A simulation approach to model cascading bankruptcy of interdependent business network

Bao Wu[1,2]*, Mingzhu Li[1,2], Zihang Ding[2]

[1] *China Institute for Small and Medium Enterprises, Zhejiang University of Technology, Hangzhou, Zhejiang, P.R.China*

[2] *Collage of Business and Administration, Zhejiang University of Technology, Hangzhou, Zhejiang, P.R.China*

Received 1 November 2014, www.cmnt.lv

Abstract

The article proposed a simulation technique which illustrates the cascading process of financial failure through interdependent business network. Based on theoretical modelling of inter-firm risk propagation, network dynamics of risk contagion was integrated into the simulation model proposed by the article. And the simulation approach was applied to two realistic cases with quite different Network Paradigms. Simulation results showed the method suggested by the article is effective.

Keywords: simulation; risk contagion; cascading bankruptcy

1 Introduction

In the context of interdependent business network, usually the failure of one firm triggers risk transmission through inter-firm financial bonds. And in many occasions, such risk spread leads to cascading bankruptcies, which is a chain or a serial of bankruptcies in independent business cycle. Literatures suggested two essential mechanisms including risk interdependence and financial accelerator are combined together to shape the financial contagion process [1][2]. Risk interdependence means the financial status of one agent is partially determined by the financial conditions. In modern economy, idiosyncratic risk of one organization is easy to spread to its counterparties through their interconnected balance sheet. In this perspective, the potential damage loss of one agent is largely depending on its location in the complex network of financial bonds. In other aspect, an idiosyncratic risk impact of one agent would be partially shared and absorbed by its neighbours. However the risk sharing that lead to deterioration of financial status of its neighbours in today would be likely to feedback on itself and further deteriorate its own financial status in tomorrow[1][3]. This effect is called as financial accelerator. In the context of complex network, feedback route of financial accelerator is tightly connected with network pathway. Network properties of local financial community (such as density, closure etc.) are obviously mediating variables in such positive feedback loops [4][5][6]. Short and closed loop is more dangerous during financial contagion process. The article try to propose a simulation approach to model cascading bankruptcy process, which is robust to different network paradigms.

The article is organized as follows: the next section briefly discusses theoretical modelling of inter-firm risk propagation by mathematical description. Section 3 proposes a simulation framework for cascading bankruptcy in the context of interdependent business network, and develops more details about simulation environments and simulation procedure. In many countries, interdependent business network is typical for its interdependence trait. And systematic crises are more often exacerbated by risk conation in such interdependent business network[7]. Popular financial bonds (such as loan guarantee) by which business network are formally or informally constructed, turns into pathway of risk spread. Characteristics about interdependent business network are integrated into the simulation framework proposed in this article. Section 4 applies the simulation approach suggested in section 3 to comparative cases that has really happened in China. Section 5 provides a discussion and conclusion about effectiveness of the simulation approach in different paradigms.

2 Theoretical modelling of risk propagation in interdependent business network

2.1 MODELLING INTERDEPENDENT BUSINESS NETWORK

Here, we define a simple network environment for analyzing financial contagion process. In many countries, there are many interdependent business cycles, in which many companies develop financial bonds or connections with other companies during daily contacts and trades. Contagious effects of cascading bankruptcy triggered by bankruptcy of a specific company are more likely to cause regional financial crisis in many developing countries, such as China mainland. The paper describes interdependent business cycle as a graph, denoted as $M = [N, D]$, in which N denotes nodes (firms) in the interdependent network (business cycle), and T denotes undirected ties (financial bonds) within the network. i.e. N nodes are interconnected

* ***Corresponding author's*** e-mail: jamesbewood@126.com

with each other with bilateral liabilities, such as loan guarantee, account receivable, sharing holding etc. let's denote $D = [d_{ij}]$ as binary relational matrix of the N nodes, and d_{ij} represents liability of node i owed to node j. For simplicity, the paper assumes network structure, tie strength and financial capacity of nodes are exogenous. Denote $\gamma_i \in (0,1)$ as financial status indicator for each company located in the business network, which is to measure its distance from bankruptcy. And defines $\gamma_i = 0$, if the node i is on the verge of bankruptcy. The company i is financially healthy is the indicator γ iis close to 1.

2.2 THEORETICAL FRAMEWORK OF CASCADING BANKRUPTCY PROCESS

In following the approach, the article employs Stochastic Differential Equation (SDE) to describe the process of inter-firm financial contagion:

$$d\gamma_i = F(\gamma(t), \gamma(t'))dt + \sigma d\varphi_i \tag{1}$$

In which, $F(\gamma(t), \gamma(t'))$ is a drift term, which depends on the past realizations of γ. $d\varphi_i$ is a Wiener stochastic process, representing stochastic dynamic of financial status. σ is the variance of idiosyncratic shocks. The article combines inter-firm risk propagation and random walk of financial status with this stochastic differential equation. Cascading bankruptcy is characterized by rapid risk propagation and destructive effects. For simplicity, the article further assumes cascading bankruptcy is progressing in discrete time: $\tau = 1, \cdots, n_\tau$. And in each discrete step, financial status of all companies in the interdependent business network is updated after risk propagation procedure has been executed. The financial status of company i is adjusted according to following rules:

$$\gamma_i(\tau) = \gamma_i(\tau-1) - \frac{1}{\sum_j d_{ij}} \sum_j \left(d_{ij} * \rho_j(\tau-1)\right) \tag{2}$$

In which, $\rho_j(\tau)$ is a discriminating function to judge whether node j is going to bankruptcy. Its specific form is

$$\rho_j(\tau-1) = \begin{cases} 1 & \text{if } \gamma_j(\tau-1) < 0, \exists \tau' \le \tau \\ 0 & \text{if } \gamma_j(\tau-1) \ge 0, \exists \tau' \le \tau \end{cases} \tag{3}$$

Denote S_0 as the fraction of companies which is knock out by at initial risk impact. And suppose the fraction, S, is the proportion of bankrupt companies after cascading bankruptcy along with risk propagation process is completed. Define P^{μ} is probability of $S \ge \mu$. This is, the probability of systematic risk, in which at least a fixed fraction $\mu \in (0,1)$ of companies of the business network is involved in cascading bankruptcy.

In following simulation, each individual nodes of a business network are selected as initial bankruptcy, and contagious effect triggered by its bankruptcy is evaluated. In this way, accumulative probability is able to get by sequencing contagious effects along with individual contagious effects.

3 Simulation framework for modelling cascading bankruptcy

3.1 INPUT OF EXOGENOUS RISK SHOCKS

We consider a very general scenario of cascading bankruptcy, and presume that one agent or a group of agents are pushed into verge of bankruptcy by an unexpected risk impact. And all total assets are suspended and clear out in the simulation procedure. These assets include money or other kinds of assets that is borrowed from other agents. In many less intensive scenario of financial contagion, the agent attacked by idiosyncratic risk is not always to be clear out. In such scenarios, following contagion process would be initiated by a partial risk loss of this agent. However, in the case of cascading bankruptcy, bankruptcy of an important company or even of a group of companies is general accidence that triggers the contagion process. Certainly, the agent that is presumed to bankrupt is critical for determining the final contagious effect. There are some agents that have relatively bigger balance sheets and more central positions in the complex financial network. These agents are called systematic-important agents. Bankruptcy of these systematic-important agents is easier to trigger a serious cascading bankruptcy. In the simulation procedure, bankrupt agent can be randomly selected and their total asset value is also randomly set.

3.2 VARIABLES FOR MEASURING CASCADING EFFECT

There are several measures about systematic risk and contagious damage in context of complex network, such as inter-bank network, complex industrial network and business network. Among these variables developed in literatures[8][9][10], probability of systematic risk and economic damage are most common variables. In this article, accumulative probability of large-scale failure of business network and the counting number of bankruptcy are used to evaluate loss of cascading effects:

1) Contagious effect of idiosyncratic bankruptcy: this is the number of companies that is going to bankruptcy in the process of cascading bankruptcy process. This indicator is a reliable for describing the devastation of cascading bankruptcy process.

2) Accumulated Distribution of contagious effect: the indicator is useful to describe probability of systematic risk in the business network. And in simulation framework, the Magnitude of large scale can be exogenously defined as policy-makers. The curve shape of this accumulated distribution is useful and offers important information about fragility.

3.3 SIMULATION FRAMEWORK FOR CASCADING BANKRUPTCY

The article proposed a sequential round-by-round algorithm, which is very close to the nature of cascading bankruptcy process. The algorithm allow for multiple failures in the beginning of the whole process. In this way, the simulation framework is also able to integrate random selection procedure for initial bankruptcy that is supposed to trigger cascading bankruptcy process.

To simulate cascading bankruptcy process, the article proposed a simulation procedure. Details of simulation framework are illustrated in Figure. 1. The simulation details are complied into MATLAB language, and operated in MATLAB 2010(a) software.

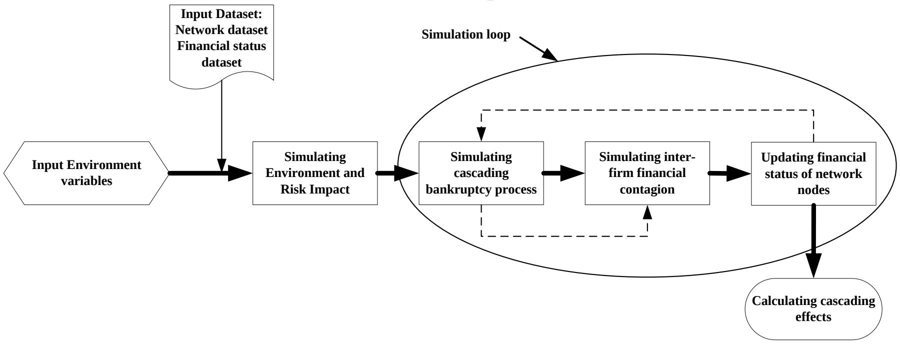

FIGURE 1 Simulation framework for cascading bankruptcy

Following the idea of literature[11][12, denote set B and set S as the sets for bankrupt companies and infected companies in the cascading bankruptcy process. And in initial time, the bankrupt companies that is knock out by idiosyncratic risk and supposed to trigger the whole process would be transfer to set B at t=0. Then, in each round of cascading process, the following steps are performed:

1) Calculate the damage loss of neighbours of new bankrupt companies, and move these neighbours into Set S and Set B according to their financial status.

2) Calculate the damage loss of effected companies which is already in Set S. and transfer new affected nodes into Set S. Repeat this step until there are no new nodes that need to transfer into Set S and B.

3) Update financial status for each nodes, and repeat rounds of cascading process until there is no new member of set B.

4 Comparative case studies

4.1 CASE SELECTION AND DATA SOURCE

The article conducted comparative case studies of regional bankruptcy crisis in Zhejiang province, China. Two real crises, happened in Keqiao County and Jiaojiang city, are selected as comparative cases for following reasons. First, both Keqiao and Jiaojiang are well-known industrial district for their typical endogenous agglomeration (Chi, 2007; Wu et.al., 2011). Second, both bankruptcy crises were influential in China, and had received lots of media attentions. And there is sufficient information released from publications, newspapers, relevant public companies and local government. Availability of research data offers continence for in-deep case studies. Third, both case happened in Zhejiang province in 2008. Similar regional culture and economic background enhanced comparability of case studies. Fourth, the final contagious effects in two bankruptcy crises were quite different, which is valuable for further comparable exploration.

The dataset used in the article was mainly gathered from annual reports, announcements and other information releases from relevant public companies. And news reports and governmental documents during the crisis periods are also provide important information. WU et.al. (2011) constructed network datasets of Keqiao and Jiaojiang business cycle during the bankruptcy crises. And the article employs the same network datasets in the comparative case studies.

4.2 COMPARATIVE ANALYSIS OF NETWORK PARADIGMS

Structural attributes of Keqiao business network and Jiaojiang business network are illustrated in table 1. Keqiao business network is denser than Jiaojiang network. The average degree and cluster coefficient of Keqiao network are respectively 3.292 and 0.196. While the average degree and cluster coefficient in Jiaojiang network are lower, respectively 2.000 and 0.115. Considering their different network scale, 48 nodes in Keqiao and 12 nodes in Jiaojiang, their discrepancy in network structure would be huger. If peripheral nodes with only 1 degree in Keqiao network are removed, average node degree of 32 nodes in core proportion increases to 4.438, and average cluster coefficient increases to 0.294. So, Keqiao network is a larger scale and dense complex network with much more closed loops, compared with Jiaojiang network.

TABLE 1 Comparison of Structural Properties of Keqiao and Jiaojiang Business Network

Structural Properties	Keqiao	Jiaojiang	Difference
Node degree	3.292	2.000	1.292**
Clustering coefficient	0.196	0.115	0.081**
Network Scale	48	12	-

Noted: ** significant at the level of 0.01

Fig.2 illustrated degree distribution of two business network, their degree distribution is similar. In Keqiao network, there are more nodes with relatively higher degree due to its larger network scale. In Keqiao case, the most powerful companies had 21 financial partners. And totally 19 companies in the network had more than 3 financial partners. Jiaojiang network is less dense due to its relative small network scale. As percentage distribution of standardized degree is concerned, the degree distribution of Jiaojiang network is skewed toward the higher standardized degree. In here, standardized degree is calculated as ratio of node degree to network scale. Network scale, as long as closed loops, provides quite different context for explaining slight variance in degree distribution.

Fig.3 illustrated distribution of clustering coefficient in the two business networks, their degree distribution is similar. In Keqiao network, there are more closed loops that is critical for risk acceleration during financial contagion process. The article observed at most 210 closed loops that were connected with a single node in Keqiao network. While, there is less closed loops in Jiaojiang network due to its small network scale and lower density. And closed loops are largely limited among 4 core nodes.

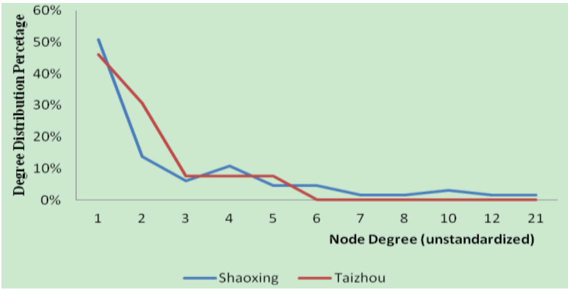

FIGURE 2 Degree distribution of Keqiao and Jiaojiang business network

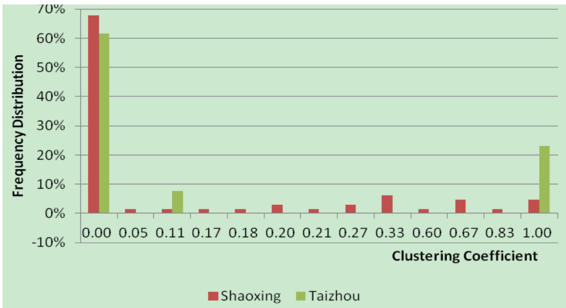

FIGURE 3 Frequency distribution of clustering coefficient in Keqiao and Jiaojiang business network

4.3 SIMULATION ANALYSIS

For simplicity, the article calculated contagious effects of cascading bankruptcy process that is triggered by single node in Keqiao and Jiaojiang business network. Fig.4 illustrated distribution of contagious effects of idiosyncratic bankruptcy in Keqiao network. There are 9 systematically important nodes. A cascading bankruptcy crisis, which would cause a clique of 91.67% nodes collapsed, would be triggered by bankruptcy of each of these 9 nodes. In this way, Keqiao network is both fragile and unstable. And there also 11 nodes, whose bankruptcy would causes local devastation equivalent to 6.25%-10.42% of total network scale.

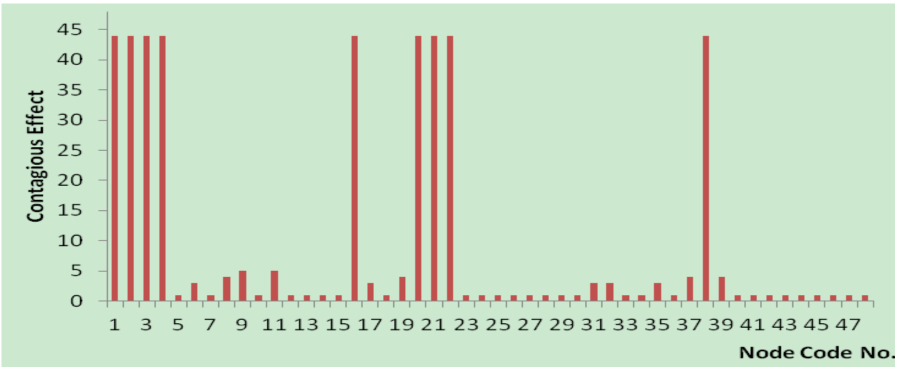

FIGURE 4 Contagious Effects of Idiosyncratic Bankruptcy in Keqiao Case

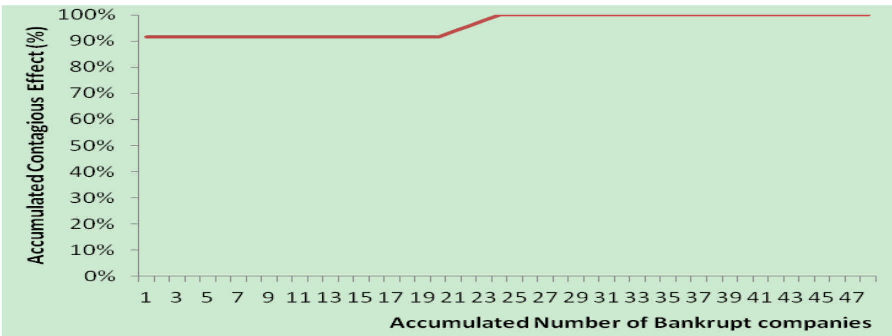

FIGURE 5 Accumulated Distribution of Contagious Effect in Keqiao Case

The fragility of Keqiao business network is also illustrated by its accumulated contagious effects (Fig.5). A systematically important node would cause cascading process, leading to bankruptcy of 91.67% nodes. If random probability of idiosyncratic bankruptcy is 1%, then there is 9% probability of systematic risk with 91.67% large-scale devastation.

Fig.6 illustrated distribution of contagious effects of idiosyncratic bankruptcy in Jiaojiang network. There are only 1 systematically important nodes. A cascading bankruptcy crisis, which would cause a clique of 66.67% nodes collapsed, would be triggered by bankruptcy of the systematically important nodes. In this way, Keqiao network is both fragile yet but relatively more robust than Keqiao network.

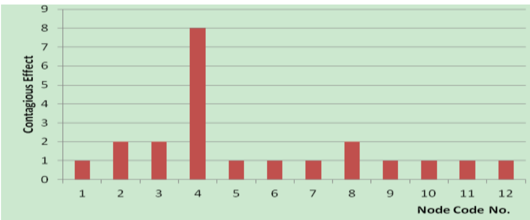

FIGURE 6 Contagious Effects of Idiosyncratic Bankruptcy in Jiaojiang Case

The fragility of Jiaojiang business network is also illustrated by its accumulated contagious effects (Fig.7). The systematically important node would cause cascading process, leading to bankruptcy of 66.67% nodes. If random probability of idiosyncratic bankruptcy is 1%, then there is only 1% probability of systematic risk with 66.67% large-scale devastation. In this way, Jiaojiang business is more robust than Keqiao network.

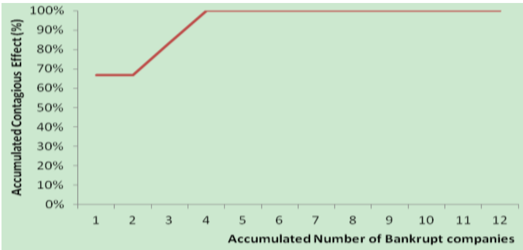

FIGURE 7 Accumulated Distribution of Contagious Effect in Jiaojiang Case

5 Discussion and Conclusion

WU et.al.(2011) analyzed inter-firm contagion process of the realistic Keqiao case and Jiaojiang case in details. In reality, bankruptcy of a core company in Keqiao business network, Zhejiang Hualian Sanxin Petrochemical Co., Ltd, caused intensive risk contagion leading to systematic risk in Keqiao County. With the strong invention by local government, the cascading bankruptcy crisis was blocked after 5 companies have declared to be bankruptcy or restructure. In its most urgent period, almost all companies in the network are threatened by unexpected risk impact. While in real case of Jiaojiang network, the bankruptcy of a local notable company, China Feiyue group, only caused limited contagious effect. There is no other company besides Feiyue group in realistic Jiaojiang Case.

And the simulation approach was applied to two realistic cases with quite different Network Paradigms. Our simulation result is very similar to reality of the two cascading bankruptcy crisis. So, the simulation approach suggested by the article is useful and its methodology is effective in assess risk loss. Furthermore, the simulation approach is adaptable to different network paradigm.

Acknowledgements

This work is supported by The National Social Science Fund of China (Grant No. 14CSH071). The research leading to this paper is also supported by grant from of MOE (Ministry of Education in China) Humanities and Social Sciences Project (Grant No. 12YJC630229). This material is also based upon work funded by Zhejiang Provincial Natural Science Foundation of China under Grant No. LQ12G03003.

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Author

Bao Wu, born Sep.1979, in Zhejiang Province, P.R.China
Current position, grades: Associate professor in China Institute for Small and Medium Enterprises and Collage of Business Administration, Zhejiang University of Technology.
University studies: PhD degree in from Zhejiang University of Technology
Scientific interest: Risk analysis, Social complex network and system

Mingzhu Li, born Mar.1991, in Liaoning Province, P.R.China
Current position, grades: Postgraduate in China Institute for Small and Medium Enterprises and Collage of Business Administration, Zhejiang University of Technology.
Scientific interest: Risk analysis, Social complex network and system

Zihang Ding, born Mar.1991, in Liaoning Province, P.R.China
Current position, grades: Postgraduate in Collage of Business Administration, Zhejiang University of Technology.
Scientific interest: Risk analysis, Social complex network and system