Sourcing and pricing strategy research of competition supply chain under supply disruption

Ling Hou[1]*, Meng Li[2], Dongyan Chen[1]

[1]*Department of Applied Mathematics, Harbin University of Science and Technology, 150080, Harbin, China*

[2]*School of Management, Harbin Institute of Technology, 150001, Harbin, China*

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Abstract

Under the environment of supply disruption, it is significant to study decision-making, because sourcing strategies of retailers impact the profit of the supply chain while the pricing strategies of suppliers affect all aspects of the supply chain. In this paper, the demand distribution function of each supply chain is obtained, which is based on the total demand of two supply chains with given distribution function, and the sourcing and pricing problems are obtained in supply chain network under the environment of supply disruption. In order to decompose the total demand with the given distribution function, customer choice theory is adopted to acquire the demand of each supply chain. By game theory and optimization theory, we obtain the sourcing strategies of two retailers and the pricing strategies of two suppliers in this system. Finally based on the assumption of a uniform demand distribution, the outcomes of the proposed models are demonstrated with a numerical example. The results show that when disruption probability or delivery cost are high, retailers will only order from the spot market although the spot market wholesale prices are a little high; but when the disruption probability is moderate or low, the retailer would rather place orders from suppliers. Specific purchasing method depends on the competition ability between suppliers.

Keywords: supply chain network, supply disruption, sourcing strategies, pricing strategies

1 Introduction

With the deepening of globalization and integration of supply chain, competition has become a hot topic in the area of supply chain management. Supply chain management, which was treated as a method for the enterprise accessing to the core competitiveness, has turn to a more complex system, namely the supply chain network management. At present, research on supply chain network mainly focuses on some simple network structures, such as one-to-many and many-to-one. Wu explored the equilibrium structure for two competing supply chains. Each chain has one manufacturer with two exclusive retailers, that is, the supply chain network structure is of 1-2 type [1]. Ha investigated the contracting problem using a two-stage game in two competing supply chains with information sharing, each consisting of one manufacturer and one retailer, that is, the supply chain network structure is of 1-1 type [2]. For more similar study, see literatures [3-5] for details. Note that above literatures do not involve competitive decision-making problems of many-to-many network. Therefore, in this paper we just discuss the case of supply network of 2-2 type which is a generalization of the above 1-2 and 1-1 type structures. It proves that this kind of network structure is fit for the actual operation state of supply chain. For example, products of Haier and ChangHong are both in Gome and Suning's supply chain and there is dramatic competition for market share between Gome and Suning.

The research of this paper is closely related to supply chain sourcing management and supply disruptions management. Early literatures on sourcing management often assume that cost, quantity and distribution ability are three important factors that wholesalers need to consider before making a decision (Dickson [6]; Verma and Pullman [7]; Weber [8] et al.). For example, Weber [8] concluded that the order quantity was the most important criteria for retailers to develop a sourcing strategy, while the cost and picking (delivery ability) followed. Talluri and Narasimhan extended the work of Weber by treating prices as an input variable and the amount of revenue and delivery ability as system outputs [9].

In the above literature, the authors did not consider the impact of the competition of several companies on the sourcing strategy, where a single supplier model easily leads to duopoly suppliers. In recent years, some scholars began to study the sourcing strategy for competitive enterprises. Parlar and Perry consider a firm that faces constant demand and sources from two identical-cost capacitated suppliers which are subject to production failure. Inter failure and repair times are exponentially distributed for both suppliers. The authors propose a suboptimal ordering policy that is solved numerically [10]. Gurler and Parlar extended the work of Parlar and Perry by considering the case of Erlang inter failure times and

*** *Corresponding author* e-mail: houling0729@163.com

general repair times. They propose that, for the same order costs and order scale without restrictions, downstream of supply chain can reduce order quantity and/or diverted to other route; Otherwise, the supply chain would use other interference management strategy, such as double sourcing and emergency purchase [11]. Xiao et al. studied the vendor selection problem which says that a supplier procures major raw materials from which raw materials suppliers [12]. Although the above two articles explore the impact of competition on the sourcing strategy and wholesale pricing strategies of supplier, they are in a stable market supply environment and do not involve the supply uncertain environment, which more reflects scenario in reality.

Supply disruptions management has become an increasing concern of the business and academia. Many companies began to realize that the supply disruption has seriously affected the ability to successfully manage their supply chain. Literature on supply disruptions management has a huge body. However, most of these studies assume a single supplier and alternative energy is not available in the system of a single supplier. While a large number of studies have shown that the effective method for weakening supply disruptions is the multi-sourcing strategy. Therefore, some scholars began to study the multi-sourcing strategy for supply disruptions. Tomlin and Wang developed a single period dual-sourcing model by two suppliers with yield uncertainty. But the information between two suppliers is not completely symmetrical: one unreliable supplier and one reliable (and thus more expensive) supplier. They focus on inventory and sourcing mitigation. They concluded that retailers can reduce interrupt risk by an appropriate purchasing strategy although the two-supplier information is not symmetrical [13]. Chopra considered the mitigation-disruption strategy when the unreliable supplier is subject to both recurrent and disruption uncertainty [14]. Further, Federgruen and Kleindorfer considered Type I service-level-related constraints in their yield management models [15, 16], while Yang et al. propose an interesting analytical approach on the multiple-sourcing random yield problem [17]. Here Yang and Dada consider the problem of a newsvendor that is served by multiple suppliers, where a supplier is defined to be either perfectly reliable or unreliable. They showed that in the optimal solution a supplier will be selected only if all less-expensive suppliers are selected, regardless of the supplier's reliability [17, 18]. Literature on supply disruptions mentioned above only investigated the retailer's strategy and assumed that the supplier's strategy is exogenously given. However, the supplier's response, for example their pricing strategies, also influences the decisions of the supply chain members. The wholesale price setting caused many scholars' attention, representative literatures give the optimal pricing strategies of suppliers in different situations, including Lariviere and Porteus [19], Wang and Gerchak [20], Bernstein and DeCroix [21], Serel [22], He [23], Cho and Tang [24], Surti C and Hassini [25] etc. However, the above literatures only discuss the many-to-one supply chain structure, without considering the many-to-many supply chain network pricing problem under supply disruptions.

This paper differs from the existing studies in the following aspects. First, the structure of the supply chain is an extension of the above supply chain structure; secondly, supply disruption is introduced into the sourcing strategy; Third, we consider how to deal with unmet demand. The above-mentioned literature assumes that every retailer only has one ordering opportunity in the entire sales process and is not allowed emergency replenishment and that unmet demand will be lost. In our paper, we assume that retailers may procure from the spot market, rather than lost, for unmet demand after observing the demand. Finally, we investigate the impacts of supply disruption on the retailer's sourcing strategy by both theoretical and computational analyses.

The remainder of this paper is as follows. In the next section, we give problem statement and model of demand function. Then, we obtain the sourcing strategies of the retailer and the supplier in Section 3 and Section 4. In Section 5, we give numerical examples to verify the validity of the results. The last section summarizes the research findings and future research directions.

2 The problem statement and model of demand function

2.1 THE PROBLEM STATEMENT

This paper analyses the competition between two supply chains with only two echelons. That is, both of two chains consist of one manufacturer and one retailer. Because members in the one supply chain often belong to the other supply chain in reality, we consider the competition between supply chains with a crossover structure in order to actually reflect competitive scenario in reality. Namely, there are cooperative relations between the two suppliers and two retailers, where two suppliers can supply to two retailers. That is 2-type 2 network structure which is shown in Figure 1.

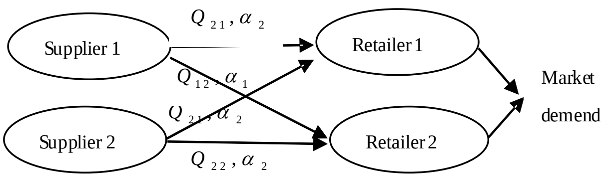

FIGURE 1 Supply chain network structure

There is lots of practical background about this model. For example, Haier and ChangHong are both in Gome and Suning's supply chain, and there is dramatic competition for market share between Suning and Gome. Similarly, two large supermarkets in a city sell the same products from two manufacturers.–This model in Figure 1 can be applied to analysis the competition decision between these enterprises. The assumptions for the 2-2 network model are listed below:

1) All parties are risk neutral. The two supply chains, suppliers and retailers are indexed by *i* and *j*, where $i, j \in \{1, 2\}$. There exists the possibility of disruption for the two suppliers. We use α_i to denote the reliability level of supplier *i*. With probability α_i, supplier *i* is "up" and fully fulfils retailer *i*'s order; with probability $1 - \alpha_i$, supplier *i* is "down" and retailer *i* receives no inventory. We say that "reliability is high" or "disruption risk is low" if α_i is high. We consider the general case where two suppliers have different reliability levels. When an interrupt occurs, we assume that the retailer's order cannot be met, which means that the retailer is completely disrupted. At the same time, we assume that there is a reliable spot market as an emergency supply points for the two retailers.

2) c_i is the unit delivery cost of the product of supplier *i* and ω_{ij} is the unit wholesale price of the product for retailer *i* offered by supplier *j*.

3) p_i is the fixed unit selling price of the product for retailer *i*. Q_{ij} is the order quantity of retailer *i* placed with supplier *j*. Q_{i3} is the inventory level after making an emergency order from the spot market. ω_{i3} is the fixed unit wholesale price of the product offered by the spot market. The unit goodwill cost of unmet demand is denoted by b_i. The surplus stock that remains unsold at the end of the period can be sold to a secondary market at a unit salvage value v_i. We always assume that $b_i < \omega_{ij}$.

(4) Marginal cost $\gamma_i c_i$ is incurred under supply disruption, where $0 < \gamma_i < 1$ denotes the total proportion of the marginal delivery cost of supply chain *i* in the event of a failure. We suppose that this part of costs is shared by the failing supplier and the retailer. The proportion of the cost incurred by the supplier *i* is η_i where $0 < \eta_i < 1$. This cost structure is different from that used in most of the literatures in which only the retailer assumes the cost in the event of a failure. However, this is not always true. In fact, before supply failures occur, both the retailer and suppliers usually have incurred some costs, which may include fixed set-up costs and variable costs. To simplify the analysis, we assume that all the setup costs are zero and all the variable costs in the event of a supply failure are proportional to the delivery cost and to the order quantity

Among the above variables, Q_{ij}, Q_{i3} and ω_{ij} is decision variables and the others exogenous variables, respectively, which are known to all the members of the supply chain. In this paper we focus on the revenues of two suppliers and two retailers. The revenue of the spot market and its delivery cost are not considered. The spot market is not a decision- maker in our paper. In addition, the retailer sells the product at a fixed price in the market in the selling season. Any unmet demand will incur a goodwill cost to the retailer. After the selling season, the residual product will be salvaged. We assume that $0 < v_i < c_i < \omega_{ij} < \omega_{i3} < p_i$.

In this article, the Nash game is obeyed by the two supply chains, while the Stackelberg game is subjected by the internal of the two supply chains. The sequence of events is as follows:

1) The two suppliers decide their individual wholesale prices simultaneously (stage 0);

2) The two retailers decide their individual order quantities with suppliers 1 and 2 simultaneously in anticipation of supply disruption and demand (stage 1);

3) The retailer *i* makes an emergency order from the spot market after a supply disruption but before demand occurs (stage 2).

2.2 MODEL OF DEMAND FUNCTION

Total demand D_i is assumed to be a positive stochastic random variable with probability density function $f(x)$ and the differentiable and strictly increasing cumulative distribution function $F(x)$, where $F_p(0) = 0$, $F_p(\infty) = 1$ and its reverse function is F_p^{-1}, $p = (p_1, p_2)$ denotes price vectors for two products, which means price influences on the demand. Obviously, F_p^{-1} is still strictly monotone increasing and second differentiable.

Due to different quality of these two kinds of products, consumer choice is influenced by their own salary level, product prices and the effect of brand. Hence, this paper always assumes that consumer demand for product *i* is influenced by product price p_i and quality level S_i. Denote consumer preference coefficient for product characteristics S_i by a. Then the consumer utility function for product *i* may be described as:

$$U_i = U_0 + aS_i - p_i , \tag{1}$$

where U_0 is the fixed utility of two kinds of product. Obviously, if $p_1 < p_2$ then $S_1 < S_2$, this shows that higher quality means higher price. Figure 2 illustrates the utility functions of two products, where a_0 is the intersection point of the utility function for two products, $a_0 \in (0, +\infty)$.

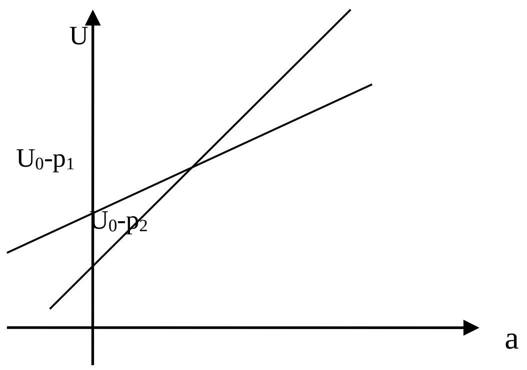

FIGURE 2 Utility function of the two products

Since different consumer has different preference coefficient in the market, consumer preference is considered to be a positive stochastic random variable with probability density function $h(a)$ and cumulative distribution function $H(a)$. From Figure 2, a_0 satisfies the following equality:

$U_0 + a_0 S_1 - p_1 = U_0 + a_0 S_2 - p_2$.

Hence:

$a_0 = (p_1 - p_2)/(S_1 - S_2)$.

Generally, consumers will choose the product with greater utility in the market. Thus two products market share λ_i can be obtained respectively:

$$\lambda_1 = \int_0^{a_0} h(a)da = H\left(\frac{p_1 - p_2}{S_1 - S_2}\right) = H(a_0),$$

$$\lambda_2 = \int_{a_0}^{+\infty} h(a)da = 1 - H(a_0) = 1 - H\left(\frac{p_1 - p_2}{S_1 - S_2}\right).$$

It is not difficult to see that if the quality standard S_i of product i increases, then the market share λ_i for the product i will increase, and therefore $\frac{d\lambda_i}{dS_i} > 0$. Similarly, if the quality standard S_j for competitor increases, then the market share λ_i for the product i will decrease, and therefore $\frac{d\lambda_i}{dp_i} < 0$. Furthermore, if the price p_i of the product i increases, then the market share λ_i for the product i will decrease, and $\frac{d\lambda_i}{dp_i} < 0$; if the price p_j for competitor increases, then the market share λ_i for the product i will decrease, which follows that $\frac{d\lambda_i}{dp_j} > 0$.

The market share of the two products will be fixed if product preferences are fixed. It is assumed that the market shares for the two products are λ_1 and λ_2 respectively. Note that $\lambda_i + \lambda_j = 1$. Hence, the demand function for product i and j can be expressed as

$$F_{ip}(x) = \mathrm{P}\{\lambda_i D \le x\} = F_p\left(\frac{x}{\lambda_i}\right), \tag{2}$$

$$f_{ip}(x) == \frac{1}{\lambda_i} f_p\left(\frac{x}{\lambda_i}\right). \tag{3}$$

It is easy to see that the demand is dependent on quality level, prices and consumer preferences of the two products. Through designing a reasonable price or improving the quality level, we can obtain a higher market share and the competitiveness for the supply chain in the market can be enhanced.

3 The sourcing strategy of the retailer

3.1 THE SOURCING STRATEGY OF THE RETAILER OF THE RETAILER i IN STAGE 2

Denote z_i as the inventory level of the supply chain before the emergency order is placed. Let $\pi_{R_i}^{II}(Q_{i3}|z_i)$ be random profit of the retailer i in stage 2, where the superscript "*II*" represents stage 2. We have:

$$\pi_{R_i}^{II}(Q_{i3}|z_i) = p_i E[\min(Q_{i3}, d_i)] + v_i E(Q_{i3} - d_i)^+ - b_i(d_i - Q_{i3})^+ - \omega_{i3}(Q_{i3} - z_i)^+$$

By a simple calculation, we can deduce the retailer i's expected profit in stage 2, denoted as $\Pi_{R_i}^{II}(Q_{i3}|z_i)$, which is given by:

$$\Pi_{R_i}^{II}(Q_{i3}|z_i) = \begin{cases} (p_i + b_i - \omega_{i3})Q_{i3} - (p_i + b_i - v_i) \times \int_0^{Q_{i3}} F_p\left(\frac{x}{\lambda_i}\right)dx + \omega_{i3} z_i - b_i E(D), & Q_{i3} \ge z_i \\ (p_i + b_i)z_i - (p_i + b_i - v_i) \times \int_0^{z_i} F_p\left(\frac{x}{\lambda_i}\right)dx - b_i E(D), & Q_{i3} \le z_i \end{cases}. \tag{4}$$

As a classical newsvendor problem, the retailer i 's sourcing problem in stage 2 is to choose the emergency order quantity to maximize its expected profit for any given initial inventory level z_i. We obtain that the order-up-to-level (OUL) policy is optimal for the retailer by using the first and second-order optimality conditions. The threshold value of the inventory level is:

$$\hat{Q}_{i3} = \lambda_i F^{-1}\left(\frac{p_i + b_i - \omega_{i3}}{p_i + b_i - v_i}\right)$$

Therefore, the optimal inventory level after the retailer placing an emergency order is as follows:

$$Q_{i3}^* = \max\{\hat{Q}_{i3}, z_i\}. \tag{5}$$

Then the maximum expected profit of retailer i in stage 2 for any given initial inventory level z_i is shown as follows:

$$\pi_{R_i}^{II*}(z_i)=\begin{cases}(p_i+b_i-\omega_{i3})\hat{Q}_{i3}-(p_i+b_i-v_i)\times\int_0^{\hat{Q}_{i3}}F_p\left(\frac{x}{\lambda_i}\right)dx+\omega_{i3}z_i-b_iE(D), & \hat{Q}_{i3}\geq z_i\\ (p_i+b_i)z_i-(p_i+b_i-v_i)\times\int_0^{z_i}F_p\left(\frac{x}{\lambda_i}\right)dx-b_iE(D), & \hat{Q}_{i3}\leq z_i\end{cases}. \qquad (6)$$

3.2 THE SOURCING STRATEGY OF THE RETAILER OF THE RETAILER i IN STAGE 1

The retailer i's sourcing problem in stage 1 is to choose the order quantities Q_{i1} and Q_{i2} from supplier i and supplier j to maximize its expected profit for any given wholesale price.

Initially, when disruptions occur at the same time to both suppliers (with probability $(1-\alpha_i)(1-\alpha_j)$ for any given wholesale price the expected profit $G_0(Q_{i1},Q_{i2})$ of the retailers is given by:

$$G_0(Q_{i1},Q_{i2})=$$
$$(1-\alpha_i)(1-\alpha_j)[\pi_{R_i}^{II*}(0)-(1-\eta)\gamma_ic_iQ_{i1}-(1-\eta)\gamma_jc_jQ_{i2}].$$

When disruption occurs solely to one supplier, the expected profit $G_1(Q_{i1},Q_{i2})$ of the retailers is given by:

$$G_1(Q_{i1},Q_{i2})=$$
$$\alpha_i(1-\alpha_j)[\pi_{R_i}^{II*}(Q_{i1})-\omega_{i1}Q_{i1}-(1-\eta)\gamma_ic_iQ_{i1}]+$$
$$(1-\alpha_i)\alpha_j\times[\pi_{R_i}^{II*}(Q_{i2})-\omega_{i2}Q_{i2}-(1-\eta)\gamma_jc_jQ_{i2}]$$

When the two supply chains do not face a disruption (with probability $\alpha_i\alpha_j$), the expected profit $G_2(Q_{i1},Q_{i2})$ of the retailers is given by:

$$G_2(Q_{i1},Q_{i2})=\alpha_i\alpha_j[\pi_{R_i}^{II*}(Q_{i1}+Q_{i2})-\omega_{i1}Q_{i1}-\omega_{i2}Q_{i2}]$$

Let $\pi_{R_i}^{I}(Q_{i1},Q_{i2})$ be the retailer i's expected profit in stage 1, where the superscript "I" represents stage 1. Then we have:

$$\pi_{R_i}^{I}(Q_{i1},Q_{i2})=G_0(Q_{i1},Q_{i2})+G_1(Q_{i1},Q_{i2})+G_2(Q_{i1},Q_{i2}). \qquad (7)$$

The optimization model, which represents the maximum of the total weighted expected profit (considering all possible combinations of disruption events on none, on one, or on both supply chains) is:

$$(p):\ (Q_{i1},Q_{i2})\in\arg\max\pi_{R_i}^{I}(Q_{i1},Q_{i2}),\ s.t\ \ Q_{i1}\geq 0, Q_{i2}\geq 0.$$

For the optimization problem (P), we have the following conclusions about the optimal sourcing strategy of the retailer i.

Theorem 1. After supply disruption has occurred, equilibrium sourcing quantity of the retailer i from the spot market satisfies the OUL policy and the threshold value of the inventory level is:

$$\hat{Q}_{i3}=\lambda_iF^{-1}[(p_i+b_i-\omega_{i3})/(p_i+b_i-v_i)].$$

The equilibrium sourcing strategies from suppliers 1 and 2 are as follows:

a) If $A<0$ and $B<0$, then two suppliers are placed with zero order quantity, where:

$$A=\alpha_i(\omega_{i3}-\omega_{i1})-(1-\alpha_i)(1-\eta_i)\gamma_ic_i,$$

$$B=\alpha_i(\omega_{i3}-\omega_{i1})-(1-\alpha_i)(1-\eta_i)\gamma_ic_i.$$

Thus, the retailer only sources from the spot market and the emergency order quantity is $\hat{Q}_{i3}$.

b) If $\alpha_i\alpha_j(\omega_{i3}-v_j)>B\geq 0>A$, then supplier 1 is placed with zero order quantity. The retailer only sources from supplier 2 and the equilibrium sources quantity from supplier 2 is $\hat{Q}_{i3}$.

c) If $\alpha_i\alpha_j(\omega_{i3}-v_j)>A\geq 0>B$, then supplier 2 is placed with zero order quantity. The retailer only sources from supplier 1 and the equilibrium quantity ordered from supplier 1 is $\hat{Q}_{i3}$.

d) When $Q_{i2}\leq\hat{Q}_{i3}\leq Q_{i1}$, if:

$$A\geq B\geq\alpha_jA\geq 0, \qquad (8)$$

then two suppliers are selected to be placed with positive orders, which are given by:

$$Q_{i1}^*=\lambda_iF_p^{-1}[\frac{p_i+b_i-\omega_{i3}}{p_i+b_i-v_i}+\frac{A-B}{\alpha_i(1-\alpha_j)(p_i+b_i-v_i)}], \qquad (9)$$

$$Q_{i2}^*=\lambda_i\{F_p^{-1}[\frac{p_i+b_i-\omega_{i3}}{p_i+b_i-v_i}+\frac{B}{\alpha_i\alpha_j(p_i+b_i-v_i)}]-$$
$$F_p^{-1}[\frac{p_i+b_i-\omega_{i3}}{p_i+b_i-v_i}+\frac{A-B}{\alpha_i(1-\alpha_j)(p_i+b_i-v_i)}]\}, \qquad (10)$$

e) When $Q_{i1}\leq\hat{Q}_{i3}\leq Q_{i2}$, if:

$$B\geq A\geq\alpha_iB\geq 0, \qquad (11)$$

then two suppliers are selected to be placed with positive orders, which are given by:

$$Q_{i2}^*=\lambda_iF_p^{-1}[\frac{p_i+b_i-\omega_{i3}}{p_i+b_i-v_i}+\frac{B-A}{\alpha_j(1-\alpha_i)(p_i+b_i-v_i)}], \qquad (12)$$

$$Q_{i1}^*=\lambda_i\{F_p^{-1}[\frac{p_i+b_i-\omega_{i3}}{p_i+b_i-v_i}+\frac{A}{\alpha_i\alpha_j(p_i+b_i-v_i)}]-$$
$$F_p^{-1}[\frac{p_i+b_i-\omega_{i3}}{p_i+b_i-v_i}+\frac{B-A}{\alpha_j(1-\alpha_i)(p_i+b_i-v_i)}]\}. \qquad (13)$$

Proof: We discuss the equilibrium sources strategies Q_{i1} and Q_{i2} of the retailers based on the following different cases.

Case1: $\hat{Q}_{i3} \ge Q_{i1} + Q_{i2}$.

In the profit function of retailer, G_0, G_1 and G_2 are respectively given by:

$$G_0(Q_{i1}, Q_{i2}) = (1-\alpha_i)(1-\alpha_j)[(p_i + b_i - \omega_{i2})\hat{Q}_{i3} - (p_i + b_i - v_i)\int_0^{\hat{Q}_{i3}} F_p\left(\frac{x}{\lambda_i}\right)dx - b_i E(D) - \gamma_i(1-\eta_i)c_i Q_{i1} - \gamma_j(1-\eta_j)c_j Q_{i2}],$$

$$G_1(Q_{i1}, Q_{i2}) = \alpha_i(1-\alpha_j)[(p_i + b_i - \omega_{i2})\hat{Q}_{i3} - (p_i + b_i - v_i)\times \int_0^{\hat{Q}_{i3}} F_p\left(\frac{x}{\lambda_i}\right)dx + \omega_{i3}Q_{i1} - b_i E(D) - \omega_{i1}Q_{i1} - \gamma_j(1-\eta_j)c_j Q_{i2}] + (1-\alpha_i)\alpha_j[(p_i + b_i - \omega_{i2})\times \hat{Q}_{i3} - (p_i + b_i - v_i)\int_0^{\hat{Q}_{i3}} F_p(\frac{x}{\lambda_i})dx + \omega_{i3}Q_{i2} - b_i E(D) - \omega_{i2}Q_{i2} - \gamma_i(1-\eta_i)c_i Q_{i1}]$$

$$G_2(Q_{i1}, Q_{i2}) = \alpha_i\alpha_j[(p_i + b_i - \omega_{i2})\hat{Q}_{i3} - (p_i + b_i - v_i)\times \int_0^{\hat{Q}_{i3}} F_p(\frac{x}{\lambda_i})dx - b_i E(D) + \omega_{i3}(Q_{i1} + Q_{i2}) - b_i E(D) - \omega_{i1}Q_{i1} - \omega_{i2}Q_{i2}]$$

It is easy to find that G_0, G_1 and G_2 are linear functions of the order quantities Q_{i1} and Q_{i2} in this case. So retailer i's expected profit $\pi^I_{R_i}(Q_{i1}, Q_{i2})$ is also a linear function of the order quantities Q_{i1} and Q_{i2}. From the equalities:

$$\frac{\partial \pi^I_{R_i}(Q_{i1}, Q_{i2})}{\partial Q_{i1}} = A, \quad \frac{\partial \pi^I_{R_i}(Q_{i1}, Q_{i2})}{\partial Q_{i2}} = B,$$

the following conclusions follow:

1) If $A \ge 0$ and $B \ge 0$ then the expected profit $\pi^I_{R_i}(Q_{i1}, Q_{i2})$ will increase as the order quantities Q_{i1} and Q_{i2} increase. Hence $Q^*_{i1} + Q^*_{i2} \ge \hat{Q}^*_{i3}$. On the other hand, we have $Q^*_{i1} + Q^*_{i2} \le \hat{Q}^*_{i3}$ from the assumption. Therefore, we have $Q^*_{i1} + Q^*_{i2} = \hat{Q}^*_{i3}$.

2) If $A < 0$ and $B < 0$, then the expected profit $\pi^I_{R_i}(Q_{i1}, Q_{i2})$ will increase as the order quantities Q_{i1} and Q_{i2} decrease. Hence $Q^*_{i1} = Q^*_{i2} = 0$.

3) If $A < 0$ and $B \ge 0$, then the expected profit $\pi^I_{R_i}(Q_{i1}, Q_{i2})$ will increase as Q_{i1} decreases or as Q_{i2} increases. Hence we have $Q^*_{i2} \ge \hat{Q}^*_{i3}$ and $Q^*_{i1} = 0$.

4) If $A < 0$ and $B \ge 0$ then the expected profit $\pi^I_{R_i}(Q_{i1}, Q_{i2})$ will increase as Q_{i2} decreases or as Q_{i1} increases. Hence $Q^*_{i1} \ge \hat{Q}^*_{i3}$ and $Q^*_{i2} = 0$.

Case 2: $\max\{Q_{i1}, Q_{i2}\} \le \hat{Q}_{i3} \le Q_{i1} + Q_{i2}$.

Similar to Case 1, we can obtain expressions of G_0, G_1 and G_2 in this case. The first-order partial derivatives of $\pi^I_{R_i}(Q_{i1}, Q_{i2})$ with respect to Q_{i1} and Q_{i2} are given by

$$\frac{\partial \pi^I_{R_i}(Q_{i1}, Q_{i2})}{\partial Q_{i1}} = \alpha_i\alpha_j[(p_i + b_i - \omega_{i3}) - (p_i + b_i - v_i)F\left(\frac{Q_{i1} + Q_{i2}}{\lambda_i}\right)] + A,$$

$$\frac{\partial \pi^I_{R_i}(Q_{i1}, Q_{i2})}{\partial Q_{i2}} = \alpha_i\alpha_j[(p_i + b_i - \omega_{i3}) - (p_i + b_i - v_i)F\left(\frac{Q_{i1} + Q_{i2}}{\lambda_i}\right)] + B.$$

But we cannot determine whether the Hessian matrix of $\pi^I_{R_i}(Q_{i1}, Q_{i2})$ is negative definite or not. So the equilibrium order quantities cannot be deduced by the first-order optimality condition.

From the assumption $\max\{Q_{i1}, Q_{i2}\} \le \hat{Q}_{i3} \le Q_{i1} + Q_{i2}$ and the analysis in Case 1, it is straightforward to deduce that $A \ge 0$. So we have:

$$\left.\frac{\partial \pi^I_{R_i}(Q_{i1}, Q_{i2})}{\partial Q_{i1}}\right|_{Q_{i1}+Q_{i2}=\hat{Q}_{i3}} = A \ge 0,$$

$$\left.\frac{\partial \pi^I_{R_i}(Q_{i1}, Q_{i2})}{\partial Q_{i1}}\right|_{Q_{i1}+Q_{i2}=+\infty} = \alpha_i\alpha_j(v_i - \omega_{i3}) + A.$$

Note that when $\alpha_i\alpha_j(\omega_{i3} - v_j) + A < 0$, $\frac{\pi^I_{R_i}(Q_{i1}, Q_{i2})}{\partial Q_{i1}}$ has a unique zero point as follows:

$$(Q_{i1} + Q_{i2})^* = \lambda_i F^{-1}\left(\frac{p_i + b_i - \omega_{i3}}{p_i + b_i - v_i} + \frac{A}{\alpha_i\alpha_j(p_i + b_i - v_i)}\right).$$

Furthermore, we deduce that

$$\frac{\partial \pi^I_{R_i}(Q_{i1}, Q_{i2})}{\partial Q_{i2}} = B - A.$$

Thus, we have the following conclusions:

1) If $B - A < 0$. i.e., $\frac{\pi^I_{R_i}(Q_{i1}, Q_{i2})}{\partial Q_{i1}} < 0$, then the expected profit will increase as Q_{i2}. Hence $Q^*_{i1} = (Q_{i1} + Q_{i2})^* \ge Q^*_{i3}$ and $Q^*_{i2} = 0$. But from the assumption $\max\{Q_{i1}, Q_{i2}\} \le \hat{Q}_{i3} \le Q_{i1} + Q_{i2}$, we have $A = 0$, $Q^*_{i1} = Q^*_{i3}$.

2) If $B-A\ge 0$, i.e. $\frac{\partial \pi_{R_i}^I(Q_{i1},Q_{i2})}{\partial Q_{i2}}<0$,then the expected profit will decrease as Q_{i2}. Hence $Q_{i1}^*+Q_{i2}^*=(Q_{i1}+Q_{i2})^*$ and $Q_{i2}^*\ge \hat{Q}_{i3}$.

Case 3: $Q_{i2}\le \hat{Q}_{i3}\le Q_{i1}$ and $Q_{i1}\le \hat{Q}_{i3}\le Q_{i2}$.

It is straightforward to verify that the Hessian matrix of $\pi_{R_i}^I(Q_{i1},Q_{i2})$ is negative definite. Hence, $\pi_{R_i}^I(Q_{i1},Q_{i2})$ is jointly concave to Q_{i1} and Q_{i2}. The equilibrium order quantity can be uniquely deduced by the first-order optimality condition.

$$\frac{\partial \pi_{R_i}^I(Q_{i1},Q_{i2})}{\partial Q_{i1}}=\alpha_i(p_i+b_i-\omega_{i3})-$$
$$\alpha_i\alpha_j(p_i+b_i-v_i)\times F_p\left(\frac{Q_{i1}+Q_{i2}}{\lambda_i}\right)- \tag{14}$$
$$\alpha_i(1-\alpha_j)\times(p_i+b_i-v_i)F_p\left(\frac{Q_{i1}}{\lambda_i}\right)+A=0,$$

$$\frac{\partial \pi_{R_i}^I(Q_{i1},Q_{i2})}{\partial Q_{i2}}=\alpha_i\alpha_j(p_i+b_i-\omega_{i3})-$$
$$\alpha_i\alpha_j(p_i+b_i-v_i)\times F_p\left(\frac{Q_{i1}+Q_{i2}}{\lambda_i}\right)+B=0. \tag{15}$$

From Equation (12) the unique optimal total order quantity is deduced as follows:

$$(Q_{i1}+Q_{i2})^*=\lambda_i F_p^{-1}\left[\frac{p_i+b_i-\omega_{i3}}{p_i+b_i-v_i}+\frac{B}{\alpha_i\alpha_j(p_i+b_i-v_i)}\right], \tag{16}$$

Substituting Equation (16) into Equation (14), it is easy to deduce Equation (9).

Moreover, since $Q_{i2}^*\ge 0\Leftrightarrow Q_{i1}^*\le(Q_{i1}^*+Q_{i2}^*)$, the equilibrium order quantity can be uniquely deduced via Equations (9) and (10) when Equation (8) holds. In the same way, we can obtain sourcing strategy of the retailer *i* when $Q_{i2}\le \hat{Q}_{i3}\le Q_{i1}$, i.e., conclusion (e) holds.

From above analysis for the three different cases, we reach the conclusions about the equilibrium sourcing strategy of the retailer.

From **Theorem 1**, we can easily obtain the conditions for both suppliers being placed with positive order quantities as follows.

Corollary 1. After supply disruption has occurred, both suppliers are placed with positive order quantities if and only if Equation (8) or Equation (9) holds.

From Theorem 1, it can be observed that:

1) Retailer *i* places with at most one supplier when $A<0$ and $B<0$. The reason is that the supplier's supply reliability is very low or its delivery cost is very high.

2) If $Q_{i1}^*+Q_{i2}^*>0$, then $Q_{i1}^*+Q_{i2}^*>\hat{Q}_{i3}^*$. This means that the total order quantity is not less than the threshold value $\hat{Q}_{i3}^*$, when any supplier is placed with a positive order quantity. This indicates that it prefers the supplier(s) to the spot market once the retailer selects one supplier or two suppliers.

(3) The sourcing strategy of the retailer is affected mainly by two key factors. They are

$$\alpha_i(\omega_{i3}-\omega_{i1})-(1-\alpha_i)(1-\eta_i)\gamma_i c_i$$

and

$$\alpha_j(\omega_{j3}-\omega_{j2})-(1-\alpha_j)(1-\eta_j)\gamma_j c_j.$$

The larger the value of a factor is, the more powerful the corresponding supplier is. So we regard these two factors as the competitiveness of the two suppliers. Furthermore, the other factors that affect supplier competitiveness consist of the unit delivery cost of the product for the supplier, the fixed wholesale price of the spot market, the total proportion of the marginal delivery cost, and the probability of delivering orders on time. The supplier can improve his competitiveness by improving his probability of delivering orders on time or decreasing his delivery cost. However, the marginal delivery cost usually increases when delivery is stable. Thus, a trade off exists between the marginal cost of delivery and the probability of on-time delivery.

In the following, we will indicate the impact of system parameters on order quantity in the case that both suppliers are placed with positive order quantities.

Corollary 2. If both suppliers are placed with positive order quantities, then the change trend of order quantity with system parameters is as follows:

1) $Q_{i1}^*(Q_{i2}^*)$ will increase as wholesale price $\omega_{i1}(\omega_{i2})$ decreases or as wholesale price $\omega_{i2}(\omega_{i1})$ increases.

2) $Q_{i1}^*(Q_{i2}^*)$ will increase as disruption probability $\alpha_i(\alpha_j)$ increase or as disruption probability $\alpha_j(\alpha_i)$ decreases.

3) $Q_{i1}^*(Q_{i2}^*)$ will increase as delivery cost $c_i(c_j)$ decreases or as delivery cost $c_j(c_i)$ increases.

4) $Q_{i1}^*(Q_{i2}^*)$ will increase as disruption probability $\gamma_i(\gamma_j)$ increase or as disruption probability $\gamma_j(\gamma_i)$ decreases.

4 Pricing strategy of the supplier

In this section, two suppliers set their individual wholesale prices simultaneously to maximize their respective expected profits before the retailer places its orders and the suppliers do not collude. This is a static non-cooperative game between two suppliers. When the order quantity cannot be met, setting of the wholesale prices will lose its meaning. So we only derive a sufficient condition for the existence of an equilibrium price strategy in the case that both suppliers are placed with positive order quantities in this section. The expected revenue function of supplier 1 in stage 0 in the case that both suppliers are placed with positive order quantities is given by:

$$\pi_{Si}(\omega_{i1},\omega_{j1}) = [\alpha_i(\omega_{i1}-c_i)-(1-\alpha_i)\eta_i\gamma_i c_i]Q_{i1}^* + [\alpha_i(\omega_{j1}-c_i)-(1-\alpha_i)\eta_i\gamma_i c_i]Q_{j1}^*,$$

$$\pi_{Sj}(\omega_{i2},\omega_{j2}) = [\alpha_j(\omega_{i2}-c_j)-(1-\alpha_j)\eta_j\gamma_j c_j]Q_{i2}^* + [\alpha_j(\omega_{j2}-c_j)-(1-\alpha_j)\eta_j\gamma_j c_j]Q_{j2}^*.$$

Therefore, the optimization model that two-supplier will set their individual wholesale prices simultaneously to maximize their respective expected profits after anticipating the order quantity of retailer is:

$$(M): \begin{cases} (\omega_{i1}^*,\omega_{j1}^*) \in \arg\max \pi_{S1}(\omega_{i1},\omega_{j1}) \\ (\omega_{i2}^*,\omega_{j2}^*) \in \arg\max \pi_{S2}(\omega_{i2},\omega_{j2}) \end{cases} .$$
$$s.t\ \ \omega_{i1}\geq 0, \omega_{j1}\geq 0, \omega_{i2}\geq 0, \omega_{j2}\geq 0$$

From above equations, we can derive the profit function of $\omega_{i1}(\omega_{j1})$ and $\omega_{i2}(\omega_{j2})$, but we cannot provide a close-form solution of $\omega_{i1}(\omega_{j1})$ and $\omega_{i2}(\omega_{j2})$. A sufficient condition for the existence of an equilibrium price strategy is as follows.

Theorem 2. In the environment of supply disruptions, if the equilibrium order quantity is a decreasing concave function of the wholesale price of its supply chain in the case that both suppliers are placed with positive order quantities, then the unique equilibrium prices satisfy the following conditions:

$$\alpha_i Q_{i1}^* + [\alpha_i(\omega_{i1}^*-c_i)-(1-\alpha_i)\eta_i\gamma_i c_i]\frac{\partial Q_{i1}^*}{\partial \omega_{i1}^*} = 0,$$

$$\alpha_i Q_{j1}^* + [\alpha_i(\omega_{j1}^*-c_i)-(1-\alpha_i)\eta_i\gamma_i c_i]\frac{\partial Q_{j1}^*}{\partial \omega_{j1}^*} = 0,$$

$$\alpha_j Q_{i2}^* + [\alpha_j(\omega_{i2}^*-c_j)-(1-\alpha_j)\eta_j\gamma_j c_j]\frac{\partial Q_{i2}^*}{\partial \omega_{i2}^*} = 0,$$

$$\alpha_j Q_{j2}^* + [\alpha_j(\omega_{j2}^*-c_j)-(1-\alpha_j)\eta_j\gamma_j c_j]\frac{\partial Q_{j2}^*}{\partial \omega_{j2}^*} = 0.$$

From **Theorem 2**, it can be observed that equilibrium price strategy exists and is unique if and only if the equilibrium order quantity is a decreasing concave function of the wholesale price of its supply chain. Many popular distributions in reality have this feature, such as uniform distribution and normal distribution.

5 Numerical examples

In this section we will use numerical example to verify the effectiveness of the conclusion. Suppose that the total demand follows the uniform distribution in the interval [300,400]. The basic parameter values are given as:

TABLE 1 System parameter values

Parameter	$\omega_{i3}(i=1,2)$	c_1	c_2	p_1	p_2	b_1	b_2	v_1	v_2	α_1	α_2	η_i	γ_i
values	16	10.5	12	18	16	5	4	3	4	0.3	0.9	0.2	0.4

Then distribution function of supplier i is given by:

$$F_{iP}(x) = F_P\left(\frac{5x}{2}\right) = \begin{cases} 0 & x<100 \\ \dfrac{x-100}{160} & x\in[100,160) \\ 1 & x\geq 160 \end{cases},$$

we can obtain equilibrium order quantity and equilibrium pricing by **Theorems 1** and **2**. They are $Q_{i1}^* = 201.60$, $Q_{i2}^* = 127.36$, $\omega_{i1}^* = 8$, $\omega_{i2}^* = 10$. This has fully demonstrated the effectiveness of our theorems in actual situation. Next we verify the validity of Corollary 2 in Figure 3-7.

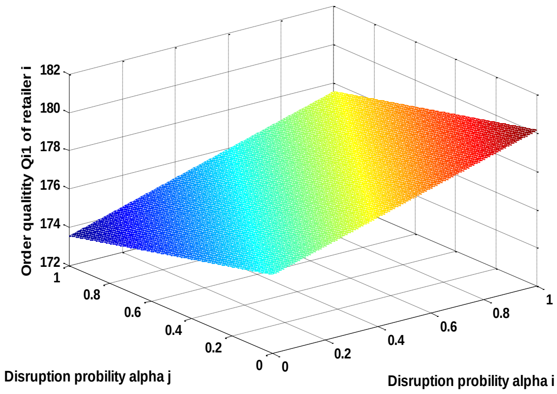

FIGURE 3 The change trend of retailer's order as α_i and α_j

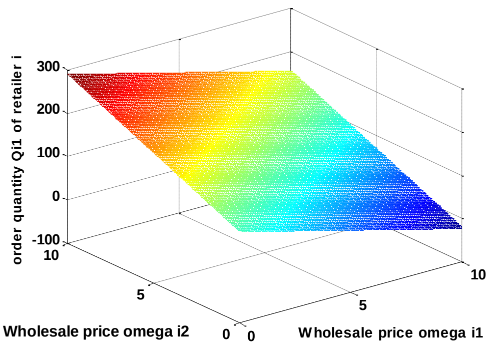

FIGURE 4 The change trend of retailer's order as ω_{i1} and ω_{i2}

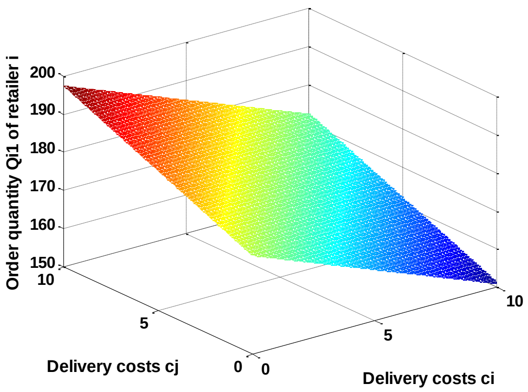

FIGURE 5 The change trend of retailer's order as c_i and c_j

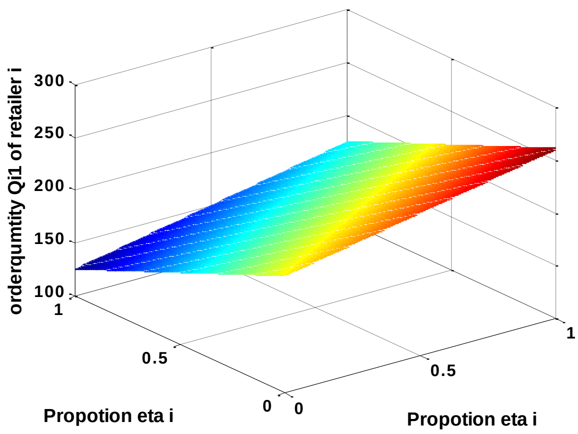

FIGURE 6 The change trend of retailer's order as η_i and η_j

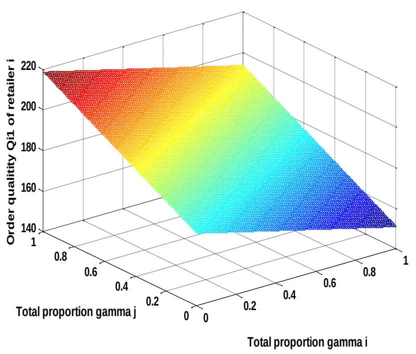

FIGURE 7 The change trend of retailer's order as γ_i and γ_j

From Figure 3-7, we can verify the validity of **Corollary 2**. In addition, the following conclusions can be obtained:

1) Supply disruption probability α_i on the order Q_{i1} of retailers i has more influence than competitive supply chain disruption probability α_j by comparing in Figure 3.

2) The wholesale price ω_{i1} on the order Q_{i1} of retailers i has more influence than competitive supply chain wholesale price ω_{i2} by comparing in Figure 4.

3) The delivery cost c_i on the order Q_{i1} of retailers i has the same influence with competitive supply chain delivery cost c_j by comparing in Figures 5.

4) On the whole, sourcing strategies Q_{i1} of the retailer i are affected mainly by disruptions probability, delivery costs and wholesale, but parameters $\eta_i(\eta_j)$ and $\gamma_i(\gamma_j)$ has less influence.

6 Conclusion

An effective sourcing strategies enhancing supply chain resilience is a necessary component of a firm's overall hedging strategy. This paper investigates sourcing strategies of the two retailers and the pricing strategies of the two suppliers in a 2-2 supply chain network under an environment of supply disruption. We obtain a sufficient condition for existence of an equilibrium sourcing and pricing strategies. The results show that equilibrium sourcing and pricing strategies are affected mainly by disruptions probability and delivery costs. Therefore, the appropriate parameters should be designed to obtain the optimal strategy in the actual operation of the market. These finding can guide suppliers to find a trade-off between the wholesale price and order quantity and a trade-off between the probability of on-time delivery and the marginal delivery cost.

We believe that several promising avenues exist for further research in this field. How to devise a mechanism to coordinate the whole channel is a potential topic for the research in the future. Furthermore, extensions can be made which includes multi-period problem or risk-averse participants.

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Authors

Ling Hou, born in June, 1978, Suihua, Heilongjiang province, China

Current position, grades: Lecturer in Department of Applied Mathematics, Harbin University of Science and Technology, China.
University studies: PhD from School of Management Harbin University of Science and Technology, China.
Scientific interest: Supply chain management, inventory, marketing science, and risk management.
Pyblications: 6

Meng Li, born in July, 1978, Harbin, Heilongjiang province, China

Current position, grades: Lecturer in Department of Applied Mathematics, Harbin University of Science and Technology, China.
University studies: PhD from University of Tsukuba, Japan.
Scientific interest: Supply chain management, mathematical finance and risk management.
Pyblications: 10 included in SCI or EI.

Dongyan Chen, born in February, 1964, Ganzhou, Jiangsu province, China

Current position, grades: Professor and doctoral tutor in Applied Science College, Harbin University of Science and Technology, China.
University studies: PhD in Harbin institute of technology, China.
Scientific interest: Supply chain management, optimization methods, robust control and time-delay system control.
Pyblications: more than 100