The dynamic modelling and simulation for vehicle suspension systems based on vector bond graph

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Abstract

A dynamic modelling and simulation procedure for vehicle suspension system based on vector bond graph is described. According to kinematic constraint relations of components, the vector bond graph model for vehicle suspension system is built. In consideration of the coupling of energy variables and coenergy variables in independent energy storage field and dependent energy storage field. The unified formulae of system state space equations which are easily generated on a computer is derived. As a result, the automatic modelling and simulation of dynamics for vehicle suspension system can be realized based on MATLAB.

Keywords: vector bond graph, vehicle suspension system, modelling and simulation, mixed causality

1 Introduction

To increase the reliability and efficiency of the dynamic modelling and simulation of complex mechanical systems, different procedures have been proposed in previous work [1, 2]. But these procedures are only suitable for a single energy domain, such as strict mechanical one, and cannot be used to deal with the problems of computer aided dynamic analysis of mechanical systems containing the coupling of multi-energy domains. Bond graphs [3] have potential applications in analysing such complex systems because of their ability to describe the dynamics of interacting systems over a multi-energy domain in a unified manner and the unification of graph and mathematical descriptions. In many fields, bond graph techniques have been used successfully [4-6]. Compared with scalar bond graph [3], vector bond graph is more suitable for modelling complex systems such as vehicle suspension system because of its more concise representation manner [7-9]. In order to make the dynamic modeling and simulation of a system be carried out automatically on a computer, it is essential that a unified formula of system state space equations be derived. In what follows, the vector bond graph model for vehicle suspension system is built. In consideration of the coupling of energy variables and coenergy variables in independent energy storage field and dependent energy storage field, The unified formulae of system state space equations which are easily generated on a computer is derived. Finally, the automatic modelling and simulation of dynamics for vehicle suspension system can be realized based on MATLAB.

2 The vector bond graph model of vehicle suspension system

The vehicle suspension system is shown in Figure 1, it consists of vehicle body, passenger and seat system, front and rear suspensions, where v_{r_1} and v_{r_2} are the driving velocity of front and rear wheel from road. From kinematic analysis [1, 2], we have:

$$V_A = V_C + \begin{bmatrix} a\sin(q) \\ -a\cos(q) \end{bmatrix} \dot{q} \,, \tag{1}$$

$$V_B = V_C + \begin{bmatrix} -b\sin(q) \\ b\cos(q) \end{bmatrix} \dot{q} \,, \tag{2}$$

$$V_D = V_C + \begin{bmatrix} d\sin(q) \\ -d\cos(q) \end{bmatrix} \dot{q} \,, \tag{3}$$

where, q and $\dot{q}$ are the pitch angle and pitch angular velocity of the vehicle body, V_A , V_B and V_D are the velocities of point *A*, *B* and *D* on vehicle body, V_C is the mass centre velocity of vehicle body, *M* is the mass of vehicle body, and *J* is the rotational inertia of vehicle body. From Equations (1)-(3), the vector bond graph model of vehicle body can be obtained and shown in Figure 3, where the modulus matrices of MTF can be obtained from Equations (1)-(3) directly.

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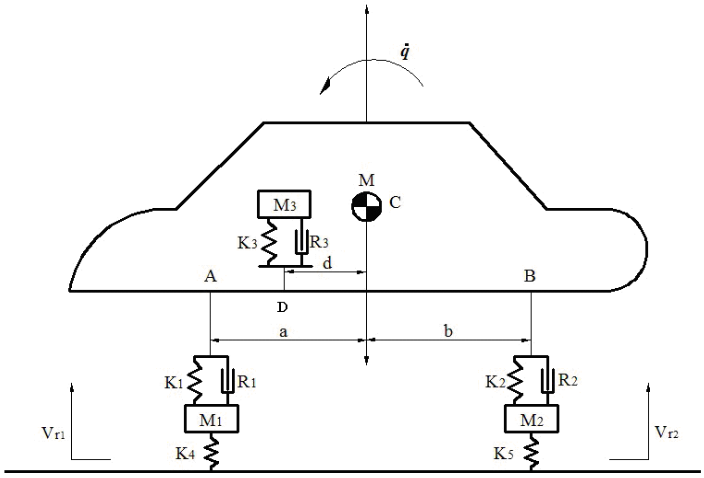

FIGURE 1 Vehicle suspension system

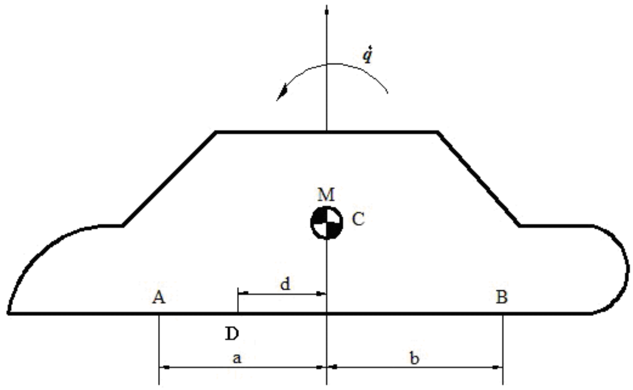

FIGURE 2 Vehicle body

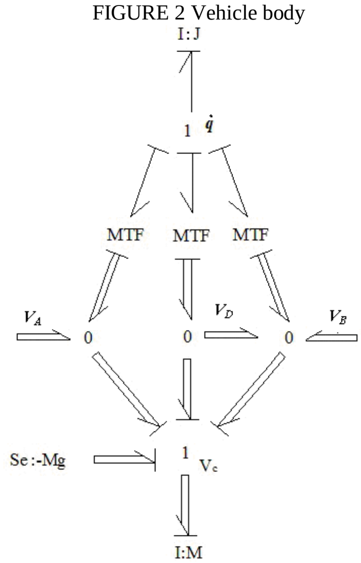

FIGURE 3 The vector bond graph model of vehicle body

The passenger and seat system is shown in Figure 4, where M_3 is the mass of passenger. K_3 and R_3 are the spring coefficient and damper coefficient of seat respectively. Thus, the vector bond graph model of the passenger and seat system can be obtained and shown in Figure 5, where V_d is the velocity of passenger.

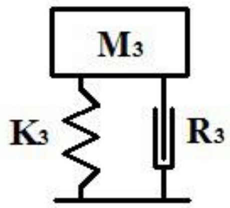

FIGURE 4 The passenger and seat system

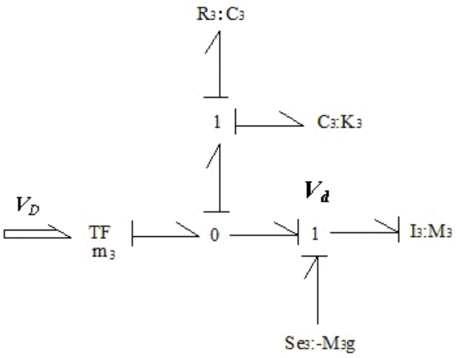

FIGURE 5 The vector bond graph model of passenger and seat system

The front and rear suspension systems are shown in Figure 6,

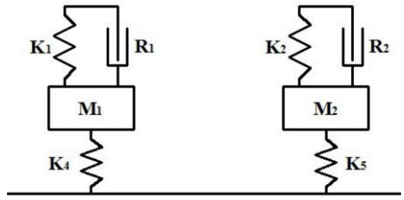

FIGURE 6 The front and rear suspension systems

where M_1 and M_2 are the mass of front wheel and rear wheel, K_1 and R_1 are the spring coefficient and damper coefficient of front suspension, K_2 and R_2 are the spring coefficient and damper coefficient of rear suspension, K_4 and K_5 are the spring coefficient of front tire and rear tire. Thus the vector bond graph model of the front and rear suspension systems can be made and shown in Figures 7a and 7b.

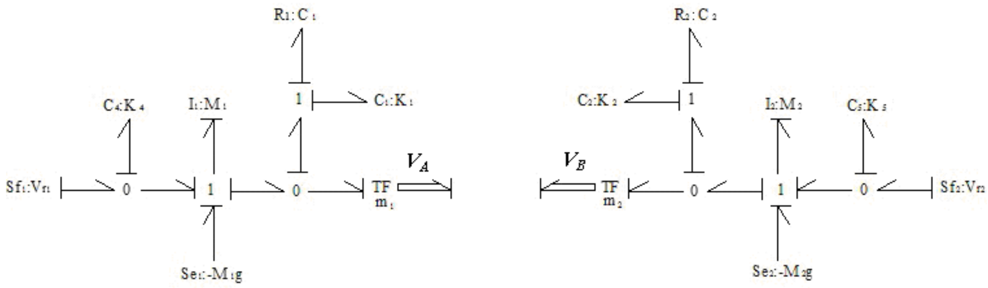

FIGURE 7 The vector bond graph model of front (a) and rear (b) suspension systems

By assembling the vector bond graph model of vehicle body, passenger and seat system, front and rear suspension systems, the overall vector bond graph model for vehicle suspension systems can be built and shown in Figure 8.

3 The unified formula of system state-space equations

The basic field and junction structure of system bond graph is shown in Figure 9, this is a basic field of system on which mixed causality is imposed [3]. In Figure 9, independent storage energy field consists of inertia element I and capacitance element *C* which posses integral causality, dependent storage energy field consists of inertia element *I* and capacitance element C which posses differential causality. Resistive field consists of resistance element *R*, and source field expresses the input to system from outer environment. Where X_i and X_d represent energy vector variables of independent and dependent storage energy field respectively, Z_i and Z_d are the corresponding coenergy vector variables respectively. D_{in}, D_{out} represent input and output vector variables in resistive field, *U*, *V* represent input and output vector variables of source field respectively.

For energy storage field, we have:

$$\begin{bmatrix} Z_i \\ Z_d \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} X_i \\ D_d \end{bmatrix}, \qquad (4)$$

The coupling relations of vector variable X_i, X_d, Z_i and Z_d are expressed by Equation (4).

For resistive field, we have:

$$D_{out} = LD_{in}, \qquad (5)$$

where *R* is a *L*×*L* matrix.

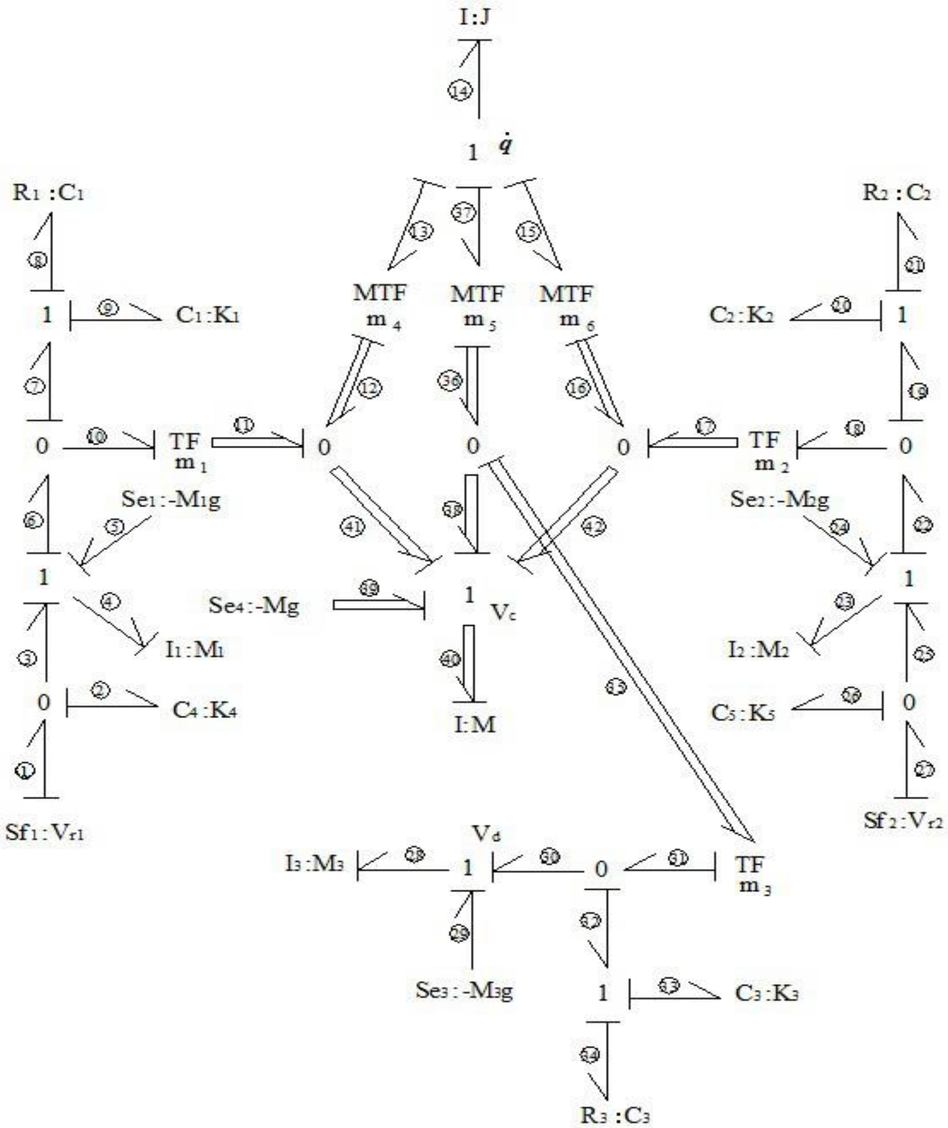

FIGURE 8 The overall vector bond graph model of vehicle suspension system

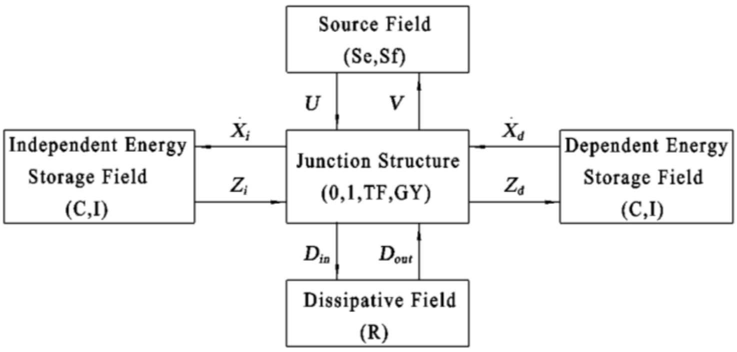

FIGURE 9 The basic field and junction structure of system

The corresponding junction structure equations can be written as:

$$\dot{X}_i = J_i Z_i + J_{id} \dot{X}_{id} + J_{iL} D_{out} + J_{iu} U \,, \tag{6}$$

$$Z_d = J_{di} Z_i + J_{du} U \,, \tag{7}$$

$$D_{in} = J_{Li} Z_i + \dot{X}_d J_{Ld} + J_{LL} D_{out} + J_{Lu} U \,, \tag{8}$$

By the algebraic manipulation from Equations (4)-(8), the system state space equations can be written as:

$$\dot{X}_i = AX_i + B_1 U + B_2 \dot{U} \,, \tag{9}$$

$$B_1 = T_1^{-1} T_3 \,,\; B_2 = T_1^{-1} T_4 \,,\; T_1 = I - J_{id} D - J_{iL} D_3 \,,$$

$$T_2 = J_{ii} (F_{11} + F_{12} D) + J_{iL} D_2 \,,$$

$T_3 = J_{ii}F_{12}D_1 + J_{iu} + J_{iL}D_4$,

$T_4 = J_{id}D_1 + J_{iL}D_5$,

$D = (I - F_{22}^{-1})^{-1}(F_{22}^{-1}J_{di}F_{11} - F_{22}^{-1}F_{21})$,

$D_1 = (I - F_{22}^{-1}J_{di}F_{12})^{-1}F_{22}^{-1}J_{du}$,

$D_2 = (I - LJ_{LL})^{-1}[LJ_{Li}(F_{11} + F_{12}D)]$,

$D_3 = (I - LJ_{LL})^{-1}LJ_{Ld}D$,

$D_4 = (I - LJ_{LL})^{-1}(LJ_{Li}F_{12}D_1 + LJ_{Lu})$,

$D_5 = (I - LJ_{LL})^{-1}LJ_{Ld}D_1$,

where I is a unit matrix.

For the system state space equations shown as Equations (9), many numerical solving algorithm that are available can be used. The corrected adaptive step size Runge-Kutta method based on MATLAB program [10] is explored here.

4 Example System

A vehicle suspension system is shown in Figure 1, the physical parameters are as followings, M=600 kg, M_1=35 kg, M_2=35 kg, M_3=80 kg, J=2500 kg·m^2, K_1=15700 N/m, K_2=15700 N/m, K_3=1000 N/m, K_4=120000 N/m, K_6=120000 N/m, R_1=1000 N·s/m, R_2=1000 N·s/m, R_3=500 N·s/m, a=1.4m, b=1.6m, d=0.8m. The driving velocities of front and rear wheels from road are as follows:

$$V_{r_1} = -0.057(t-3)\exp\left(\frac{-(t-3)2}{1.75}\right) \text{m/s},$$

$$V_{r_2} = -0.057(t-3-0.216)\exp\left(\frac{-(t-3-0.216)2}{1.75}\right) \text{m/s}.$$

From the vector bond graph model shown in Figure 8, the energy vector variables of independent and dependent storage energy field X_i and X_d, the corresponding coenergy vector variables Z_i and Z_d, the input and output vector variables in resistive field D_{in} and D_{out}, the input and output vector variables of source field U and V can be defined.

Inputting the initial values of state variable vector, the physical parameters of system, the coefficient matrices of Equations (4)-(8) into the program associated with the procedure presented here based on MATLAB [10], the system responses are obtained and shown in Figures 10-13.

For this example, the Newton-Euler method [1, 2] was used to determine the corresponding responses of the system, the results are in good agreement with that obtained by the procedure in this paper.

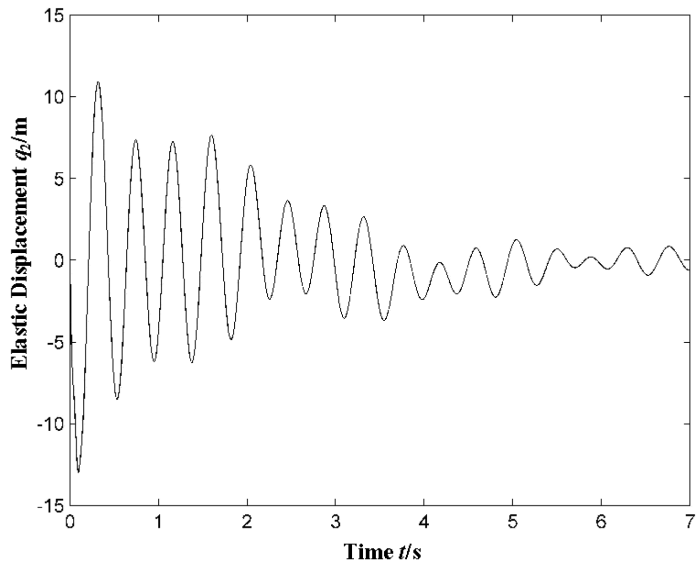

FIGURE 10 The elastic displacement of front tire

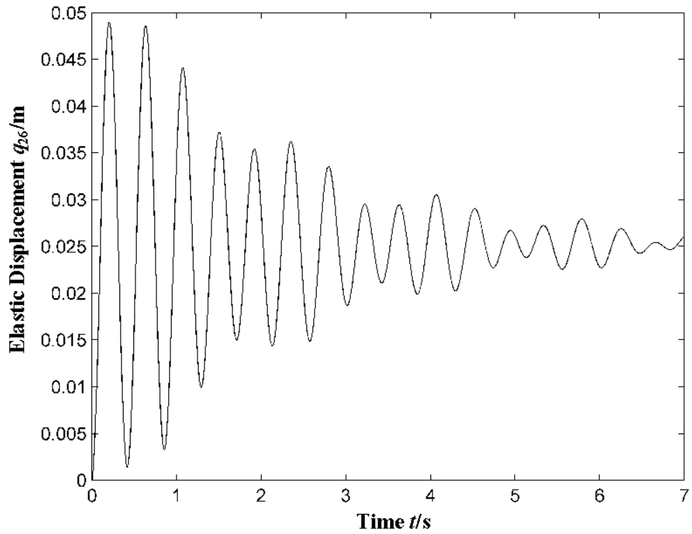

FIGURE 11 The elastic displacement of rear tire

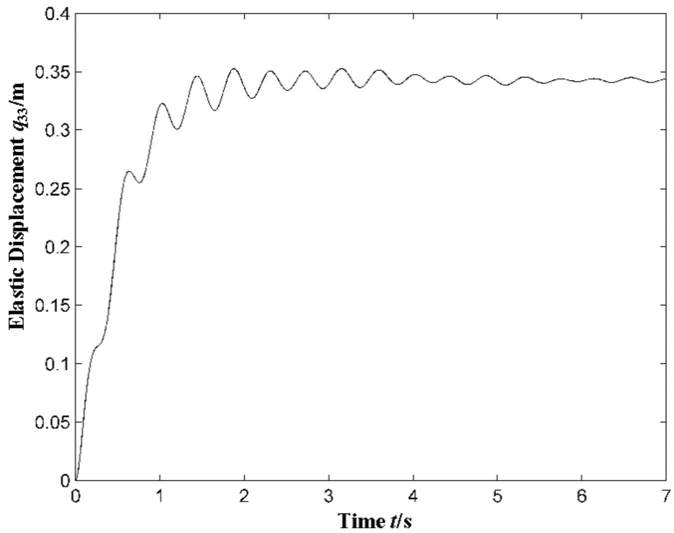

FIGURE 12 The elastic displacement of passenger and seat system

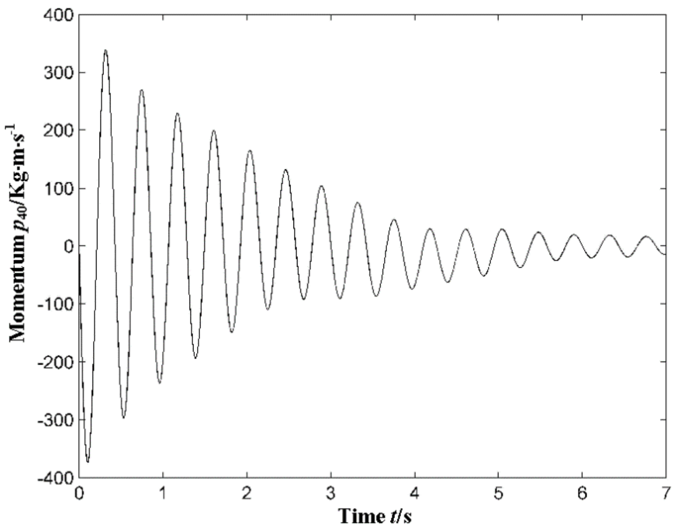

FIGURE 13 The momentum of vehicle body in vertical direction

5 Conclusions

A vector bond graph procedure was presented for modelling and simulation of vehicle suspension system. Compared with standard scalar bond graph model, the procedure presented here is more compact and clear. In consideration of the coupling of energy variables and coenergy variables in independent energy storage field and dependent energy storage field, the unified formulae of system state space equations are derived. These lead to a more efficient and practical automated procedure for modelling and simulation of complex large-scale systems over a multi-energy domains in a unified manner. The validity of this procedure is illustrated by a practical example.

Acknowledgments

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