Two-sided matching considering the preferences of agents and intermediary

Qi Yue[1], Yuhua Li[1,2*]

[1]*School of Information Management, Jiangxi University of Finance and Economics, Nanchang, Jiangxi 330013, China*

[2]*School of International Trade and Economics, Zhongshan Jiahede Investment Company, Zhongshan, Guangdong 528400, China*

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Abstract

This paper proposes a novel method for solving the two-sided matching problem, in which the preferences provided by agents are ordinal numbers, and the preferences provided by intermediary is the expense standard on ordinal numbers. In this paper, the description of the considered two-sided matching problem is given. Then the concepts of satisfaction degrees and expense are introduced. Furthermore, a multi-objective optimization model can be set up consider satisfaction degrees of agents and expense of intermediary. The method of weighted sums based on membership function is used to convert the multi-objective optimization model into a single-objective model. The matching result is obtained by solving the model.

Keywords: two-sided matching, ordinal number, satisfaction degree, expense

1 Introduction

There are plenty of two-sided matching problems in many fields of real life, such as marriage assignment [1], college admission [2], employee selection [3], personnel assignment [4] and trading partner selection [5]. Therefore two-sided matching is a research topic with extensive application backgrounds.

Study on the two-sided matching problem originates from the problem of college admission and marriage assignment [6]. Gale and Shapley initially investigate the concept, existence, optimality and solution algorithm of stable assignment [7]. Then, Roth explicitly proposes the concept of two-sided matching [3]. Following that, various methods, techniques and algorithms have been proposed for solving the two-sided matching problem with different forms of information. For example, Teo et al. derive the optimal cheating strategy in Gale-Shapley stable marriage model, and then apply it to Singapore school admissions problem [8]. Ehlers studies truncation strategies in matching markets using the deferred acceptance algorithm (i.e., Gale-Shapley algorithm), and show that truncation strategies are also applicable to all priority mechanisms and all linear programming mechanisms [9]. Azevedo gives a simple equilibrium model of an imperfectly competitive matching market, in which an infinite number of firms is matched to a continuum of workers [10]. Teo and Sethuraman study the classical stable marriage and stable roommate problems using a polyhedral approach [11]. Fleiner gives a linear characterization of the bipartite stable b-matching polytope [12]. Manlove et al. give a 2-approximation algorithm for the stable marriage problem with incomplete lists and ties of finding a stable matching of maximum or minimum size [13]. Iwama et al. give a $(2-c/\sqrt{N})$-approximation algorithm to solve the stable marriage problem of finding a stable matching of maximum size when both ties and unacceptable partners are allowed in preference lists, where c is an arbitrary positive constant that satisfies $c \le 1/4\sqrt{6}$, and N is the instance size [14].

Prior studies have made significant contributions to solving the two-sided matching problems. However, on the one hand, most of the existing studies focus on obtaining stability matching(s). However, in some cases, comparing with stability matching, each agent is more concerned about his/her own satisfaction degree over his/her partner. On the other hand, in real world problems, the determination of the stability matching(s) is usually suggested by intermediary [5, 15, 16]. The intermediary could also be the type of pursuing the benefit, and yet the existing studies seldom consider this type of intermediary. Therefore, how to consider the satisfaction degree of agents and the profit of intermediary in the two-sided matching problem is a valuable research topic. These are the motivation of this paper.

The purpose of this paper is to propose a novel method for solving the two-sided matching problem considering the preferences of agents and intermediary. In the method, the concepts of satisfaction degree and expense are introduced. Furthermore, a multi-objective optimization model is built, where the considered sub-objectives are to maximize the satisfaction degree of each agent and expense of intermediary. By solving the multi-objective optimization model, the matching result can be determined.

The rest of this paper is provided as follows: Section 2 describes the two-sided matching problem considering the preferences of agents and intermediary. Section 3 gives the concepts of satisfaction degrees and expense. Section 4 builds a multi-objective optimization model considering satisfaction degrees and expense. Section 5 develops an

* ***Corresponding author's*** e-mail: liyuhuajx@sina.com

algorithm for solving the model. Section 6 summarizes and highlights the main features of the proposed method.

2 The problem

The related concept and notation on two-sided matching can be found in [17, 18]. The two-sided matching problem considering the preferences of agents and intermediary is described below.

For convenience, let $M=\{1,2,\ldots,m\}$, $N=\{1,2,\ldots,n\}$, and suppose $m\le n$. Let $A=\{A_1,A_2,\ldots,A_m\}$ be the set of agents of side A, where A_i denotes the *i*-th agent of side A, $i\in M$; $B=\{B_1,B_2,\ldots,B_n\}$ be the set of agents of side B, where B_j denotes the *j*-th agent of side B, $j\in N$. Let $R_i=(r_{i1},r_{i2},\ldots,r_{in})$ be the ordinal number vector provided by agent A_i over the agents of side B, where r_{ij} denotes that agent A_i ranks B_j in the r_{ij}-th position, $r_{ij}\in N$. Let $T_j=(t_{1j},t_{2j},\ldots,t_{mj})^T$ be the ordinal number vector provided by agent B_j over the agents of side A, where t_{ij} denotes that agent B_j ranks A_i in the t_{ij}-th position, $t_{ij}\in M$. The expense standard of agents is usually determined by intermediary. Here, the intermediary refers to a single person, an organization or a decision system that makes a matching between agents of sides A and B.

The problem concerned in this paper is how to obtain the reasonable matching result based on ordinal number vectors $R_i\ (i\in M)$ and $T_j\ (j\in N)$ such that the satisfaction degrees of each agents and the profit of intermediary are as large as possible.

3 The satisfaction degrees and expense

In the two-sided matching problem, without loss of generality, if agent A_i ranks B_j in the first position, then the satisfaction degree of agent A_i over B_j is the highest; if agent A_i ranks B_k in the last position, then the satisfaction degree of agent A_i over B_k is the lowest. Usually, satisfaction degrees of one agent over potential partners are in interval [0, 1]. Hence, we give the following definitions of satisfaction degrees.

Definition 1. Let $r_{ij}\in N$ be the ordinal number provided by agent A_i over B_j. Then the function l_i used to obtain the satisfaction degree of agent A_i over B_j (noted as α_{ij}) is defined as:

$$l_i: N\to[0,1],$$

$$l_i(r_{ij})=\alpha_{ij},$$

where l_i is a monotone decreasing function and satisfies $0\le l_i(n)$, $l_i(1)\le 1$.

Definition 2. Let $t_{ij}\in M$ be the ordinal number provided by agent B_j over A_i. Then the function g_j used to obtain the satisfaction degree of agent B_j over A_i (noted as β_{ij}) is defined as:

$$g_j: M\to[0,1],$$

$$g_j(t_{ij})=\beta_{ij},$$

where g_j is a monotone decreasing function and satisfies $0\le g_j(m)$, $g_j(1)\le 1$.

The expressions of functions l_i and g_j could be different due to the different consideration of agents. The typical expressions of functions l_i and g_j are expressed by:

$$l_i(r_{ij})=\left[\frac{n+1-r_{ij}}{n}\right]^2,\ i\in M \quad (1)$$

and

$$g_j(t_{ij})=\left[\frac{m+1-t_{ij}}{m}\right]^2,\ j\in N. \quad (2)$$

By Equations (1) and (2), the satisfaction degree matrices $\tilde{A}=[\alpha_{ij}]_{m\times n}$ and $\tilde{B}=[\beta_{ij}]_{m\times n}$ are constructed, respectively.

Definition 3. Let $r_{ij}\in N$ be the ordinal number provided by agent A_i over B_j. Then the function $y_A(r)$ used to obtain the expense that agent A_i provides to the intermediary if agent A_i is matched with agent B_j (noted as λ_{ij}) is defined as:

$$y_A(r): N\to R,$$

$$y_A(r_{ij})=\lambda_{ij},$$

where $y_A(r)$ is a monotone decreasing function and satisfies $y_A(n)>0$.

Definition 4. Let $t_{ij}\in M$ be the ordinal number provided by agent B_j over A_i. Then the function $y_B(t)$ used to obtain the expense that agent B_j provides to the intermediary if agent B_j is matched with agent A_i (noted as π_{ij}) is defined as:

$$y_B(t): M\to R,$$

$$y_B(t_{ij})=\pi_{ij},$$

where $y_B(t)$ is a monotone decreasing function and satisfies $y_B(m)>0$.

Usually, the expressions of function $y_A(r)$ and $y_B(t)$ are various. In this paper, we consider the following formats:

$$y_A(r)=\begin{cases} p_1, & r=1, \\ p_2, & r=2, \\ \ldots, & \\ p_n, & r=n, \end{cases} \tag{3}$$

$$y_B(t)=\begin{cases} q_1, & t=1, \\ q_2, & t=2, \\ \ldots, & \\ q_m, & t=m, \end{cases} \tag{4}$$

where $p_1, p_2, \ldots, p_n$ are the expense values and satisfy $p_1 > p_2 > \ldots > p_n > 0$, and $q_1, q_2, \ldots, q_m$ are also the expense values and satisfy $q_1 > q_2 > \ldots > q_m > 0$. By Equations (3) and (4), expense matrices $\Gamma = [\lambda_{ij}]_{m\times n}$ and $\Pi = [\pi_{ij}]_{m\times n}$ are constructed, respectively.

4 The two-sided matching model

Let x_{ij} be an 0-1 variable, where $x_{ij}=0$ denotes $\mu(A_i)\neq B_j$, $x_{ij}=1$ denotes $\mu(A_i)=B_j$. To maximize the satisfaction degrees of agents and the expense of intermediary, a multi-objective optimization model (5) can be established as follows:

$$\max \quad Z(A_i)=\sum_{j=1}^{n}\alpha_{ij}x_{ij}\ ,\ i\in M\ , \tag{5a}$$

$$\max \quad Z(B_j)=\sum_{i=1}^{m}\beta_{ij}x_{ij}\ ,\ j\in N\ , \tag{5b}$$

$$\max \quad Z(T)=\sum_{i=1}^{m}\sum_{j=1}^{n}(\lambda_{ij}+\pi_{ij})x_{ij}\ , \tag{5c}$$

$$\text{s.t.} \quad \sum_{j=1}^{n}x_{ij}=1,\ i\in M\ , \tag{5d}$$

$$\sum_{i=1}^{m}x_{ij}\leq 1,\ \ j\in N\ , \tag{5e}$$

$$x_{ij}=0 \text{ or } 1,\ i\in M\ ,\ \ j\in N\ . \tag{5f}$$

In the model (5), the meaning of Equation (5d) is that agent A_i must match only one agent of side B. The meaning of Equation (5e) is that agent B_j matches at most one agent of side A.

Generally, each agent of one side has equal priority, thus model (5) can be further transformed into the following optimization model (6):

$$\max \quad Z(A)=\sum_{i=1}^{m}\sum_{j=1}^{n}\alpha_{ij}x_{ij}\ , \tag{6a}$$

$$\max \quad Z(B)=\sum_{i=1}^{m}\sum_{j=1}^{n}\beta_{ij}x_{ij}\ , \tag{6b}$$

$$\max \quad Z(T)=\sum_{i=1}^{m}\sum_{j=1}^{n}(\lambda_{ij}+\pi_{ij})x_{ij}\ , \tag{6c}$$

$$\text{s.t.} \quad \sum_{j=1}^{n}x_{ij}=1,\ i\in M\ , \tag{6d}$$

$$\sum_{i=1}^{m}x_{ij}\leq 1,\ \ j\in N\ , \tag{6e}$$

$$x_{ij}=0 \text{ or } 1,\ i\in M\ ,\ \ j\in N\ . \tag{6f}$$

5 The algorithm

In order to solve model (6), the method of weighted sums based on membership function is adopted [19]. In the followings, we give the detailed analysis on the solution of model (6). Firstly, let $Z_{\min}(A)$ and $Z_{\max}(A)$ be the minimum value and the maximum value for objective function $Z(A)$. Let $Z_{\min}(B)$ and $Z_{\max}(B)$ be the minimum value and the maximum value for objective function $Z(B)$. Let $Z_{\min}(T)$ and $Z_{\max}(T)$ be the minimum value and the maximum value for objective function $Z(T)$. Then, three membership functions can be respectively described with the followings:

$$\mu(Z(A))=\frac{Z(A)-Z_{\min}(A)}{Z_{\max}(A)-Z_{\min}(A)}\ , \tag{7}$$

$$\mu(Z(B))=\frac{Z(B)-Z_{\min}(B)}{Z_{\max}(B)-Z_{\min}(B)}\ , \tag{8}$$

$$\mu(Z(T))=\frac{Z(T)-Z_{\min}(T)}{Z_{\max}(T)-Z_{\min}(T)}\ , \tag{9}$$

obviously, $0\leq\mu(Z(A))\leq 1$, $0\leq\mu(Z(B))\leq 1$ and $0\leq\mu(Z(T))\leq 1$.

Let w_A, w_B and w_T be the weight of objectives functions $Z(A)$, $Z(B)$ and $Z(T)$, respectively, such that $w_A+w_B+w_T=1$, $0<w_A, w_B, w_T<1$, then model (6) is transformed into the single-objective optimization model (10):

$$\max \quad Z=w_A\mu(Z(A))+w_B\mu(Z(B))+w_T\mu(Z(T))\ , \tag{10a}$$

$$\text{s.t.} \quad \sum_{j=1}^{n}x_{ij}=1,\ i\in M\ , \tag{10b}$$

$$\sum_{i=1}^{m}x_{ij}\leq 1,\ \ j\in N\ , \tag{10c}$$

$$x_{ij}=0 \text{ or } 1,\ i\in M\ ,\ \ j\in N\ , \tag{10d}$$

where weight $w_D\ (D=A,B,T)$ reflects the importance degree of objective function $Z(D)$ in practical decision problems. Usually, to guarantee the fairness of the agents of sides A and B, we have $w_A = w_B$.

Model (10) the can be solved using the existing mathematical optimization software. Then the matching result is determined based on the obtained optimal solution.

In sum, we give an algorithm for solving the two-sided matching problem and its steps are presented as follows:

Step 1: Set up the satisfaction degree matrices $\tilde{A} = [\alpha_{ij}]_{m\times n}$ and $\tilde{B} = [\beta_{ij}]_{m\times n}$ by Equations (1) and (2).

Step 2: Set up the expense matrices $\Gamma = [\lambda_{ij}]_{m\times n}$ and $\Pi = [\pi_{ij}]_{m\times n}$ by Eqs. (3) and (4).

Step 3: Set up the multiple-objective optimization model (5) based on satisfaction degree matrices $\tilde{A}$ and $\tilde{B}$, and expense matrices Γ and Π.

Step 4: Determine $Z_{\min}(D)$ and $Z_{\max}(D)$, $D = A, B, T$.

Step 5: Transform model (6) into model (10) by Eqs. (7)-(9).

Step 6: Determine the matching result by solving model (10).

6 Conclusion

The two-sided matching problem arises from a wide range of real-world situations. Although many researchers have paid attention to the two-sided matching problem, there are few methods considering the satisfaction degrees of agents and the profit of intermediary. This paper has presented a novel method for solving the two-sided matching problem with ordinal numbers.

Comparing with the existing methods, the proposed method has two characteristics as discussed below. Firstly, the agents' satisfaction degrees and intermediary's profit are considered. This is usually absent in the existing methods. Secondly, the proposed method is simple and is a supplement or extension of the existing methods. In terms of future research, the proposed method can be extended to support situations in which the information data are in other formats.

References

[1] Irving R W, Manlove D F, O'Malley G 2009 Stable marriage with ties and bounded length preference lists *Journal of Discrete Algorithms* **7**(2) 213–9

[2] Klaus B, Klijn F 2006 Median stable matching for college admissions. *International Journal of Game Theory* **34**(1) 1–11

[3] Roth A E 1985 Common and conflicting interests in two-sided matching markets *European Economic Review* **27**(1) 75–96

[4] Korkmaz I, Gökçen H, Çetinyokuş T 2008 An analytic hierarchy process and two-sided matching based decision support system for military personnel assignment *Information Sciences* **178**(14) 2915–27

[5] Bloch F, Ryder H 2000 Two-sided search, marriage and matchmakers *International Economic Review* **41**(1) 93–115

[6] Alkan A, Gale D 2003 Stable schedule matching under revealed preference *Journal of Economic Theory* **112**(2) 289–306

[7] Gale D, Shapley L 1962 College admissions and the stability of marriage *American Mathematical Monthly* **69**(1) 9–15

[8] Teo C P, Sethuraman J, Tan W P 2001 Gale-Shapley stable marriage problem: Revisited strategic issues and applications *Management Science* **47**(9) 1252–67

[9] Ehlers L 2008 Truncation strategies in matching markets *Mathematics of Operations Research* **33** (2) 327–35

[10] Azevedo E M 2014 Imperfect competition in two-sided matching markets *Games and Economic Behavior* **83**(1) 207-23

[11] Teo C P, Sethuraman J 1998 The geometry of fractional stable matchings and its applications *Mathematics of Operations Research* **23**(4) 874–91

[12] Fleiner T 2003 On the stable *b*-matching polytope. *Mathematical Social Sciences* **46**(2) 149–58

[13] Manlove D F, Irving R W, Iwama K, Miyazaki S, Moritab Y 2002 Hard variants of stable marriage *Theoretical Computer Science* **276**(1-2) 261–79

[14] Iwama K, Miyazaki S, Yamauchi N 2008 A $(2-c/\sqrt{N})$ – approximation algorithm for the stable marriage problem *Algorithmica* **51**(3) 342–56

[15] Ehlers L 2007 Von Neumann–Morgenstern stable sets in matching problems *Journal of Economic Theory* **134**(1) 537–47

[16] Kang N, Han S Y 2003 Agent-based e-marketplace system for more fair and efficient transaction *Decision Support Systems* **34**(2) 157–65

[17] Echenique F 2008 What matchings can be stable? The testable implications of matching theory *Mathematics of Operations Research* **33**(3) 757–68

[18] Hałaburda H 2010 Unravelling in two-sided matching markets and similarity of preferences *Games and Economic Behavior* **69**(2) 365–93

[19] Chen Y W, Wang C H, Lin S J 2008 A multi-objective geographic information system for route selection of nuclear waste transport *Omega* **36**(3) 363–72

Authors

Qi Yue, born in November 1983, Jiangxi, China.

Current position, grades: lecturer at Jiangxi University of Finance and Economics, China.
University studies: MSc at Guangxi University of Nanning in China. DSc at the Northeast University of Liaoning in China.
Scientific interests: decision analysis and operations research.
Publications: more than 40 papers.
Experience: teaching experience of 3 years, 5 scientific research projects.

Yuhua Li, 1984.6, Shangdong, China.

Current position, grades: lecturer at Jiangxi University of Finance and Economics, China.
University studies: BSc and MSc at the School of Business Administration at Northeastern University of Shenyang in China. DSc at Kyushu University of Fukuoka in Japan.
Scientific interests: multinational banking and foreign direct investment.
Publications: more than 20 papers.
Experience: teaching experience of 1 years, 4 scientific research projects.