Chaos control within finite time of the chaotic financial system based on CLF theory

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Abstract

In this paper, we deal with the finite-time chaos control in the chaotic financial system. The control law are proposed to drive chaos to equilibria within finite time based on the control Lyapunov function (CLF) theory. Numerical simulations are provided to show the effectiveness of the proposed controller.

Keywords: Finite-time control, Control Lyapunov function, chaotic financial system

1 Introduction

Chaos synchronization has been a hot topic since the pioneering work of Pecocra and Carroll [1] variety of method and techniques have been proposed for the control and synchronization of chaotic systems [2-14]. These methods can stabilize the chaotic system asymptotically. In practical engineering process, we may want to stabilize the system as quickly as possible. Therefore, more and more people began to realize the important role of synchronization time. To attain fast convergence speed, many effective methods have been introduced and finite-time control is one of them.

In [8], Wang et al. proposed a continuous controller to realize finite-time synchronization of the unified system with bounded uncertain disturbed parameters based on CLF. In [9], Feng et al. proposed finite-time synchronization of two chaotic systems by using a terminal sliding mode controller and they adopted the Genesio chaotic system as their example. In [10], Gilles et al. proposed finite-time global chaos synchronization for piecewise linear maps and discrete chaotic systems. In this paper, we present a controller to realize finite-time chaos control of the chaos financial system model based on CLF. The controller is robust to noise and simple to be constructed.

2 The chaos financial system model

In [10], Ma et al. have studied the bifurcation and chaotic structure of the chaotic financial system around the equilibrium referring to formula (1):

$$\begin{cases} \dot{x} = z + (y-a)x \\ \dot{y} = 1 - by - x^2 \\ \dot{z} = -x - cz \end{cases} \tag{1}$$

Where $a(\geq 0)$ is the saving amount, $b(\geq 0)$ is the cost of per-investment, and $c \geq 0$ is the demand elasticity. In the model, the interest rate expresses as variable x, the investment demand expresses as variable y, and the price exponent expresses as variable z. The result of their study shows: System (1) has only one balance point $P = (0, \frac{1}{b}, 0)$ when $c - b - abc \leq 0$, and it will appear stable sink, saddle, un-hyperbolic unstable balance point, bifurcation, chaos attractor, Hopf bifurcation, and periodic solution family at the balance point $P = (0, \frac{1}{b}, 0)$ when $c - b - abc \leq 0$, and its stability is decided by the sign of R:

$$R = \left(\frac{1}{8}\right)\left\{\frac{\sqrt{1-c^2}[2(a+2c)+b]b}{4(1-c^2)+b^2} - a - c - \frac{2(1-c^2)[2(a+2c)+b}{4(1-c^2)+b^2}\right\} \tag{2}$$

For system (1) when the parameter a or c satisfies:

$$d = \alpha'(0) = -1, \mu_2 = -\frac{R}{d} = R, c_0 + a_0 - \frac{1}{b_0} = 0,$$

Hopf bifurcation occurs in the system. It is bifurcated into an unstable limit cycle near the balance point when $R > 0$. The system is bifurcated into a stable limit cycle near the balance point when $R < 0$.

3 Preliminary definitions and lemmas

The meaning of finite-time chaos control is that the state of the dynamic system converges to a desired target

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within a finite time. First, some necessary definitions and lemmas are introduced.

Definition 1. For the following controlled chaotic system:

$$\dot{x} = f(x) + u, \tag{3}$$

Where $x \in R^n$, u is a controller. If there exists a constant $T > 0$ (T may depend on the initial state $x(0)$), such that

$$\lim_{t \to T} \|x(t) - E\| = 0,$$

and $\|x(t) - E\| \equiv 0$, if $t > T$, such that the chaotic trajectories of the original system is driven to the equilibrium E, then chaos control of the system (2) is achieved within finite time, where E stand for equilibria of the original chaotic system $\dot{x} = f(x)$.

Definition 2[11] let $V(x): R^n \to R^+$ be a continuous function. V is positive definite if $V(0) = 0$ and $V(x) > 0$ for $x \neq 0$; $V(x)$ is proper if $V(x) \to \infty$ as $\|x\| \to \infty$.

Definition 3[11] Consider the nonlinear affine system described by

$$\dot{x} = f(x) + g(x)u, \tag{4}$$

Where $x \in R^n$ and $u \in R^m$ are the state and the input of the system, respectively. A positive-definite function $V(x): R^n \to R^+$ is a control Lyapunov function (CLF) of system (4) if it is smooth, proper, and satisfies:

$$L_g V(x) = 0,\ x \neq 0 \Rightarrow L_f V(x) < 0,$$

where $f(x)$ and $g(x)$ are two vector function from R^n to R^n, $L_f V(x)$ and $L_g V(x)$ denote the Lie derivative of $V(x)$ along $f(x)$ and $g(x)$, i.e.,

$$L_f V(x) = \frac{\partial V}{\partial x} f(x) = \frac{\partial V}{\partial x_1} f_1(x) + \frac{\partial V}{\partial x_2} f_2(x) + \cdots \frac{\partial V}{\partial x_n} f_n(x),$$

$$L_g V(x) = \frac{\partial V}{\partial x} g(x) = \frac{\partial V}{\partial x_1} g_1(x) + \frac{\partial V}{\partial x_2} g_2(x) + \cdots \frac{\partial V}{\partial x_n} g_n(x).$$

$L_f V(x)$ And $L_g V(x)$ are still two functions from R^n to R.

An alternative definition of CLF is as follows:[13]

$$\inf_u \left\{ \frac{\partial V}{\partial x} (f(x) + g(x)u) \right\} < 0.$$

It is exactly, from the definition, an extension of the Lyapunov function in a controlled system

Lemma 1[9] Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -cV^{\eta}(t)\ \ \forall t \geq t_0, V(t_0) \geq 0,$$

where $c > 0$, $0 < \eta < 1$ all constants are then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t - t_0),\ t_0 \leq t \leq t_1,$$

And $V(t) \equiv 0$ $\forall t \geq t_1$ with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}.$$

We can obtain the following conclusion from Lemma 1.

$$\inf_u \left\{ \frac{\partial V}{\partial x} (f(x) + g(x)u) \right\} < -cV^{\eta}(x),$$

where $x \in R^n \setminus 0$. We call such a $V(x)$ is a finite-time control Lyapunov function (f-CLF). An equivalent description of f-CLF is that if there exists a positive function $V(x)$ satisfying

$$L_g V(x) = 0,\ x \neq 0 \Rightarrow L_f V(x) < -cV^{\eta}(x).$$

4 Finite-time controlling chaos to the equilibrium base on CLF

In this section, effective feedback controllers are designed to drive the chaotic trajectories to equilibria within finite time.

Consider the following controlled system of system (1):

$$\begin{cases} \dot{x} = z + (y - a)x + u_1 \\ \dot{y} = 1 - by - x^2 + u_2 \\ \dot{z} = -x - cz + u_3 \end{cases} \tag{5}$$

Let

$$\begin{cases} x_1 = x - m_1, \\ y_1 = y - m_2, \\ z_1 = z - m_3, \end{cases}$$

where (m_1, m_2, m_3) stand for the equilibrium of system (1).

Then system (5) yields the following system

$$\begin{cases} \dot{x}_1 = z_1 + m_3 + (y_1 + m_2 - a)(x_1 + m_1) + u_1, \\ \dot{y}_1 = 1 - b(y_1 + m_2) - (x_1 + m_1)^2 + u_2, \\ \dot{z}_1 = -(x_1 + m_1) - c(z_1 + m_3) + u_3. \end{cases} \tag{6}$$

Firstly, we design

$$u_1 = -m_3 - (y_1 + m_2 - a)(x_1 + m_1) - \beta(\frac{1}{2} x_1^2)^{\eta} \Big/ x_1,$$

where $\beta > 0, 0 < \eta < 1$, β and η can be chosen suitably to regulate the synchronization time. Then system (6) can be written in a compact form with this u_1 as

$$\dot{Y} = f(Y) + B\tilde{u},$$

$$Y = \begin{pmatrix} x_1 & y_1 & z_1 \end{pmatrix}^T, \tag{7}$$

where $f(Y) = \begin{pmatrix} z_1 - \beta(\frac{1}{2}x_1^2)^\eta \big/ x_1 \\ 1 - b(y_1 + m_2) - (x_1 + m_1)^2 \\ -(x_1 + m_1) - c(z_1 + m_3) \end{pmatrix}$,

$$B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\tilde{u} = \begin{pmatrix} u_2 \\ u_3 \end{pmatrix}.$$

Consider the following positive function for system (7)

$$V(Y) = \frac{1}{2}x_1^2 + \frac{1}{2}y_1^2 + \frac{1}{2}z_1^2. \tag{8}$$

Then $\frac{\partial V}{\partial Y}B = 0,\ Y \neq 0 \Rightarrow y_1 = z_1 = 0,\ x_1 \neq 0.$

So when $\frac{\partial V}{\partial Y}B = 0,\ Y \neq 0$, we have:

$$\frac{\partial V}{\partial Y}f(Y) = -z_1^2 - \beta(\frac{1}{2}x_1^2)^\eta < -\beta(\frac{1}{2}x_1^2)^\eta.$$

Then the positive function (8) can be an f-CLF of system (7).

Proposition1. The system (7) can be finite-timely stabilized by the following feedback

$$\tilde{u}(Y) = \begin{cases} -(L_B V)^T \dfrac{L_f V + \beta V^\eta}{\|L_B V\|^2}, & L_B V \neq 0, \\ 0, & L_B V = 0. \end{cases} \tag{9}$$

Proof. Consider the following f-CLF for the system (7)

$$V(Y) = \frac{1}{2}x_1^2 + \frac{1}{2}y_1^2 + \frac{1}{2}z_1^2.$$

At arbitrary nonzero Y, calculating the derivation of $V(Y)$ along the trajectory of system (6) , we can get:

$$\frac{dV(Y)}{dt} = \frac{\partial V}{\partial Y}f(Y) + \frac{\partial V}{\partial Y}B\tilde{u} = L_f V - L_B V (L_B V)^T \frac{L_f V + \beta V^\eta}{\|L_B V\|^2} = L_f V - L_f V - \beta V^\eta = -\beta V^\eta$$

From Lemma 1, we know that the system (6) can be finite-timely stabilized by feedback (7); that is, the system (6) can finite-timely stabilize to equilibria by the designed controller.

We can obtain the following designed controller from Proposition 1 $u_2 = -ky_1,\ u_3 = -kz_1,$ where

$$k = [x_1 z_1 + m_3 x_1 + (y_1 + m_2 - a)(x_1 + m_1)x_1 - \beta(0.5x_1^2)^\eta + y_1 - by_1(y_1 + m_2) - (x_1 + m_1)^2 y_1 - \\ -(x_1 + m_1)z_1 - cz_1(z_1 + m_3) + \beta(V(Y))^\eta] \big/ (y_1^2 + z_1^2)$$

For this numerical simulation, fourth-order Runge-Kutta method is used to our simulation with time step being equal to 0.001. In this control scheme, the initial value of system (1) is $(3,1,6)$, when $a = 0.45$, $b = 0.25$, $c = 1.1$, the system (1) has a chaotic attractor, and the equilibrium of system (1) are:

$$P_1(0,4,0),\ P_2(0.8125, 1.3591, -0.7386),\ P_3(-0.8125, 1.3591, 0.7386).$$

We assume $\beta = 4,\ \eta = 1/2$. Figs.1-3 show that chaos are finite-timely suppressed to equilibrium point P_1, P_2, P_3.

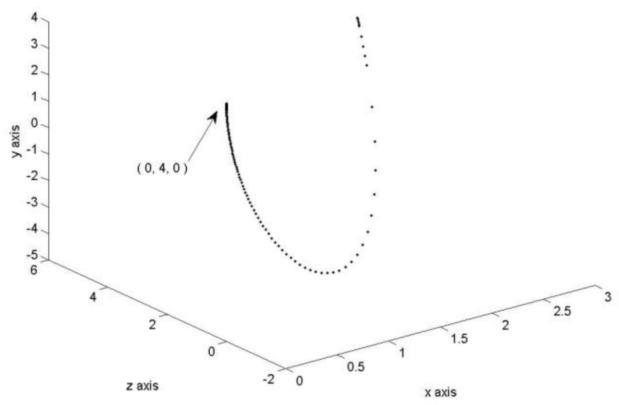

FIGURE 1 Finite-time stabilizing chaos to equilibria $P_1(0,4,0)$

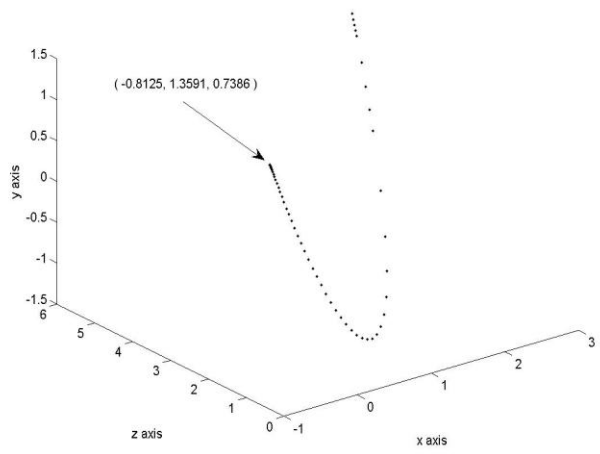

FIGURE 2 Finite-time stabilizing chaos to equilibria $P_2(0.8125, 1.3591, -0.7386)$

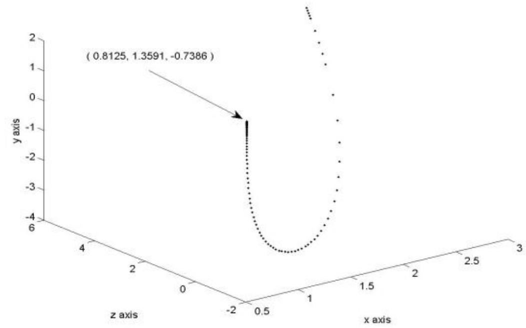

FIGURE.3. Finite-time stabilizing chaos to equilibria $P_3(-0.8125, 1.3591, 0.7386)$

5 Conclusions

In this paper, we have proposed the control law to realize finite-time chaos control of the chaotic financial system based on CLF. Of course, these methods also can be applied to the other chaotic system from the proof process.

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