (13) can be changed to the follow s:

$$\begin{bmatrix} y_{1R} \\ \theta_{1R} \\ M_{1R} \\ Q_{1R} \end{bmatrix} = [C]_{4\times4} \begin{bmatrix} y_{1L} \\ \theta_{1L} \\ M_{1L} \\ Q_{1L} \end{bmatrix} = \begin{bmatrix} C_{12}^1 & C_{14}^1 \\ C_{22}^1 & C_{24}^1 \\ C_{32}^1 & C_{34}^1 \\ C_{43}^1 - K_i C_{12}^1 & C_{44}^1 - K_i C_{14}^1 \end{bmatrix} \begin{bmatrix} M_{1L} \\ Q_{1L} \end{bmatrix}. \quad (15)$$

By means of the method used in deriving formula (13), the recursion relation of bearing internal force of this kind of pipeline supported by mixture supports can be obtained.

$$\begin{bmatrix} y_{iR} \\ \theta_{iR} \\ M_{iR} \\ Q_{iR} \end{bmatrix} = \begin{bmatrix} C_{12}^{i-1} & C_{14}^{i-1} \\ C_{22}^{i-1} & C_{24}^{i-1} \\ C_{32}^{i-1} & C_{34}^{i-1} \\ C_{43}^{i-1} - K_i C_{12}^{i-1} & C_{44}^{i-1} - K_i C_{14}^{i-1} \end{bmatrix} \begin{bmatrix} M_{1L} \\ Q_{1L} \end{bmatrix}. \quad (16)$$

The natural circular frequencies of the multi-span pipeline with rigid supports are the values that make the determinant value of matrix C zero. The determinants of the matrix C is depended on support condition of the pipeline. The determinants of matrix C for two kinds of supports are listed as follows: $\begin{vmatrix} C_{12}^i & C_{14}^i \\ C_{32}^i & C_{24}^i \end{vmatrix} = 0$. The *Fixed* support at the right end $\begin{vmatrix} C_{32}^i & C_{34}^i \\ C_{43}^i - K_i C_{12}^i & C_{44}^i - K_i C_{14}^i \end{vmatrix} = 0$. The *Elastic* support at the right end Test-value method is also used to determine the natural circular frequencies of the multi-span continuous pipeline.

3 Modal function for multi-span continuous flow pipeline

Since each span of the multi-span pipeline has its vibration modal function, therefore the modal function analysis on the whole pipeline structure is to determine the four constants of the vibration modal function of each span. It is noted that the multi-span pipeline is continuous. Thus, the four constants of the vibration modal function for a span can be obtained by introducing the boundary condition and constraint condition between the span and its adjacent span into the modal function.

The modal function for the first span can be written as:

$$y_1(x) = A_1 S(kx) + B_1 T(kx) + C_1 U(kx) + D_1 V(kx). \quad (17)$$

It is assumed that the left end of the first span is supported by fixed bearing (see figure 2). Then the boundary condition for the left end of the first span can be described by the following equations

$$\begin{cases} A_1 = 0 \\ B_1 = 0 \\ C_1 U_1(kL_1) + D_1 V_1(kL_1) = 0 \end{cases}. \tag{18}$$

If the support between the i[th] span and (i+1)[th] span is hinge bearing, the constraint condition between the two spans can be expressed as follows

$$\begin{cases} A_i = 0 \\ B_{i+1} = B_i S_i(kL_i) + C_i T_i(kL_i) + D_i U_i(kL_i) \\ C_{i+1} = B_i V_i(kL_i) + C_i S_i(kL_i) + D_i T_i(kL_i) \\ B_i T_i(kL_i) + C_i U_i(kL_i) + D_i V_i(kL_i) = 0 \end{cases}. \tag{19}$$

If the right end of the pipeline is supported by fixed bearing, then the boundary condition can be written as

$$\begin{cases} A_n = 0 \\ B_n = B_{n-1} S_{n-1}(kL_{n-1}) + C_{n-1} T_{n-1}(kL_{n-1}) + D_{n-1} U_{n-1}(kL_{n-1}) \\ B_n S_n(kL_n) + C_n T_n(kL_n) + D_n U_n(kL_n) = 0 \\ B_n T_n(kL_n) + C_n U_n(kL_n) + D_n V_n(kL_n) = 0 \end{cases}. \tag{20}$$

The modal computing can be accomplished by means of the recursive method. In the method, the four constants for the modal function of the first span can be determined by assuming $C_1 = 1$ and introducing it into the equation (18). Base on the calculation for the first span, the other four constants for the second span can be obtained by using the equation (19). The similar recursive process is repeated until the four constants for the modal function of the last span is computed by using the equation (20).

Based on the theory presented by the above sections, the computer code of the method is developed using the FORTRAN language and the validity of the method will be proved in the next section by using the finite elements software ABAQUS.

4 Example

In order to verify the computer code, computations were carried out for multi-span flow pipelines with different type of supports.

4.1 MULTI-SPAN CONTINUOUS OIL PIPELINE WITH RIGID SUPPORTS

A multi-span oil pipeline with two ends fixed and middle part supported by rigid bearing is shown in Figure 5. The geometry and physical parameters of the pipeline are listed in Table 1.

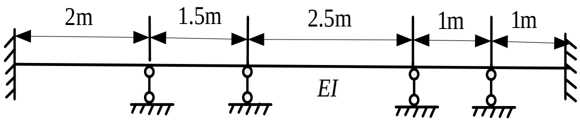


FIGURE 5 Unequal-span oil pipeline with rigid supports

TABLE 1 Geometry and physical parameters of the unequal-span pipeline with rigid supports

Elasticity modulus (GPa)	Pipeline density (kg/m^3)	Oil density (kg/m^3)	Pipe outside diameter (m)	Pipeline wall thickness (m)
200	7850	900	0.16	0.01

The computer code developed in this paper is used to analysis the vibration modal of the pipeline. A comparison between the calculation result of the first five order modal of the pipeline and that from ABAQUS (in the ABAQUS 1000 B21 elements are used) is carried out. The frequencies are listed in Table 2 and the vibration modals are shown in Figure 6.

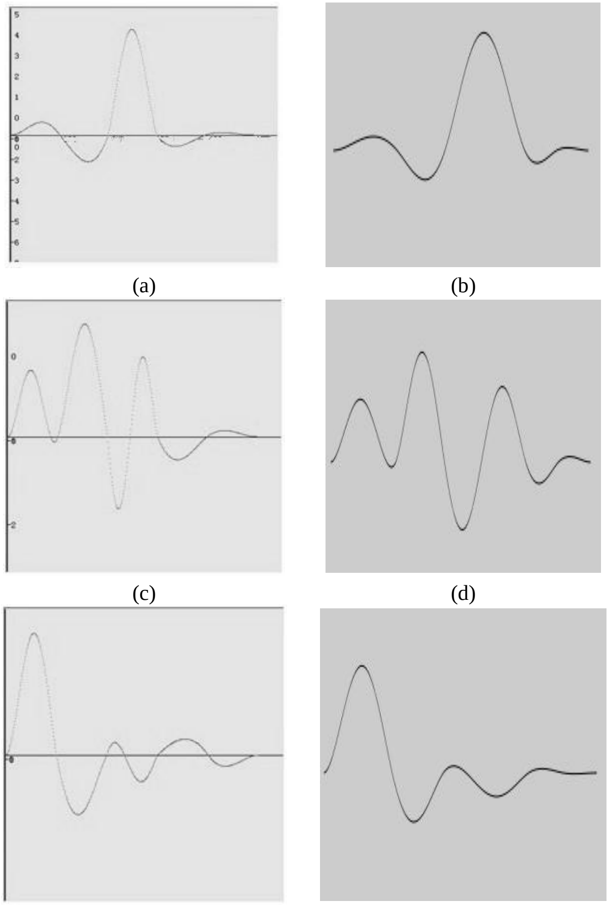
TABLE 2 The first five natural frequencies of the unequal-span pipeline with rigid supports

Frequency order	Current method	ABAQUS
1	81.585	80.452
2	138.937	136.93
3	203.391	205.99
4	263.632	263.83
5	421.436	422.52
6	494.519	508.73
7	592.239	607.44
8	666.580	656.59
9	795.180	818.83
10	866.966	858.19
11	953.093	1009.5
12	1004.330	166.8
13	1292.535	1248.9
14	1308.915	1261.9
15	1408.384	1321.4

It is shown in Table 2 that the first five order frequencies respectively from the current method and ABAQUS are closed and the maximum error between them is less than 5%. In the Figure 6, it appears that the vibration modals from the model presented in this paper are essentially identical to those from ABAQUS. Therefore, the current model is efficient in the vibration analysis on unequal-span pipeline with rigid support.

4.2 MULTI-SPAN CONTINUOUS OIL PIPELINE WITH ELASTIC SUPPORTS

In this section, the vibration modal analysis on a multi-span oil pipeline with two ends fixed and middle part supported by elastic bearings is carried out. The geometry and physical parameters of the pipeline are the same as those listed in Table 1. The stiffness coefficients of all the four elastic bearings are 43 kN/m.



(a) (b) (c) (d) (e) (f)

FIGURE 6 Vibration modals of unequal-span oil pipeline with rigid supports: (a) First-order modal in current model; (b) First-order modal in ABAQUS; (c) Second-order modal in current model; (d) Second-order modal in ABAQUS; (e) Third-order modal in current model; (f) Third-order modal in ABAQUS.

The computer code is used to investigate the vibration characteristics of the pipeline. A comparison between the calculation result of the first three-order modal of the pipeline and that from ABAQUS (in the ABAQUS 1000 B21 elements are used) is carried out. The first fifteen orders frequencies are listed in Table 3 and the vibration modals are shown in Figure 7.

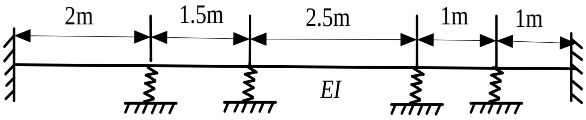
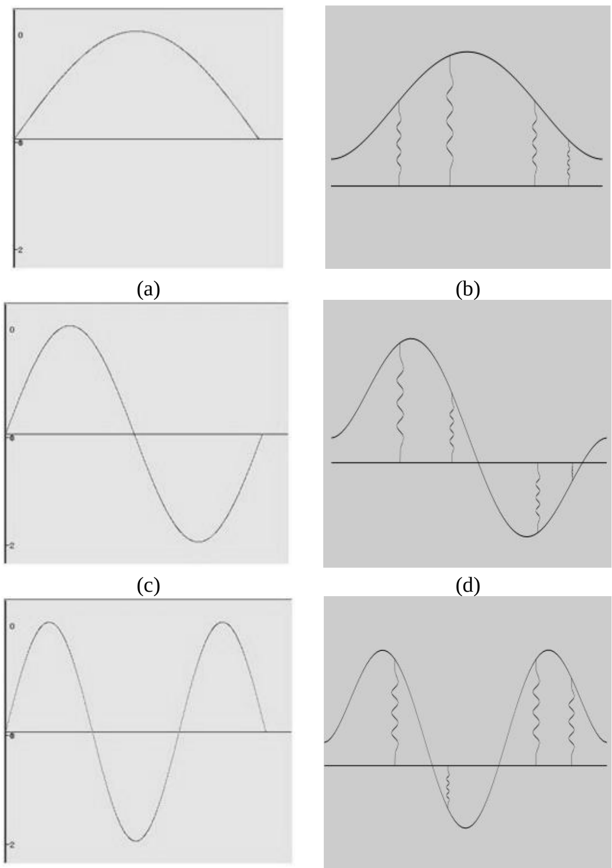


FIGURE 7 Unequal-span oil pipeline with elastic supports

It is shown in Table 3 that the first seven order frequencies respectively from the current method are very closed to those from ABAQUS and the maximum error between them is less than 4%. In Figure 6 the vibration modals from current method are also closed to those from ABAQUS. Compared the Table 2 with Table 3, it can be found that the natural frequencies of the pipeline with elastic supports are lower than those of the pipeline with rigid supports. This is consistent to the actual situation and easy to be understood because the stiffness of the later is greater than the former. Thus, it can be seen the method in this paper is also efficient in the vibration modal analysis on unequal-span pipeline with elastic supports.

TABLE 3 The first five-order natural frequencies of the unequal-span pipeline with elastic supports

Frequency order	Current method	ABAQUS
1	14.821	15.289
2	41.371	40.764
3	80.798	78.866
4	133.388	128.70
5	199.214	189.61
6	270.408	260.78
7	300.408	315.45
6	270.261	260.78
7	300.841	315.45
8	370.408	341.26
9	457.750	430.26
10	594.268	526.89
11	643.151	630.78
12	726.448	739.83
13	870.557	845.57
14	954.677	945.87
15	959.739	973.71



(a) (b) (c) (d) (e) (f)

FIGURE 8 Vibration modals of unequal-span oil pipeline with elastic supports: (a) First-order modal in current model; (b) First-order modal in ABAQUS;(c) Second-order modal in current model; (d) Second-order modal in ABAQUS;(e) Third-order modal in current model; (f) Third-order modal in ABAQUS

5 Conclusions

An efficient vibration modal analysis model for unequal-span pipeline with different kinds of supports has been developed. The model is based on the earlier concepts of the matrix transfer, but has been extended to the pipeline with different spans and different kinds of supports. Two examples of unequal-span oil pipeline with respectively elastic supports and rigid supports are used to test the effectiveness of the model. The principal conclusions are as follows.

(i) Compared with the ABAQUS software, the model not only shows good precise in the analysis on the low-order dynamic property, but also appears capability of computing high-order vibration modal.

(ii) Commercial software based on finite element method may be more precise but is hard to be grasped by the ordinary designers. From this point, the model in this paper is more convenient for practical application than commercial software, since there are not many parameters needed to be input in the model.

(iii) Vibration characteristics analysis on multi-span line is a very important step in many engineering problems such as “Fluidelastic instability”, “Vibration induced by turbulence”. The realization of the model in this paper provides a choice concerned the vibration characteristics analysis model for these problems.

Acknowledgments

This research has been supported by the National Natural Science Foundation of China (Nos. 51105319 and 51274171), National Significant Science and Technology Special Sub-project of China (2011ZX05026-001-07) and National Key Laboratory of Nuclear Reactor System Design Technology Foundation of China.

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Authors

Liu Jun, born on May 8, 1980, Sichuan Province, PRC

Current position, grades: PhD, associate professor
Scientific interest: Engineering mechanics, structural dynamics, computational method and its application in engineering

He Xing, born on March 15, 1990 ,Sichuan Province, PRC

Current position, grades: Graduate student
University studies: School of Civil Engineering and Architecture, Southwest Petroleum University, Chengdu,610500, China
Scientific interest: Structural engineering, Structural dynamics

Qingyou Liu, born on October 12, 1965, Sichuan Province, PRC

Current position, grades: PhD, professor
Research interests: Engineering Mechanics Petroleum machinery, Pipe string mechanics, Rock breaking with bit and computer simulation

Jiang Naibin, born on March 5, 1980, Sichuan Province, PRC

Current position, grades: PhD, senior engineer
University studies: graduated at Nuclear Power Institute of China, Ph.D degree in June 2013.
Scientific interest: Structural mechanics in reactor technology

Huang Chen, born on April 2, 1989, Sichuan Province, PRC

Current position, grades: Graduate student
University studies: School of Civil Engineering and Architecture, Southwest Petroleum University, Chengdu,610500, China
Scientific interest: Architectural and civil engineering, Structural dynamics