

# Research on seismic response of a large liquid storage tank using equivalent load method

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## Abstract

Based on the principle of added mass method, a method of calculating the dynamic response of the large liquid storage tank under seismic loads is proposed in this paper i.e. equivalent load method. The main idea of this method is converting the added mass to dynamic surface pressure. Compared with added mass method, equivalent load method not only simplifies calculation process but also improves computational efficiency. In order to verify the feasibility of the proposed method, a large liquid storage tank (36m in diameter and 19m in height) was simulated in ABAQUS with added mass method and equivalent load method respectively. The user subroutine UEL was introduced to build added mass element and the different distributions of dynamic pressure was derived in equivalent load method. The results of added mass method and equivalent load method show that they are numerically close. It illustrates the difference and relationship between equivalent load method and added mass method in calculating the dynamic response of the large liquid storage tank under seismic loads, at the same time demonstrates the feasibility of equivalent load method.

*Keywords:* liquid storage tank; added mass; equivalent load; dynamic response

## 1 Introduction

Large liquid storage tank is usually used to store flammable, explosive or toxic liquid, once destroyed, the consequence is destructive. Hence the safety performance of the tank becomes more and more important. The liquid storage tank is thin-walled container with large diameter. Such structural features lead to poor resistance to deformation under dynamic loads. The bottom of the liquid storage tank suffers large hoop tensile stress and axial compressive stress. Plastic deformation concentrates mostly in this part of the tank, resulting in the failure modes called "elephant foot effect" and "diamond effect".

Haroun and Housner [1-2] used shell element to simulate the wall of the tank and used added mass method to simulate the liquid's dynamic pressure. They also derived the added mass matrix with the boundary integral theory. Hoop shells with four degrees of freedom were used to discretize the wall of the tank into four parts to reduce the total number of degrees of freedom. Not only did they analyze the vibration characteristics of the elastic liquid storage tank, but they discussed the influence of the sloshing of the liquid free surface and the initial environmental stress on the liquid storage tank wall. In 1974, Veletsos [3] put forward an assumed mode method, the idea of this method is simplifying the liquid storage tank into a single degree of freedom system, the inertia force of fluid to the tank wall is equivalent to added mass attached to the wall on assuming that the wall vibrates in accordance with the pre-assumed deformation vibration model. This method was developed into Veletsos Yang model [4] in 1976 and its main idea is simplifying the model into a one-dimensional cantilever, which is one of the most commonly used methods in the calculation of the dynamic response of liquid storage tank. Veletsos also analysed the elastic liquid storage tank with Flugge's shell element theory. Displacement of any point on the wall was represented by the superimposed inherent vibration modes of the cantilever. Part of the liquid attached

to the tank wall was built to simulate the contact interaction of the liquid and the wall. Zhuang Zhuo and Xiaochuan You [5-7] using finite element software ABAQUS simulated the dynamic response of large liquid storage tank under seismic load. Basing on Housner and Veletsos added mass formula, they achieved added mass method in ABAQUS by introducing user subroutine UEL and analysed the occurrence mechanism of the "elephant foot effect" and "diamond effect" by energy principle.

Based on the existed liquid storage tank model, the first part of this paper calculates the value and distribution of the added mass by Housner and Veletsos's formula. Finite element analysis software ABAQUS [8] was used to calculate the dynamic response of liquid storage tank under seismic loads. In addition, a method is put forward in this paper that transfers the dynamic pressure of the liquid on the wall to a time-varying surface pressure from the idea of Housner and Veletsos who decoupling the dynamic pressure of the liquid. The value and distribution of the surface pressure was obtained by Housner and Veletsos formula. The dynamic response under seismic load was calculated. The second part of this paper mainly compares the results calculated by the two methods, and analyses the internal force changes quantitatively under the dynamic load. A new simple and efficient method is proposed in this paper to analyse the dynamic response of the large liquid storage tank. The simulation results can provide theoretical basis for structural design.

## 2 Model parameters of liquid storage tank

### 2.1 GEOMETRIC PARAMETERS

Large liquid storage tank mainly works under fluid pressure increasing gradually from top to bottom. Based on the strength design concept, liquid storage tank consists of several tubes in different thickness [9], and the wall thickness decreases gradually from the bottom. The model in this

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paper is 36m in diameter, 19m in height and the bottom plate is 15mm in thickness. Geometric parameters of the model are shown in Figure 1.

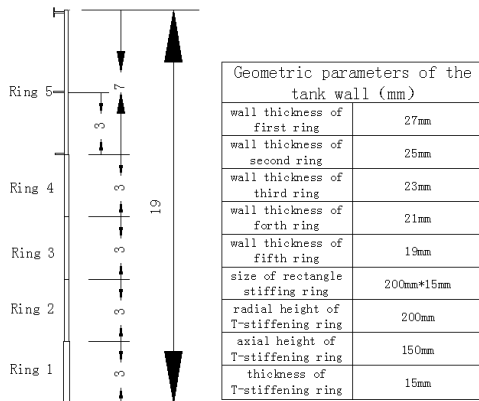


FIGURE 1 Geometric parameters of the liquid storage tank wall

2.2 PARAMETERS OF SEISMIC LOAD

EI Centro seismic wave of USA California earthquake in 1940 is selected to be the seismic load in this simulation. Recording time of this earthquake is 30s and the acceleration is in north-south direction, peak accelerator appears at 2.12s and the value is 3.417 m/s<sup>2</sup>. In this simulation, the record of the relatively small amplitude after 16s was ignored. The maximum value of the earthquake acceleration time history curve curves in 8 degree seismic fortification intensity is 1.1m/s<sup>2</sup> in code for seismic design of buildings. Compressing the acceleration amplitude to 0.322 times of the original value, the actual earthquake acceleration time history curve for this simulation is shown in Figure 2.

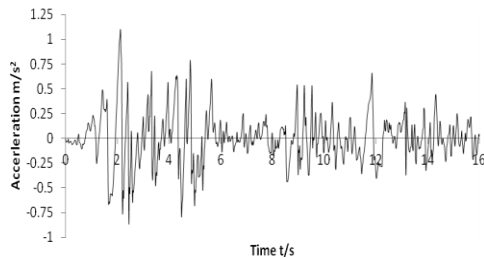


FIGURE 2 Seismic acceleration curve

3 Added mass method and finite element model

3.1 ADDED MASS METHOD

In Housner's theory, liquid is assumed to be ideal fluid without stickiness and compressibility. Equivalent spring-mass model is selected to calculate flowing pressure of liquid. The flowing pressure is divided into pulsating pressure

and convection pressure in this method [10], and pulsating pressure is the main part. Figure 3 shows some parameters of the liquid storage tank.

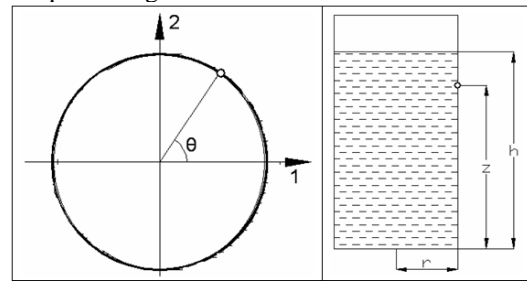


FIGURE 3 Liquid storage tank coordinates and parameters

When the cylindrical liquid storage tank subjected to impulse acceleration  $a(t)$  in x-axis direction, the fluid will also obtain an acceleration  $a_0$  in x-axis direction. Supposing that accelerations in the vertical and tangential direction are both zero, fluid can be simplified into a lot of fluid beam parallel with x-axis, there's no relative displacement between adjacent fluid beams. The dynamic pressure of the liquid on the tank wall obtained by Housner method is shown as Eq.(1).

$$P^I = -\sqrt{3}\rho \frac{d^2u}{dx^2} h \left( \frac{h-z}{h} - \frac{1}{2} \left( \frac{h-z}{h} \right)^2 \right) \cos \varphi \tanh \frac{\sqrt{3}R}{h} \quad (1)$$

The force can be seen as a inertial force that caused by an equivalent added mass, the equivalent added mass is shown as Eq.(2).

$$m = -\sqrt{3}\rho h \left( \frac{h-z}{h} - \frac{1}{2} \left( \frac{h-z}{h} \right)^2 \right) \cos \varphi \tanh \frac{\sqrt{3}R}{h} \quad (2)$$

Veletsos (1970s) took the elasticity of liquid storage tank wall into account and obtained the distribution of dynamic liquid pressure. Based on Chopra's research of dam-liquid storage tank system dynamic pressure coupling effect [11], Veletsos assumed the system to be single degree of freedom. He supposed that longitudinal profile shape was variable during the deformation of the container. The deflection profile

used in this paper is  $\varphi(z) = \frac{z}{h}$ .

Dynamic liquid pressure of any point  $(\theta, z)$  on the liquid storage tank wall can be obtained ,

$$p(\psi(z), \theta, z, t) = \rho h \cdot \cos \theta \cdot a_1(t) \cdot A(\psi(z), z), \quad (3)$$

where

$$A(\psi(z), z) = \frac{4}{\pi} \sum_{n=1}^{\infty} d_n \cos \left[ (2n-1) \frac{\pi z}{2h} \right]. \quad (4)$$

The Veletsos added mass formula can be simplified into Eq. (5).

$$p[\varphi(z), z, t] = \frac{4}{\pi} \cdot \rho \cdot h \cdot \left[ \frac{\sqrt{2}}{6} \cos \frac{z\pi}{2h} - \frac{\sqrt{2}}{18} \cos \frac{3z\pi}{2h} + \frac{\sqrt{2}}{30} \cos \frac{5z\pi}{2h} + \frac{\sqrt{2}}{42} \cos \frac{7z\pi}{2h} \right] \cos \theta \quad (5)$$

### 3.2 FINITE ELEMENT MODEL OF LIQUID STORAGE TANK

4 nodes reduced integrated element S4R is selected to simulate the tank, 2 nodes beam element is selected to simulate the reinforcing ring. All the elements size is 0.5m. There are totally 14266 elements and 13702 nodes. Added mass element is simulated by the user subroutines UEL in ABAQUS. Its principle is adding a user element with mass on the liquid storage tank wall element, the new added element share nodes with the original shell element.

According to Housner and Veletsos added mass theory, the value of added mass at any point of liquid storage tank wall is cosine of the added mass on the line of  $\theta = 0$ .

$$\varphi(z) = \frac{z}{h}$$

theory. When the liquid storage tank is full of liquid (17m), values of added mass on the line  $\theta = 0$  distribute along the height according to Housner method and Veletsos method are shown in Figure 4.

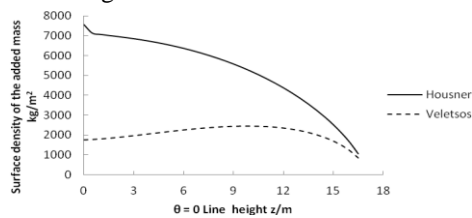


FIGURE 4 Distribution of added mass along height

### 3.3 SEISMIC RESPONSE ANALYSIS STEP

The simulation process for model with added mass is divided into two steps, the hydrostatic pressure step and the seismic load step. 3 degrees of freedom of translation for bottom of the tank are restrained. When acceleration in 1 direction is applied, restraint in 1 direction will be released.

When using equivalent load method (applying dynamic load on surface) to simulate the response of liquid storage tank under seismic loading, the analysis process is also divided into two steps, the application of hydrostatic pressure, and the application of time-varying dynamic load on surface in the form of Housner or Veletsos distribution. 3 degrees of freedom of translation on tank bottom are restrained.

## 4 Results and discussions

### 4.1 STRESS DISTRIBUTION OF LIQUID STORAGE TANK

On the condition that liquid storage tank is full of liquid, analyses for seismic response of liquid storage tank with added mass method and equivalent load method were conducted respectively. The results are shown in Figure 5-8 and listed in Table 1. It can be seen in table 1 that the maximum Mises stress in the four methods are in little difference. Among them, the biggest is 78.6MPa obtained from Housner added mass method and the smallest is 60.2MPa obtained from Veletsos equivalent load method. The tank wall is supposed to be rigid in Housner added mass method, which

resulted in a bigger stress during the calculation. The position of the biggest Mises stress occurred in the same position except the equivalent load method. It can be concluded that the stress is bigger at the point of (0, 1.5) and it is necessary to pay more attention on it. The biggest Mises stress occurs in added mass method at about 4.90s, while in equivalent load method it is about 2.46s. Because of the added mass element, the dynamic response of the whole structure lags behind the original structure [12].

TABLE 1 stress results

Reservoir	Housner added mass	Veletsos added mass	Housner equivalent load method	Veletsos equivalent load method
Max Mises stress (MPa)	78.6	62.8	65.2	60.2
Max Mises stress position (θ, Z)	(0,1.5)	(0,1.5)	(118,1.5)	(0,1.5)
Time of max Mises stress (s)	4.92	4.88	2.46	2.46

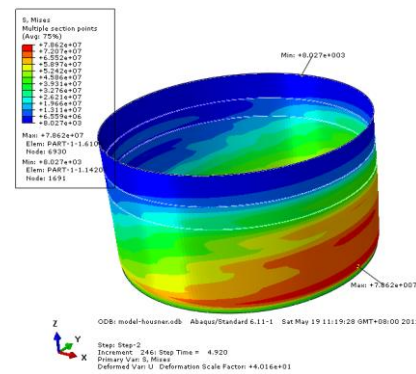


FIGURE 5 Maximum Mises stress (Housner added mass method)

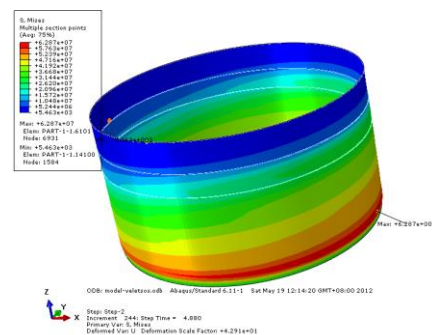


FIGURE 6 Maximum Mises stress (Veletsos added mass method)

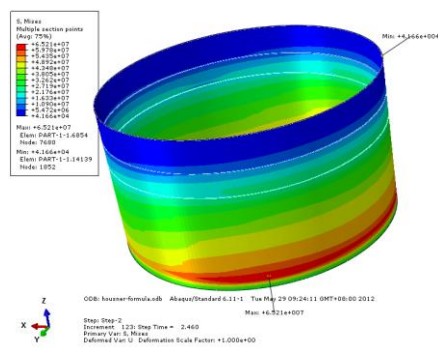


FIGURE 7 Maximum Mises stress (Housner equivalent load method)

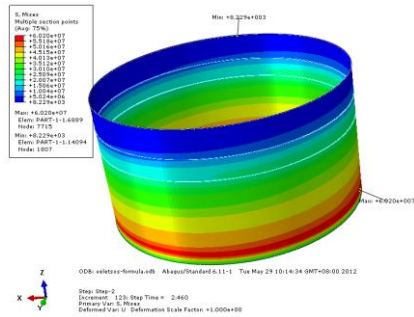


FIGURE 8 Maximum Mises stress (Veletsos equivalent load method)

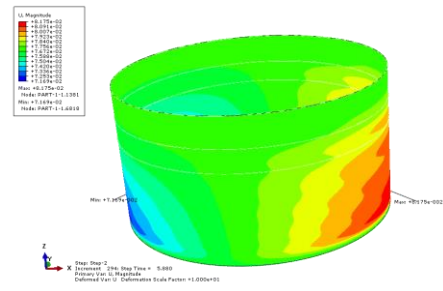


FIGURE 10 Maximum displacement (veletsos added mass)

### 4.2 LIQUID STORAGE TANK DEFORMATION ANALYSIS

Displacement results of the liquid storage tank under seismic load are shown in Figure 9-12 and listed in Table 2. It can be seen in the table that the maximum displacement in four methods are similar. Among them, the biggest is 6.9 mm obtained by Housner added mass method and the smallest is 5.2mm obtained by Veletsos equivalent load method. Comparing the position where the biggest displacement came out, displacement of added mass method distribution is near the line  $\theta = 0^\circ$  and equivalent load method distribution is near the line  $\theta = 45^\circ$ . Three of the biggest displacements appear 1.5m above bottom of the tank, near the position of the biggest Mises stress. Comparing the moments when the biggest displacement occurs, time of added mass method is behind that of the equivalent load method due to the added mass element.

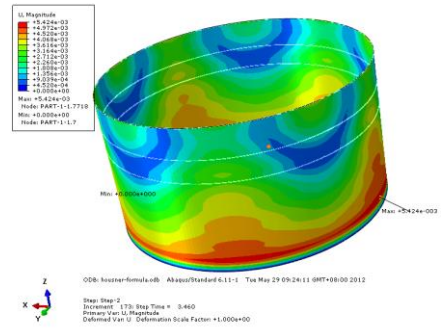


FIGURE 11 Maximum displacement (Housner equivalent load method)

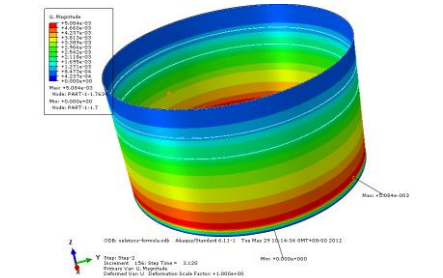


FIGURE 12 Maximum displacement (Veletsos equivalent load method)

TABLE 2 displacement results

	Housner added mass	Veletsos added mass	Housner equivalent load method	Veletsos equivalent load method
Max displacement (mm)	6.9	5.4	5.6	5.2
Position of max displacement ( $\theta, Z$ )	(0, 6.5)	(0, 1.5)	(44, 1.5)	(45, 1.5)
Time of max displacement (s)	5.92	5.88	3.46	3.12

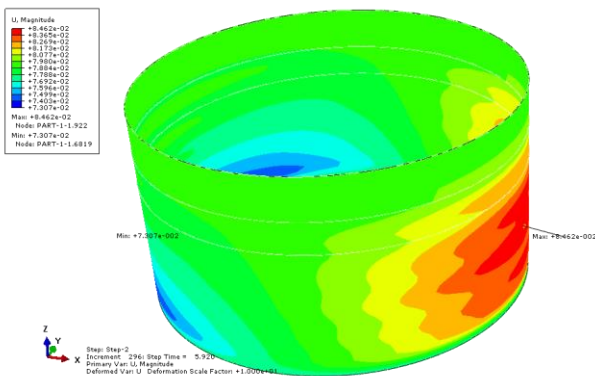


FIGURE 9 Maximum displacement (Housner added mass)

Figure 13 shows the maximum displacement of the liquid storage tank wall obtained by the added mass method at each moment. It can be seen that the response of the liquid storage tank under seismic loads obtained by the Housner added mass method stronger than the response obtained by Veletsos added mass method. The deformation from Housner added mass method changes in a wide range while the results from Veletsos added mass method is relatively stable. Figure 14 shows the contrasts of wall's deformation between two kinds of equivalent load method in every moment. The comparison shows that the results of the the Housner distribution and Veletsos distribution of equivalent load method have the same trend and the wall's deformation is larger in Housner distribution. From Figure 13 and Figure 14, it can be inferred that the deformation of the liquid storage tank is mainly caused by the hydrostatic pressure of the liquid.

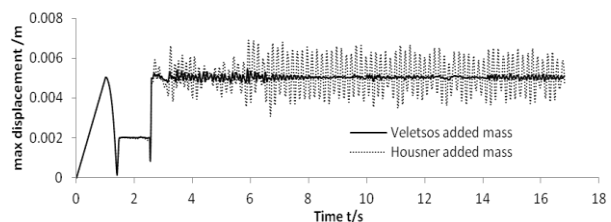


FIGURE 13 Each moment of maximum displacement of added mass method



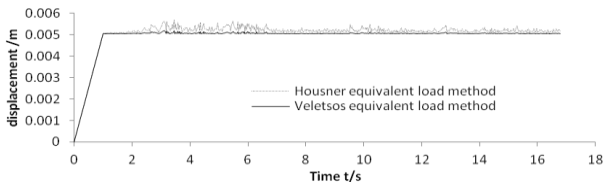


FIGURE 14 Each moment of maximum displacement of equivalent load method

**5 Conclusions**

A new method of equivalent load method is proposed for the research on dynamic response of large liquid storage tank under seismic load. Compared with added mass method, it has several advantages. The using of ABAQUS user subroutine UEL is avoided, and instead of the added mass element method, dynamic surface pressure method is adopted. In this way the simulation process simplifies greatly, and the hardware requirement is reduced as well. The efficiency of operation improves drastically.

Added mass method and equivalent load method are used

**References**

[1] Haroun M A, Housner G W. Dynamic Characteristics of Liquid Storage Tanks. Journal of the Engineering Mechanics Division, ASCE 1982 108(EM5) 783-800

[2] Haroun M A, Housner G W. Complications in Free Vibration Analysis of Tanks. Journal of the Engineering Mechanics Division, ASCE 1982 108(EM5) 801-818

[3] Veletsos, A.S.: Seismic effects in flexible liquid storage tanks, Proc. 5th WCEE. Rome, Italy. 1974, 1:630-639.

[4] Veletsos A S, Yang J Y. Earthquake response of liquid storage tanks. Proceedings of 2nd Annual Emd Specialty Conf, ASCE, New York. 1977 1-24

[5] Zhuang Zhuo, Liu Huanzhong, Li Qing. The development of added mass model for the analysis on dynamic response of liquid storage tank. Modern Manufacturing Engineering. 2006(1) 69-73

[6] Liu Huanzhong, Li Qing, Zhuang Zhuo. The analysis on dynamic response of liquid storage tank by the using of added mass model. Engineering Mechanics. 2005 161-171

[7] Hu Yinghui, Zhuang Zhuo, You Xiaochuan. Added mass method on seismic response of a large liquid storage tank. Pressure Vessel Technology. 2009(8) 1-6

[8] ABAQUS theory manual, ABAQUS, Version 6.10 Documentation. 2010

[9] Zhao Huilei. Study of stability of large tank subjected to external pressure. Beijing, Beijing University of Chemical Technology 2009

[10] Wang Yi, Chen Weimin, Lin Mian. Variation of added mass and its application to the calculation of amplitude response for a circular cylinder. China Ocean Engineering 2007(3) 429-437

[11] Su Li, Li Shujuan, Tang Guoan. Numerical analysis of fluid flow and added mass induced by vibration of structure. Applied Mathematics and Mechanics (English Edition) 2005(2) 252-260

[12] Mu Peng, Zhao Shuai, Miao Hui. The effect on vibration performance of cylindrical shell of added mass. Technological Development of Enterprise. 2011(24):27-28

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