A modified non-monotone method with 3-1 piecewise NCP function for nonlinear complementary problem

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Abstract

In this paper, we proposed a modified nonmonotone method for nonlinear complementarity problem, different from the existed methods, we transform the original problem to a semi-smooth equation by using a piecewise NCP function, and combined with the nonmonotone line search. Only one nonlinear equations need to be solved per iteration so that the computational costs are reduced. Under some suitable assumptions, we give the convergence properties of the proposed method and the numerical results to show that our method is efficient.

Keywords:

nonlinear complementary problem,
piecewise NCP function,
nonmonotone,
global convergence

1 Introduction

We consider the following nonlinear complementarity problem NCP: find $x \in R^n$, to satisfy:

$$x \geq 0, F(x) \geq 0, x^T F(x) = 0, \tag{1}$$

where $F : R^n \to R^n$ is continuously differentiable P_0 -function, i.e., for all $x, y \in R^n$ with $x \neq y$, there exists an index i_0 such that

$$x_{i_0} \neq y_{i_0}, (x_{i_0} - y_{i_0})[F_{i_0}(x) - F_{i_0}(y)] \geq 0 \cdot \tag{2}$$

The nonlinear complementarity problem is one of the important types of variational inequalities, mainly comes from the actual problem in the field of economy (such as balance problem) and relevant issues in the field of physics, mechanics and engineering (such as the discretization of infinite dimensional problem [1-3]). Therefore, the problem NCP (1) has attracted great attention due to its various application. One way of solving the nonlinear complementarity problem (1) is to construct a Newton method. This method is to solve a system of nonlinear equations:

$$H(x,s) = \begin{pmatrix} s - F(x) \\ \varphi(x,s) \end{pmatrix} = 0,$$

which is equivalent to (1). Among them, $\phi(x,s)$ is a kind of NCP function satisfies:

$$\varphi(x,s) = \begin{cases} 3x - \dfrac{x^2}{s} & if\ s \geq x > 0, or\ 3s > -x \geq 0, \\ 3s - \dfrac{s^2}{x} & if\ x > s > 0, or\ 3x > -s \geq 0, \\ 9x + 9s & if\ x \leq 0\ and - x \geq 3s, or\ s \leq -3x \leq 0. \end{cases}$$

$F : R^n \to R^n$ is continuously differentiable P_0 -function.

Chen et al [4] investigate a semismooth Newton algorithm for P_0 -NCP. In order ro improve the numerical results, Zhang et al [5] replace the monotone line search with a non-monotone line search when they fulfill the algorithm. Qi H D and Qi L Q [6] propose a new QP-free method which ensures the strict feasibility of all iterates based on the Fischer-Burmister NCP function. They also prove that the method has global convergence without isolatedness of accumulation point and strict complementarity condition. D.G.Pu et al [7] minimizes a smooth function subject to smooth inequality constraints. This iterative method is to solve a nonsmooth equations that are obtained by the multiplier and the Fischer-Burmeister NCP function. Liu and Pu [8] present 3-1 piecewise NCP function for new nonmonotone QP-free infeasible method. This method proved globally convergent without a linear independence constraint qualification.

Motivated by the above ideas, we construct a Newton method based on the solution of nonlinear equations obtained by the 3-1 piecewise NCP function for the P_0 -NCP function. The acceptance of a trial step is more flexible by means of nonmonotone techniques. In this algorithm, we only need to solve one nonlinear equations per iteration so that the computational costs are reduced. The method has proved to be implementable and globally convergent without a strict complementarity.

This paper is organized as follows. In the next section, we introduce the 3-1 piecewise linear NCP function and the properties of it. The nonlinear complementary problem is transformed into equivalent system of nonlinear equations. In Sect.3 introduces the algorithm. In Sect.4 proves the algorithm to be implementable and presents the algorithm's convergence theory.

2 Preliminaries

Function $\phi: R^2 \to R$ is called an NCP function when $\phi(a,b)=0$ if and only if $a \ge 0, b \ge 0$ and $ab=0$. The 3-1 piecewise linear NCP function is defined as:

$$\varphi(a,b)=\begin{cases} 3a-\dfrac{a^2}{b} & if\ b \ge a > 0, or\ 3b > -a \ge 0, \\ 3b-\dfrac{b^2}{a} & if\ a > b > 0, or\ 3a > -b \ge 0, \\ 9a+9b & if\ a \le 0\ and -a \ge 3b, or\ b \le -3a \le 0 \end{cases} s \cdot (3)$$

If $(a,b) \ne (0,0)$, then

$$\nabla\varphi(a,b)=\begin{cases} \begin{pmatrix} 3-\dfrac{2a}{b} \\ \dfrac{a^2}{b^2} \end{pmatrix} & if\ b \ge a > 0, or\ 3b > -a \ge 0, \\ \begin{pmatrix} \dfrac{b^2}{a} \\ 3-\dfrac{2b}{a} \end{pmatrix} & if\ a > b > 0, or\ 3a > -b \ge 0, \\ \begin{pmatrix} 9 \\ 9 \end{pmatrix} & if\ a \le 0\ and -a \ge 3b, or\ b \le -3a \le 0 \end{cases} . \quad (4)$$

Detailed Property and application of piecewise NCP function see [9].

It is easy to check the following Proposition.

Proposition 2.1 For the function $\phi(a,b)$ the following holds.

1. $\varphi(a,b)=0s \Leftrightarrow a \ge 0, b \ge 0$ and $ab=0$;a
2. the square of φ is continuously differentiable;
3. φ is twice continuously differentiable everywhere except at the origin, but it is strongly semismooth at the origin and is a pseudo-smooth NCP function.

Constuct function: $H: R^{2n} \to R^{2n}$

$$H(x,s)=\begin{pmatrix} s-F(x) \\ \varphi(x,s) \end{pmatrix}. \quad (5)$$

For the update of s, we require it infinitely close to F(x), So we order s-F(x)=0. Therefore, contacting the first line of Proposition 3, we know that nonlinear complementarity problem (1) is equivalent to solving the minimization problem:

$$\begin{gathered} \min \ \psi(x,s) \\ \psi(x,s)=\| H(x,s) \| \end{gathered}. \quad (6)$$

3 Algorithm

For solving (6), we need to introduce the following symbols

$$(\xi_i^k \eta_i^k)=\begin{cases} (1, 1) & (x,s)=(0,0) \\ \nabla\varphi(x,s) & otherwise \end{cases}, \quad (7)$$

$i=1,2\cdots n$, obviously, $\xi_i^k > 0$ and $\eta_i^k > 0$.

Compute the Jacobian matrix $V(x^k,s^k)$ of $H(x^k,s^k)$, we get

$$V(x^k,s^k)=\begin{pmatrix} -F'(x^k) & I \\ diag(\xi_i^k) & diag(\eta_i^k) \end{pmatrix},$$

where I is identity matrix of $n \times n$, $diag(\xi_i^k)$ or $diag(\eta_i^k)$ denotes the diagonal matrix whose i th diagonal element is ξ_i^k or η_i^k, respectively.

We now present the algorithm combining a Newton method with the nonmonotone line search, the following algorithm is obtained d and λ by calculating system of nonlinear equations, which from the Hessian of H. In order to solve: min $\psi(x,s)$, we adopt the nonmonotone line search based on [8], so that the trial step is more flexible.

Algorithm 3.1

Step 0. Initialization:

Given initial point:

$$x^0, s^0 \in R^n,\ \tau \in (0,1),\ 0 < \theta < 1,\ k=0.$$

Step1. If $\psi(x^k,s^k)=0$ then stop. Otherwise, calculation of the search direction:

Calculate d^k and λ^k by solving the following linear system in (d,λ):

$$V_k\begin{pmatrix} d \\ \lambda \end{pmatrix}=\begin{pmatrix} F(x^k)-s^k \\ -\varphi(x^k,s^k) \end{pmatrix}, \quad (8)$$

Step 2. Nonmonotone line search.

Step 2.1. If

$$\psi(x^k+d^k, s^k+\lambda^k) \le \theta\psi(x^k,s^k), \quad (9)$$

$$\phi(x^k+d^k, s^k+\lambda^k) \le \theta \max_{0 \le r \le m(k)-1} \| \phi^{k-r} \|, \quad (10)$$

where $m(0)=0,\ 0 \le m(k) \le \min\{m(k-1)+1, M\}$, M is a positive constant.

Then let $x^{k+1}=x^k+d^k$, $s^{k+1}=s^k+\lambda^k$, go to step 3.

Otherwise go to step 2.2.

Step 2.2. Let $x^{k+1}=x^k+\alpha_k d^k$, $s^{k+1}=s^k+\alpha_k\lambda^k$.

Where $\alpha_k=\tau^j$ $(0<\tau<1)$ and j is the smallest non-negative integer and satisfied: (10).

Step 3. Update: let $k=k+1$ and go to step 1.

4 Convergence

In this section, we discuss the global convergence property of algorithm with the nonmonotone line search. In order to achieve the convergence of the algorithm, we give some Assumptions as follows:

Assumption 4.1

A. F: $R^n \to R^n$ is continuously differentiable P_0 -function, so that $\nabla F(x)$ is positive semidefinite.

B. F is Lipschitz continuously differentiable, namely, there exists a constant L such that for all $x_1, x_2 \in R^n$, $y_1, y_2 \in R^{2n}$

$$\| F(x_1)-F(x_2) \| \le L \| x_1 - x_2 \|,$$

$\| H(y_1) - H(y_2) \| \le L \| y_1 - y_2 \|$.

Lemma 4.1 If $\phi^k \neq 0$ then given any $\varepsilon > 0$ there is a $\bar{t} > 0$, such that for any $0 < t \le \bar{t}$ and any k,

$$\left\|\varphi^k\right\|^2 - \left\|\varphi(x^k + td^k, s^k + t\lambda^k)\right\|^2 \ge (2-\varepsilon)t\left\|\varphi^k\right\|^2$$

Proof: If $\varphi^k \neq 0$ implies

$$diag(\xi^k)\cdot d^k + diag(\eta^k)\cdot\lambda^k = -\varphi(x^k, s^k). \tag{11}$$

We define that if $(x,s) \neq (0,0)$, then $(\bar{\xi}_i^k, \bar{\eta}_i^k) = (\xi_i^k, \eta_i^k)$.

Otherwise, $\bar{\xi}_i^k d^k + \bar{\eta}_i^k \lambda^k = \phi_i'((x^k, s^k),(d^k, \lambda^k))$.

Where $\varphi_i'((x^k, s^k),(d^k, \lambda^k))$ is the direction derivative of $\varphi_i(x,s)$ at (x^k, s^k) in the direction (d^k, λ^k). Let $diag(\bar{\xi}^k)$ or $diag(\bar{\eta}^k)$ denote the diagonal matrix whose ith diagonal element is $\bar{\xi}_i^k$ or $\bar{\eta}_i^k$, respectively.

Clearly, for all i,

$$\varphi_i(x^k + td^k, s^k + t\lambda^k) - \varphi_i^k - t(\bar{\xi}_i^k d^k + \bar{\eta}_i^k \lambda^k) = o(t). \tag{12}$$

It follows by the definition of above, we have

$$\left\|\varphi^k + t(diag(\bar{\xi}^k)d^k + diag(\bar{\eta}^k)\lambda^k)\right\|^2 = (1-2t)\left\|\varphi^k\right\|^2 + t\left\|diag(\bar{\xi}^k)d^k + diag(\bar{\eta}^k)\lambda^k\right\|^2. \tag{13}$$

It follows from (12) and (13) that, given any $\varepsilon > 0$, there is a $t > 0$, such that for any $0 < t \le t$,

$$\left\|\varphi^k\right\|^2 - \left\|\varphi(x^k + td^k, s^k + t\lambda^k)\right\|^2 \ge (2-\varepsilon)t\left\|\varphi^k\right\|^2.$$

Hence, this lemma holds.

Lemma 4.2 For all k, there is an $\alpha_{\min} > 0$ such that $\alpha_k \ge \alpha_{\min} > 0$.

Proof: Assume $\phi^k \neq 0$ for sufficiently large k, it follows by Lemma 4.1 that, for all k, $\phi^k \neq 0$ and any $\alpha \le \min\{\frac{1-\theta}{2-\varepsilon}, \bar{t}\}$.

$$\left\|\varphi(x^k + \alpha d^k, s^k + \alpha\lambda^k)\right\|^2 \le [1-(2-\varepsilon)\alpha]\left\|\varphi^k\right\|^2 \le \theta\left\|\varphi^k\right\|^2 \le \theta^2 \max_{0\le r\le m(k)-1}\left\|\varphi^{k-r}\right\|^2.$$

Lemma 4.3 If $H(x^k, s^k) \neq 0$ then V^k is nonsingular.

Proof: Assume $H(x^k, s^k) \neq 0$, If $V^k(u,v)^T = 0$ for some $(u,v)^T \in R^{2n}$, where $u = (u_1, u_2 \cdots u_n)$, $v = (v_1, v_2 \cdots v_n)$, then

$$-F'(x^k)u + Iv = 0, \tag{14}$$

$$diag(\xi^k)u + diag(\eta^k)v = 0. \tag{15}$$

From the definitions of ξ_i^k and η_i^k we know that $\xi_i^k > 0$ and $\eta_i^k > 0$ for all i.

So, $diag(\eta^k)$ is nonsingular. We have

$$v = -(diag(\eta^k))^{-1} diag(\xi^k)u. \tag{16}$$

Puting (16) into (14), and multiplying by u^T, we have

$$-u^T F'(x^k)u - u^T(diag(\eta^k))^{-1} diag(\xi^k)u = 0.$$

By the fact that $F(x)$ is the P_0-function, so all the principal minor determinant of $\nabla F(x)$ is non-negative, that is to say, $\nabla F(x)$ is positive semidefinite. And matrix $(diag(\eta^k))^{-1} diag(\xi^k)$ is positive definite. Therefore $u = 0$. It follows from (16) that $v = 0$. Hence, V^k is nonsingular.

Lemma 4.4 If V^* is an accumulation matrix of $\{V^k\}$, then V^* is nonsingular.

Proof: It is clear that V^k is nonsingular for all $k = 0,1,2....$ Since ξ_i^k and η_i^k are bounded without loss of generality, let $\xi_i^k \to \xi_i^*$, $\eta_i^k \to \eta_i^*$ and let $x^k \to x^*$, then

$$V^k \to V^* = \begin{pmatrix} -\nabla F(x^*) & I \\ diag(\xi_i^*) & diag(\eta_i^*) \end{pmatrix}$$

Let $(u,v) \in R^{2n}$ be the solution of $V^*(u,v)^T = 0$

$$-\nabla F(x^*)u + Iv = 0, \tag{17}$$

$$diag(\xi^*)u + diag(\eta^*)v = 0. \tag{18}$$

In the next section, V^* is proven to be nonsingular, which is equivalent to showing that $(u,v) = (0,0)$

First, consider such an $j \in J$ for which $\xi_j^* = 0$. From the definition of the 3-1 piecewise NCP function, it is only possible in the second area and $x > s > 0$ or $3x > -s \ge 0$,

$$\xi_j^k = \left(\frac{s}{x}\right)^2 \to 0.$$

Hence $\eta_j^k = 3 - \frac{2s}{x} \to 3 \neq 0$. Then for such an $j \in J$, we deduce that the matrix $diag(\eta_j^*)$ is nonsingular, and $v_j = 0,\ j \in J$ by (18).

For $j \notin J$ such that $\xi_j^* \neq 0$, substituting (18) into (17) and multiplyng (17) by v_j^T, then

$$v_j^T \cdot \nabla F(x^*) \cdot \sum_{j:\xi_j^* \neq 0} \frac{\eta_j^*}{\xi_j^*} \cdot v_j + v_j^T I v_j = 0.$$

$\nabla F(x^*)$ is positive semidefinite together with the $\xi^* > 0, \eta^* \ge 0$ implies $v_j = 0,\ j \notin J$.

This proves $(u,v)=(0,0)$ and hence V^* is nonsingular.

Lemma 4.5 Suppose the Assumption 4.1 holds, $\phi(x^k,s^k)\to 0$, as $k\to\infty$.

Proof: In view of convenience, if for all sufficiently large k (10) holds, define $\|\phi^{l(k)}\|=\max\limits_{0\le r\le m(k)-1}\|\phi^{k-r}\|$, where $k-m(k)+1\le l(k)\le k$.

Since $m(k+1)\le m(k)+1$, then

$$\begin{aligned}\|\varphi^{l(k+1)}\| &= \max_{0\le r\le m(k+1)-1}\|\varphi^{k+1-r}\| \\ &\le \max_{0\le r\le m(k)}\|\varphi^{k+1-r}\| \qquad 1 \\ &= \max\{\|\varphi^{l(k)}\|,\|\varphi^{k+1}\|\} \\ &= \|\varphi^{l(k)}\|\end{aligned}$$

So, $\|\phi^{l(k)}\|$ is monotone decreasing, which implies that the $\{\|\phi^{l(k)}\|\}$ converges.

It follows from (10) that $\|\phi^{l(k)}\|\le\theta\|\phi^{l(l(k)-1)}\|$.

Since $\theta\in(0,1)$, therefore $\{\|\phi^{l(k)}\|\}\to 0\,(k\to\infty)$.

Therefore $\|\phi^{k+1}\|\le\theta\|\phi^{l(k)}\|\to 0$ holds by the Algorithm 3.1.

That is, $\lim\limits_{k\to\infty}\|\varphi^k\|=0$.

Lemma 4.6 Suppose the Assumption 4.1 holds, $d^k\to 0,\lambda^k\to 0,H^k\to 0$, as $k\to\infty$.

Proof: Suppose the contrary that exists $\varepsilon_1>0,\ \varepsilon_2>0$ for a subsequence (x^k,s^k) such that $\|d^k\|\ge\varepsilon_1>0,\ \|\lambda^k\|\ge\varepsilon_2>0$. If $\phi^k\ne 0$, then (d^k,λ^k) is the decreasing direction of $\|\phi^k\|$ by lemma 4.1, which contradict $\lim\limits_{k\to\infty}\|\phi^k\|=0$. Hence, $d^k\to 0,\lambda^k\to 0$.

V^* is nonsingular from lemma 4.4 together with $V^*\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}F(x^*)-s^*\\0\end{pmatrix}$. It is seen that $F(x^*)-s^*=0$ and $\phi(x^*,s^*)=0$, namely, $\psi(x^*,s^*)=0$, so $(x^k,s^k)\to(x^*,s^*)$ is the solving of NCPs.

5 Numerical tests

In this section, we implemented Algorithm 3.1 for solving NCP. All experiments were performed on a personal computer with 2.0 GB memory and Intel(R) Core(TM)2 Duo CPU 2.93 GHz. The operating system was Windows 7 and the computer codes were all written in Matlab 7.1.

In the following tables, IT denotes the number of iterations. CPU denotes the CPU time in seconds. x^k is the final value of x, FV denotes the value of $\|H(x,s)\|$ when the algorithm terminates. We considered the following 3 examples.

Example 5.1: Consider (1), where $x\in R^3$ and $F(x):R^3\to R^3$ given by $F(x)=\begin{pmatrix}x_2\\x_3\\-x_2+x_3+1\end{pmatrix}$.

This problem is from Example 4.4 in [11], which has infinitely many solutions $(0,\lambda,0)$, where $\lambda\in[0,1]$. The initial point x^0,s^0 is randomly generated whose elements are in the interval (0,10). The termination criterion is $\|H(x,s)\|\le 10^{-6}$. Parameters are chosen as follows: $\theta=0.6,\ \tau=0.9$. The test results are listed in Table 1 by using different starting points.

TABLE 1

x^0	s^0	IT	CPU	x^k
(9.5013,2.3114,6.0684)	(6.582,3.782,2.478)	6	0.018834	(-0.0000,0.5000,-0.0000)
(6.8128,3.7948,8.3180)	(8.459,5.248,6.254)	6	0.017559	(-0.0000,0.1285,-0.0000)
(4.4470,6.1543,7.9194)	(5.791,3.896,8.412)	4	0.015375	(-0.0000,0.9998,-0.0000)
(8.4622,5.2515,2.0265)	(7.685,3.365,2.489)	5	0.017096	(-0.0000,1.0000,-0.0000)
(3.0462,1.8965,1.9343)	(4.235,1.226,2.742)	4	0.010422	(-0.0000,0.8585,-0.0000)

Example 5.2: Consider (1), where $x\in R^3$ and $F(x):R^3\to R^3$ given by $F(x)=\begin{pmatrix}x_1-5\\x_2^3+x_2-x_3-3\\x_2+2x_3^3+x_3-3\end{pmatrix}$

This problem has infinitely many solutions. The initial point x^0,s^0 is randomly generated. The termination criterion is $\|H(x,s)\|\le 10^{-6}$. Parameters are chosen as follows: $\theta=0.6,\ \tau=0.9$. The test results are listed in Table 2 by using different starting points.

TABLE 2

x^0	s^0	IT	CPU	FV	x^k
(2,3,9)	(1,1,2)	14	0.016404	2.4007×10^{-7}	(5.0000,1.3428,0.7643)
(8,13,9)	(3,4,2)	14	0.012882	1.7533×10^{-7}	(5.0000,1.3428,0.7643)
(9,14,18)	(4,17,12)	16	0.014060	1.2517×10^{-7}	(5.0000,1.2027,0.7944)
(11,7,8)	(6,9,13)	14	0.015661	2.8667×10^{-7}	(5.0000,1.3428,0.7643)
(5,7,3)	(4.9,3)	12	0.023225	4.6498×10^{-7}	(5.0000,1.2027,0.7944)

Example 5.3: Consider (1), where $x \in R^4$ and $F(x): R^4 \to R^4$ given by $F(x) = \begin{pmatrix} x_1^3 - 8 \\ x_2 + x_2^3 - x_3 + 3 \\ x_2 + x_3 + 2x_3^3 - 3 \\ x_4 + 2x_4^3 \end{pmatrix}$

This problem is from Example 1 in [12]. The initial point x^0, s^0 select the following. The termination criterion is $\|H(x,s)\| \le 10^{-4}$. Parameters are chosen as follows: $\theta = 0.8$, $\tau = 0.6$. The test results are listed in Table 3 by using different starting points.

TABLE 3

x^0	s^0	IT	CPU	FV	x^k
(3,1,1,1)	(1,2,2,5)	5	0.033747	2.4217×10^{-5}	(2.0000,-0.0000,1.0000,0.0000)
(3,1,2,1)	(1,2,6,2)	5	0.014731	5.8588×10^{-6}	(2.0000,-0.0000,1.0000,0.0000)
(1,1,2,1)	(1,2,5,1)	5	0.016660	6.8160×10^{-6}	(2.0000,-0.0000,1.0000,0.0000)
(2,1,1,1)	(1,1,4,2)	5	0.026659	2.7528×10^{-5}	(2.0000,-0.0000,1.0000,0.0000)

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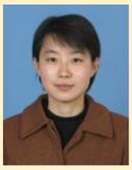

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