# Numerical analysis and improvement of torsional vibration of shaft systems for engine with cylinder deactivation

# Liu Xiaoyong<sup>\*</sup>, Su Tiexiong, Zhang Yi

School of Mechanical and Power Engineering North University of China, Taiyuan City, Shanxi Province, China, 030051

Received 1 January 2014, www.tsi.lv

# Abstract

Cylinder deactivation is one of effective ways to improve fuel economy of engine, but will lead to changes in torsional vibration characteristics of shaft systems for engine. A lumped parameter model of torsional vibration of shaft systems for engine with cylinder deactivation was established, the numerical computing method was determined, harmonic analysis was engaged for the excitation torque of single cylinder. Based on these studies, torsional vibration of a V8-engine was analysed, the natural frequency results was verified by comparing with that of utilizing software AMESIM. The forced vibration results show that the torsional angle displacement of crankshaft under cylinder deactivation increases obviously, which mainly consists of the 2rd order rolling vibrations, but torsional stress decreases little. In order to control the rolling vibration, the measure of increasing the rotational inertia of the flywheel was adopted. The results after the adjustment show that the vibration of crankshaft was under control. In a word, the method is feasible and referred.

Keywords: Cylinder Deactivation, Torsional Vibration, Natural Frequency, Forced Vibration, Rolling Vibration, Excitation Torque

# **1** Introduction

In recent years, cylinder deactivation is widely used to save fuel on some of the advanced internal combustion engine [1]. Cylinder deactivation, when engine working in partial loads, is that the fuel for several cylinders is cut off by some mechanism so that the remainder cylinder for normal combustion can run efficiently in the high load region [2]. Cylinder deactivation would not only reduces pumping losses and mechanical loss, but also improves fuel economy of engine, which the reason is that a low concentrated gas mixture can stabilize combustion due to increasing the mixture gas and reducing residual exhaust gas in the remainder cylinder. When vehicle accelerating fast and climbing, all cylinders will start work to enhance engine power output. Currently, the measure for the deactivating cylinder is now widely used by cutting off fuel and closing value [3].

The fuel will be saved as much as 20% by employing the engine with cylinder deactivation. Nevertheless, under cylinder deactivation, the firing interval angle of remainder cylinder and the unevenness of running will increase. There will be changes in torsional vibration characteristics of rotating shaft systems for engine. The rotating crankshaft will produce torsional vibration due to the effects of periodic excitation torque, which torsional vibration is a common vibration form of rotating machinery. When excitation torque frequency and natural frequency of crankshaft are equal, the resonance in shafting be evoked. The resonance would have invoked a great stress of torsional vibration that can cause damage to the crankshaft [4]. The situation must be under control. Consequently, numerical analysis of torsional vibration for engine is very necessary.

Torsional vibration characteristics of shaft systems for engine with several cylinders deactivation was studied in this study. For the aim of providing a theoretical reference for the applications of cylinder deactivation in engines by taking a kind of V8-engine for example, which the engine would run in 8 cylinder, or 4 cylinder, The impact of torsional vibration for engine with and without cylinder deactivation were comparatively analysed. Improvement measures were proposed.

# 2 Modelling and numerical computing methods

# 2.1 LUMPED PARAMETER PHYSICAL MODELS

The actual structure of rotating shafts of engine is very complex elastic motion bodies, as shown in Figure 1. According to the principle of energy conservation, they have been addressed as a lumped parameter physical model and simplified as discretized systems with several lumped mass discs in which consists connected massless elastic shaft elements [5]. Simplified equivalent model for torsional vibration of shaft systems was shown as Figure 2, which the lumped mass number represents different part in shaft systems. Mm represents excitation torque of crank.

<sup>\*</sup> Corresponding author - Tel: +86-351-392-0980; fax: +86-351-392-0980; E-mail: liuxiaoyong0354@nuc.edu.cn



FIGURE 1 Sketch of shafts systems



FIGURE 2 Lumped parameter model

#### 2.2 MATHEMATICAL MODEL

The muli-mass lumped parameter model is a multidegree of freedom torsional vibration system. The kinetic equations of the m-th mass is expressed according to d'Alembert principle as follows.

$$I_{m}\ddot{\varphi}_{m}(t) + C_{m}\dot{\varphi}_{m}(t) + C_{m-1,m}(\dot{\varphi}_{m}(t) - \dot{\varphi}_{m-1}(t)) + C_{m,m+1}(\dot{\varphi}_{m}(t) - \dot{\varphi}_{m-1}(t)) + K_{m-1,m}(\varphi_{m}(t) - \varphi_{m-1}(t)), \qquad (1) + K_{m,m+1}(\varphi_{m}(t) - \varphi_{m+1}(t)) = M_{m}$$

where  $\varphi_m(t)$ ,  $\varphi_m(t)$ ,  $\varphi_m(t)$ ,  $I_m$ ,  $C_m$ ,  $M_m$ ,  $\psi_v$  are respectively torsional angle displacement, angular velocity ,angular acceleration, rotational inertia, viscous damping factor excitation torque, excitation torque amplitude, initial phase angle for the *m*-th lumped mass;  $K_{m-1,m}$  and  $C_{m-1,m}$  are respectively torsional stiffness and damping coefficient of shaft element *m*-1, *m*; *t* is the time.

Each lumped mass has the same mathematical expression, and n lumped masses can compose n differential equations. It can be written in matrix form.

$$I\varphi(t) + C\varphi(t) + K\varphi(t) = M(t).$$
<sup>(2)</sup>

#### Xiaoyong Liu, Tiexiong Su, Yi Zhan

where *I* is the inertia matrix; *C* is the damping matrix; *K* is the stiffness matrix; M(t) is the excitation torque vector;  $\varphi(t)$  is the torsional angle displacement vector.

#### 2.3 EXCITATION TORQUE MODEL FOR ENGINE

Excitation torque with the periodically change is the energy source of torsional vibration resulted from the tangential force imposed on the crankpin of crank.

## 2.3.1 Excitation torque for normal combustion cylinder

Excitation torque of single cylinder under normal combustion cylinder is

$$M = M_p + M_j + M_g, \qquad (3)$$

where  $M_p$ ,  $M_j$  and  $M_g$  is the excitation torque on crank of single cylinder pressure, reciprocating inertial force and moving bodies gravity, respectively. written by

$$M_{p} = \frac{\pi D^{2}}{4} pR \frac{\sin(\alpha + \beta)}{\cos \beta},$$
(4)

$$M_{j} = -m_{j}\omega^{2}R^{2}(\cos\alpha + \lambda\cos 2\alpha)\frac{\sin(\alpha + \beta)}{\cos\beta}, \quad (5)$$

$$M_g = m_j g R \frac{\sin(\alpha + \beta)}{\cos \beta}.$$
 (6)

In Figure 3, where *p* is the cylinder pressure, which is obtained by the experiment or the simulation. *D* is the cylinder bore, *R* is the crank radius,  $\alpha$  is the crank angle,  $\beta$  is the angle between the rod and the centre line of cylinder,  $m_j$  is the bodies mass with the reciprocating motion,  $\lambda$  is the crank and rod ratio,  $\omega$  is the crank angular velocity.



FIGURE 3 Sketch of crank rod system

# COMPUTER MODELLING & NEW TECHNOLOGIES 2014 **18**(2) 76-82 2.3.2 Excitation torque for deactivating cylinder

# Only compression and expansion of initial air took place in deactivating cylinder due to the close of intake and exhaust valve. The excitation torque on the crank imposed by cylinder pressure is very small and could be neglected compared to explosive pressure in normal combustion cylinder. Nevertheless, the excitation torque produced by reciprocating inertial force and moving bodies gravity still remains with piston mechanism of deactivating cylinder reserved. Thus, excitation torque of

# Xiaoyong Liu, Tiexiong Su, Yi Zhan

single cylinder for deactivating cylinder would be written by

$$M = M_{i} + M_{g}. \tag{7}$$

The excitation torque curve of single cylinder in a working cycle of a V8 engine under declared working condition was showed in Figure 4. Figure 4(a) and Figure 4(b) are respectively excitation torque for normal combustion cylinder and deactivating cylinder.



FIGURE 4 Excitation torque of single cylinder in a working cycle: a) Normal combustion cylinder, b) Deactivating cylinder

# 2.3.3 Harmonic analysis for excitation torque

The periodic excitation torque of single cylinder can be expressed as Fourier series [6], which is equal to the sum of average torque and harmonic torque with different amplitude and different frequency, written by

$$M = a_0 + \sum_{\nu}^{\infty} M^{\nu} \sin(\nu \omega t + \psi_{\nu}), \qquad (8)$$

where  $a_0$  is the average torque, v is the order,  $\psi_v$  is the phase angle,  $M^v$  is the excitation torque amplitude under the *v*-th order.

Each cylinder on the crankshaft in multi-cylinder engine works in a regular firing order. The crankshaft would be affected by several excitation torque, which have a certain amount of phase angle difference between each other. There are two cylinders fixed on one single crank on V-engine, so excitation torque of *v*-th order on one single crank is written by:

$$M^{\nu} = M^{\nu}_{\mathsf{R}} \sin(\nu\omega t + \nu\beta_{\mathsf{l},\mathsf{R}}) + M^{\nu}_{\mathsf{L}} \sin(\nu\omega t + \nu\beta_{\mathsf{l},\mathsf{L}}), \qquad (9)$$

where  $M_{R}^{\nu}$  and  $M_{L}^{\nu}$  is respectively the excitation torque of the *v*-th order of left cylinder and right cylinder on this single crank. where  $\beta_{1,L}$  and  $\beta_{1,R}$  is respectively the firing interval angle of left and right cylinders compared to the first cylinder.

## 2.4 NUMERICAL COMPUTING METHODS

# 2.4.1 Numerical computing method of free vibration

Free vibration is a vibration without excitation torque. The computation aims to solve the natural frequency and mode shape. Assuming that each lump mass functions a harmonic vibration, then the torsional angle displacement vector can be represented as:

$$\varphi(t) = A e^{\omega t} \quad , \tag{10}$$

$$A = [A_1, A_2, \cdots A_n]^T, \qquad (11)$$

where A is mode shape vector,  $A_1$ ,  $A_2$ ,  $A_n$  are the amplitude of torsional angle displacement for the first, second and *n*-th lumped mass.

Substitute equation (10) into equation (2) and let the excitation torque and damping coefficient in the equation (2) equals to zero, which damping is neglected thanks to little influence to free vibration. The problem can be changed into solving the problem of eigenvalue of matrix [7].

$$(\mathbf{K} - \omega^2 I)Ae^{\omega t} = 0.$$
<sup>(12)</sup>

By solving the eigenvalue and eigenvector of matrix K/I, The natural frequency and mode shape can be obtained.

## 2.4.2 Numerical computing method of forced vibration

The computation of forced vibration aims to solve the torsional vibration response of shaft systems to excitation torque. The solution of the m-th lumped mass under the influence of harmonic excitation torque of the v-th order is written by

$$\varphi_m^v = A_m^v e^{iv\omega t + \varepsilon_m^r} , \qquad (13)$$

where  $A_K^v$  and  $\mathcal{E}_K^r$  is respectively the amplitude and phase angle of torsional angle displacement for lumped mass *m* of *v*-th order. Substitute the solution for each lumped mass into equation (2) to get *n* united complex number equations. Decompose each complex number into the real part and the imaginary part and get the 2nunited real number linear equations.  $A_m^v$  and  $\mathcal{E}_K^r$  could be obtained after solving these equations.

According to the principle of linear superposition, the torsional angle displacement synthesis amplitude for the *m*-th lumped mass is written by

$$A_m = \sum_{\nu} A_m^{\nu} e^{i\nu\omega t + \varepsilon_m^{\nu}} = \sum_{\nu} A_m^{\nu} \sin(\nu w t + \varepsilon_m^{\nu}).$$
(14)

The torsional stress synthesis amplitude for shaft element m, m+1 is got by

$$A\tau_{m,m+1} = K_{m,m+1} \sum_{\nu} (A_m^{\nu} - A_{m+1}^{\nu}) / W_{m,m+1}, \qquad (15)$$

where  $W_{m,m+1}$  is the section modulus in torsion for shaft element m, m+1.

#### 3 Numerical analysis and simulations

Torsional vibration of shaft systems was analysed with a V8 four-stroke engine under normal combustion of all cylinders and 4-cylinder deactivation. The V firing angle of the engine is 90<sup>0</sup>. The firing order is R1-L4-L2-R2-L3-R3-R4-L1.Considered the uniformity of the output torque of the engine, deactivating cylinders are R1, L2, L3 and R4 under the 4-cylinder deactivation. The lumped parameter model of shaft systems of the engine is established as shown in Table 1.

Xiaoyong Liu, Tiexiong Su, Yi Zhan

TABLE 1 Lumped parameter model of shaft systems

Lumped mass number	Rotational inertia (kg.m2)	Shaft number	torsional stiffness (N.m/rad)
1	0.479	1,2	0.22e6
2	0.185	2,3	5.404e6
3	0.246	3,4	3.192e6
4	0.246	4,5	3.192e6
5	0.246	5,6	3.192e6
6	0.246	6,7	6.606e6
7	5.786		

# **3.1 COMPUTATION OF NATURAL FREQUENCY**

The natural frequency of shaft systems are same with normal combustion of all cylinders and under cylinder deactivation since there is no change in the structure of shaft systems. The natural frequency results are shown in Table 2. To verify the accuracy of the results, the simulation software AMESIM, which is widely used to simulate in the mechatronics and hydraulics domain, was used for the calculation of natural frequency of shaft systems for the engine. As shown in Table 2, the calculation error is less than 3%, consequently the results' reasonability and credibility of the calculating program was verified.

TABLE 2 Lumped parameter model of shaft systems

Model No.	Based on this study (r/min)	Based on AMESIM (r/min)	Error (%)
1	5890	5888	0.003
2	13741	13634	0.7
3	34610	34020	1.9
4	54117	53524	1.1
5	67245	66414	1.3
6	74829	74214	0.8

# 3.2 COMPUTATION OF FORCED VIBRATION

# 3.2.1 Engine with normal combustion of all cylinders

Numerical computation of forced vibration for engine with normal combustion is displayed in Figure 5.From Figure 5(a),the synthesis amplitude of torsional angle displacement on the free end of crankshaft (the second lumped mass) can reach its maximum as much as 1700r/min. The resonance will happen when the frequency of excitation torque of the order 7, 7.5, 8, 8.5, 9, 9.5, 10.5, 11 and 12 equals to the 2rd natural frequency of shaft systems. The peak amplitude of the order 8 is the highest among all orders.

From Figure 5(b), the torsional stress synthesis amplitude on shaft element 6,7 (between crank 4 and flywheel) reaches its maximum value when this crankshaft speeds up to 1700r/min. The peak torsional stress of the order 8 is the highest among all orders.

# COMPUTER MODELLING & NEW TECHNOLOGIES 2014 **18**(2) 76-82 3.2.2 Engine under cylinder deactivation

From Figure 6(a), under 4-cylinder deactivation, the synthesis amplitude of torsional angle displacement on the free end of crankshaft sharply increases. The peak amplitude 1.1 is 2.4 times as large as that under normal combustion in the corresponding speed. The main cause is there appears rolling vibration of the order 2 in low speed region. Compared with that under normal combustion, the torsional angle displacement of all other orders except for the order 2 and 10 decreases.

# Xiaoyong Liu, Tiexiong Su, Yi Zhan

From Figure 6(b), even though rolling vibration forces a augmentation to the amplitude of torsional angle displacement on free-end, the torsional stress amplitude in the order 2 is not very big because the amplitude difference of torsional angle displacement between two lumped mass at the two ends of shaft element 6,7 has not increased a lot. Compared with that under normal combustion, the torsional stress amplitude of the synthesis and all other orders except for the order 2 and 10 will decrease because of going up of even firing interval angle under 4-cylinder deactivation and reduce the number of normal combustion cylinders.



FIGURE 5 Torsional vibration of engine with normal combustion: a) Torsional angle displacement amplitude on the free end of crankshaft ,b) Torsional stress amplitude on shaft element 6,7



a) Torsional angle displacement amplitude on the free end of crankshaft, b) Torsional stress amplitude on shaft element 6,7

# **4** Improvement measures

Rolling vibration is that torsional angle displacement amplitude of all lumped masses swings with the same value and direction. According to equation (15), Rolling vibration produce no torsional stress and is harmless to strength of shafts, but excessive amplitude of rolling vibration will worsen the motion characteristics and dynamic load for the valve train driven by the crankshaft and ultimately cause damage to it. The situation must be under control. From Figure 6(a), the amplitude of the 2 order decreases with increase of excitation frequency (the product of the order and the speed of crankshaft). The rolling vibration amplitude of the order 2 is smaller when excitation frequency is located closer to the  $1^{st}$  natural frequency (5900r/min). However, rolling vibration amplitude is not obvious in the order 0.5, 1, 1.5, the reason is that excitation torque vector sum of all the cylinders in the above orders is relatively small. It indicates that rolling vibration amplitude have

## Xiaoyong Liu, Tiexiong Su, Yi Zhan

relationship with natural frequency of shaft systems and vector sum of all the cylinders. Above all, rolling vibration amplitude can be controlled by reducing natural frequency and excitation torque vector sum of all the cylinders.

In point of the structural characteristics of the V8engine, the rolling vibration amplitude can be controlled by reducing the natural frequency through increasing the rotational inertia of the flywheel. The rotational inertia can be changed by adjusting its thickness. When the rotational inertia of the flywheel increase to 10.786kgm<sup>2</sup>, numerical computation of forced vibration for engine under 4-cylinder deactivation is shown in Figure 7.

Compared the result before the adjustment, from Figure 7(a), the torsional angle displacement amplitude of the synthesis and the 2-th order will decrease at the speed of 800r/min. Torsional stress amplitude changes little from Figure 7(b). The amplitude of the 2 order is lower than allowable amplitude  $5^{\circ}$  [8]. It indicates that the method of reducing the rolling vibration amplitude by adjusting the inertia of the flywheel is feasible.



FIGURE 7 Torsional vibration of engine under 4-cylinder deactivation after the adjustment: a) Torsional angle displacement amplitude on the free end of crankshaft, b) Torsional stress amplitude on shaft element 6,7

#### **5** Conclusion

Through the numerical computation for torsional vibration of shaft systems for engine with deactivating cylinder, conclusions drawn as below: The natural frequency characteristics of shaft systems do not change under cylinder deactivation but only change of that of forced vibration. For a V8-engine under the normal combustion of all cylinders, the torsional angle displacement amplitude of order 8 are the biggest of all orders, there is no rolling vibration within the working speed. Under 4-cylinder deactivation, the torsional angle displacement amplitude had an obvious augmentation, which mainly consists of order 2 rolling vibrations, but the torsional stress synthesis amplitude decreased a little. In order to reduce the rolling vibration amplitude, the measure of reducing the natural frequency by increasing the rotational inertia of the flywheel was adopted. The results after the adjustment showed that the rolling vibration decreases and torsional stress amplitude changes little. This study contribute to comprehend the torsional vibration characteristics and provide improvement measures to control vibration for engine with cylinder deactivation.

#### References

- Kai W, Tingfang Z, Zhijun W, Jun D, Tao W 2010 Optimizing on the simulation of the gasoline engine with cylinder deactivation *Small Internal Combustion Engine Motorcycle* 39(1) 84-7
- [2] Osman A K, Hikmet A, Alper T C 2005 Methods to improve efficiency of four stroke, spark ignition engines at part load *Energy Conversion and Management* 46(20) 3202-20
- [3] Dengpan Z, Yinnan Y, Yong C 2007 Technology of cylinder deactivation of automotive gasoline engine *Small Internal Combustion Engine Motorcycle* 36(6) 89-93
- [4] Li H, Stone B J 1999 Time domain modelling of a reciprocating engine Mechanical Systems and Signal Processing 13(1)169-78
- [5] Qing H, Dongmei D 2010 Modelling and calculation analysis of torsional vibration for turbine generator shafts *Journal of Information & Computational Science* 10(7) 2174-82
- [6] Fuchun Y, Zhenxing S, Jianfeng M 2013 Nonlinear dynamics and load sharing of double-mesh helical gear train *Journal of Engineering Science and Technology* 6(2) 29-34
- [7] Jianhua X, Ridong L, Weizhen Z 2007 Structural dynamic modification of crankshaft torsional vibration system of IC engine base on sensitivity analysis *Chinese Internal Combustion Engine Engineering* 28(6) 66-9 (in Chinese)
- [8] Zhaohan W 1990 Internal combustion engine design Press: Beijing, 175-95 (in Chinese)

Xiaoyong Liu, Tiexiong Su, Yi Zhan

