The improvement of particle swarm optimization algorithm based on stability analysis

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Received 1 March 2013, www.cmnt.lv

Abstract

Particle swarm optimization(PSO) is a very important swarm intelligence algorithm which plays an effective role in searching the optimum piont of space systems. The key to search for the optimum piont is the behavior of each particle as well as the entire swarm. During their searching, the stability of the particles is the premise to ensure the convergence of the system. Only under the condition that the whole searching process is of stable convergence does pso algorithm effectively find the global optimum. This paper analyzes the relationship between PSO parameters from the aspect of stability and achieves the goal of ensuring the stable convergence of the algorithm.

Keywords: (PSO, swarm intelligence ,Lyapunov theory, constraint parameters)

1 Introduction

The particle swarm optimization is an intelligence optimazation algorithm proposed by Kennedy and Eberhart in 1995[1,2]. PSO algorithm, inspired by the swarming behavior of social groups, simulates the foraging behavior of flocks of birds which collaborate to achieve optimal state in their search for food. PSO is based on the iteration of the swarm, where particles, members of the swarm, work in collaboration to search in the space towards the optimal point determined by the best particle with the full use of the group intelligence[3,4]. Owning to its characterristics of being simple and easy to be implemented, PSO has great advantage in many engineering applications and can be applied to many problems such as nonlinear continuous optimization, combinatorial optimization and others. It is also widely used in classification recognition, pattern recognition, multi-objective optimization, signal processing, nonlinear control, automation control, detection technology, artificial intelligence, self-adaptive learning, data fusion and other aspects [5-8]. However, the standard PSO model shows defects in the fact that the particle swarm is prone to local optimum and premature convergence.

There is much literature on improving the PSO algorithm.In 2000, Eberhart and Shi employed constriction factor in the PSO iterative equation to constrain the velocity and position of the particles, resulting in the improvement of the PSO performance [9]. In 2001, Shi further applied fuzzy self-adaptive regulation w for the better result in the unimodal function [10]. Angeline considered the selection operator in his research which transferred characteristics of good particles selected in each iteration to the next generation, so that the particles maintained a good performance [11]. Clerc and Kennedy considered a deterministic approximation of the swarm dynamics by treating the random coefficients as constants, and studied stable and limit cyclic behavior of the dynamics for the settings of appropriate values to its parameters [12]. Trelea adopted the discrete-time dynamic system theory to study the convergence and robustness of standard PSO algorithm, and emphasized the importance of parameter selection on the convergence of the system [13]. Kong Ying adopted a hybrid learning algorithm combining Particle Swarm Optimization with BP. Through the comparison between predictive and the experimental data and the scroll efficiency experiment, the proposed prediction method is validated and can be successfully used to improve Pneumatic conversion efficiency[14]. Alfi incorporated an adaptive mutation mechanism and a dynamic inertia weight into the standard PSO method to enhance global search ability and to increase accuracy [15]. Ghosh, S. et al. proposed a state-space model of the lbest PSO, and concluded that the use of control theory can ensure the stability and convergence of the particle dynamics [16].

The motion of PSO particles tends to be stable and convergent during their searching towards the optimal point in wide space. These improvements are directly or indirectly related to choosing the parameters of the standard PSO algorithm. So the mutual dependence and the range of PSO parameters are crucial to the convergence of the algorithm and the search for the optimal point. In this paper, we focus on the relationship between parameters.

2 Lyapunov Theorem And Pso Stability

2.1 LYAPUNOV THEORY

The equation of state is $\dot{\chi} = A\chi$,where χ is the state vector in n-dimensional space, and A is $n \times n$ constant coefficient and non-singular matrix. The system is at the stable equilibrium point, i.e. the convergence point when χ becomes zero. The necessary and sufficient condition of the asymptotic stability in a wide range is that if given a real symmetric positive definite matrix Ω, there exists a real symmetric positive definite matrix G. If the two matrices satisfy the equation $\Omega = -(A^T G + GA)$, the scalar function equals to $V(\chi,t) = \chi_t^T G \chi_t$. The scalar function $V(\chi,t)$ is a Lyapunov function, where t is iterative time step, describing the change of the system, and A is the coefficient matrix of the internal parameters of the system, which shows the relationship between the state variables of the space and the variation of the state variables in their movement in the space, and is influenced by the system model and mechanism, the spatial structure of the model, and etc.

Using the scalar function $V(\chi) = \chi^T G \chi$, where: χ^T is the transpose of χ. If $V(\chi) > 0$, then G is positive definite. Suppose that Ω is positive definite, and satisfies the following relationship:

$$\begin{aligned} \dot{V}(\chi) &= \dot{\chi}^T G\chi + \chi^T G\dot{\chi} = \chi^T A^T G\chi + \chi^T GA\chi \\ &= \chi^T (A^T G + GA)\chi = -\chi^T \Omega \chi < 0 \end{aligned}, \tag{1}$$

$$\Omega = -(A^T G + GA) \tag{2}$$

The system tends to be asymptotically stable. The way to decide whether Ω is a positive-definite matrix or not can accord to Sylvester criteria, which defines the necessary and sufficient condition of a positive-definite matrix is that its each leading principal minor is positive.

2.2 CONVERGENCE AND STABILITY ANALYSIS

The problem of convergence can be transformed into that of stability. In other words, the stability theory can be used to ensure the convergence of the system. Under the condition that $\Gamma|\chi_n|^\gamma < \infty$, $\Gamma|\chi^*|^\gamma < \infty$, and $\gamma \forall C$, we have the equation: $\lim_{n\to\infty} \Gamma|\chi_n - \chi^*|^\gamma = 0$. Then γ order of $\{\chi_n\}$ is converged at χ^*.

Where: $\{\chi_n\}$ is a random variable domain; χ^* is a target value; Γ represents a transformation; C is a constant domain.

The relationship of any optimization can be defined by: $f(\chi) \ge f(\chi^*)$, $\forall \chi \in R^n$.

If in continuous systems, $\frac{\partial f(\chi^*)}{\partial t} = \Phi(\chi^*) = 0$ and in discrete systems, $\nabla f(\chi^*) = \Phi(\chi^*) = 0$. Then χ^* here is the stable point of the system. Using an arbitrarily small real number ε which satisfies $\varepsilon \subseteq \mathrm{R}^n$, $\varepsilon > 0$, and $\psi(\chi_{k,t}^D) < \varepsilon$. Where; $\chi_{k,t}^D$ is an acceptable expected value achieved through t times of iteration by χ_k in dimension D. In the entire search domain, if the value range of χ is $\chi \subseteq [a,b] \subseteq R^n$ (A, B refers to the boundary of the particular domain), and if there exist $\liminf_{t\to\infty} \|\Phi(\chi_{k,t})\| = 0$ and $\limsup_{t\to\infty} \|\Phi(\chi_{k,t})\| = \varepsilon$, the convergence can be reached at $\chi_{k,t}^D$ of D-dimensional space state.

The stability is the precondition of convergence. We discover because Lyapunov function can guarantee the convergence of state parameters during their movement in the space, the PSO algorithm applies the state space. In given search domain, Lyapunov function is used to constrain the motion of each particle of the swarm, which can ensure the PSO algorithm tends to be stable and converged, and finally achieve the convergence in the global search.

3 The relationship between pso parameters

3.1 THE DESCRIPTION OF STANDARD PSO

PSO algorithm is a very effective swarm intelligence algorithm. The particles gradually accumulate at the global optimum during their search in a given space, continuously adjusting their position and speed, and taking into account the change of the best position in the possible search domain. Their motion is gradually converged and stable, which may well be analyzed by using Lyapunov theory. Based on the analysis of the convergence above, we analyze the convergence of the standard PSO algorithm, and further propose the guideline on parameter selection.

The standard PSO[1] is defined by the following (3) and (4):

$$\begin{aligned} \upsilon_i(t+1) &= \omega \cdot \upsilon_i(t) + c_1 r_1 (P_{Pbest,t} - \chi_i(t)) \\ &+ c_2 r_2 (G_{gbest,t} - \chi_i(t)) \end{aligned}, \tag{3}$$

$$\chi_i(t+1) = \chi_i(t) + \upsilon_i(t+1). \tag{4}$$

For the standard PSO algorithm, every particle, when searching in the space, adjusts its optimal position by P_{pbest} and G_{gbest}, and strengthens the communication and interaction with other particles by G_{gbest}, and as a result, all particles can approach the global optimum more quickly. So that P_{pbest}, G_{gbest} can ensure particles are interrelated, and at the same time independent on each dimension of the space.

In order to accurately characterize the search of the particle swarm on D-dimensional space, and maintain the generality of our analysis, we deal with the analysis of convergence and stability in a one-dimensional space. Analyzing the behaviour of a particle in one-dimensional space, we can indirectly analyze the trends of the swarm in the space search. According to the analysis of the literature, the iterative method is adopted in the implementation of the PSO algorithm. So long as the particles remain relatively stable in each iteration, the state in each dimension is mutually stable in D-dimensional space.

3.2 DETERMINING PSO PARAMETERS ON THE BASIS OF LYAPUNOV

Derivation of the basic formula of the PSO algorithm:

$$\hat{\upsilon}_i(t+1) = \omega \cdot \hat{\upsilon}_i(t) + \xi \cdot (\psi - \hat{\chi}(t)), \tag{5}$$

$$\hat{\chi}_i(t+1)=\hat{\chi}_i(t)+\hat{\upsilon}_i(t+1)\,, \tag{6}$$

where: $\xi=\xi_1+\xi_2$ and $\xi_1=r_1\cdot c_1$, $\xi_2=r_2\cdot c_2$

Suppose:

$$\psi=\frac{(\xi_1\cdot P_{pbest}+\xi_2\cdot G_{gbest})}{\xi}\,. \tag{7}$$

According to the recurrence relation:

$$\hat{\upsilon}_i(t+2)=\omega\cdot\hat{\upsilon}_i(t+1)+\xi\cdot(\psi-\hat{\chi}(t+1))\,. \tag{8}$$

According to the equation of the state of discrete systems, we choose the state variables $\varphi(t)$ to satisfy the following relationship:

$$\Phi(t+1)=\begin{bmatrix}\varphi_1(t+1)\\ \varphi_2(t+1)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -\omega & \omega+1-\xi\end{bmatrix}\cdot\begin{bmatrix}\varphi_1(t)\\ \varphi_2(t)\end{bmatrix}. \tag{9}$$
$$=\mathrm{A}\cdot\Phi(t)$$

Because of their programming needs, the researchers usually discredited the continuous systems and conduct recursive analysis. For a system object, the motion of the particles is within the scope of linear space. In other words, there should not be any changes to their own characteristics. The motion of the particles is continuous.

In accordance with Lyapunov Function[18,19] ,let $G=\begin{bmatrix}g_{11} & g_{12}\\ g_{21} & g_{22}\end{bmatrix}$ Especially, $\Omega=-\frac{1}{2}\mathrm{I}$. I is the identity matrix.

In light of the definition of the Lyapunov Function, the following are required to guarantee the stability of the particles in the wide range of n-dimensional space.

If $g_{11}>0$ and $g_{11}\cdot g_{22}-g_{12}\cdot g_{21}>0$, then there can be deduced the constraint condition $\omega+1-\xi<0$, i.e. $\omega+1<\xi$, which can ensure the stability of the particle swarm in space search, its good convergence and tendency to a stable point.

In addition, based on the limit superior of the parameters derived in Frans Van den Bergh's doctoral thesis (Van den Bergh Frans,2001)[20]:

$$\xi_{\text{criteria}}=\sup\xi\ \left|0.5\xi-1<\omega,\xi\in(0,c_1+c_2]\right.. \tag{10}$$

It can be deduced that there exists $\sup(\xi)=2\cdot(\omega+1)$, so ξ has a closed range of values, i.e. $\xi\in(\omega+1,2\omega+2)$.

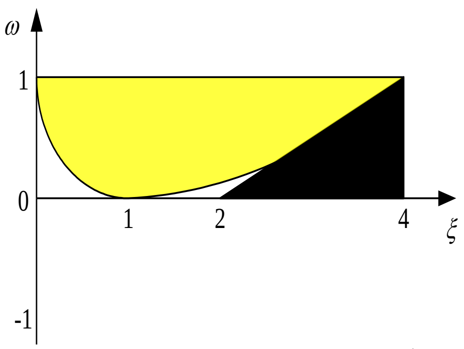

FIGURE 1 The relationship between ω and ξ in Frans Van den Bergh' dissertation.

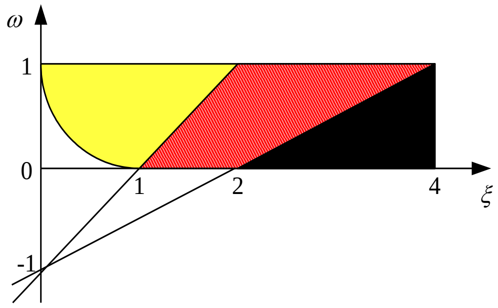

FIGURE 2The relationship between ω and ξ based on Lyapunov theory.

Figure 1 depicts the relationship between ω and ξ set forth in the dissertation. In the black area, the particles are less likely to converge, and algorithm as a whole is not easy to find the optimal solution. The red region in Figure 2 shows the following relationship derived in this paper:

$$\omega+1<\xi<2\cdot(\omega+1)\,, \tag{11}$$

where: $0<\omega\leq 1$ and $0<\xi\leq 4$. The relationship expressed in the red region improves the computing of the algorithm to a certain extent, and can ensure that the particles achieve stable convergence in space search.

When parameters vary within the range of Eq. (11), the flight speed and positions of the particles change in each iteration. This improvement helps the algorithm to overcome the defect of local optimum; for the system as a whole, it not only takes into account the randomness of the PSO algorithm, but also guarantees the stability of the particles during their motion in the space,and further ensures the convergence of the algorithm.

4 The improvement of the pso algorithm and simulation

4.1 THE CONSTRAINT PSO ALGORITHM

Based on the analysis above, the improvement of the standard PSO algorithm is on the constraint of parameters (referred to as the constraint PSO algorithm),which is presented here.

Constraint PSO Algorithm

```
Begin initializes PSO
 Sub_fun ( )
   { Set  ω , c1 , c2
Random r1 , r2
        Calculate ξ , ψ  Eq. (7)
     If  ( Satisfy  ξ ∈ (ω+1, 2ω+2) )
          Updata  Eq.(5)(6)          }
 While (not satisfy termination conditions)
   { Calculate each particle's fitness
     Calculate  G_gbest
     Call Sub fun ( )                 }
 Output fitness ( χ_gbest )
```

4.2 FUNCTION TEST AND ANALYSIS

Rastrigrin function, having approximately $10n$ local minimum. Because it is a typical non-linear multi-modal function, and its peaks are variable, going ups and downs, it is not easy to find the global optimum.

The description of the function:

$$f(x_i)=\sum_{i=1}^{n}[x_i^2-10\cdot\cos(2\cdot\pi\cdot x_i)+10]$$

$$-5.12\le x_i\le 5.12$$

its global minimum: $f_{\min}(x)=0$

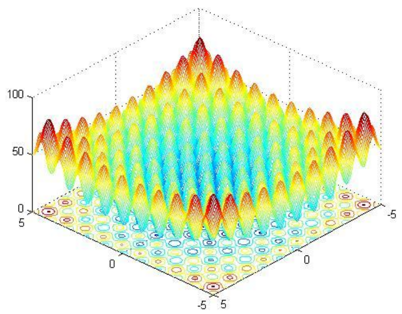

FIGURE 3 The space effect of the Rastrigrin function.

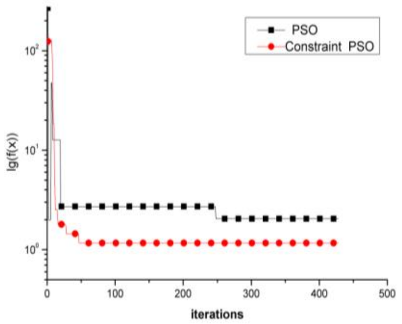

FIGURE 4 The change of the log value of the absolute positions of the particles in the iteration of standard PSO and constraint PSO

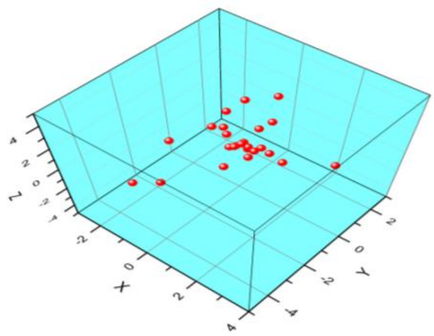

FIGURE 5 The degree of convergence of the standard PSO particles.

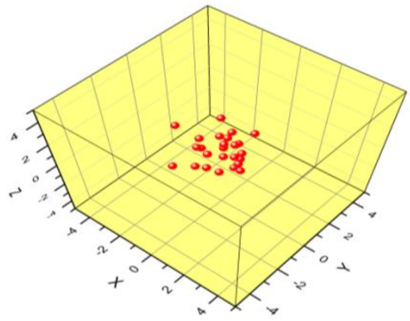

FIGURE 6 The degree of convergence of the constraint PSO particles.

In Figure 3 shows the multimodal Rastrigrin function, compared with the unimodal function, is more likely to cause PSO to fall into local optima in the search for global optimum because there are more peaks in the search domain Figure 4, as the change of the log value of the absolute positions of the particles in the iteration of standard PSO and Constraint PSO, illustrates the constraint PSO is better converged in multimodal and complex case. In Figure 6, the particles of the constraint PSO are better converged in iteration, while in Figure 5, many particles scatter away from the centre. Through a comparative analysis of Figure 5 and Figure 6, the PSO algorithm based on Lyapunov function enables the particles to effectively gather at the convergence point and have better convergence.

5 Conclusion

Analyzing the behaviour of PSO particles, we discover that there exists strong binding and coupling between the parameters of the standard PSO model, and the different range of parameters keeps the system in different state. Therefore, parameter selection is essential for the stable convergence of the system and is able to directly affect the convergence and performance of the PSO algorithm. In this paper, we transform the standard PSO model into the iterative model, describe the motion of the particles with the equation of state, determine the relationship between PSO parameters, and finally test the standard PSO and Constraint PSO with the use of Rastrigrin functions. From the test results, the Constraint PSO is proved to effectively improve the convergence of the system by restricting the parameters, whose effect is clearly shown in the simulation graphics. Therefore, the Constraint PSO is an improvement with good performance in guaranteeing the convergence.

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